# LIMITS AND CONTINUITY

## Limit Definition

A limit of a function is a number that a function reaches as the independent variable of the function reaches a given value. The value (say a) to which the function f(x) gets close arbitrarily as the value of the independent variable x becomes close arbitrarily to a given value "A" symbolized as f(x) = A.

### Points to remember:

- If $\lim_{x \to a-} f(x)$ is the expected value of f at x = a given the values of 'f' near x to the left of a. This value is known as the **left-hand limit** of 'f' at a.
- If $\lim_{x \to a+} f(x)$ is the expected value of f at x = a given the values of 'f' near x to the right of a. This value is known as the **right-hand limit** of f(x) at a.
- If the right-hand and left-hand limits coincide, we say the common value as the limit of f(x) at x = a and denote it by $\lim_{x \to a} f(x)$.

### One-Sided Limit

The limit that is based completely on the values of a function taken at x -value that is slightly greater or less than a particular value. A two-sided limit $\lim_{x \to a} f(x)$ takes the values of x into account that are both larger than and smaller than *a*. A one-sided limit from the left $\lim_{x \to a^-} f(x)$ or from the right $\lim_{x \to a^-} f(x)$ takes only values of x smaller or greater than *a* respectively.

### Properties of Limit

- The limit of a function is represented as f(x) reaches L as x tends to limit a, such that; $\lim_{x \to a} f(x) = L$
- The limit of the sum of two functions is equal to the sum of their limits, such that: $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
- The limit of any constant function is a constant term, such that, $\lim_{x \to a} C = C$
- The limit of product of the constant and function is equal to the product of constant and the limit of the function, such that: $\lim_{x \to a} m\, f(x) = m \lim_{x \to a} f(x)$
- Quotient Rule: $\lim_{x \to a}[f(x)/g(x)] = \lim_{x \to a} f(x)/\lim_{x \to a} g(x)$; if $\lim_{x \to a} g(x) \neq 0$

## Continuity

Many functions have the property that they can trace their graphs with a pencil without lifting the pencil from the paper's surface. These types of functions are called continuous. Intuitively, a function is continuous at a particular point if there is no break in its graph at that point. A precise definition of continuity of a real function is provided in terms of a limit's idea. First, a function f with variable x is continuous at the point "a" on the real line, if the limit of f(x), when x approaches the point "a", is equal to the value of f(x) at "a", i.e., f(a). Second, the function (as a whole) is continuous, if it is continuous at every point in its domain.

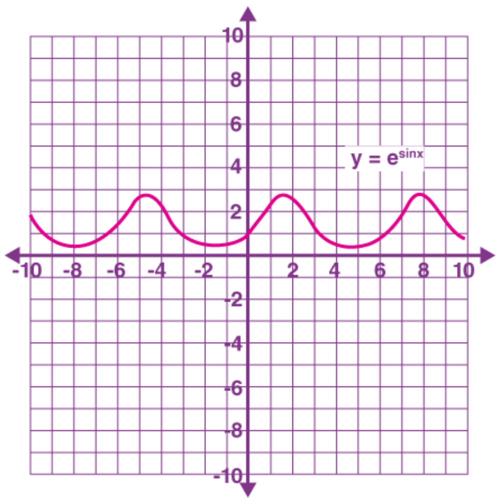
Mathematically, continuity can be defined as given below:

A function is said to be continuous at a particular point if the following three conditions are satisfied.

1. f(a) is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a^+} f(x) = \lim_{x \to a^-} f(x) = f(a)$

As mentioned before, a function is said to be continuous if you can trace its graph without lifting the pen from the paper. But a function is said to be discontinuous when it has any gap in between.

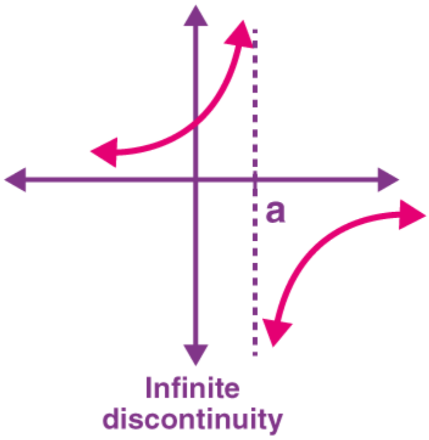
Below figure shows the graph of a continuous function.




## Types of Discontinuity

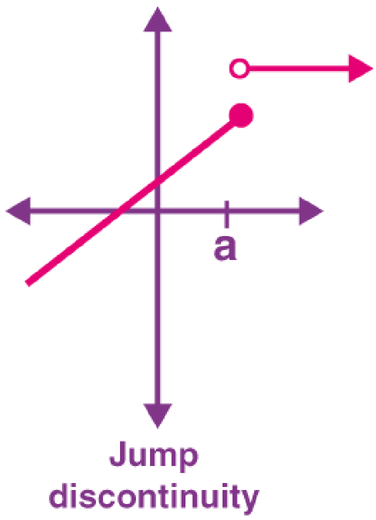
### Infinite Discontinuity

A branch of discontinuity wherein, a vertical asymptote is present at x = a and f(a) is not defined. This is also called Asymptotic Discontinuity. If a function has values on both sides of an asymptote, then it cannot be connected, so it is discontinuous at the asymptote. This can be shown using the graph as given below.




## Jump Discontinuity

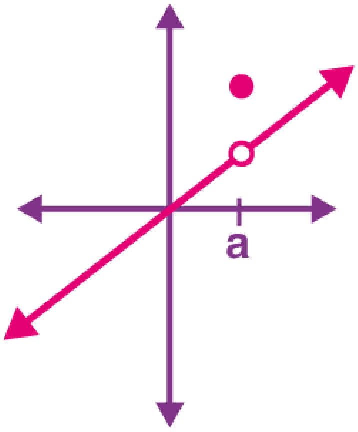
A branch of discontinuity wherein $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-} f(x)$, but both the limits are finite. This is also called simple discontinuity or continuities of the first kind. The graphical representation of jump discontinuity is given below.




## Positive Discontinuity

A branch of discontinuity wherein a function has a predefined two-sided limit at x = a, but either f(x) is undefined at *a,* or its value is not equal to the limit at *a*. This is also called a removable discontinuity.

Graphically, this can be shown as:




Positive or removable discontinuity

## Solved Examples

**1) Compute lim $(3x^2+5x-9)$**
***$x\to-2$***

**Solution:**

First, use property 2 to divide the limit into three separate limits. Then use property 1 to bring the constants out of the first two. This gives,

$$\lim_{x\to-2} (3x^2+5x-9) = \lim_{x\to-2} (3x^2) + \lim_{x\to-2} (5x) - \lim_{x\to-2} (9)$$

$= 3(-2)^2 + 5(-2) - (9)$

$= 12 - 10 - 9$

$= -7$

**2) Find the value of $\lim_{x\to3}$ [x(x+2)].**

**Solution:**

$\lim_{x\to3} [x(x+2)] = 3(3+2) = 3 \times 5 = 15$