

LOGARITHMS

- A logarithm of a number with a base is equal to another number. A logarithm is just the opposite function of exponentiation. For example, if $10^2 = 100$ then $\log_{10} 100 = 2$.
- Hence, we can conclude that,

$$\text{Log}_b x = n \text{ or } b^n = x$$

Where b is the base of the logarithmic function.

- A logarithm is defined as the power to which number must be raised to get some other values. It is the most convenient way to express large numbers. A logarithm has various important properties that prove multiplication and division of logarithms can also be written in the form of logarithm of addition and subtraction.
- The logarithm of a positive real number a with respect to base b , a positive real number not equal to $1^{[nb\ 1]}$, is the exponent by which b must be raised to yield a .

i.e $b^y = a$ and it is read as “the logarithm of a to base b .”

- In other words, the logarithm gives the answer to the question “How many times a number is multiplied to get the other number?”.

Example: How many 3's are multiplied to get the answer 27?

If we multiply 3 for 3 times, we get the answer 27.

Therefore, the logarithm is 3.

The logarithm form is written as follows:

$$\text{Log}_3 (27) = 3 \dots (1)$$

Therefore, the base 3 logarithm of 27 is 3.

The above logarithm form can also be written as:

$$3 \times 3 \times 3 = 27$$

$$3^3 = 27 \dots (2)$$

Thus, the equations (1) and (2) both represent the same meaning.

Logarithm Types

In most cases, we always deal with two different types of logarithms, namely

- Common Logarithm
- Natural Logarithm

Common Logarithm

The common logarithm is also called the base 10 logarithms. It is represented as \log_{10} or simply \log . For example, the common logarithm of 1000 is written as a $\log (1000)$. The common logarithm defines how many times we have to multiply the number 10, to get the required output.

For example, $\log (100) = 2$

If we multiply the number 10 twice, we get the result 100.

Natural Logarithm

The natural logarithm is called the base e logarithm. The natural logarithm is represented as \ln or \log_e . Here, "e" represents the Euler's constant which is approximately equal to 2.71828. For example, the natural logarithm of 78 is written as $\ln 78$. The natural logarithm defines how many we have to multiply "e" to get the required output.

For example, $\ln(78) = 4.357$.

Thus, the base e logarithm of 78 is equal to 4.357.

Logarithm Rules and Properties

There are certain rules based on which logarithmic operations can be performed. The names of these rules are:

- Product rule
- Division rule
- Power rule/Exponential Rule
- Change of base rule
- Base switch rule
- Derivative of log
- Integral of log

Let us have a look at each of these properties one by one

Product Rule

In this rule, the multiplication of two logarithmic values is equal to the addition of their individual logarithms.

$$\log_b(mn) = \log_b m + \log_b n$$

$$\text{For example: } \log_3(2y) = \log_3(2) + \log_3(y)$$

Division Rule

The division of two logarithmic values is equal to the difference of each logarithm.

$$\log_b(m/n) = \log_b m - \log_b n$$

$$\text{For example, } \log_3(2/y) = \log_3(2) - \log_3(y)$$

Exponential Rule

In the exponential rule, the logarithm of m with a rational exponent is equal to the exponent times its logarithm.

$$\log_b(m^n) = n \log_b m$$

$$\text{For example: } \log_b(2^3) = 3 \log_b 2$$

Change of Base Rule

$$\log_b m = \log_a m / \log_a b$$

For example: $\log_b 2 = \log_a 2 / \log_a b$

Base Switch Rule

$$\log_b (a) = 1 / \log_a (b)$$

For example: $\log_b 8 = 1 / \log_8 b$

Derivative of log

If $f(x) = \log_b(x)$, then the derivative of $f(x)$ is given by;

$$f'(x) = 1/(x \ln(b))$$

For example: Given, $f(x) = \log_{10}(x)$

Then, $f'(x) = 1/(x \ln(10))$

Integral of Log

$$\int \log_b(x) dx = x(\log_b(x) - 1/\ln(b)) + C$$

$$\text{Example: } \int \log_{10}(x) dx = x \cdot (\log_{10}(x) - 1/\ln(10)) + C$$

Other Properties

Some other properties of logarithmic functions are:

- $\log_b b = 1$
- $\log_b 1 = 0$
- $\log_b 0 = \text{undefined}$

Logarithmic Formulas

$$\log_b(mn) = \log_b(m) + \log_b(n)$$

$$\log_b(m/n) = \log_b(m) - \log_b(n)$$

$$\log_b(xy) = y \log_b(x)$$

$$\log_b m \sqrt[n]{n} = \log_b n/m$$

$$m \log_b(x) + n \log_b(y) = \log_b(x^m y^n)$$

$$\log_b(m+n) = \log_b m + \log_b(1+n/m)$$

$$\log_b(m-n) = \log_b m + \log_b(1-n/m)$$

Solved Examples

Question 1: Solve $\log_2(64) = ?$

Solution:

since $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, 6 is the exponent value and $\log_2(64) = 6$.

Question 2: What is the value of $\log_{10}(100)$?

Solution: In this case, 10^2 yields you 100. So, 2 is the exponent value, and the value of $\log_{10}(100) = 2$

Question 3: Use of the property of logarithms, solve for the value of x for $\log_3 x = \log_3 4 + \log_3 7$

Solution: By the addition rule, $\log_3 4 + \log_3 7 = \log_3 (4 \times 7)$

$\log_3 (28)$. Thus, $x = 28$.

Question 4: Solve for x in $\log_2 x = 5$

Solution: This logarithmic function can be written in the exponential form as $2^5 = x$

Therefore, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, $x = 32$.

Logarithmic Function Definition

The logarithmic function is defined as an inverse function to exponentiation. The logarithmic function is stated as follows

For x , $a > 0$, and $a \neq 1$,

$$y = \log_a x, \text{ if } x = a^y$$

Then the logarithmic function is written as:

$$f(x) = \log_a x$$

The most common bases used in logarithmic functions are base e and base 10. The log function with base 10 is called the common logarithmic function and it is denoted by \log_{10} or simply \log .

$$f(x) = \log_{10} x$$

The log function to the base e is called the natural logarithmic function and it is denoted by \log_e .

$$f(x) = \log_e x$$

To find the logarithm of a number, we can use the logarithm table instead of using a mere calculation. Before finding the logarithm of a number, we should know about the characteristic part and mantissa part of a given number:

- **Characteristic Part** – The whole part of a number is called the characteristic part. The characteristic of any number greater than one is positive, and if it is one less than the number of digits to the left of the decimal point in a given number. If the number is less than one, the characteristic is negative and is one more than the number of zeros to the right of the decimal point.
- **Mantissa Part** – The decimal part of the logarithm number is said to be the mantissa part and it should always be a positive value. If the mantissa part is in a negative value, then convert into the positive value.

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

How to Use the Log Table?

The procedure is given below to find the log value of a number using the log table. First, you have to know how to use the log table. The log table is given for the reference to find the values.

Step 1: Understand the concept of the logarithm. Each log table is only usable with a certain base. The most common type of logarithm table is used is log base 10.

Step 2: Identify the characteristic part and mantissa part of the given number. For example, if you want to find the value of $\log_{10}(15.27)$, first separate the characteristic part and the mantissa part.

Characteristic Part = 15

Mantissa part = 27

Step 3: Use a common log table. Now, use row number 15 and check column number 2 and write the corresponding value. So the value obtained is 1818.

Step 4: Use the logarithm table with a mean difference. Slide your finger in the mean difference column number 7 and row number 15, and write down the corresponding value as 20.

15.27												Mean Difference						
N	0	1	2	3	4	5	6	7	8	9		1	2	3	4	5	6	7
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430		3	6	10	13	16	19	23
14	1431	1492	1523	1553	1584	1614	1644	1673	1703	1732		3	6	9	12	15	18	21
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014		3	6	8	11	14	17	20
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279		3	5	8	11	13	16	7
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529		2	5	7	9	12	6	7

Step 5: Add both the values obtained in step 3 and step 4. That is $1818 + 20 = 1838$. Therefore, the value 1838 is the mantissa part.

$$\begin{array}{c}
 15.27 \\
 \swarrow \quad \searrow \\
 \text{Row 15 + column 2} \quad \text{Row 15 + column 7} \\
 1818 + 20 \\
 = 1838 \\
 \quad \quad \quad \nearrow \\
 \quad \quad \quad \text{Mantissa}
 \end{array}$$

Step 6: Find the characteristic part. Since the number lies between 10 and 100, (10^1 and 10^2), the characteristic part should be 1.

Step 7: Finally combine both the characteristic part and the mantissa part, it becomes 1.1838.

$$\begin{array}{ccc}
 10 & (10^1) & \leftarrow \text{Characteristic} \\
 \updownarrow 15 & & \\
 100 & (10^2) & \\
 \log_{10} 15.27 = & \text{Characteristic} & \text{Mantissa} \\
 & 1 & .1838 \\
 & \text{Characteristic} & \text{Mantissa}
 \end{array}$$

Example: Find the value of $\log_{10} 2.872$

Solution:

Step 1: Characteristic Part= 2 and mantissa part= 872

Step 2: Check the row number 28 and column number 7. So the value obtained is 4579.

Step 3: Check the mean difference value for row number 28 and mean difference column 2. The value corresponding to the row and column is 3

Step 4: Add the values obtained in step 2 and 3, we get 4582. This is the mantissa part.

Step 5: Since the number of digits to the left side of the decimal part is 1, the characteristic part is less than 1. So, the characteristic part is 0

Step 6: Finally combine the characteristic part and the mantissa part. So, it becomes 0.4582.

Therefore, the value of $\log 2.872$ is 0.4582.

Logarithmic Differentiation

Logarithmic differentiation is a method to find the derivatives of some complicated functions, using logarithms. There are cases in which differentiating the logarithm of a given function is simpler as compared to differentiating the function itself. By the proper usage of properties of logarithms and chain rule finding, the derivatives become easy. This concept is applicable to nearly all the non-zero functions which are differentiable in nature.

Therefore, in calculus, the differentiation of some complex functions is done by taking logarithms and then the logarithmic derivative is utilized to solve such a function.

Logarithmic Differentiation Formula

The equations which take the form $y = f(x) = [u(x)]^{v(x)}$ can be easily solved using the concept of logarithmic differentiation. The formula for log differentiation of a function is given by;

$$d/dx(x^x) = x^x(1+\ln x)$$

For differentiating functions of this type we take on both the sides of the given equation.

Therefore, taking log on both sides we get, $\log y = \log[u(x)]^{v(x)}$

$$\log y = v(x)\log u(x)$$

Now, differentiating both the sides w.r.t. x by implementing chain rule, we get

$$\begin{aligned}\frac{1}{y} \frac{dy}{dx} &= v(x) \times \frac{1}{u(x)} \times u'(x) + \log u(x) \times v'(x) \\ \Rightarrow \frac{dy}{dx} &= y \left[v(x) \times \frac{1}{u(x)} \times u'(x) + \log u(x) \times v'(x) \right]\end{aligned}$$

The only constraint for using logarithmic differentiation rules is that $f(x)$ and $u(x)$ must be positive as logarithmic functions are only defined for positive values.

The basic properties of real logarithms are generally applicable to the logarithmic derivatives.

For example: $(\log uv)' = (\log u + \log v)' = (\log u)' + (\log v)'$

Method to Solve Logarithm Functions

Follow the steps given here to solve find the differentiation of logarithm functions.

- Find the natural log of the function first which is needed to be differentiated.
- Now by the means of properties of logarithmic functions, distribute the terms that were originally gathered together in the original function and were difficult to differentiate.
- Now differentiate the equation which was resulted.
- At last, multiply the available equation by the function itself to get the required derivative.

Now, as we are thorough with logarithmic differentiation rules let us take some logarithmic differentiation examples to know a little bit more about this.

Example: Find the value of $\frac{dy}{dx}$ if, $y = e^{x^4}$

Solution: Given the function $y = e^{x^4}$

Taking natural logarithm of both the sides we get,

$$\ln y = \ln e^{x^4}$$

$$\ln y = x^4 \ln e$$

$$\ln y = x^4$$

Now, differentiating both the sides w.r.t we get,

$$\frac{1}{y} \frac{dy}{dx} = 4x^3$$

$$\Rightarrow \frac{dy}{dx} = y \cdot 4x^3$$

$$\Rightarrow \frac{dy}{dx} = e^{x^4} \times 4x^3$$

Therefore, we see how easy and simple it becomes to differentiate a function using logarithmic differentiation rules.

Example: Find the value of $\frac{dy}{dx}$ if $y = 2x^{\cos x}$.

Solution: Given the function $y = 2x^{\cos x}$

Taking logarithm of both the sides, we get

$$\log y = \log(2x^{\cos x})$$

$$\Rightarrow \log y = \log 2 + \log x^{\cos x} \quad \text{((As } \log(mn) = \log m + \log n))$$

$$\Rightarrow \log y = \log 2 + \cos x \times \log x \quad \text{((As } \log m^n = n \log m))$$

Now, differentiating both the sides w.r.t by using the chain rule we get,

$$\frac{1}{y} \frac{dy}{dx} = \frac{\cos x}{x} - (\sin x)(\log x)$$

Log and Ln Definition

Log: In Maths, the logarithm is the inverse function of exponentiation. In simpler words, the logarithm is defined as a power to which a number must be raised in order to get some other number. It is also called the logarithm of base 10, or common logarithm. The general form of a logarithm is given as:

$$\log_a (y) = x$$

The above-given form is written as:

$$a^x = y$$

Rules of Logarithm

There are four major rules or properties of the logarithm.

- $\log_b (mn) = \log_b m + \log_b n$
- $\log_b (m/n) = \log_b m - \log_b n$
- $\log_b (m^n) = n \log_b m$
- $\log_b m = \log_a m / \log_a b$

Ln: Ln is called the natural logarithm. It is also called the logarithm of the base e. Here, e is a number which is an irrational and transcendental number and is approximately equal to 2.718281828459... The natural logarithm (ln) is represented as **ln x or $\log_e x$**

Key Differences Between Log and Ln

Log	Ln
Log refers to a logarithm to the base 10	Ln refers to a logarithm to the base e
This is also called as a common logarithm	This is also called as a natural logarithm
The common log is represented as $\log_{10} (x)$	The natural log is represented as $\log_e (x)$
The exponent form of the common logarithm is $10^x = y$	The exponent form of the natural logarithm is $e^x = y$
The interrogative statement for the common logarithm is "At which number should we raise 10 to get y?"	The interrogative statement for the natural logarithm is "At which number should we raise Euler's constant number to get y?"
It is more widely used in physics when compared to ln	As logarithms are usually taken to the base in physics, ln is used much lesser
Mathematically, it is represented as log base 10	Mathematically, this is represented as log base e

Antilog Table

The Antilog which is also known as “Anti- Logarithms”, of a number is the inverse technique of finding the logarithm of the same number. Consider, if x is the logarithm of a number y with base b , then we can say y is the antilog of x to the base b . It is defined by

$$\text{If } \log_b y = x \qquad \text{Then, } y = \text{antilog } x$$

Both logarithm and antilog have their base as 2.7183. If the logarithm and antilogarithm are having their base 10, that should be converted into natural logarithm and antilog by multiplying it by 2.303.

How to Calculate Antilog?

Before finding the antilog of a number, we should know about the parts like the characteristic and mantissa part.

- **Characteristic Part** – The whole part is called the characteristic part. If the characteristic of logarithm of any number greater than one is positive and is one less than the number of digits in the left side of the decimal point.
- **Mantissa Part** – The decimal part of the logarithm number for a given number is called the mantissa part, and it should always be a positive value. If the mantissa part is in a negative value, convert into the positive value.

Procedure to Find the Antilog of a Number

Method 1: Using an Antilog Table

Consider a number, 2.6452

Step 1: Separate the characteristic part and the mantissa part. From the given example, the characteristic part is 2, and the mantissa part is 6452.

Step 2: To find a corresponding value of the mantissa part uses the antilog table. Using the antilog table, find the corresponding value. Now, find the row number that starts with .64, then the column for 5. Now, you get the corresponding value as 4416.

Step 3: From mean difference columns find the value. Again use the same row number .64 and find the value for column 2. Now, the value corresponding to this is 2.

Step 4: Add the values obtained in step 2 and 3, we get $4416 + 2 = 4418$.

Step 5: Now insert the decimal point. The decimal point always goes the designated place. For this, you have to add 1 to the characteristic value. Now you get 3. Then add the decimal point after 3 digits, we get 441.8

So, the antilog value of 2.6452 is 441.8.

Method 2: Antilog Calculation

Step 1: Separate the characteristic part and the mantissa part. From the above example given, the characteristic part is 2, and the mantissa part is 6452.

Step 2: Know the base. For numerical computations, the base is always 10. Therefore for computing the antilog use base 10.

Step 3: Calculate the 10^x . X is the number which you are using. If the mantissa of the number is 0, then the computation is easy. Calculate the value $10^{2.6452}$. Use a calculator to find the value. Finally, it comes 441.7

Both methods will give the same result.

Common Antilog Table

Below table helps to find the values of Characteristic Part and Mantissa Part of the number.

COMMON ANTILOGARITH TABLE

	0	1	2	3	4	5	6	7	8	9	Mean difference								
											1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	2
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	2
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	2	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	2	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	2	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	2	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	2	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	2	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	2	2	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	2	2	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	1	2	2	2	2	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	1	2	2	2	2	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	1	2	2	2	2	3
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	1	2	2	2	2	3
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	1	2	2	2	2	3
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	1	2	2	2	2	3
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	1	2	2	2	2	3
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	1	2	2	2	2	3
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	1	2	2	2	2	3
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	1	2	2	2	2	3
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	1	2	2	2	2	3
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	1	2	2	2	2	3
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	1	2	2	2	2	3
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	2	3	3	4	4
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	2	3	3	4	4
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	2	3	3	4	4
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	2	3	3	4	4
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	2	3	3	4	4
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	2	3	3	4	4
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	2	3	3	4	4
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	2	3	3	4	4
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	2	3	3	4	4
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	2	2	3	3	4	4
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	2	2	3	3	4	4
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	2	2	3	3	4	4
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	2	2	3	3	4	4
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	2	2	3	3	4	4
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	2	2	3	3	4	4
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	2	2	3	3	4	4
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	2	2	3	3	4	4
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	2	2	3	3	4	4
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	2	2	3	3	4	4
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	2	2	3	3	4	4
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	2	2	3	3	4	4
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	2	2	3	3	4	4
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	2	2	2	3	3	4	4
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	2	2	2	3	3	4	4
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	2	2	2	3	3	4	4
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	2	2	2	3	3	4	4
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	2	2	2	3	3	4	4
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	2	2	2	3	3	4	4
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	2	2	2	3	3	4	4
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	2	2	2	3	3	4	4
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	2	2	2	3	3	4	4
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	2	2	2	3	3	4	4
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	2	2	2	3	3	4	4
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	2	2	2	3	3	4	4
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	2	2	2	3	3	4	4
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	2	2	2	3	3	4	4
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	2	2	2	3	3	4	4
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	2	2	2	3	3	4	4
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358									

Step 4: Add the values obtained in step 2 and 3 , $2000 + 0 = 2000$.

Step 5: Now insert the decimal place. We know that the characteristic part is 3 and we have to add it with 1. Therefore, we get the value 4. Insert the decimal point after 4 places, and we get 2000.

Therefore, the solution of the antilog 3.3010 is 2000.