L.C.M AND H.C.F

The full forms of H.C.F. and L.C.M. are, Highest Common factor and Least Common Multiple, respectively. The H.C.F. defines the greatest factor present in between given two or more numbers, whereas L.C.M. defines the least number which is exactly divisible by two or more numbers. H.C.F. is also called the greatest common factor (GCF) and LCM is also called the Least Common Divisor.

HCF (Highest Common Factor)

As the rules of mathematics dictate, the greatest common divisor or the gcd of two or more positive integers happens to be the largest positive integer that divides the numbers without leaving a remainder. For example, take 8 and 12. The H.C.F. of 8 and 12 will be 4 because the highest number that can divide both 8 and 12 is 4.

LCM (Least Common Multiple)

In arithmetic, the least common multiple or LCM of two numbers say a and b, is denoted as LCM (a,b). And the LCM is the smallest or least positive integer that is divisible by both a and b. For example, let us take two positive integers 4 and 6.

Multiples of 4 are: 4,8,12,16,20,24…

Multiples of 6 are: 6,12,18,24….

The common multiples for 4 and 6 are 12,24,36,48…and so on. The least common multiple in that lot would be 12. Let us now try to find out the LCM of 24 and 15.

2	24, 15
2	12, 15
2	6, 15
3	3, 15
5	1, 5
	1, 1

LCM of 24 and 15 = 2 × 2 × 2 × 3 × 5 = 120

LCM of Two Numbers

Suppose there are two numbers, 8 and 12, whose LCM we need to find. Let us write the multiples of these two numbers.

8 = 16, 24, 32, 40, 48, 56, …

12 = 24, 36, 48, 60, 72, 84,…

You can see, the least common multiple or the smallest common multiple of two numbers, 8 and 12 is 24.

HCF and LCM Formula

The formula which involves both HCF and LCM is:

Product of Two numbers = (HCF of the two numbers) x (LCM of the two numbers)

Say, A and B are the two numbers, then as per the formula;

$A \times B = H.C.F.(A,B) \times L.C.M.(A,B)$

We can also write the above formula in terms of HCF and LCM, such as:

H.C.F. of Two numbers = Product of Two numbers/L.C.M of two numbers

L.C.M of two numbers = Product of Two numbers/H.C.F. of Two numbers

HCF and LCM Relation

The followings are the relation between HCF and LCM. Go through the relation between HCF and LCM, solve the problem using the relations in an easy way.

(i) The product of LCM and HCF of the given natural numbers is equivalent to the product of the given numbers.

From the given property, LCM × HCF of a number = Product of the Numbers

Consider two numbers A and B, then.

Therefore, LCM (A , B) × HCF (A , B) = A × B

Example: Show that the LCM (6, 15) × HCF (6, 15) = Product (6, 15)

Solution: LCM and HCF of 6 and 15:

$6 = 2 \times 3$

$15 = 3 \times 5$

LCM of 6 and 15 = 30

HCF of 6 and 15 = 3

LCM (6, 15) × HCF (6, 15) = 30 × 3 = 90

Product of 6 and 15 = 6 × 15 = 90

Hence, LCM (6, 15) × HCF (6, 15)=Product(6, 15) = 90

(ii) The LCM of given co-prime numbers is equal to the product of the numbers since the HCF of co-prime numbers is 1.

So, LCM of Co-prime Numbers = Product Of The Numbers

Example: 17 and 23 are two co-prime numbers. By using the given numbers verify that, LCM of given co-prime Numbers = Product of the given Numbers

Solution: LCM and HCF of 17 and 23:

17 = 1 x 7

23 = 1 x 23

LCM of 17 and 23 = 391

HCF of 17 and 23 = 1

Product of 17 and 23 = 17 × 23 = 391

Hence, LCM of co-prime numbers = Product of the numbers

(iii) H.C.F. and L.C.M. of Fractions

LCM of fractions = LCM of Numerators / HCF of Denominators

HCF of fractions = HCF of Numerators / LCM of Denominators

Example: Find the LCM of the fractions 1/2 , 3/8, 3/4
Solution:

LCM of fractions = LCM of Numerators/HCF of Denominators

LCM of fractions = LCM (1,3,3)/HCF(2,8,4) = 3/2

Example: Find the HCF of the fractions 3/5, 6/11, 9/20

HCF of fractions HCF of Numerators/LCM of Denominators

HCF of fractions = HCF (3,6,9)/LCM (5,11,20) = 3/220

How to Find LCM and HCF?

We can find HCF and LCM of given natural numbers by two methods i.e., by prime factorization method or division method. In the **prime factorization method**, given numbers are written as the product of prime factors. While in the division method, given numbers are divided by the least common factor and continue still remainder is zero.

Note: *Prime numbers are numbers which have only two factors i.e. one and the number itself.*

LCM by Prime Factorization Method

Here, given natural numbers are written as the product of prime factors. The lowest common multiple will be the product of all prime factors with the highest degree (power).

Example: Find the LCM of 20 and 12 by prime factorization method.

Solution:

Step 1: To find LCM of 20 and 12, write each number as a product of prime factors.

20= 2 × 2 × 5 = 22 × 5

12 = 2 × 2 × 3 = 22 × 3

Step 2: Multiply all the prime factors with the highest degree.

Here we have 2 with highest power 2 and other prime factors 3 and 5. Multiply all these to get LCM.

LCM of 20 and 12 = 2 × 2 × 3 × 5 = 2^2 × 3 × 5 = 60

LCM by Division Method

In this method, divide the given numbers by common prime number until the remainder is a prime number or one. LCM will be the product obtained by multiplying all divisors and remaining prime numbers.

Example: Find the LCM of 24 and 15 by the division method.

Solution:

Step 1: Divide the given numbers by the least prime number.

Here, 2 is the least number which will divide 24.

$$\begin{array}{r|l} 2 & 24,\ 15 \\ \hline \end{array}$$

Step 2: Write the quotient and the number which is not divisible by the above prime number in the second row.

In the second row, write the quotient we get after the division of 24 by 2. Since 15 is not divisible by 2, write 15 in the second row as it is.

Step 3: Divide the numbers with another least prime number.

$$\begin{array}{r|l} 2 & 24,\ 15 \\ \hline 2 & 12\ ,\ 15 \\ \hline \end{array}$$

Step 4: Continue division until the remainder is a prime number or 1.

$$\begin{array}{c|l} 2 & 24, 15 \\ \hline 2 & 12, 15 \\ \hline 2 & 6, 15 \\ \hline 3 & 3, 15 \\ \hline 5 & 1, 5 \\ \hline & 1, 1 \end{array}$$

Step 5: Multiply all the divisors and remaining prime number (if any) to obtain the LCM.

LCM of 24 and 15= $2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3 \times 5 = 120$

HCF By Prime Factorization Method

Given natural numbers to be written as the product of prime factors. To obtain the highest common factor multiply all the common prime factors with the lowest degree (power).

Example: Find the HCF of 20 and 12 by prime factorization method.

Solution:

Step 1: To find HCF of 20 and 12, write each number as a product of prime factors.

$20 = 2 \times 2 \times 5 = 2^2 \times 5$

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

Step 2: Multiply all the common prime factors with the lowest degree.

Here we have only 2 as a common prime factor with the lowest power of 2.

HCF of 20 and 12 = $2^2 = 4$

HCF by Division Method

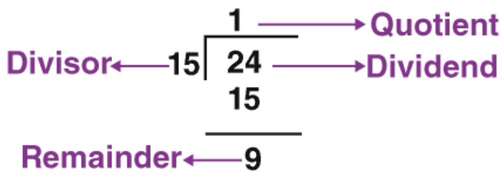
In this method divide the largest number by the smallest number among the given numbers until the remainder is zero. The last divisor will be the HCF of given numbers.

Example: Find the LCM of 24 and 15 by the division method.

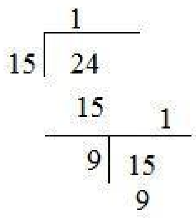
Solution:

Step 1: Divide the largest number by the smallest number.

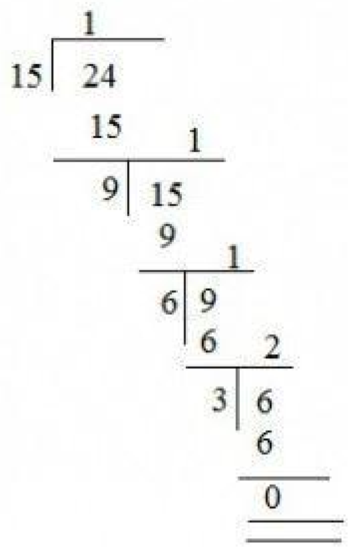
Here, the largest number is 24 and the smaller one is 15. Divide 24 by 15



Step 2: Take divisor as new dividend and remainder as the new divisor, i.e. divide the first divisor by the first remainder.



Step 3: Proceed till the remainder is zero and the last divisor will be the HCF of the given numbers.



Therefore, HCF of 24 and 15 is 3.

Alternatively, we can divide both the numbers by the least common prime factor, still there is no more common prime factors. Multiply all divisors to get the HCF of given numbers.

Consider the above example, HCF of 24 and 15 can also be calculated using the following steps:

Step 1: Divide the given numbers by the least common prime factor.

Here, 3 is the least common prime factor of 24 and 15.

$$\begin{array}{r|l} 3 & 24, 15 \\ \hline \end{array}$$

Step 2: Continue still there is no more common prime factor. Then multiply all the divisors.

$$\begin{array}{r|l} 3 & 24, 15 \\ \hline & 8, 5 \\ \hline & \end{array}$$

Division of 24 and 15 by 3 will leave 8 and 5 as their remainders respectively. 8 and 5 do not have a common prime factor.

Hence, the HCF of 24 and 15 is 3.

Solved Examples

Example 1: Find the Highest Common Factor of 25, 35 and 45.

Solution: Given, three numbers as 25, 35 and 45.

We know, 25 = 5 × 5

35 = 5 × 7

45 = 5 × 9

From the above expression, we can say 5 is the only common factor for all the three numbers.

Therefore, 5 is the HCF of 25, 35 and 45.

Example 2: Find the Least Common Multiple of 36 and 44.

Solution: Given, two numbers 36 and 44. Let us find out the LCM, by division method.

	36, 44
2	18, 22
2	9, 11
3	3, 11
3	1, 11
11	1, 1

Therefore, LCM (36, 44) = 2 × 2 × 3 × 3 × 11 = 396

Example 3: What is the L.C.M. of 25, 30, 35 and 40?

Solution: L.C.M. of 25, 30, 35 and 40

Let us find LCM by prime factorisation.

Prime factorisation of 25 = 5 x 5 = 5^2

Prime factorisation of 30 = 2 x 3 x 5

Prime factorisation of 35 = 5 x 7

Prime factorisation of 40 = 2 x 2 x 2 x 5 = 2^3 x 5

Thus, LCM (25, 30, 35, 40) = 2 x 2 x 2 x 3 x 5 x 5 x 7 = 4200

Example 4: The HCF of two numbers is 29 & their sum is 174. What are the numbers?

Solution: Let the two numbers be 29x and 29y.

Given, 29x + 29y = 174

29(x + y) = 174

x + y = 174/29 = 6

Since x and y are co-primes, therefore, possible combinations would be (1,5), (2,4), (3,3)

For (1,5): 29 x = 29 x 1 and 29 y = 29 (5) = 145

Therefore, the required numbers are 29 and 145.