INDEFINITE INTEGRALS

Indefinite Integrals Definition

An integral which is not having any upper and lower limit is known as an indefinite integral.

Mathematically, if F(x) is any anti-derivative of f(x) then the most general antiderivative of f(x) is called an indefinite integral and denoted,

$\int f(x)\,dx = F(x) + C$

We mention below the following symbols/terms/phrases with their meanings in the table for better understanding.

Symbols/Terms/Phrases	**Meaning**
$\int f(x)\,dx$	Integral of f with respect to x
f(x) in $\int f(x)\,dx$	Integrand
x in $\int f(x)\,dx$	Variable of integration
An integral of f	A function F such that $F'(x) = f(x)$
Integration	The process of finding the integral
Constant of Integration	Any real number C, considered as constant function

Anti-derivatives or integrals of the functions are not unique. There exist infinitely many antiderivatives of each of certain functions, which can be obtained by choosing C arbitrarily from the set of real numbers. For this reason, C is customarily referred to as an arbitrary constant. C is the parameter by which one gets different antiderivatives (or integrals) of the given function.

Indefinite Properties

Property 1: The process of differentiation and integration are inverses of each other in the sense of the following results:

$$\frac{d}{dx}\int f(x)\,dx = f(x)$$

And

$$\int f'(x)\,dx = f(x) + C$$

where C is any arbitrary constant.

Let us now prove this statement.

Proof: Consider a function f such that its anti-derivative is given by F, i.e.

$$\frac{d}{dx}\mathrm{F}(x) = f(x)$$

Then

$$\int f(x)\,dx = \mathrm{F}(x) + \mathrm{C}$$

On differentiating both the sides with respect to x we have,

$$\frac{d}{dx}\int f(x)\,dx = \frac{d}{dx}(\mathrm{F}(x)+\mathrm{C})$$

As we know, the derivative of any constant function is zero. Thus,

$$\frac{d}{dx}\int f(x)\,dx = \frac{d}{dx}(\mathrm{F}(x)+\mathrm{C})$$

$$= \frac{d}{dx}\mathrm{F}(x)$$

$$= f(x)$$

The derivative of a function f in x is given as f'(x), so we get;

$$f'(x) = \frac{d}{dx}\ f(x)$$

Therefore,

$$\int f'(x)\,dx = f(x) + \mathrm{C}$$

Hence, proved.

Property 2: Two indefinite integrals with the same derivative lead to the same family of curves, and so they are equivalent.

Proof: Let f and g be two functions such that

$$\frac{d}{dx}\int f(x)\,dx = \frac{d}{dx}\int g(x)\,dx$$

or

$$\frac{d}{dx}\left[\int f(x)\,dx - \int g(x)\,dx\right] = 0$$

Now,

$$\int f(x)\,dx - \int g(x)\,dx = C$$

or

$$\int f(x)\,dx = \int g(x)\,dx + C$$

where C is any real number.

From this equation, we can say that the family of the curves of [∫ f(x)dx + C_3, $C_3 \in R$] and [∫ g(x)dx + C_2, $C_2 \in R$] are the same.

Therefore, we can say that, ∫ f(x)dx = ∫ g(x)dx

Property 3: The integral of the sum of two functions is equal to the sum of integrals of the given functions, i.e.,

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

Proof:

From the property 1 of integrals we have,

$$\frac{d}{dx}\left[\int [f(x) + g(x)]\,dx\right] = f(x) + g(x) \quad \text{... (1)}$$

Also, we can write;

$$\frac{d}{dx}\left[\int f(x)\,dx + \int g(x)\,dx\right] = \frac{d}{dx}\int f(x)\,dx + \frac{d}{dx}\int g(x)\,dx = f(x) + g(x) \quad \text{... (2)}$$

From (1) and (2),

$$\int [f(x) + g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

Hence proved.

Property 4: For any real value of p,

$$\int pf(x)\,dx = p\int f(x)\,dx$$

Proof: From property 1 we can say that

$$\frac{d}{dx}\int pf(x)\,dx = pf(x)$$

Also,

$$\frac{d}{dx}\left[p\int f(x)\,dx\right] = p\frac{d}{dx}\int f(x)\,dx = pf(x)$$

From property 2 we can say that

$$\int pf(x)\,dx = p\int f(x)\,dx$$

Property 5:

For a finite number of functions f_1, f_2.... f_n and the real numbers p_1, p_2...p_n,

$\int[p_1f_1(x) + p_2f_2(x)....+p_nf_n(x)]dx = p_1\int f_1(x)dx + p_2\int f_2(x)dx + ... + p_n\int f_n(x)dx$

Indefinite Integral Formulas

The list of indefinite integral formulas are:

- $\int 1\,dx = x + C$
- $\int a\,dx = ax + C$
- $\int x^n\,dx = ((x^{n+1})/(n+1)) + C$; $n \neq 1$
- $\int \sin x\,dx = -\cos x + C$
- $\int \cos x\,dx = \sin x + C$
- $\int \sec^2x\,dx = \tan x + C$
- $\int \text{cosec}^2x\,dx = -\cot x + C$
- $\int \sec x \tan x\,dx = \sec x + C$
- $\int \text{cosec}\,x \cot x\,dx = -\text{cosec}\,x + C$
- $\int (1/x)\,dx = \ln|x| + C$
- $\int e^x\,dx = e^x + C$
- $\int a^x\,dx = (a^x/\ln a) + C$; $a > 0$, $a \neq 1$

Example 1: Evaluate the given indefinite integral problem: $\int 6x^5 - 18x^2 + 7\,dx$

Solution:

Given,

$\int 6x^5 - 18x^2 + 7\, dx$

Integrate the given function, it becomes:

$\int 6x^5 - 18x^2 + 7\, dx = 6(x^6/6) - 18\,(x^3/3) + 7x + C$

Note: Don't forget to put the integration constant "C"

After simplification, we get the solution

Thus, $\int 6x^5 - 18x^2 + 7\, dx = x^6 - 6x^3 + 7x + C$

Example 2: Evaluate f(x), given that $f'(x) = 6x^8 - 20x^4 + x^2 + 9$

Solution:

Given,

$f'(x) = 6x^8 - 20x^4 + x^2 + 9$

We know that, the inverse process of differentiation is an integration.

Thus, $f(x) = \int f'(x)\, dx = \int [6x^8 - 20x^4 + x^2 + 9]\, dx$

$f(x) = (2/3)x^9 - 4x^5 + (1/3)x^3 + 9x + C$

Indefinite Integral vs Definite Integral

An indefinite integral is a function that practices the antiderivative of another function. It can be visually represented as an integral symbol, a function, and then a dx at the end. The **indefinite integral** is an easier way to signify getting the antiderivative. The indefinite integral is similar to the definite integral, yet the two are not the same. The below figure shows the difference between definite and indefinite integral.