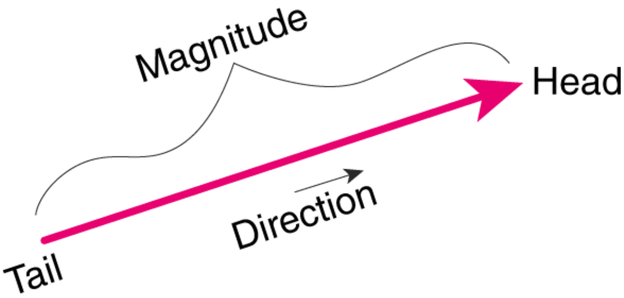
VECTORS

What is a vector?

Vector is a physical quantity that has both direction and magnitude. In other words, the vectors are defined as an object comprising both magnitude and direction. It describes the movement of the object from one point to another. Below figure shows the vector with head, tail, magnitude and direction.



The length of the segment of the directed line is called the magnitude of a vector and the angle at which the vector is inclined shows the direction of the vector. The starting point of a vector is called "Tail" and the ending point (having an arrow) is called "Head."

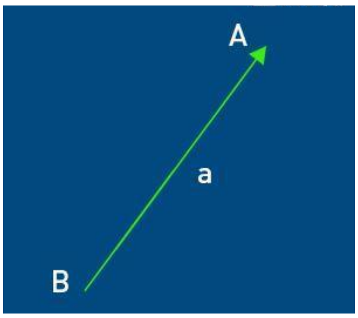
Along with the term vector, we have heard the term scalar. A scalar actually represents the "real numbers". In simpler words, a vector of "n" dimensions is an ordered collection of n elements called "components".

The most common examples of the vector are Velocity, Acceleration, Force, Increase/Decrease in Temperature etc. All these quantities have directions and magnitude both. Therefore, it is necessary to calculate them in their vector form.

Also, speed is a quantity that has magnitude but no direction. This is the basic difference between speed and velocity.

Notation

As we know already, a vector has both magnitude and direction. In the below figure, the length of the line AB is the magnitude and head of the arrow points towards the direction.



Therefore, vectors between two points A and B is given $\overrightarrow{AB}$ as,or vector a. The arrow over the head of the vector shows the direction of the vector.

Types of Vectors List

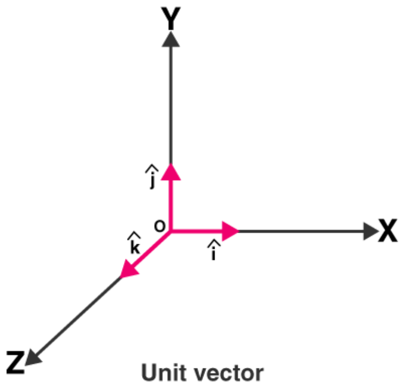
Zero Vector

A zero vector is a vector when the magnitude of the vector is zero and the starting point of the vector coincides with the terminal point. In other words, for a vector $AB\rightarrow$ the coordinates of the point **A** are the same as that of the point **B** then the vector is said to be a zero vector and is denoted by 0**.** This follows that the magnitude of the zero vector is zero and the direction of such a vector is indeterminate.

Unit Vector

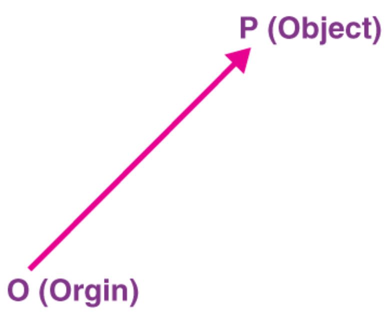
A vector which has a magnitude of unit length is called a unit vector. Suppose if $x\rightarrow$ is a vector having a magnitude x then the unit vector is denoted by $\hat{\mathbf{x}}$ in the direction of the vector $x\rightarrow$ and has the magnitude equal to 1.

$$\therefore \hat{\mathbf{x}} = \frac{x\rightarrow}{|x|}$$



Position Vector

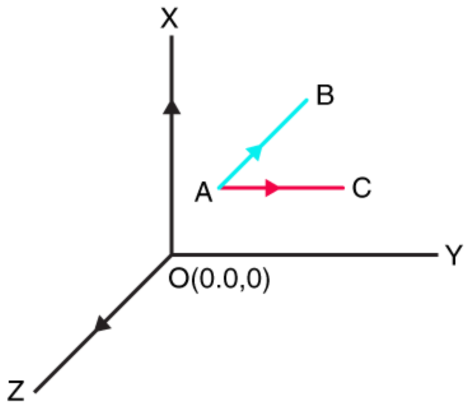
If **O** is taken as reference origin and **P** is an arbitrary point in space then the vector $OP\rightarrow$ is called as the position vector of the point. Position vector simply denotes the position or location of a point in the three-dimensional Cartesian system with respect to a reference origin.



Co-initial Vectors

The vectors which have the same starting point are called co-initial vectors.

The vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are called co-initial vectors as they have the same starting point.



Like and Unlike Vectors

The vectors having the same direction are known as like vectors. On the contrary, the vectors having the opposite direction with respect to each other are termed to be unlike vectors.

Co-planar Vectors

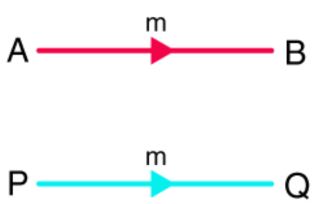
Three or more vectors lying in the same plane or parallel to the same plane are known as co-planar vectors.

Collinear Vectors

Vectors which lie along the same line or parallel lines are known to be collinear vectors. They are also known as parallel vectors.

Equal Vectors

Two or more vectors are said to be equal when their magnitude is equal and also their direction is the same.



Displacement Vector

If a point is displaced from position A to B then the displacement AB represents a vector $\overrightarrow{AB}$ which is known as the displacement vector.

Negative of a Vector

If two vectors are the same in magnitude but exactly opposite in direction then both the vectors are negative of each other. Assume there are two vectors **a** and **b,** such that these vectors are exactly the same in magnitude but opposite in direction then these vectors can be given by

$\mathbf{a} = -\mathbf{b}$

Magnitude of a Vector

The magnitude of a vector formula is used to calculate the length for a given vector (say **v**) and is denoted as $|\mathbf{v}|$.

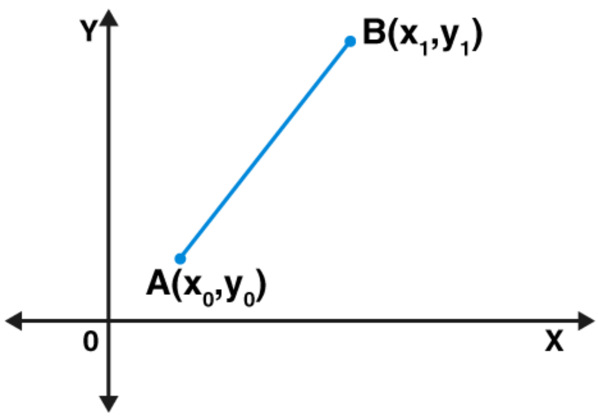
Magnitude of a Vector Formula

Suppose, AB is a vector quantity that has magnitude and direction both. To calculate the magnitude of the vector $\overrightarrow{AB}$, we have to calculate the distance between the initial point A and endpoint B. In XY – plane, let A has coordinates (x_0, y_0) and B has coordinates (x_1, y_1). Therefore, by distance formula, the magnitude of vector $\overrightarrow{AB}$, can be written as;

$$|\overrightarrow{AB}| = \sqrt{(x_1-x_0)^2 + (y_1-y_0)^2}$$

Now if the starting point is at (x, y) and the endpoint is at the origin, then the magnitude of a vector formula becomes;

$$|\overrightarrow{AB}| = \sqrt{x^2+y^2}$$



Direction of a vector

The direction of a vector is nothing but the measurement of the angle which is made with the horizontal line. One of the methods to find the direction of the vector $AB\rightarrow$ is;
tan α = y/x; endpoint at 0.

Where x is the change in horizontal line and y is the change in a vertical line.

Or $\tan \alpha = \frac{y_1 - y_0}{x_1 - x_0}$; where (x_0, y_0) is initial point and (x_1, y_1) is the endpoint.

Components of a Vector

The components of a vector in two dimension coordinate system are usually considered to be x-component and y-component. It can be represented as, $V = (v_x, v_y)$, where V is the vector. These are the parts of vectors generated along the axes. In this article, we will be finding the components of any given vector using formula both for two-dimension and three-dimension coordinate system.

Suppose a vector V is defined in a two-dimensional plane. The vector V is broken into two components such as v_x and v_y

Now let an angle θ, is formed between the vector V and x-component of vector. The vector V and its x-component (v_x) form a right-angled triangle if we draw a line parallel to y-component (v_y).

By trigonometric ratios, we know,

$\cos \theta$ = Adjacent Side/Hypotenuse = v_x/V

$\sin \theta$ = Opposite Side/Hypotenuse = v_y/V

where V is the magnitude of the vector V.

Components of vector formula

Since, in the previous section we have derived the expression:

$\cos \theta = v_x/V$

$\sin \theta = v_y/V$

Therefore, the formula to find the components of any given vector becomes:

$\mathbf{v_x = V \cos \theta}$

$\mathbf{v_y = V \sin \theta}$

Where V is the magnitude of vector V and can be found using Pythagoras theorem;

$\mathbf{|V| = \sqrt{(v_x^2, v_y^2)}}$

Orthogonal vectors

Vectors can be easily represented using the co-ordinate system in three dimensions. Before getting into the representation of vectors, let us understand what orthogonal representation is.

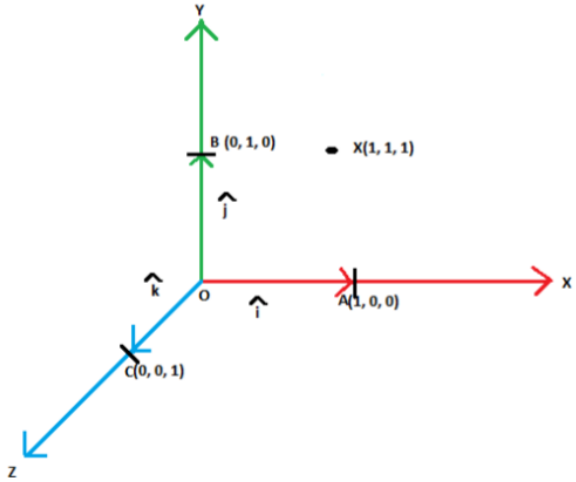
In terms of coordinate geometry, by orthogonal representation, we mean parameters that are at right angles to each other. In orthogonal three dimensional system, we have three axes perpendicular to each other, which represent x,y and z axis.

Unit vectors: are the vectors which have magnitude of unit length.

$$\hat{x} = \frac{\vec{x}}{|\vec{x}|}$$

Here, $\hat{x}$ represents a unit vector, $x\rightarrow$ represents the vector and represents the magnitude of the vector.

In orthonormal or orthogonal systems, we can have three different unit vectors with one in each direction. It can be represented as follows:



The point X(1, 1, 1) can be represented using the three mutually perpendicular axes as points A(1, 0, 0), B(0, 1, 0) and C(0, 0, 1) on the and axes respectively.

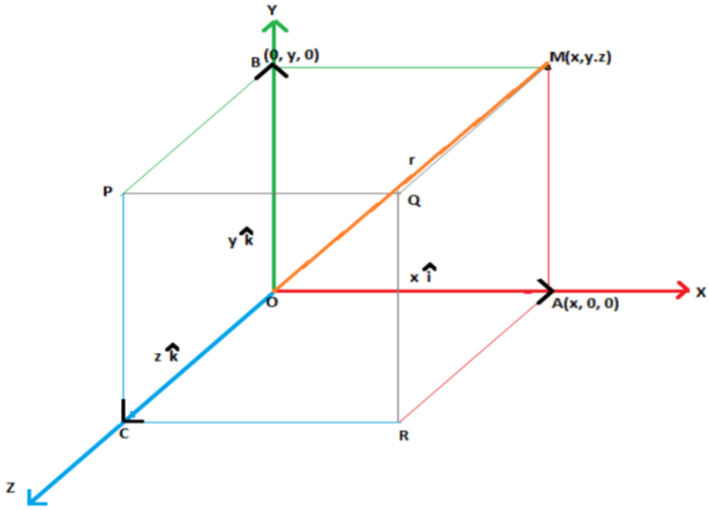
The magnitude of the vector $OA\rightarrow$ along the axis is 1. Similarly, that of vectors $OB\rightarrow$ and $OC\rightarrow$ is also 1 along the y and z axes respectively. These vectors are the unit vectors along x, y and z axis and are represented by $\hat{i}$, $\hat{j}$ and $\hat{k}$ respectively.

Now, with the help of unit vectors we can represent any vector in the three-dimensional coordinate system.

Components of vector in 3D

To represent a vector in space, we resolve the vector along the three mutually perpendicular axes as shown below.

The vector OM can be resolved along the three axes as shown. With OM as the diagonal, a parallelepiped is constructed whose edges OA, OB and OC lie along the three perpendicular axes.



From the above figure, we can say that

$\overrightarrow{OA} = x\hat{i}$

$\overrightarrow{OB} = y\hat{j}$

$\overrightarrow{OC} = z\hat{k}$

The vector can be represented as

r = $\overrightarrow{OM} = x\hat{i} + y\hat{j} + z\hat{k}$

This is known as the component form of a vector.

Thus, the vector r can be resolved in the directions i, j and k respectively. This represents the position of given vectors in terms of the three co-ordinate axes.

If a vector is given in a form as shown above, then the magnitude of such a vector can be found out by using the Pythagoras theorem in the given figure as,

r = $\overrightarrow{OM} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow |r| = \sqrt{(x^2+y^2+z^2)}$

The sum of two vectors a = $a_{1\hat{i}} + a_{2\hat{j}} + a_{3\hat{k}}$ and b = $b_{1\hat{i}} + b_{2\hat{j}} + b_{3\hat{k}}$ is given by adding the components of the three axes separately.

i.e., $a + b = a_{1\hat{i}} + a_{2\hat{j}} + a_{3\hat{k}} + b_{1\hat{i}} + b_{2\hat{j}} + b_{3\hat{k}}$

$\Rightarrow a + b = (a_1+b_1)\,\hat{i} + (a_2+b_2)\,\hat{j} + (a_3+b_3)\,\hat{k}$

Similarly, the difference can be given as:

$a - b = (a_1-b_1)\,\hat{i} + (a_2-b_2)\,\hat{j} + (a_3-b_3)\,\hat{k}$