# Algebra & Trig Review

This review was originally written for my Calculus I class, but it should be accessible to anyone needing a review in some basic algebra and trig topics. The review contains the occasional comment about how a topic will/can be used in a calculus class. If you aren't in a calculus class, you can ignore these comments. I don't cover all the topics that you would see in a typical Algebra or Trig class, I've mostly covered those that I feel would be most useful for a student in a Calculus class although I have included a couple that are not really required for a Calculus class. These extra topics were included simply because they do come up on occasion and I felt like including them. There are also, in all likelihood, a few Algebra/Trig topics that do arise occasionally in a Calculus class that I didn't include.

Because this review was originally written for my Calculus students to use as a test of their algebra and/or trig skills it is generally in the form of a problem set. The solution to the first problem in a set contains detailed information on how to solve that particular type of problem. The remaining solutions are also fairly detailed and may contain further required information that wasn't given in the first problem, but they probably won't contain explicit instructions or reasons for performing a certain step in the solution process. It was my intention in writing the solutions to make them detailed enough that someone needing to learn a particular topic should be able to pick the topic up from the solutions to the problems. I hope that I've accomplished this.

So, why did I even bother to write this?

The ability to do basic algebra is absolutely vital to successfully passing a calculus class. As you progress through a calculus class you will see that almost every calculus problem involves a fair amount of algebra. In fact, in many calculus problems, 90% or more of the problem is algebra.

So, while you may understand the basic calculus concepts, if you can't do the algebra you won't be able to do the problems. If you can't do these problems you will find it very difficult to pass the course.

Likewise, you will find that many topics in a calculus class require you to be able to basic trigonometry. In quite a few problems you will be asked to work with trig functions, evaluate trig functions and solve trig equations. Without the ability to do basic trig you will have a hard time doing these problems.

Good algebra and trig skills will also be required in Calculus II or Calculus III. So, if you don't have good algebra or trig skills you will find it very difficult to complete this sequence of courses.

Most of the following set of problems illustrates the kinds of algebra and trig skills that you will need in order to successfully complete any calculus course here at Lamar University. The algebra and trig in these problems fall into three categories :

- Easier than the typical calculus problem,
- similar to a typical calculus problem, and
- harder than a typical calculus problem.

Which category each problem falls into will depend on the instructor you have. In my calculus course you will find that most of these problems falling into the first two categories.

Depending on your instructor, the last few sections (Inverse Trig Functions through Solving Logarithm Equations) may be covered to one degree or another in your class. However, even if your instructor does cover this material you will find it useful to have gone over these sections. In my course I spend the first couple of days covering the basics of exponential and logarithm functions since I tend to use them on a regular basis.

This problem set is not designed to discourage you, but instead to make sure you have the background that is required in order to pass this course. If you have trouble with the material on this worksheet (especially the Exponents - Solving Trig Equations sections) you will find that you will also have a great deal of trouble passing a calculus course.

Please be aware that this problem set is NOT designed to be a substitute for an algebra or trig course. As I have already mentioned I do not cover all the topics that are typically covered in an Algebra or Trig course. Most of the topics covered here are those that I feel are important topics that you MUST have in order to successfully complete a calculus course (in particular my Calculus course). You may find that there are other algebra or trig skills that are also required for you to be successful in this course that are not covered in this review. You may also find that your instructor will not require all the skills that are listed here on this review.

Here is a brief listing and quick explanation of each topic covered in this review.

**Algebra**

**Exponents** – A brief review of the basic exponent properties.
**Absolute Value** – A couple of quick problems to remind you of how absolute value works.
**Radicals** – A review of radicals and some of their properties.
**Rationalizing** – A review of a topic that doesn't always get covered all that well in an algebra class but is required occasionally in a Calculus class.
**Functions** – Function notation and function evaluation.
**Multiplying Polynomials** – A couple of polynomial multiplication problems illustrating common mistakes in a Calculus class.
**Factoring** – Some basic factoring.
**Simplifying Rational Expressions** – The ability to simplify rational expressions can be vital in some Calculus problems.
**Graphing and Common Graphs** – Here are some common functions and how to graph them. The functions include parabolas, circles, ellipses, and hyperbolas.
**Solving Equations, Part I** – Solving single variable equations, including the quadratic formula.
**Solving Equations, Part II** – Solving multiple variable equations.
**Solving Systems of Equations** – Solving systems of equations and some interpretations of the solution.
**Solving Inequalities** – Solving polynomial and rational inequalities.
**Absolute Value Equations and Inequalities** – Solving equations and inequalities that involve absolute value.

# Algebra Review

## Exponents

Simplify each of the following as much as possible.

1. $2x^4y^{-3}x^{-19}+y^{\frac{1}{3}}y^{-\frac{3}{4}}$

2. $x^{\frac{3}{5}}x^2x^{-\frac{1}{2}}$

3. $\dfrac{xx^{-\frac{1}{3}}}{2x^5}$

4. $\left(\dfrac{2x^{-2}x^{\frac{4}{5}}y^6}{x+y}\right)^{-3}$

5. $\left(\dfrac{x^{\frac{4}{7}}x^{\frac{9}{2}}x^{-\frac{10}{3}}-x^2x^{-9}x^{\frac{1}{2}}}{x+1}\right)^0$

# Absolute Value

1. Evaluate $\left|5\right|$ and $\left|-123\right|$

2. Eliminate the absolute value bars from $\left|3-8x\right|$

3. List as many of the properties of absolute value as you can.

# Radicals

Evaluate the following.

1. $\sqrt[3]{125}$

2. $\sqrt[6]{64}$

3. $\sqrt[5]{-243}$

4. $\sqrt[2]{100}$

5. $\sqrt[4]{-16}$

Convert each of the following to exponential form.

6. $\sqrt{7x}$

7. $\sqrt[5]{x^2}$

8. $\sqrt[3]{4x+8}$

Simplify each of the following.

9. $\sqrt[3]{16x^6y^{13}}$ Assume that $x \geq 0$ and $y \geq 0$ for this problem.

10. $\sqrt[4]{16x^8y^{15}}$

# Rationalizing

Rationalize each of the following.

1. $\dfrac{3xy}{\sqrt{x}+\sqrt{y}}$

2. $\dfrac{\sqrt{t+2}-2}{t^{2}-4}$

# Functions

1. Given $f(x) = -x^2 + 6x - 11$ and $g(x) = \sqrt{4x-3}$ find each of the following.

(a) $f(2)$ (b) $g(2)$ (c) $f(-3)$ (d) $g(10)$

(e) $f(t)$ (f) $f(t-3)$ (g) $f(x-3)$ (h) $f(4x-1)$

2. Given $f(x) = 10$ find each of the following.

(a) $f(7)$ (b) $f(0)$ (c) $f(-14)$

3. Given $f(x) = 3x^2 - x + 10$ and $g(x) = 1 - 20x$ find each of the following.

(a) $(f-g)(x)$ (b) $\left(\dfrac{f}{g}\right)(x)$ (c) $(fg)(x)$

(d) $(f \circ g)(5)$ (e) $(f \circ g)(x)$ (f) $(g \circ f)(x)$

## Multiplying Polynomials

Multiply each of the following.

1. $(7x-4)(7x+4)$

2. $(2x-5)^2$

3. $2(x+3)^2$

4. $(2x^3-x)\left(\sqrt{x}+\dfrac{2}{x}\right)$

5. $(3x+2)(x^2-9x+12)$

# Factoring

Factor each of the following as much as possible.

1. $100x^2 - 81$

2. $100x^2 + 81$

3. $3x^2 + 13x - 10$

4. $25x^2 + 10x + 1$

5. $4x^5 - 8x^4 - 32x^3$

6. $125x^3 - 8$

## Simplifying Rational Expressions

Simplify each of the following rational expressions.

1. $\dfrac{2x^2-8}{x^2-4x+4}$

2. $\dfrac{x^2-5x-6}{6x-x^2}$

## Graphing and Common Graphs

Sketch the graph of each of the following.

1. $y = -\frac{2}{5}x + 3$

2. $y = (x+3)^2 - 1$

3. $y = -x^2 + 2x + 3$

4. $f(x) = -x^2 + 2x + 3$

5. $x = y^2 - 6y + 5$

6. $x^2 + (y+5)^2 = 4$

7. $x^2 + 2x + y^2 - 8y + 8 = 0$

8. $\frac{(x-2)^2}{9} + 4(y+2)^2 = 1$

9. $\frac{(x+1)^2}{9} - \frac{(y-2)^2}{4} = 1$

10. $y = \sqrt{x}$

11. $y = x^3$

12. $y = |x|$

# Solving Equations, Part I

Solve each of the following equations.

1. $x^3 - 3x^2 = x^2 + 21x$

2. $3x^2 - 16x + 1 = 0$

3. $x^2 - 8x + 21 = 0$

# Solving Equations, Part II

Solve each of the following equations for *y*.

1. $x = \dfrac{2y-5}{6-7y}$

2. $3x^2\left(3-5y\right)+\sin x = 3xy+8$

3. $2x^2+2y^2=5$

# Solving Systems of Equations

1. Solve the following system of equations. Interpret the solution.

$$\begin{aligned} 2x - y - 2z &= -3 \\ x + 3y + z &= -1 \\ 5x - 4y + 3z &= 10 \end{aligned}$$

2. Determine where the following two curves intersect.

$$\begin{aligned} x^2 + y^2 &= 13 \\ y &= x^2 - 1 \end{aligned}$$

3. Graph the following two curves and determine where they intersect.

$$\begin{aligned} x &= y^2 - 4y - 8 \\ x &= 5y + 28 \end{aligned}$$

# Solving Inequalities

Solve each of the following inequalities.

1. $x^2 - 10 > 3x$

2. $x^4 + 4x^3 - 12x^2 \le 0$

3. $3x^2 - 2x - 11 > 0$

4. $\dfrac{x-3}{x+2} \ge 0$

5. $\dfrac{x^2 - 3x - 10}{x-1} < 0$

6. $\dfrac{2x}{x+1} \ge 3$

## Absolute Value Equations and Inequalities

Solve each of the following.

1. $\left|3x+8\right|=2$

2. $\left|2x-4\right|=10$

3. $\left|x+1\right|=-15$

4. $\left|7x-10\right|\le 4$

5. $\left|1-2x\right|<7$

6. $\left|x-9\right|\le -1$

7. $\left|4x+5\right|>3$

8. $\left|4-11x\right|\ge 9$

9. $\left|10x+1\right|>-4$

# Trig Review

## Trig Function Evaluation

One of the problems with most trig classes is that they tend to concentrate on right triangle trig and do everything in terms of degrees. Then you get to a calculus course where almost everything is done in radians and the unit circle is a very useful tool.

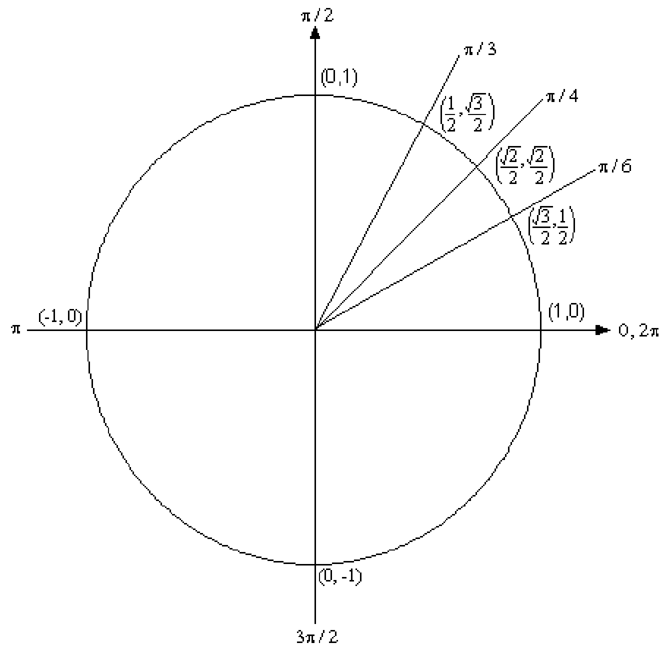
So first off let's look at the following table to relate degrees and radians.

| Degree | 0 | 30 | 45 | 60 | 90 | 180 | 270 | 360 |
|---|---|---|---|---|---|---|---|---|
| Radians | 0 | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ | $\frac{3\pi}{2}$ | $2\pi$ |

Know this table! There are, of course, many other angles in radians that we'll see during this class, but most will relate back to these few angles. So, if you can deal with these angles you will be able to deal with most of the others.

**Be forewarned, everything in most calculus classes will be done in radians!**

Now, let's look at the unit circle. Below is the unit circle with just the first quadrant filled in. The way the unit circle works is to draw a line from the center of the circle outwards corresponding to a given angle. Then look at the coordinates of the point where the line and the circle intersect. The first coordinate is the cosine of that angle and the second coordinate is the sine of that angle. There are a couple of *basic* angles that are commonly used. These are $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2},$ and $2\pi$ and are shown below along with the coordinates of the intersections. So, from the unit circle below we can see that $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$ and $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.




Remember how the signs of angles work. If you rotate in a counter clockwise direction the angle is positive and if you rotate in a clockwise direction the angle is negative.

Recall as well that one complete revolution is $2\pi$ , so the positive *x*-axis can correspond to either an angle of 0 or $2\pi$ (or $4\pi$ , or $6\pi$ , or $-2\pi$ , or $-4\pi$ , *etc*. depending on the direction of rotation). Likewise, the angle $\frac{\pi}{6}$ (to pick an angle completely at random) can also be any of the following angles:

$\frac{\pi}{6}+2\pi=\frac{13\pi}{6}$ (start at $\frac{\pi}{6}$ then rotate once around counter clockwise)

$\frac{\pi}{6}+4\pi=\frac{25\pi}{6}$ (start at $\frac{\pi}{6}$ then rotate around twice counter clockwise)

$\frac{\pi}{6}-2\pi=-\frac{11\pi}{6}$ (start at $\frac{\pi}{6}$ then rotate once around clockwise)

$\frac{\pi}{6}-4\pi=-\frac{23\pi}{6}$ (start at $\frac{\pi}{6}$ then rotate around twice clockwise)

*etc.*

In fact, $\frac{\pi}{6}$ can be any of the following angles $\frac{\pi}{6}+2\pi\,n$, $n=0,\pm 1,\pm 2,\pm 3,\ldots$ In this case *n* is the number of complete revolutions you make around the unit circle starting at $\frac{\pi}{6}$. Positive values of *n* correspond to counter clockwise rotations and negative values of *n* correspond to clockwise rotations.

So, why did I only put in the first quadrant? The answer is simple. If you know the first quadrant then you can get all the other quadrants from the first. You'll see this in the following examples.

Find the exact value of each of the following. In other words, don't use a calculator.

1. $\sin\left(\frac{2\pi}{3}\right)$ and $\sin\left(-\frac{2\pi}{3}\right)$

2. $\cos\left(\frac{7\pi}{6}\right)$ and $\cos\left(-\frac{7\pi}{6}\right)$

3. $\tan\left(-\frac{\pi}{4}\right)$ and $\tan\left(\frac{7\pi}{4}\right)$

4. $\sin\left(\frac{9\pi}{4}\right)$

5. $\sec\left(\frac{25\pi}{6}\right)$

6. $\tan\left(\frac{4\pi}{3}\right)$

**Trig Evaluation Final Thoughts**

As we saw in the previous examples if you know the first quadrant of the unit circle you can find the value of ANY trig function (not just sine and cosine) for ANY angle that can be related back to one of those shown in the first quadrant. This is a nice idea to remember as it means that you only need to memorize the first quadrant and how to get the angles in the remaining three quadrants!

In these problems I used only "basic" angles, but many of the ideas here can also be applied to angles other than these "basic" angles as we'll see in Solving Trig Equations.

# Graphs of Trig Functions

There is not a whole lot to this section. It is here just to remind you of the graphs of the six trig functions as well as a couple of nice properties about trig functions.

Before jumping into the problems remember we saw in the Trig Function Evaluation section that trig functions are examples of *periodic* functions. This means that all we really need to do is graph the function for one periods length of values then repeat the graph.

Graph the following function.

1. $y = \cos(x)$

2. $y = \cos(2x)$

3. $y = 5\cos(2x)$

4. $y = \sin(x)$

5. $y = \sin\left(\frac{x}{3}\right)$

6. $y = \tan(x)$

7. $y = \sec(x)$

8. $y = \csc(x)$

9. $y = \cot(x)$

# Trig Formulas

This is not a complete list of trig formulas. This is just a list of formulas that I've found to be the most useful in a Calculus class. For a complete listing of trig formulas you can download my Trig Cheat Sheet.

Complete the following formulas.

1. $\sin^2\left(\theta\right)+\cos^2\left(\theta\right)=$

2. $\tan^2\left(\theta\right)+1=$

3. $\sin\left(2t\right)=$

4. $\cos\left(2x\right)=$ (Three possible formulas)

5. $\cos^2(x)=$ (In terms of cosine to the first power)

6. $\sin^2(x)=$ (In terms of cosine to the first power)

# Solving Trig Equations

Solve the following trig equations. For those without intervals listed find ALL possible solutions. For those with intervals listed find only the solutions that fall in those intervals.

1. $2\cos(t)=\sqrt{3}$

2. $2\cos(t)=\sqrt{3}$ on $[-2\pi,2\pi]$

3. $2\sin(5x)=-\sqrt{3}$

4. $2\sin(5x+4)=-\sqrt{3}$

5. $2\sin(3x)=1$ on $[-\pi,\pi]$

6. $\sin(4t)=1$ on $[0,2\pi]$

7. $\cos(3x)=2$

8. $\sin(2x)=-\cos(2x)$

9. $2\sin(\theta)\cos(\theta)=1$

10. $\sin(w)\cos(w)+\cos(w)=0$

11. $2\cos^2(3x)+5\cos(3x)-3=0$

12. $5\sin(2x)=1$

13. $4\cos\left(\dfrac{x}{5}\right)=-3$

14. $10\sin(x-2)=-7$

# Inverse Trig Functions

One of the more common notations for inverse trig functions can be very confusing. First, regardless of how you are used to dealing with exponentiation we tend to denote an inverse trig function with an "exponent" of "-1". In other words, the inverse cosine is denoted as $\cos^{-1}(x)$. It is important here to note that in this case the "-1" is NOT an exponent and so,

$$\cos^{-1}(x) \neq \frac{1}{\cos(x)}.$$

In inverse trig functions the "-1" looks like an exponent but it isn't, it is simply a notation that we use to denote the fact that we're dealing with an inverse trig function. It is a notation that we use in this case to denote inverse trig functions. If I had really wanted exponentiation to denote 1 over cosine I would use the following.

$$\left(\cos(x)\right)^{-1} = \frac{1}{\cos(x)}$$

There's another notation for inverse trig functions that avoids this ambiguity. It is the following.

$$\cos^{-1}(x) = \arccos(x)$$
$$\sin^{-1}(x) = \arcsin(x)$$
$$\tan^{-1}(x) = \arctan(x)$$

So, be careful with the notation for inverse trig functions!

There are, of course, similar inverse functions for the remaining three trig functions, but these are the main three that you'll see in a calculus class so I'm going to concentrate on them.

To evaluate inverse trig functions remember that the following statements are equivalent.

$$\theta = \cos^{-1}(x) \qquad \Leftrightarrow \qquad x = \cos(\theta)$$
$$\theta = \sin^{-1}(x) \qquad \Leftrightarrow \qquad x = \sin(\theta)$$
$$\theta = \tan^{-1}(x) \qquad \Leftrightarrow \qquad x = \tan(\theta)$$

In other words, when we evaluate an inverse trig function we are asking what angle, $\theta$, did we plug into the trig function (regular, not inverse!) to get *x*.

So, let's do some problems to see how these work. Evaluate each of the following.

1. $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

3. $\sin^{-1}\left(-\frac{1}{2}\right)$

4. $\tan^{-1}(1)$

5. $\cos\left(\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)$

6. $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)$

7. $\tan\left(\tan^{-1}(-4)\right)$.

# Exponential & Logarithm Review

## Basic Exponential Functions

First, let's recall that for $b > 0$ and $b \neq 1$ an exponential function is any function that is in the form

$$f\left(x\right) = b^x$$

We require $b \neq 1$ to avoid the following situation,

$$f\left(x\right) = 1^x = 1$$

So, if we allowed $b = 1$ we would just get the constant function, 1.

We require $b > 0$ to avoid the following situation,

$$f\left(x\right) = \left(-4\right)^x \qquad \Rightarrow \qquad f\left(\frac{1}{2}\right) = \left(-4\right)^{\frac{1}{2}} = \sqrt{-4}$$

By requiring $b > 0$ we don't have to worry about the possibility of square roots of negative numbers.

1. Evaluate $f\left(x\right) = 4^x$, $g\left(x\right) = \left(\frac{1}{4}\right)^x$ and $h\left(x\right) = 4^{-x}$ at $x = -2, -1, 0, 1, 2$.

2. Sketch the graph of $f\left(x\right) = 4^x$, $g\left(x\right) = \left(\frac{1}{4}\right)^x$ and $h\left(x\right) = 4^{-x}$ on the same axis system.

3. List as some basic properties for $f\left(x\right) = b^x$.

4. Evaluate $f\left(x\right) = \mathbf{e}^x$, $g\left(x\right) = \mathbf{e}^{-x}$ and $h\left(x\right) = 5\mathbf{e}^{1-3x}$ at $x = -2, -1, 0, 1, 2$.

5. Sketch the graph of $f\left(x\right) = \mathbf{e}^x$ and $g\left(x\right) = \mathbf{e}^{-x}$.

## Basic Logarithm Functions

1. Without a calculator give the exact value of each of the following logarithms.

(a) $\log_2 16$ (b) $\log_4 16$ (c) $\log_5 625$

(d) $\log_9 \dfrac{1}{531441}$ (e) $\log_{\frac{1}{6}} 36$ (f) $\log_{\frac{3}{2}} \dfrac{27}{8}$

2. Without a calculator give the exact value of each of the following logarithms.

(a) $\ln \sqrt[3]{\mathbf{e}}$ (b) $\log 1000$ (c) $\log_{16} 16$

(d) $\log_{23} 1$ (e) $\log_2 \sqrt[7]{32}$

## Logarithm Properties

Complete the following formulas.

1. $\log_b b =$

2. $\log_b 1 =$

3. $\log_b b^x =$

4. $b^{\log_b x} =$

5. $\log_b xy =$

6. $\log_b \left(\dfrac{x}{y}\right) =$

7. $\log_b \left(x^r\right) =$

8. Write down the change of base formula for logarithms.

9. What is the domain of a logarithm?

10. Sketch the graph of $f\left(x\right) = \ln\left(x\right)$ and $g\left(x\right) = \log\left(x\right)$.

## Simplifying Logarithms

Simplify each of the following logarithms.

1. $\ln x^3 y^4 z^5$

2. $\log_3\left(\dfrac{9x^4}{\sqrt{y}}\right)$

3. $\log\left(\dfrac{x^2+y^2}{(x-y)^3}\right)$

## Solving Exponential Equations

In each of the equations in this section the problem is there is a variable in the exponent. In order to solve these, we will need to get the variable out of the exponent. This means using Property 3 and/or 7 from the Logarithm Properties section above. In most cases it will be easier to use Property 3 if possible. So, pick an appropriate logarithm and take the log of both sides, then use Property 3 (or Property 7) where appropriate to simplify. Note that often some simplification will need to be done before taking the logs.

Solve each of the following equations.

1. $2\mathbf{e}^{4x-2}=9$

2. $10^{t^2-t}=100$

3. $7+15\mathbf{e}^{1-3z}=10$

4. $x-x\mathbf{e}^{5x+2}=0$

## Solving Logarithm Equations

Solving logarithm equations are similar to exponential equations. First, we isolate the logarithm on one side by itself with a coefficient of one. Then we use Property 4 from the Logarithm Properties section with an appropriate choice of *b*. In choosing the appropriate *b*, we need to remember that the *b* MUST match the base on the logarithm!

Solve each of the following equations.

1. $4\log\left(1-5x\right)=2$

2. $3+2\ln\left(\frac{x}{7}+3\right)=-4$

3. $2\ln\left(\sqrt{x}\right)-\ln\left(1-x\right)=2$

4. $\log x+\log\left(x-3\right)=1$