# ST5209/X Assignment 4

Due 1 Apr, 11.59pm

## Set up

1. Make sure you have the following installed on your system: LaTeX, R4.2.2+, RStudio 2023.12+, and Quarto 1.3.450+.
2. Pull changes from the course repo.
3. Create a separate folder in the root directory of the repo, label it with your name, e.g. `yanshuo-assignments`
4. Copy the assignment1.qmd file over to this directory.
5. Modify the duplicated document with your solutions, writing all R code as code chunks.
6. When running code, make sure your working directory is set to be the folder with your assignment .qmd file, e.g. `yanshuo-assignments`. This is to ensure that all file paths are valid.[1]

## Submission

1. Render the document to get a .pdf printout.
2. Submit both the .qmd and .pdf files to Canvas.

## 1. AR polynomial

Consider the AR(2) model

$$X_t = 4 + 0.5X_{t-1} - 0.25X_{t-2} + W_t. \tag{1}$$

a. What is the autoregressive polynomial?

b. What are its roots?

c. Is this model causal? Why?

d. What is the period of the sinusoidal portion of its ACF?

[1] You may view and set the working directory using `getwd()` and `setwd()`.

e. Given the representation

$$X_t = \sum_{j=0}^{\infty} \psi_j W_{t-j},$$

solve for $\psi_0, \psi_1, \psi_2, \psi_3$.

## 2. Likelihood

Consider the AR(1) model

$$X_t = 1 - 0.6X_{t-1} + W_t,$$

where $W_t \sim WN(0, 0.25)$ is Gaussian white noise. We are given observations $x_1 = 0.2, x_2 = -0.3, x_3 = 0.4$.

a. What is the mean of $X_t$?

b. Write the full likelihood of this model.

c. Write the conditional likelihood of this model, when conditioning on the value of $x_1$.

## 3. Reversibility

a. Create a sample trajectory of length $n = 200$ from the AR(2) model from Problem 1 using `arima.sim()`.

b. Reverse the time index of the vector you obtain using `rev()`.

c. Fit an AR(2) model to the reversed time series using `fable`. Hint: You may use the code snippet `model(AR(X ~ order(2)))`.

d. Inspect the model parameters using `tidy()`. Why are they similar to the those in Equation 1?

e. Make a forecast with $h = 10$. What does this correspond to in terms of the original time series?

## 4. Yule-Walker

a. Write the Yule-Walker equations for the AR(2) model from Problem 1.

b. Arrange the equations in the following matrix form (i.e. fill in the missing entries):

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} \gamma(0) \\ \gamma(1) \\ \gamma(2) \end{bmatrix} = \begin{bmatrix} \sigma^2 \\ 0 \\ 0 \end{bmatrix}.$$

c. Solve the system from part b) for $\gamma(0), \gamma(1), \gamma(2)$ numerically using `solve()`.

d. Given

$$\Gamma_2 = \begin{bmatrix} \gamma(0) & \gamma(1) \\ \gamma(1) & \gamma(0) \end{bmatrix},$$

what is the top left entry of $\Gamma_2^{-1}$ in terms of $\gamma(0)$ and $\gamma(1)$?

e. Write a 95% confidence interval for $\phi_1$ using your answers for d) and 3d).

## 5. Real data analysis

a. Load the dataset `astsa::globtemp` and convert it into a tsibble.

b. Filter the time series to include measurements only up to 1960.

c. Fit an AR model using `model(AR(X))`.

d. What is order of the model selected? What are the fitted parameters?

e. Inspect the model residuals using `gg_tsresiduals()`. What can you say about the model fit?