EQUATION IN A CIRCLE

What is the Equation of a Circle?

A circle is a closed curve that is drawn from the fixed point called the centre, in which all the points on the curve are having the same distance from the centre point of the centre. The equation of circle with (h,k) center and r radius is given by:

$(x-h)^2 + (y-k)^2 = r^2$

Thus, if we know the coordinates of the center of the circle and its radius as well, we can easily find its equation.

Example: Say point (1,2) is the center of the circle and radius is equal to 4 cm. Then the equation of this circle will be:

$(x-1)^2 + (y-2)^2 = 4^2$

$(x^2-2x+1) + (y^2-4y+4) = 16$

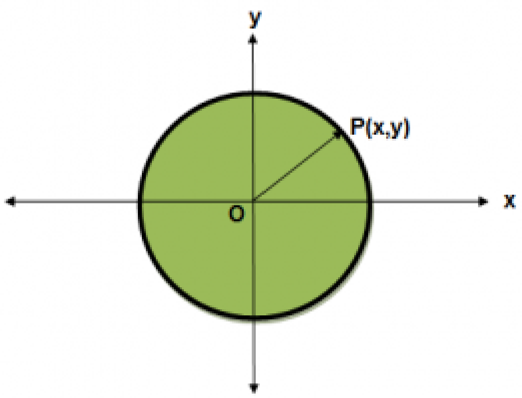
$X^2 + y^2 - 2x - 4y - 11 = 0$

Function or Not

We know that there is a question that arises in case of circle whether being a function or not. It is clear that a circle is not a function. Because, a function is defined by each value in the domain is exactly associated with one point in the codomain, but a line that passes through the circle, intersects the line at two points on the surface.

The mathematical way to describe the circle is an equation. Here, the equation of the circle is provided in all the forms such as general form, standard form along with the examples.

Equation of a Circle When the Centre is Origin



Consider an arbitrary point P(x, y) on the circle. Let 'a' be the radius of the circle which is equal to OP.

We know that the distance between the point (x, y) and origin (0,0) can be found using the distance formula which is equal to-

$\sqrt{[x^2 + y^2]} = a$

Therefore, the equation of a circle, with the centre as the origin is,

$\mathbf{x^2 + y^2 = a^2}$

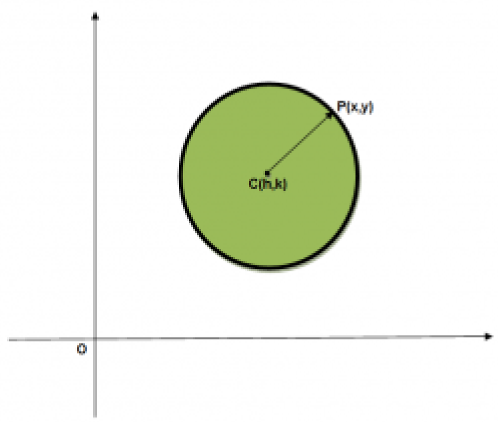
Where "a" is the radius of the circle.

Alternative Method

Let us derive in another way. Suppose (x,y) is a point on a circle, and the center of the circle is at origin (0,0). Now if we draw a perpendicular from point (x,y) to the x-axis, then we get a right triangle, where radius of the circle is the hypotenuse. The base of the triangle is distance along x-axis and height is the distance along the y-axis. Thus, by applying the Pythagoras theorem here, we get:

$\mathbf{x^2 + y^2 = radius^2}$

Equation of a Circle When the Centre is not an Origin



Let C(h, k) be the centre of the circle and P(x, y) be any point on the circle.

Therefore, the radius of a circle is CP.

By using distance formula,

$(x-h)^2 + (y-k)^2 = CP^2$

Let radius be 'a'.

Therefore, the equation of the circle with centre (h, k)and the radius 'a' is,

$\mathbf{(x-h)^2 + (y-k)^2 = a^2}$

which is called the **standard form for the equation of a circle.**

Equation of a Circle in General Form

The general equation of any type of circle is represented by:

$x^2 + y^2 + 2gx + 2fy + c = 0$, for all values of g, f and c.

Adding $g^2 + f^2$ on both sides of the equation gives,

$x^2 + 2gx + g^2 + y^2 + 2fy + f^2 = g^2 + f^2 - c$ ………………(1)

Since, $(x+g)^2 = x^2 + 2gx + g^2$ and $(y+f)^2 = y^2 + 2fy + f^2$ substituting the values in equation (1), we have

$(x+g)^2 + (y+f)^2 = g^2 + f^2 - c$ ……………(2)

Comparing (2) with $(x-h)^2 + (y-k)^2 = a^2$, where (h, k) is the centre and 'a' is the radius of the circle.

$H = -g$, $k = -f$

$a^2 = g^2 + f^2 - c$

Therefore,

$x^2 + y^2 + 2gx + 2fy + c = 0$, represents the circle with centre $(-g, -f)$ and radius equal to $a^2 = g^2 + f^2 - c$.

- If $g^2 + f^2 > c$, then the radius of the circle is real.
- If $g^2 + f^2 = c$, then the radius of the circle is zero which tells us that the circle is a point which coincides with the centre. Such type of circle is called a point circle.
- $g^2 + f^2 < c$, then the radius of the circle become imaginary. Therefore, it is a circle having a real centre and imaginary radius.

Solved Example

Example 1: Consider a circle whose centre is at the origin and radius is equal to 8 units.

Solution:

Given: Centre is (0, 0), radius is 8 units.

We know that the equation of a circle when the centre is origin:

$\mathbf{x^2 + y^2 = a^2}$

For the given condition, the equation of a circle is given as

$x^2 + y^2 = 8^2$

$x^2 + y^2 = 64$, which is the equation of a circle

Example 2: Find the equation of the circle whose centre is (3,5) and the radius is 4 units.

Solution:

Here, the centre of the circle is not an origin.

Therefore, the general equation of the circle is,

$(x-3)^2 + (y-5)^2 = 4^2$

$x^2 - 6x + 9 + y^2 - 10y + 25 = 16$

$x^2 + y^2 - 6x - 10y + 18 = 0$

Example 3: Equation of a circle is $x^2+y^2-12x-16y+19=0$. Find the centre and radius of the circle.

Solution:

Given equation is of the form $x^2 + y^2 + 2gx + 2fy + c = 0$,

$2g = -12$, $2f = -16$, $c = 19$

$g = -6$, $f = -8$

Centre of the circle is (6,8)

Radius of the circle = $\sqrt{[(-6)^2 + (-8)^2 - 19]} = \sqrt{[100 - 19]}$ =

= $\sqrt{81}$ = 9 units.

Therefore, the radius of the circle is 9 units.