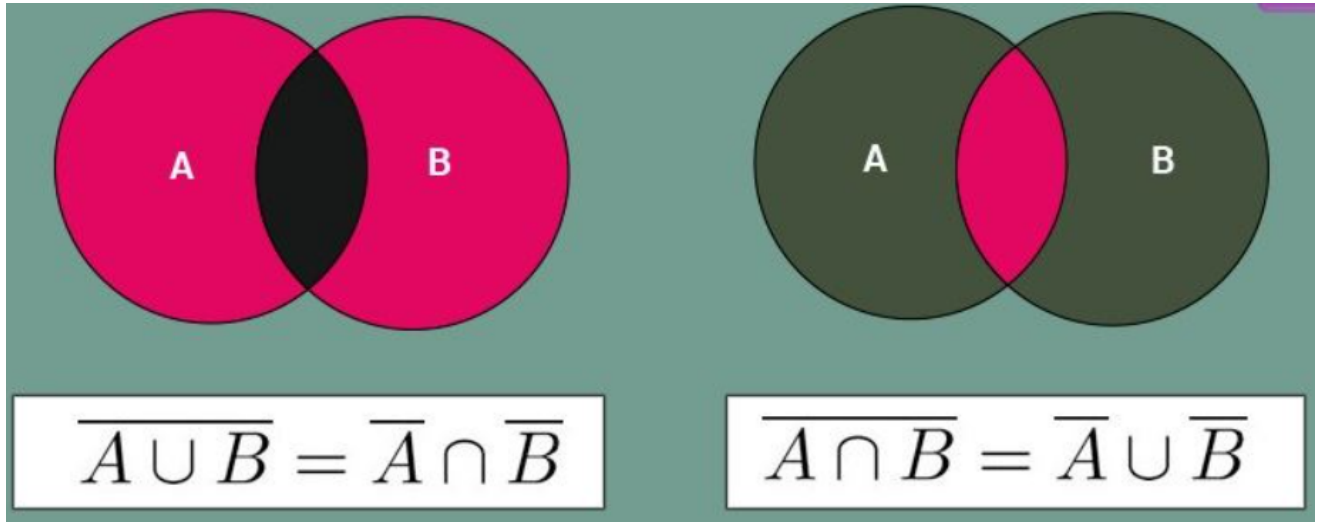# DE MORGAN'S LAW

De Morgan's law states that 'The complement of the union of two sets A and B is equal to the intersection of the complement of the sets A' and B'. Also, according to De Morgan's law, the complement of the intersection of two sets A and B is equal to the union of the complement of the sets A and B i.e.,

(A∪B)' = A' ∩ B'

And (A ∩ B)' = A' ∪ B'

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

$$\overline{A \cap B} = \overline{A} \cup \overline{B}$$

What is the complement of a Set?

The complement of a set is defined as

A' = {x : x ∈ U and x ∉ A}

where A' denotes the complement.

Complement of Sets Properties

i) **Complement Laws:** The union of a set A and its complement A' gives the universal set U of which, A and A' are a subset.

A ∪ A' = U

Also, the intersection of a set A and its complement A' gives the empty set Ø.

A ∩ A' = Ø

For Example: If U = {1 , 2 , 3 , 4 , 5 } and A = {1 , 2 , 3 } then A' = {4 , 5}. From this it can be seen that

A ∪ A' = U = { 1 , 2 , 3 , 4 , 5}

Also

A ∩ A' = Ø

ii) Law of Double Complementation: According to this law if we take the complement of the complemented set A' then, we get the set A itself.

(A')' = A

In the previous example we can see that, if U = {1 , 2 , 3 , 4 , 5} and A = {1 , 2 ,3} then A' ={4 , 5}. Now if we take the complement of set 'A' we get,

(A')' = {1 , 2 , 3} = A

This gives back the set A itself.

iii) Law of empty set and universal set:

According to this law the complement of the universal set gives us the empty set and vice-versa i.e.,

Ø' = U And U' = Ø

This law is self-explanatory.

Solved Problem

Example:

A universal set U which consists of all the natural numbers which are multiples of 3, less than or equal to 20. Let A be a subset of U which consists of all the even numbers and the set B is also a subset of U consisting of all the prime numbers. Verify De Morgan Law.

Solution: We have to verify (A ∪ B)' = A' ∩ B' and (A ∩ B)' = A'∪B'. Given that,

U = {3 , 6 , 9 , 12 , 15 , 18}

A = {6 , 12 , 18}

B = {3}

The union of both A and B can be given as,

A ∪ B = {3 , 6 , 12 , 18}

The complement of this union is given by,

(A ∪ B)'={9 , 15}

Also, the intersection and its complement are given by:

A ∩ B = Ø

(A ∩ B)' = {3 , 6 , 9 , 12 , 15 ,18}

Now, the complement of the sets A and B can be given as:

A' = {3 , 9 , 15}

B' = {6 , 9 , 12 , 15 , 18}

Taking the union of both these sets, we get,

A'∪B' = {3 , 6 , 9 , 12 , 15 ,18}

And the intersection of the complemented sets is given as,

A' ∩ B' = {9 , 15}

We can see that:

(A ∪ B)′ = A′ ∩ B′ = {9 , 15}

And also,

(A ∩ B)′ = A′ ∪ B′ = {3 , 6 , 9 , 12 , 15 ,18}

Hence, the above result is true in general and is known as De Morgan Law.

De Morgan's Laws Statement and Proof

A well-defined collection of objects or elements is known as a set. Various operations like complement of a set, union and intersection can be performed on two sets. These operations and their usage can be further simplified using a set of laws known as De Morgan's Laws. These are very easy and simple laws.

Any set consisting of all the objects or elements related to a particular context is defined as a universal set. Consider a universal set U such that A and B are the subsets of this universal set.

According to De Morgan's first law, the complement of the union of two sets A and B is equal to the intersection of the complement of the sets A and B.

$(A∪B)' = A'∩ B'$ ——(1)

Where complement of a set is defined as

$A' = \{x : x \in U$ and $x \notin A\}$

Where A' denotes the complement.

This law can be easily visualized using Venn Diagrams.

The L.H.S of the equation 1 represents the complement of union of two sets A and B. First of all, union of two sets A and B is defined as the set of all elements which lie either in set A or in set B. It can be visualized using Venn Diagrams as shown:
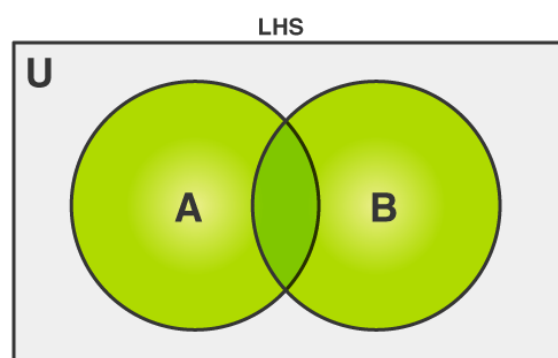
Figure 1 Union of Sets

The highlighted or the green coloured portion denotes AUB. The complement of union of A and B i.e., (AUB)'is set of all those elements which are not in AUB. This can be visualized as follows:
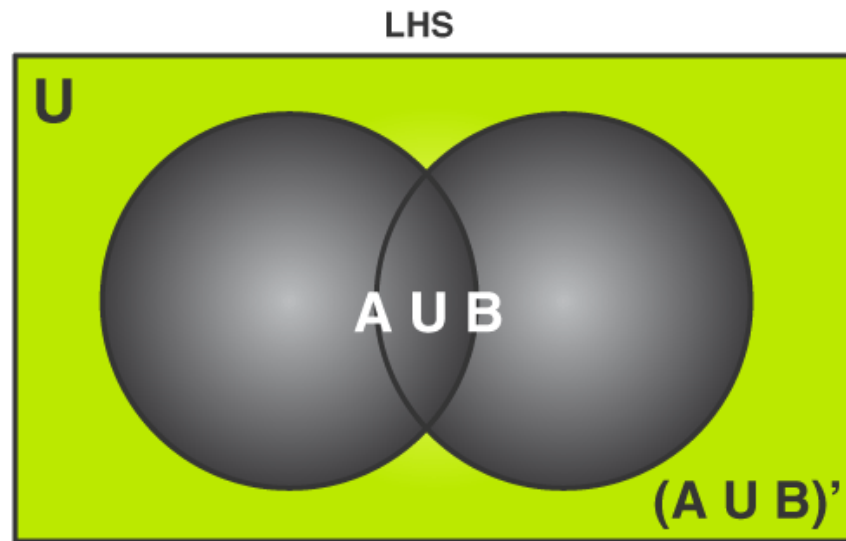
LHS

Figure 2 Complement of sets

Similarly, R.H.S of equation 1 can be represented using Venn Diagrams as well, the first part i.e., A' can be depicted as follows:
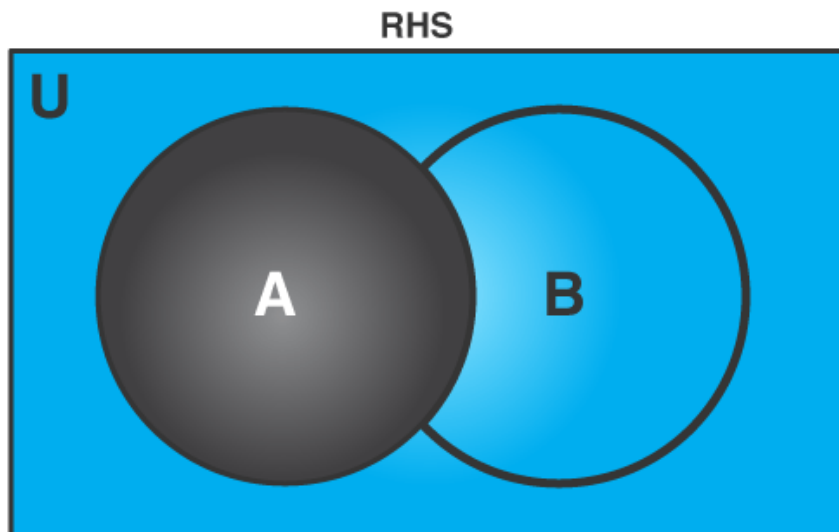
RHS

Figure 3 Complement of set A

The portion in black indicates set A and the blue part denotes its complement i.e., A'. Similarly, B' is represented as:
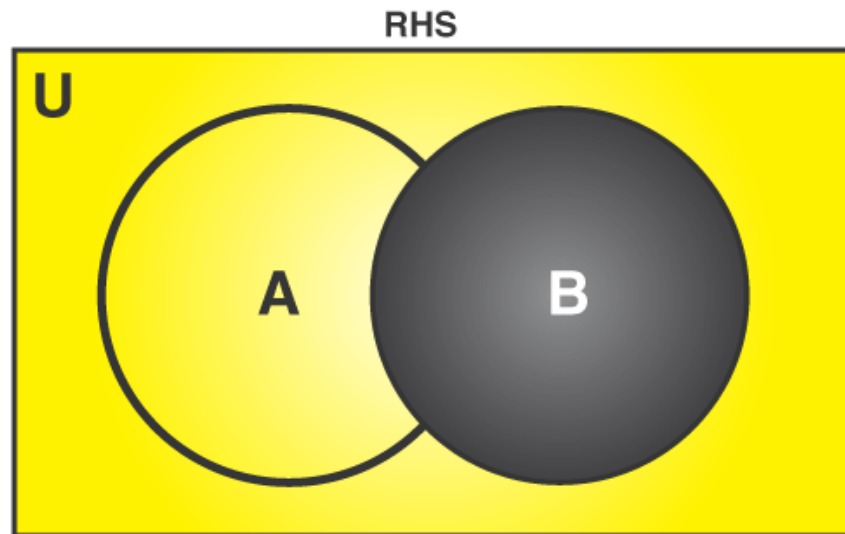
Figure 4 Complement of set B

The portion in black indicates set B and the yellow part denotes its complement i.e., B'.

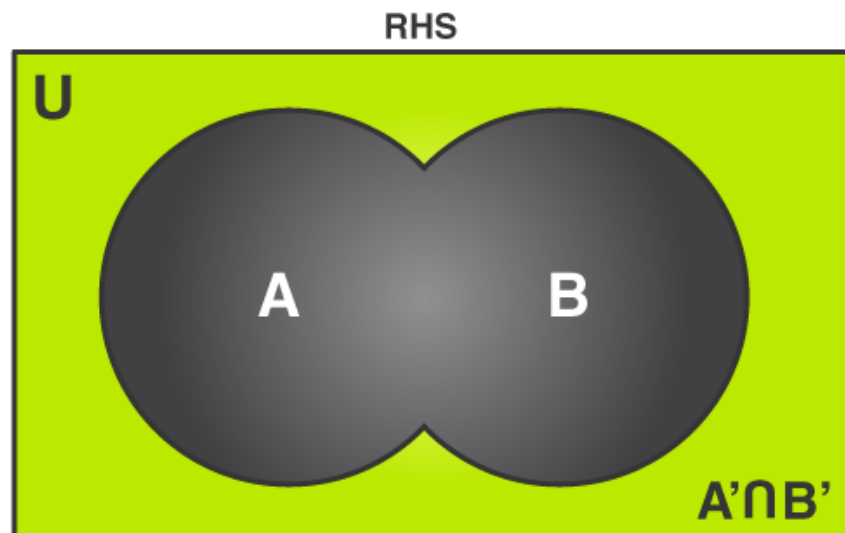If fig. 3 and 4 are superimposed on one another, we get the figure similar to that of the complement of sets.

Figure 5 Intersection of complements of sets

Hence L.H.S = R.H.S

Mathematically,

As, A∪B= either in A or in B

(A∪B)'= L.H.S = neither in A nor in B

Also, A'= Not in A

B'= Not in B

A'∩ B'= Not in A and not in B

⇒(A∪B)'= A'∩ B'

Thus, by visualizing the Venn Diagrams and analyzing De Morgan's Laws by writing it down, its validity can be justified.

We may apply **De Morgan's theorem** to negating a dis-junction or the negation of conjunction in all or part of a formula. This theorem explains that the complement of all the terms' product is equal to the sum of each term's complement. Similarly, the complement of the sum of all the terms is equal to the product of the complement of each term. Also, this theorem is used to solve different problems in boolean algebra.