Collision

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We investigate the collision term effects in the neutrino flavor oscillation.

I. INTRODUCTION

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II. MODEL

In this work, we consider the two-flavor neutrino flavor oscillation between ν_e and ν_τ, where ν_τ represents a linear combination of the physical μ and τ flavor neutrino. On the other hand, we only consider monochromatic neutrino to simplify the model.

Denoting the (anti-)neutrino flavor density matrix and the corresponding polarization vector as

$$\rho = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \tag{1}$$

$(\bar{\rho})$and $\mathbf{P}(\bar{\mathbf{P}})$ respectively, or

$$\rho_{ex} \equiv \rho_{12} = \frac{1}{2}\left(P_x + iP_y\right) \tag{2}$$

$$\bar{\rho}_{ex} \equiv \bar{\rho}_{12} = \frac{1}{2}\left(\bar{P}_x + i\bar{P}_y\right) \tag{3}$$

, where $P_i(\bar{P}_i)$ is the i component of $P(\bar{P})$.

Considering the collision terms, we have equations of motion (EOM) for polarization vectors as follows:

$$\dot{\mathbf{P}} = \omega\mathbf{B}\times\mathbf{P} + \mu(\mathbf{P}-\bar{\mathbf{P}})\times\mathbf{P} - \Gamma_+^{CC}\mathbf{P}_T + \Gamma_+^{AE}(\mathbf{P}^{AE}-\mathbf{P}) + \Gamma_-^{AE}(P_0^{AE}-P_0)\mathbf{z} \tag{4}$$

$$\dot{\bar{\mathbf{P}}} = -\omega\mathbf{B}\times\bar{\mathbf{P}} + \mu(\mathbf{P}-\bar{\mathbf{P}})\times\bar{\mathbf{P}} - \bar{\Gamma}_+^{CC}\bar{\mathbf{P}}_T + \bar{\Gamma}_+^{AE}(\bar{\mathbf{P}}^{AE}-\bar{\mathbf{P}}) + \bar{\Gamma}_-^{AE}(\bar{P}_0^{AE}-\bar{P}_0)\mathbf{z} \tag{5}$$

$$\dot{P_0} = \Gamma_+^{AE}(P_0^{AE}-P_0) + \Gamma_-^{AE}(P_z^{AE}-P_z) \tag{6}$$

$$\dot{\bar{P_0}} = \bar{\Gamma}_+^{AE}(\bar{P}_0^{AE}-\bar{P}_0) + \bar{\Gamma}_-^{AE}(\bar{P}_z^{AE}-\bar{P}_z) \tag{7}$$

We then choose the parameters

$$\begin{bmatrix} \omega \\ \mathbf{B} \\ \mu \\ \theta \end{bmatrix} = \begin{bmatrix} 0.3\ km^{-1} \\ -sin2\theta\ \mathbf{e}_x^f + cos2\theta\ \mathbf{e}_z^f \\ 3\times10^5\ km^{-1} \\ 10^{-6} \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \Gamma_+^{CC} \\ \Gamma_+^{AE} \\ \Gamma_-^{AE} \end{bmatrix} = \begin{bmatrix} 0.5/11.4\ km^{-1} \\ 0.5/0.417\ km^{-1} \\ 0.5/0.417\ km^{-1} \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \bar{\Gamma}_+^{CC} \\ \bar{\Gamma}_+^{AE} \\ \bar{\Gamma}_-^{AE} \end{bmatrix} = \begin{bmatrix} 0.5/37.2\ km^{-1} \\ 0.5/4.36\ km^{-1} \\ 0.5/4.36\ km^{-1} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \mathbf{P}^{AE} \\ P_0^{AE} \\ \bar{\mathbf{P}}^{AE} \\ \bar{P}_0^{AE} \end{bmatrix} = \begin{bmatrix} 2\ \mathbf{e}_z^f \\ 4 \\ 1.5\ \mathbf{e}_z^f \\ 3.5 \end{bmatrix} \tag{11}$$

and theinitial condition

$$\begin{bmatrix} \mathbf{P} \\ P_0 \\ \bar{\mathbf{P}} \\ \bar{P}_0 \end{bmatrix} = \begin{bmatrix} 2\ \mathbf{e}_z^f \\ 4 \\ 1.5\ \mathbf{e}_z^f \\ 3.5 \end{bmatrix} \tag{12}$$

III. RESULTS

A. Linearlized EOM

When $P_x,\ P_y \ll P_z \sim 1$, nonlinear terms could be ignored, we have

$$i\partial_t \rho_{ex} = \omega\ sin2\theta P_z + \left[-\omega\ cos2\theta - \sqrt{2}G_F(n_{\bar{\nu_e}} - n_{\bar{\nu_x}}) - i\Gamma\right]\rho_{ex} + \sqrt{2}G_F(n_{\nu_e} - n_{\nu_x})\bar{\rho}_{ex} \tag{13}$$

$$i\partial_t \bar{\rho}_{ex} = -\omega\ sin2\theta \bar{P}_z + \left[+\omega\ cos2\theta + \sqrt{2}G_F(n_{\nu_e} - n_{\nu_x}) - i\bar{\Gamma}\right]\bar{\rho}_{ex} - \sqrt{2}G_F(n_{\bar{\nu_e}} - n_{\bar{\nu_x}})\rho_{ex} \tag{14}$$

, where

$$\begin{cases} B \equiv (sin2\theta, 0, cos2\theta) \\ \rho_{ex} \equiv \rho_{21} = P_x + iP_y \\ \bar{\rho}_{ex} \equiv \bar{\rho}_{21} = \bar{P}_x + i\bar{P}_y \\ \Gamma \equiv \Gamma^{AE} + \Gamma^{CC} \\ \bar{\Gamma} \equiv \bar{\Gamma}^{AE} + \bar{\Gamma}^{CC} \end{cases} \tag{15}$$

Let

$$\begin{cases} \mu_+ \equiv \left(\frac{S+D}{2}\right)\mu \\ \mu_- \equiv \left(\frac{S-D}{2}\right)\mu \end{cases} \tag{16}$$

and

$$A = \begin{bmatrix} -\omega\ cos2\theta - \mu_- - i\Gamma & \mu_+ \\ -\mu_- & \omega\ cos2\theta + \mu_+ - i\bar{\Gamma} \end{bmatrix} \tag{17}$$

, we have

$$i\partial_t \begin{bmatrix} \rho_{ex} \\ \bar{\rho}_{ex} \end{bmatrix} = \begin{bmatrix} \omega\ sin2\theta \\ -\omega\ sin2\theta \end{bmatrix} + A \begin{bmatrix} \rho_{ex} \\ \bar{\rho}_{ex} \end{bmatrix} \tag{18}$$

Notice that, P_z and $\bar{P}_z$ are both close to 1 under this limitation condition.

B. Analytic Solution

Assume

$$\begin{cases} \rho_{ex}(t) = \rho^0_{ex} + Qe^{-i\Omega t} \\ \bar{\rho}_{ex}(t) = \bar{\rho}^0_{ex} + \bar{Q}e^{-i\Omega t} \end{cases} \tag{19}$$

, where ρ^0_{ex} and $\bar{\rho}^0_{ex}$ satisfy

$$0 = \begin{bmatrix} \omega\ sin2\theta \\ -\omega\ sin2\theta \end{bmatrix} + A \begin{bmatrix} \rho^0_{ex} \\ \bar{\rho}^0_{ex} \end{bmatrix} \tag{20}$$

Then, from equation 18, we have

$$\Omega \begin{bmatrix} Q \\ \bar{Q} \end{bmatrix} = A \begin{bmatrix} Q \\ \bar{Q} \end{bmatrix} \tag{21}$$

IV. DISCUSSION

V. CONCLUSIONS

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