

LOGARITHMS

- A logarithm of a number with a base is equal to another number. A logarithm is just the opposite function of exponentiation. For example, if $10^2 = 100$ then $\log_{10} 100 = 2$.
- Hence, we can conclude that,

$$\text{Log}_b x = n \text{ or } b^n = x$$

Where b is the base of the logarithmic function.

- A logarithm is defined as the power to which number must be raised to get some other values. It is the most convenient way to express large numbers. A logarithm has various important properties that prove multiplication and division of logarithms can also be written in the form of logarithm of addition and subtraction.
- The logarithm of a positive real number a with respect to base b, a positive real number not equal to 1^[nb 1], is the exponent by which b must be raised to yield a.

i.e $b^y = a$ and it is read as “the logarithm of a to base b.”

- In other words, the logarithm gives the answer to the question “How many times a number is multiplied to get the other number?”.

Example: How many 3's are multiplied to get the answer 27?

If we multiply 3 for 3 times, we get the answer 27.

Therefore, the logarithm is 3.

The logarithm form is written as follows:

$$\text{Log}_3 (27) = 3 \dots (1)$$

Therefore, the base 3 logarithm of 27 is 3.

The above logarithm form can also be written as:

$$3 \times 3 \times 3 = 27$$

$$3^3 = 27 \dots (2)$$

Thus, the equations (1) and (2) both represent the same meaning.

Logarithm Types

In most cases, we always deal with two different types of logarithms, namely

- Common Logarithm
- Natural Logarithm

Common Logarithm

The common logarithm is also called the base 10 logarithms. It is represented as \log_{10} or simply \log . For example, the common logarithm of 1000 is written as $\log (1000)$. The common logarithm defines how many times we have to multiply the number 10, to get the required output.

For example, $\log (100) = 2$

If we multiply the number 10 twice, we get the result 100.

Natural Logarithm

The natural logarithm is called the base e logarithm. The natural logarithm is represented as \ln or \log_e . Here, "e" represents the Euler's constant which is approximately equal to 2.71828. For example, the natural logarithm of 78 is written as $\ln 78$. The natural logarithm defines how many we have to multiply "e" to get the required output.

For example, $\ln(78) = 4.357$.

Thus, the base e logarithm of 78 is equal to 4.357.

Logarithm Rules and Properties

There are certain rules based on which logarithmic operations can be performed. The names of these rules are:

- Product rule
- Division rule
- Power rule/Exponential Rule
- Change of base rule
- Base switch rule
- Derivative of log
- Integral of log

Let us have a look at each of these properties one by one

Product Rule

In this rule, the multiplication of two logarithmic values is equal to the addition of their individual logarithms.

$$\log_b(mn) = \log_b m + \log_b n$$

For example: $\log_3(2y) = \log_3(2) + \log_3(y)$

Division Rule

The division of two logarithmic values is equal to the difference of each logarithm.

$$\log_b(m/n) = \log_b m - \log_b n$$

For example, $\log_3(2/y) = \log_3(2) - \log_3(y)$

Exponential Rule

In the exponential rule, the logarithm of m with a rational exponent is equal to the exponent times its logarithm.

$$\log_b(m^n) = n \log_b m$$

For example: $\log_b(2^3) = 3 \log_b 2$

Change of Base Rule

$$\log_b m = \log_a m / \log_a b$$

For example: $\log_b 2 = \log_a 2 / \log_a b$

Base Switch Rule

$$\log_b (a) = 1 / \log_a (b)$$

For example: $\log_b 8 = 1 / \log_8 b$

Derivative of log

If $f(x) = \log_b(x)$, then the derivative of $f(x)$ is given by;

$$f'(x) = 1/(x \ln(b))$$

For example: Given, $f(x) = \log_{10}(x)$

Then, $f'(x) = 1/(x \ln(10))$

Integral of Log

$$\int \log_b(x) dx = x(\log_b(x) - 1/\ln(b)) + C$$

Example: $\int \log_{10}(x) dx = x \cdot (\log_{10}(x) - 1 / \ln(10)) + C$

Other Properties

Some other properties of logarithmic functions are:

- $\log_b b = 1$
- $\log_b 1 = 0$
- $\log_b 0 = \text{undefined}$

Logarithmic Formulas

$$\log_b(mn) = \log_b(m) + \log_b(n)$$

$$\log_b(m/n) = \log_b(m) - \log_b(n)$$

$$\log_b(xy) = y \log_b(x)$$

$$\log_b m \sqrt[n]{n} = \log_b n/m$$

$$m \log_b(x) + n \log_b(y) = \log_b(x^m y^n)$$

$$\log_b(m+n) = \log_b m + \log_b(1+nm)$$

$$\log_b(m-n) = \log_b m + \log_b(1-n/m)$$

Solved Examples

Question 1: Solve $\log_2(64) = ?$

Solution:

since $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 64$, 6 is the exponent value and $\log_2(64) = 6$.

Question 2: What is the value of $\log_{10}(100)$?

Solution: In this case, 10^2 yields you 100. So, 2 is the exponent value, and the value of $\log_{10}(100) = 2$

Question 3: Use of the property of logarithms, solve for the value of x for $\log_3 x = \log_3 4 + \log_3 7$

Solution: By the addition rule, $\log_3 4 + \log_3 7 = \log_3 (4 \times 7)$

$\log_3 (28)$. Thus, $x = 28$.

Question 4: Solve for x in $\log_2 x = 5$

Solution: This logarithmic function can be written in the exponential form as $2^5 = x$

Therefore, $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$, $x = 32$.

Logarithmic Function Definition

The logarithmic function is defined as an inverse function to exponentiation. The logarithmic function is stated as follows

For $x, a > 0$, and $a \neq 1$,

$$y = \log_a x, \text{ if } x = a^y$$

Then the logarithmic function is written as:

$$f(x) = \log_a x$$

The most common bases used in logarithmic functions are base e and base 10. The log function with base 10 is called the common logarithmic function and it is denoted by \log_{10} or simply log.

$$f(x) = \log_{10}$$

The log function to the base e is called the natural logarithmic function and it is denoted by \log_e .

$$f(x) = \log_e x$$

To find the logarithm of a number, we can use the logarithm table instead of using a mere calculation. Before finding the logarithm of a number, we should know about the characteristic part and mantissa part of a given number:

- **Characteristic Part** – The whole part of a number is called the characteristic part. The characteristic of any number greater than one is positive, and if it is one less than the number of digits to the left of the decimal point in a given number. If the number is less than one, the characteristic is negative and is one more than the number of zeros to the right of the decimal point.
- **Mantissa Part** – The decimal part of the logarithm number is said to be the mantissa part and it should always be a positive value. If the mantissa part is in a negative value, then convert into the positive value.

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

How to Use the Log Table?

The procedure is given below to find the log value of a number using the log table. First, you have to know how to use the log table. The log table is given for the reference to find the values.

Step 1: Understand the concept of the logarithm. Each log table is only usable with a certain base. The most common type of logarithm table is used is log base 10.

Step 2: Identify the characteristic part and mantissa part of the given number. For example, if you want to find the value of $\log_{10}(15.27)$, first separate the characteristic part and the mantissa part.

Characteristic Part = 15

Mantissa part = 27

Step 3: Use a common log table. Now, use row number 15 and check column number 2 and write the corresponding value. So the value obtained is 1818.

Step 4: Use the logarithm table with a mean difference. Slide your finger in the mean difference column number 7 and row number 15, and write down the corresponding value as 20.

15.27																	
N	0	1	2	3	4	5	6	7	8	9	Mean Difference						
											1	2	3	4	5	6	7
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23
14	1431	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	7
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	9	12	6	7

Step 5: Add both the values obtained in step 3 and step 4. That is $1818 + 20 = 1838$. Therefore, the value 1838 is the mantissa part.

$$\begin{array}{c}
 15.27 \\
 \text{Row 15 + column 2} \rightarrow 1818 \quad \text{Row 15 + column 7} \rightarrow 20 \\
 1818 + 20 \\
 = 1838 \\
 \text{Mantissa}
 \end{array}$$

Step 6: Find the characteristic part. Since the number lies between 10 and 100, (10^1 and 10^2), the characteristic part should be 1.

Step 7: Finally combine both the characteristic part and the mantissa part, it becomes 1.1838.

$$\begin{array}{c}
 10 \quad (10^1) \leftarrow \text{Characteristic} \\
 \uparrow 15 \\
 100 \quad (10^2) \\
 \text{Characteristic} \rightarrow 1 \quad \text{Mantissa} \rightarrow .1838 \\
 \log_{10} 15.27 = 1.1838
 \end{array}$$

Example: Find the value of $\log_{10} 2.872$

Solution:

Step 1: Characteristic Part= 2 and mantissa part= 872

Step 2: Check the row number 28 and column number 7. So the value obtained is 4579.

Step 3: Check the mean difference value for row number 28 and mean difference column 2. The value corresponding to the row and column is 3

Step 4: Add the values obtained in step 2 and 3, we get 4582. This is the mantissa part.

Step 5: Since the number of digits to the left side of the decimal part is 1, the characteristic part is less than 1. So, the characteristic part is 0

Step 6: Finally combine the characteristic part and the mantissa part. So, it becomes 0.4582.

Therefore, the value of $\log 2.872$ is 0.4582.

Practice Questions

1. Express $5^3 = 125$ in logarithm form.

Solution:

$$5^3 = 125$$

As we know,

$$a^b = c \Rightarrow \log_a c = b$$

Therefore;

$$\log_5 125 = 3$$

2. Express $\log_{10} 1 = 0$ in exponential form.

Solution:

$$\text{Given, } \log_{10} 1 = 0$$

By the rule, we know;

$$\log_a c = b \Rightarrow a^b = c$$

Hence,

$$10^0 = 1$$

3. Find the log of 32 to the base 4.

Solution: $\log_4 32 = x$

$$4^x = 32$$

$$(2^2)^x = 2 \times 2 \times 2 \times 2 \times 2$$

$$2^{2x} = 2^5$$

$$2x=5$$

$$x=5/2$$

Therefore,

$$\log_4 32 = 5/2$$

4. Find x if $\log_5(x-7)=1$.

Solution: Given,

$$\log_5(x-7)=1$$

Using logarithm rules, we can write;

$$5^1 = x-7$$

$$5 = x-7$$

$$x=5+7$$

$$x=12$$

5. If $\log_a m=n$, express a^{n-1} in terms of a and m.

Solution:

$$\log_a m=n$$

$$a^n=m$$

$$a^n/a=m/a$$

$$a^{n-1}=m/a$$

6. Solve for x if $\log(x-1)+\log(x+1)=\log_2 1$

Solution: $\log(x-1)+\log(x+1)=\log_2 1$

$$\log(x-1)+\log(x+1)=0$$

$$\log[(x-1)(x+1)]=0$$

Since, $\log 1 = 0$

$$(x-1)(x+1) = 1$$

$$x^2-1=1$$

$$x^2=2$$

$$x = \pm \sqrt{2}$$

Since, log of negative number is not defined.

Therefore, $x = \sqrt{2}$

7. Express $\log(75/16) - 2\log(5/9) + \log(32/243)$ in terms of log 2 and log 3.

Solution: $\log(75/16) - 2\log(5/9) + \log(32/243)$

Since, $n\log_a m = \log_a m^n$

$$\Rightarrow \log(75/16) - \log(5/9)^2 + \log(32/243)$$

$$\Rightarrow \log(75/16) - \log(25/81) + \log(32/243)$$

Since, $\log_a m - \log_a n = \log_a (m/n)$

$$\Rightarrow \log[(75/16) \div (25/81)] + \log(32/243)$$

$$\Rightarrow \log[(75/16) \times (81/25)] + \log(32/243)$$

$$\Rightarrow \log(243/16) + \log(32/243)$$

Since, $\log_a m + \log_a n = \log_a mn$

$$\Rightarrow \log(32/16)$$

$$\Rightarrow \log 2$$

8. Express $2\log x + 3\log y = \log a$ in logarithm free form.

Solution: $2\log x + 3\log y = \log a$

$$\log x^2 + \log y^3 = \log a$$

$$\log x^2 y^3 = \log a$$

$$x^2 y^3 = \log a$$

9. Prove that: $2\log(15/18) - \log(25/162) + \log(4/9) = \log 2$

Solution: $2\log(15/18) - \log(25/162) + \log(4/9) = \log 2$

Taking L.H.S.:

$$\Rightarrow 2\log(15/18) - \log(25/162) + \log(4/9)$$

$$\Rightarrow \log(15/18)^2 - \log(25/162) + \log(4/9)$$

$$\Rightarrow \log(225/324) - \log(25/162) + \log(4/9)$$

$$\Rightarrow \log[(225/324)(4/9)] - \log(25/162)$$

$$\Rightarrow \log[(225/324)(4/9)]/(25/162)$$

$$\Rightarrow \log(72/36)$$

$$\Rightarrow \log 2 \text{ (R.H.S)}$$

10. Express $\log_{10}(2+1)$ in the form of $\log_{10}x$.

Solution: $\log_{10}(2+1)$

$$= \log_{10}2 + \log_{10}1$$

$$= \log_{10}(2 \times 10)$$

$$= \log_{10}20$$