AVERAGE

What is the first thing that comes to your mind after hearing average?

In simple words we can say that average is that common value which may be assigned to all and after doing this the end result will be the same.

The average of the number of quantities of observations of the same kind is their sum divided by their number. The average is also called average value or mean value or arithmetic mean.

Average= $\frac{Sum\ of\ Terms}{Number\ of\ Terms}$

- The result obtained by adding several quantities together and then dividing this total by the number of quantities is called Average.
- The main term of average is equal distribution of a value among all which may distribute persons or things. We obtain the average of a number using formulae that is the sum of observations divided by Number of observations.
- Here are Average based on some facts and formulas and some shortcut tricks with examples. Below are some more examples for practicing.

Formula:

- Average = (Sum of observations / Number of observations).

Find the Average Speed

- If a person travels a distance at a speed of x km/hr and the same distance at a speed of y km/hr then the average speed during the whole journey is given by- $\frac{2xy}{x+y}$
- If a person covers A km at x km/hr and B km at y km/hr and C km at z km/hr, then the average speed in covering the whole distance is $\frac{A+B+C}{\frac{A}{X}+\frac{B}{Y}+\frac{C}{2}}$

Note-

- If the average age is increased, Age of new person = Age of separated person + (Increase in average × total number of persons)
- If the average age is decreased, Age of new person = Age of separated person - (Decrease in average × total number of persons)

When a person joins the group- In case of increase in average

- Age of new member = Previous average + (Increase in average × Number of members including new member)

In case of decrease in average

- Age of new member = Previous average - (Decrease in average × Number of members including new member)

In the Arithmetic Progression, there are two cases when the number of terms is odd and the second one is when the number of terms is even. So, when the number of terms is odd the average will be the middle term

- When the number of terms is even then the average will be the average of two middle terms.

Average

An average or an arithmetic mean of given data is the sum of the given observations

divided by number of observations

Important Formulae Related to Average of numbers

1. Average of first n natural number=(n+1)/2
2. Average of first n even number = (n+1)
3. Average of first n odd number = n
4. Average of consecutive number = (First number + Last number)/2
5. Average of 1 to n odd numbers = (Last odd number+1)/2
6. Average of 1 to n even numbers = (Last even number+2)/2
7. Average of squares of first n natural numbers = [(n+1) (2n+1)]/6
8. Average of the cubes of first n natural number =[n(n+1)2]/4
9. Average of n multiples of any number =[Number×(n+1)]/2

Concept 1 If the average of n1 observations is a1; the average of n2 observations is a2 and so on, then Average of all the observations = (n1× a_1+n2 ×a2+.....)/(n1+n2+.....)

Concept 2 If the average of m observations is a and the average of n observations taken out of is b, then Average of rest of the observations=(ma-n(2)/(m-n)

Example1:

A man bought 20 cows in RS. 200000. If the average cost of 12 cows is Rs. 12500, then what will be the average cost of remaining cows?

Solution:

Here m = 20, n = 12, a = 10000, b = 12500

average cost of remaining cows (20-8) cows = (2010000 -12×12500)/ (20-8) =Rs 6250

Concept 3

If the average of n students in a class is a, where average of passed students is x and average of failed students is y, then

Number of students passed= [Total Students (Total Average-Average of failed students)]/ (Average of passed students-Average of failed students) = [n(a-y)]/(x-y)

Example2:

In a class, there are 75 students whose average marks in the annual examination is 35. If the average marks of passed students is 55and average marks of failed students is 30, then find out the number of students who failed.

Solution:

Here, n = 75, a = 35, x = 55, y = 30

Number of students who passed = 75(35-30)/ (55-30) = 15

Number of students who failed = 75-15 = 60

Concept 4

If the average of total components in a group is a, where average of n components (1st part) is b and average of remaining components (2nd part) is c, then Number of remaining components (2nd part) =[n(a-(2)]/(c-(1)

Example3:

The average salary of the entire staff in an office is Rs. 200 per day. The average salary of officers is Rs. 550 and that of non-officers is Rs. 120. If the number of officers is 16, then find the numbers of non-officers in the office.

Solution:

Here n= 16, a = 200, b = 550, c = 120

Number of non -officer = 16(200-550)/ (120-200) = 70

Average Speed

Average speed is defined as total distance travelled divided by total time taken.

Average speed=Total distance travelled/ Total time taken

Case 1

If a person covers a certain distance at a speed of A km/h and again covers the same distance at a speed of B km/h, then the average speed during the whole journey will be 2AB/A+B

Case 2

If a person covers three equal distances at the speed of A km/h, B km/h and C km/h respectively, then the average speed during the whole Journey will be 3ABC/ (AB+BC+C (1)

Case 3

If distance P is covered with speed x, distance Q is covered with speed y and distance R is covered with speed z, then for the whole journey, Average speed= (P+Q+R+....)/ (P/x+Q/y+R/z+...)

Example 4:

A person covers 20 km distance with a speed of 5 km/h, then he covers the next 15 km with a speed of 3 km/h and the last 10 km is covered by him with a speed of 2 km/h. Find out his average speed for the whole journey.

Solution:

Average speed = (20 +15 +10)/ (20/5+15/3+10/2) = 3(3/14)

Case 4

If a person covers P part of his total distance with speed of x, Q part of total distance with speed of y and R part of total distance with speed of z,then Average speed=1/(P/x+Q/y+R/z+.....)

Example 5: The average of 6 consecutive even numbers is 21. Find the largest number?

Solution:

Largest no. = A + (n−1)

A = average

n = no. of terms Largest no.

= 21 + (6 -1) = 26

Example 6: The average of 6 consecutive odd numbers is 22. Find the smallest number?

Solution:

Smallest no. = A - (n - 1)

A = average

n = no. of terms

Smallest no. = 22 - (6 - 1) = 17

Example 7: The average of 5 consecutive even numbers is 46. Find the smallest number?

Solution:

Smallest no. = A - (n - 1)

A = average

n = no. of terms

Smallest no. = 46 -(5 -1) = 42

Example8: Find the average of the first 100 natural numbers?

Solution:

Average = $\frac{(n+1)}{2} = \frac{(100+2)}{2}$ = 50.5

Example 9: The average of 5 numbers is 29. If one number is excluded, the average becomes 27. Find the excluded number?

Solution:

Excluded no. = (5×29 - 4×27)

= (145-108) = 37

Example 10: The average age of 36 students is 15 years. When the teacher's age is included in it, the average increases by 1. What is the teacher's age?

Solution:

Teacher's age = (37×16 - 36×15)

= (592-540) = 52

Example 11: **The average weight of 8 persons increases by 2.5 kg when a new person comes in place of one of them weighing 40 kg. What is the weight of a new person?**

Solution:

Total weight increased = 8 x 2.5 = 20 kg weight of the new person = 40 + 20 = 60 kg

Example 12: The average weight of 10 persons decreases by 2.5 kg when a new person comes in place of one of them weighing 70 kg. What is the weight of a new person?

Solution:

Total weight decreased = 10 x 2.5 = 25 kg Weight of the new person = 70 - 25 = 45 kg

Example 13: A batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3 runs. Find his average after the 17th inning.

Solution:

Let the average after 17th inning = X and average after 16th inning = (X - 3)

16(X - 3) + 87 = 17X

16X - 48 +87 = 17X

X = 39

Example 14: The average of 11 results is 60. If the average of first 6 results is 58 and that of last 6 results is 63. Find the 6th result?

Solution:

A11 = 60

Average of first 6 (A6) = 58

Average of last 6 (A6) = 63

6th result = (58 × 6 + 63 × 6 - 60 × 11)

= (348+378) - 660

= 726 -660

= 66

Example 15: **The average of a, 11,23 and 17 is 15 and the average of a, b,12 and 25 is 16. Find the value of a: b?**

Solution: a + 11 + 23 +17 = 15 x 4

a = 9

a + b + 12 + 25 = 16 x 4

a + b = 27

9 + b = 27

b = 18

a: b = 9: 18

= 1: 2

Example 16: The average age of all the 100 employees in an office is 29 years, where 2/5 employees are female and the ratio of average age of male to female is 5: 7. Find the average age of female employees?

Solution:

60 × 5x + 40 × 7x = 29 ×100

300x + 280x = 2900

x = 5

average age of female employees

= 7x

= 7 x 5

= 35 years

Example 17: The average of two numbers A & B is 20, an average of B & C is 19 and average of C & A is 21, So find the value of A?

Solution:

A + B = 40

B + C = 40

C + A = 42

On adding above three

2(A + B + (3) =40 + 38 + 42= 120

= A + B + C = 60

A = (A + B + (3) – (B + (3)

= 60 – 38= 22

Example 18: Three years ago, the average age of a family of 5 members was 17 years. A baby having been born; the average age of the family is the same today. The present age of the baby is:

Solution:

Total age of 5 members, 3 years ago

= (17×5) = 85 years

Total age of 5 members, now = [85+(3×5)]

= 85 +15 = 100 years

Total age of 6 members now = (17×6)

= 102 years

The age of the baby = (102-100) = 2 years.

Example 19: **The average temperature of a town in the first four days of a month was 58 degrees. The average for the second, third, fourth and fifth days was 60 degrees. The temperature of the first and fifth days was in the ratio 7:8, then what is the temperature on the fifth day?**

Solution:

First four days average Temperature =580

1,2,3, 4th days total temp. = 58×4 = 232

Then 2,3,4,5 days total temp. = 60×4 = 240

Let the unknown temp, be x

5th day – 1st day = 240-232

=8(2,3,4 days temp. is common)

Given the ratio of first and fifth day is 7: 8

8x-7x=8

x=8

Fifth day's temperature = 8x=8x8=64

Practice Questions:

Q1. Shubh was conducting an experiment in which the average of 11 observations came to be 90, while the average of first five observations was 87, and that of the last five was 84. What was the measure of the 6th observation?

(1)165

(2)150

(3)145

(4)135

(5)125

Ans-(4) 135

Q2. A batsman has a certain average of runs for 12 innings. In the 13th innings he scored 96 runs, thereby increasing his average by 5 runs. What is his average after the 13th innings?

(1)64

(2)48

(3)36

(4)72

(5)89

Ans-(3) 36

Q3. There was one mess for 30 boarders in a certain hostel. If the number of boarders was increased by 10, the expenses of the mess increased by Rs. 40 per month, while the average expenditure per head diminished by Rs. 2. Find the original monthly expenses.

(1) Rs. 390

(2) Rs. 360

(3) Rs. 410

(4) Rs. 480

(5) Rs. 450

Ans-(2) Rs 360

Q4. The mean monthly salary paid to 75 workers in a factory is Rs. 5680. The mean salary of 25 of them is Rs. 5400 and that of 30 others is Rs. 5700. The mean salary of remaining workers is:

(1) Rs. 5000

(2) Rs. 7000

(3) Rs. 6000

(4) Rs. 8000

(5) Rs. 9000

Ans-(3) Rs. 6000

Q5. Of the three numbers, the first is twice the second and the second is twice the third. The average of these three numbers is 21. Find the largest number.

(1) 36

(2) 38

(3)47

(4) 48

(5) 35

Ans-(1) 36

Q6. In a mathematics exam, a student scored 30% marks in the first paper out of a total of 180. How much should he score in the second paper out of a total of 150, if he is to get an overall average of 50%?

(1) 74%

(2) 76%

(3)70%

(4) 80%

(5) 75%

Ans-(1) 74%

Q7. The average marks of a student in 8 subjects are 87. Of these, the highest marks are 2 more than the next in value. If these two subjects are eliminated, the average marks of the remaining subjects are 85. What is the highest score?

(1) 91

(2) 94

(3)89

(4) 96

(5) 92

Ans-(2) 94

Q8. An officer's pension on retirement from service is equal to half the average salary during the last 36 months of his service. His salary from 1 January, 2014 is Rs. 3800 per month with increment of Rs. 400 on 1 October 2014, October 2015 and 1 October, 2016. If he retires on 1 January, 2017, what pension does he draw?

(1) Rs. 2100

(2) Rs. 2150

(3) Rs. 2200

(4) Rs. 2250

(5) Rs. 2300

Ans-(2) Rs. 2150

Q9. In a one-day cricket match, Virat, Sehwag, Sachin, Dhoni and Yuvraj scored an average of 39 runs. Dhoni scored 7 more than Yuvraj. Yuvraj scored 9 fewer than Virat. Sehwag scored as many as Dhoni and Yuvraj combined; and Sehwag and Sachin together scored 110 runs between them. How many runs did Sachin score?

(1) 47

(2) 51

(3)53

(4) 49

(5) 57

Ans-(5) 57

Q10. The average of marks obtained by 120 candidates was 35. If the average of the passed candidates was 39 and that of the failed candidates was 15, then the number of those candidates, who passed the examination was:

(1) 100

(2) 110

(3)120

(4) 150

(5) 115

Ans-(1) 100

Q11. A train travels from A to B at the rate of 20kmper hour and from B to A at the rate of 30 km/hr. What is the average rate for the whole journey?

(1) 24 km/hr

(2) 25 km/hr

(3) 26 km/hr

(4) 28 km/hr

(5) None of these

Ans-(1) 24km/hr

Q12. The average salary of the entire staff in an office is Rs 120 per month. The average salary of officers is Rs 460 and that of non- officers is Rs 110. If the number of officers is 15, then find the number of non –officers in the office.

(1) 500

(2) 510

(3) 520

(4) 550

(5) None of these

Ans-(2) 510

Q13. There were 35 students in a hostel. If the number of students increases by 7, the expenses of the mess increase by Rs. 42 per day while the average expenditure per head diminishes by Rs1. Find the original expenditure of the mess.

(1) Rs. 400

(2) Rs. 340

(3) Rs. 420

(4) Rs. 450

(5) Rs. 300

Ans-(3) Rs. 420

Q14. The average age of a jury of 5 is 40, if a member aged 35 resigns and a man aged 25 becomes a member, then the average age of the new jury is

(1) 30

(2) 38

(3)40

(4) 42

(5) 36

Ans-(2)38

Q15. The average weight of 8 person is increased by 2.5 kg when one of them whose weight is 56 kg is replaced by a new man. The weight of the new man is:

(1) 58.5 kg

(2) 76 kg

(3)20 kg

(4) 64 kg

(5) None of these

Ans-(2)76kg