CONTINUITY AND DISCONTINUITY

Continuity Definition

A function is said to be continuous in a given interval if there is no break in the graph of the function in the entire interval range. Assume that "f" be a real function on a subset of the real numbers and "c" be a point in the domain of f. Then f is continuous at c if

$\lim_{x \to c} f(x) = f(c)$

In other words, if the left-hand limit, right-hand limit and the value of the function at x = c exist and are equal to each other, i.e.,

$\lim_{x \to c-} f(x) = f(c) = \lim_{x \to c+} f(x)$,

then f is said to be continuous at x = c

Conditions for Continuity

- A function "f" is said to be continuous in an open interval (a, b) if it is continuous at every point in this interval.
- A function "f" is said to be continuous in a closed interval [a, b] if
  - f is continuous in (a, b)
  - $\lim_{x \to a+} f(x) = f(a)$
  - $\lim_{x \to b-} f(x) = f(b)$

Discontinuity Definition

The function "f" will be discontinuous at x = a in any of the following cases:

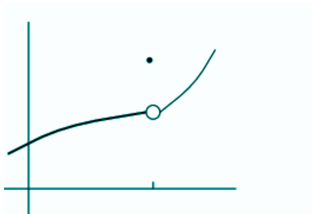
- f (a) is not defined.
- $\lim_{x \to a-} f(x)$ and $\lim_{x \to a+} f(x)$ exist but are not equal.
- $\lim_{x \to a-} f(x)$ and $\lim_{x \to a+} f(x)$ exist and are equal but not equal to f (a).

Types of Discontinuity

Removable Discontinuity

In removable discontinuity, a function which has well- defined two-sided limits at x = a, but either f(a) is not defined or f(a) is not equal to its limits. The removable discontinuity can be given as:

$\boldsymbol{\lim_{x \to a} f(x) \neq f(a)}$



Removable Discontinuity

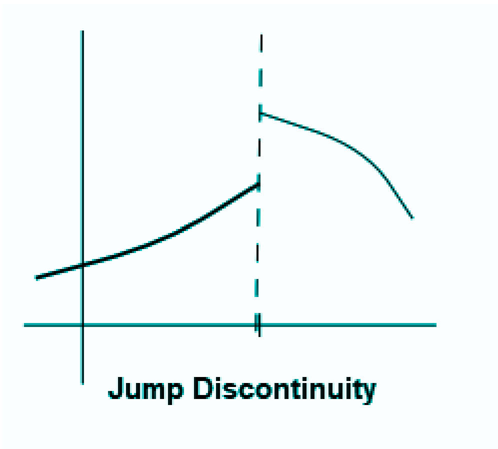
This type of discontinuity can be easily eliminated by redefining the function in such a way that

$f(a) = \lim_{x \to a} f(x)$

Jump Discontinuity

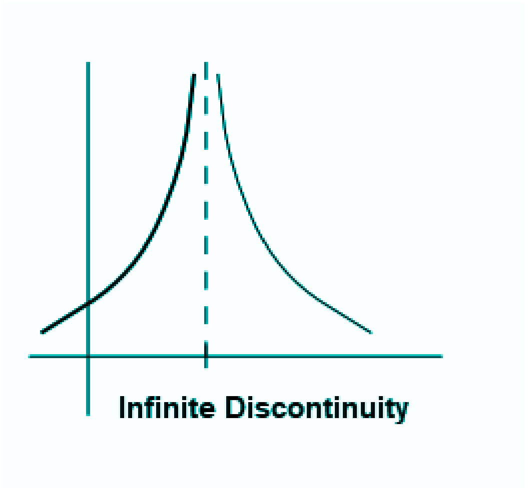
Jump Discontinuity is a type of discontinuity, in which the left-hand limit and right-hand limit for a function x = a exists, but they are not equal to each other. The jump discontinuity can be represented as:

$\lim_{x \to a+} f(x) \neq \lim_{x \to a-} f(x)$



Infinite Discontinuity

In infinite discontinuity, the function diverges at x =a to give a discontinuous nature. It means that the function f(a) is not defined. Since the value of the function at x = a does not approach any finite value or tends to infinity, the limit of a function x ⟶ a are also not defined.



Solved Examples

Example 1: Discuss the continuity of the function f(x) = sin x . cos x.

Solution:

We know that sin x and cos x are the continuous function, the product of sin x and cos x should also be a continuous function.

Hence, f(x) = sin x . cos x is a continuous function.

Example 2: Prove that the function f is defined by *f(x)* = $\begin{cases} x\sin\frac{1}{x} & x\neq 0 \\ 0 & x = 0 \end{cases}$ **is continuous at x =0**

Solution:

Left hand limit at x = 0 is given by

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x\sin\frac{1}{x} = 0$

Similarly, $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} x\sin\frac{1}{x} = 0$, [f(0) =0]

Thus, $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$.

Hence, the function f(x) is continuous at x =0.