Stream Ciphers
–
An Overview

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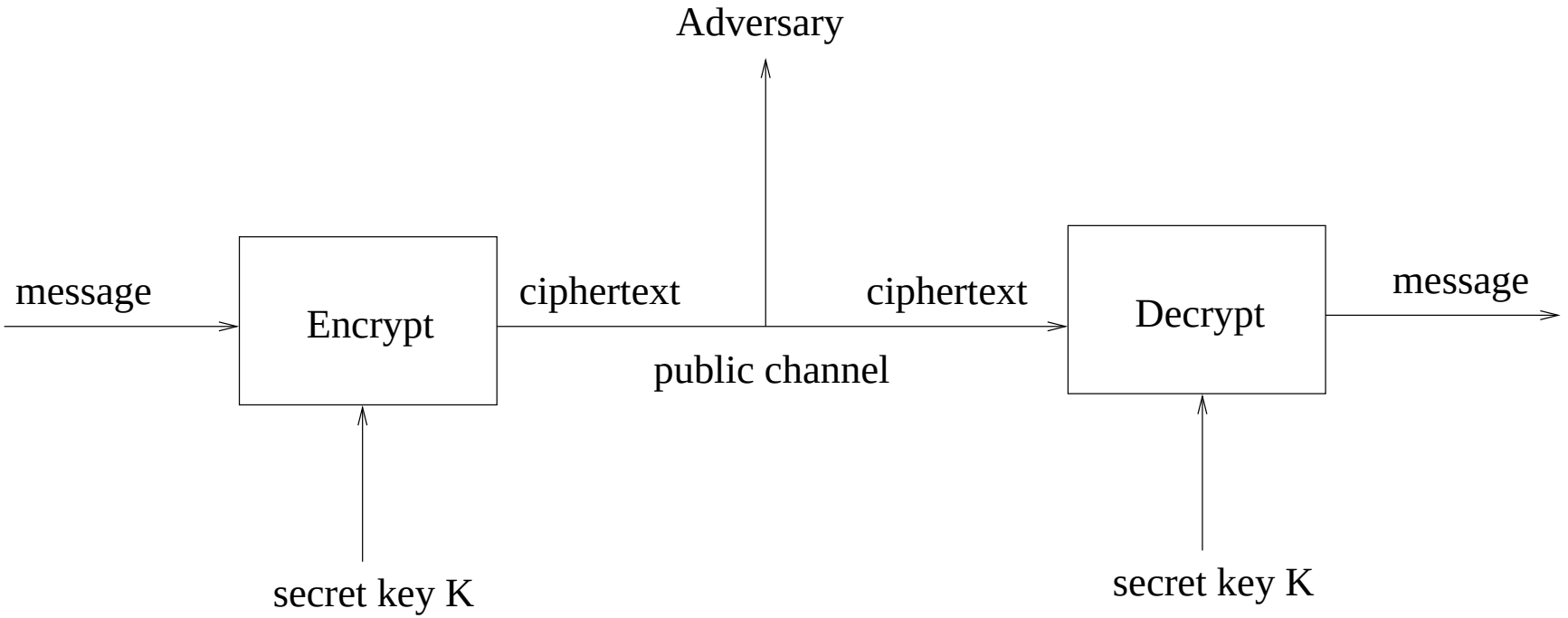
Classical Encryption



Adversary's goal: To obtain message or secret key.

One Time Pad

- Let the message be a sequence of bits: $m_0, m_1, \ldots, m_\ell$
- Let $r_0, \ldots, r_\ell$ be a sequence of **random** bits.
- Ciphertext is $c_0, c_1, \ldots, c_\ell$ where $c_i = m_i \oplus r_i$.
- Perfect Secrecy: Given a ciphertext, the message can be any binary string of length ℓ.
- Impractical:
  (a) Difficult to get random bits.
  (b) Sequence must be pre-distributed to both sender and receiver.
  (c) random sequence as long as the message.

Additive Stream Ciphers

- Use a pseudorandom generator PRG to produce the sequence $r_0, \ldots, r_\ell$.
- The PRG extends a short "seed" into a long "pseudorandom" string, i.e., $\text{PRG}(K) = r_0, \ldots, r_\ell$.
- The seed is the secret key K.
- Security depends on the design of PRG.

Security of PRG

- PRG is a broad concept.
- For cryptographic use, a PRG must be unpredictable:
- **Next bit test:** Given an initial segment, it should not be possible to **efficiently** guess the next bit.
- **Statistical Tests:** The generated pseudorandom sequence should pass **all polynomial** time statistical tests.
- The above notions are equivalent.

Block Ciphers

Let

$$E : \{0,1\}^n \times \{0,1\}^k \rightarrow \{0,1\}^n.$$

- For each fixed $K \in \{0,1\}^k$, $E_K()$ is a permutation of $\{0,1\}^n$.
- $E(K, M)$ is written as $E_K(M)$.
- $E_K()$ is called an n-bit block cipher.
- K is the secret key.
- Security: Pseudorandom permutation (PRP).

Stream and Block Ciphers

- **Block Ciphers:** Applies a fixed permutation to all n-bit blocks.
- **Stream Ciphers:**
  - Applies a time-varying map.
  - Maintains a state vector, which allows the application of a time-varying map.
- This difference is not very important.
  - Some block cipher modes of operations are actually stream ciphers.
  - Modern day stream ciphers are becoming more and more block oriented.
- The difference between PRP and PRG is more fundamental.

Attacks

General Attack Models

- Known (or chosen) plaintext attack: The adversary learns a segment of the pseudorandom sequence.
- Chosen ciphertext attack: Adversary submits ciphertexts and gets back the corresponding plaintexts.

Attack Goals

Distinguishing Attacks: The adversary should be able to distinguish the output of the PRG from a random string.

Key Recovery Attacks: The adversary has to find the "seed" of the PRG from the output of the PRG.

Based on Hard Problems

- **Blum, Blum and Shub (BBS):**
  - Let $N = pq$, where $p, q \equiv 3 \bmod 4$.
  - Seed: x_0. Iterates $x_{i+1} = x_i^2 \bmod N$.
  - Bit extraction: $b_i = \text{parity}(x_i)$.
  - Security: If determining quadratic residuacity modulo N is difficult, then guessing b_i is also difficult.
  - Efficiency: Slow; one squaring per bit.
  - Recent criticism: Menezes and Koblitz have argued that security guarantee is not realistic.
- Elliptic curve based construction also known.

Classical Models

- Hardware based – well studied – based on old ideas (almost four decades).
- Uses Linear Feedback Shift Registers (LFSR) and Boolean Functions.
- LFSR: Fast generation of bit sequences (satisfying a linear recurrence) in hardware.
- Boolean function: Maps an n-bit string to one bit.
- S-box: Maps an n-bit string to an m-bit string.

LFSR

- Let $\mathbf{a} = a_0, a_1, a_2, \ldots$ be a bit sequence.
- Suppose $a_{t+\ell} = c_{\ell-1}a_{t+\ell-1} \oplus \cdots \oplus c_0 a_t$.
- Then $\mathbf{a}$ is called a linear recurring sequence.
- $\mathbf{a}$ can be produced in hardware using an LFSR.
- Let $p(x) = x^\ell \oplus c_{\ell-1}x^{\ell-1} \oplus \cdots \oplus c_1 x \oplus c_0$.
- Then $p(x)$ is called the characteristic polynomial of $\mathbf{x}$.
- If $p(x)$ is primitive over $GF(2)$, then $\mathbf{x}$ has the maximum possible period of $2^\ell - 1$.

Two Standard Models

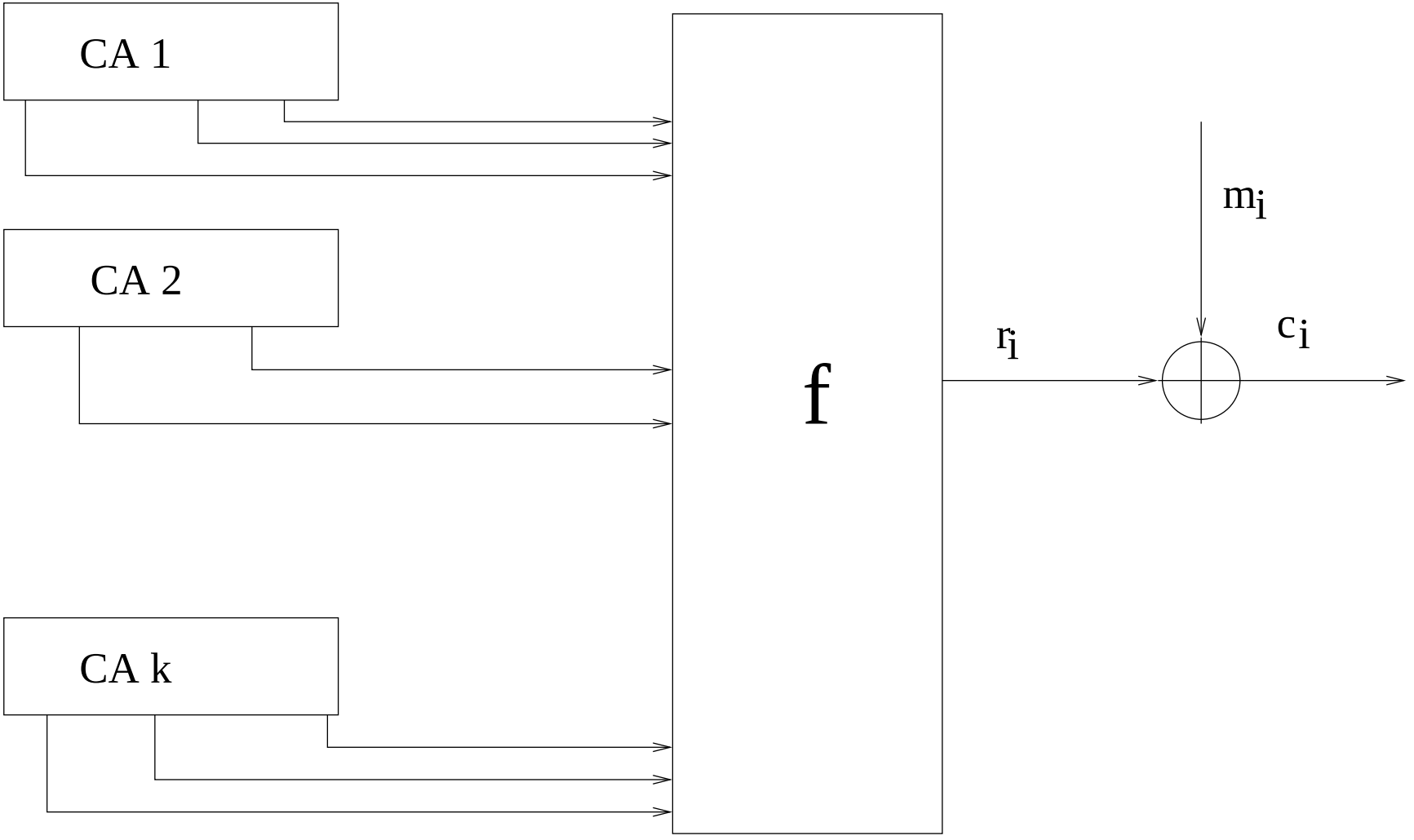
Nonlinear Filter Model: Boolean function combines outputs of different cells of a single LFSR.

Nonlinear Combiner Model: Boolean function combines outputs of several LFSRs.

Filter-Combiner model (Sarkar, Crypto 2002) combines the best features of the above two models.

- Uses linear cellular automata (CA) instead of LFSRs.
- Suitable Boolean functions are required to realize FC-model.

Filter-Combiner Model



Necessary Properties

- High linear complexity of the output sequence.
- Suitable choice of the Boolean function.
  - Balanced.
  - Correlation immune.
  - High algebraic degree.
  - High nonlinearlity.
  - Resistance to algebraic attacks.

Linear Complexity

- Let $\mathbf{k} = k_0, \ldots, k_t$ be a sequence (possibly used to encrypt a message).
- The linear complexity of $\mathbf{k}$ is the length of a minimum length LFSR that can produce $\mathbf{k}$.
- Berlekamp-Massey algorithm can be used to find the linear complexity and a minimal length LFSR producing $\mathbf{k}$.
- The time complexity of the algorithm is $O(t^2)$.
- The BM algorithm was originally introduced for decoding BCH codes.
- If $\mathbf{k}$ is a random sequence, then its expected linear complexity is $k/2$.

Linear Complexity (contd.)

Nonlinear-Filter Model:

- Partial results are known – due to Rueppel.
- Depends on the choice of the Boolean function.
- For certain Boolean functions, the linear complexity can be large.

Nonlinear-Combiner Model:

- Let $f(x_1, \ldots, x_n)$ be the combining Boolean function.
- Let $L_1, \ldots, L_n$ be the lengths of the LFSRs which are taken to be distinct.
- Then the linear complexity is $f(L_1, \ldots, L_n)$ where the arithmetic is over the integers (Rueppel-Staffelbach).

Correlation Attack

Introduced by Siegenthaler.
Assume the nonlinear-combiner model.

- Let $x_1^{(i)}, \ldots, x_t^{(i)}$ be the sequence obtained from the ith LFSR $\mathcal{L}_i$.
- Let $k_0, \ldots, k_t$ be the output of the Boolean function.
- Suppose $\mathsf{Prob}[k_j = x_j^{(i)}] = \alpha_i \neq \frac{1}{2}$.
- Then the output $k_0, \ldots, k_t$ is biased with respect to the sequence $x_1^{(i)}, \ldots, x_t^{(i)}$.

Correlation Attack (contd.)

- Assume that the sequence $\mathbf{k} = k_0, \ldots, k_t$ is known. (Also possible to work with the ciphertext.)
- For each of the $2^{L_i} - 1$ nonzero initial states of $\mathcal{L}_i$, generate the sequence $x_1^{(i)}, \ldots, x_t^{(i)}$.
- Compute $\widehat{\alpha_i} = \mathsf{Prob}[k_j = x_j^{(i)}]$ for each such sequence.
- If the choice of initial condition is correct, then $\widehat{\alpha_i} \approx \alpha_i$ else, $\widehat{\alpha_i} \approx 1/2$.
- If α_i and $1/2$ are "sufficiently" separate and t is "sufficiently" large, then we can find the correct initial state for $\mathcal{L}_i$.

Correlation Attack (contd.)

- If $\alpha_i = 1/2$, then this does not work for $\mathcal{L}_i$.
- If $\alpha_i = 1/2$ for all i, then $f()$ is said to be correlation immune (CI).
- One can then look for correlation between $\mathbf{k}$ and XORs of several $\mathcal{L}_i$'s.
- If $f()$ is kth order CI, then this method will not succeed if we take the XORs of upto k LFSRs.
- A balanced k-CI function is called k-resilient.

Correlation Attack (contd.)

Fast Correlation Attack: Avoid considering all possible initial states (Meier-Staffelbach).

- Each LFSR sequence satisfies a linear recurrence.
- Let $p(x)$ be the characteristic polynomial of the recurrence.
- Then the sequence also satisfies a recurrence given by any multiple of $p(x)$.
- Collection of all such recurrences (with "low span") constitute a set of parity check equations.
- An iterative decoding method can be applied to obtain the original sequence from these parity check equations.

Multiples of Polynomials

- Multiples of $p(x)$ provide parity check equations.
- It is helpful for the attack if these parity check equations have few terms.
- For example, if $p(x)$ has a trinomial multiple, then the obtained parity check has three terms.
- A "good" polynomial $p(x)$ will not have low degree sparse multiples.
- Determining whether a polynomial is "good" can be difficult. Work by Wagner (Crypto 2002) based on the generalized birthday attack.

Algebraic Attack

Courtois-Meier: Consider the nonlinear-filter model, i.e., there is a single LFSR $\mathcal{L}$.

- Let S_0 be the initial state of the LFSR.
- The ith state S_i is obtained as $S_i = \mathcal{L}^i(S_0)$.
- Let $f()$ be the combining Boolean function of degree d.
- The key bits of $\mathbf{k}$ can be written as

$$f(\mathcal{L}^i(S_0)) = k_i.$$

- For any i, the map $\mathcal{L}^i()$ is a linear map.
- Thus, for any i, we obtain a nonlinear equation in the bits of S_0 of maximum degree d.

Algebraic Attack (contd.)

- Linearization: Replace each monomial of degree greater than one by a new variable.
- Let $D = \sum_{i=0}^{d} \binom{n}{i}$.
- We obtain a system of linear equations in at most D variables.
- If there are D "independent" equations, then we can solve for these D variables and use back substitution to obtain the bits of S_0.
- Time required is $O(D^3)$.
- Becomes infeasible if D is large, which can be ensured by having d to be large.

Algebraic Attack (contd.)

- Suppose f has large degree d, but there are low degree polynomials $g(x)$ and $h(x)$ such that $h(x) = g(x)f(x)$.
- Then a modification of the above idea can be applied to $h()$ instead of $f()$.
- If $h(x)$ is zero, then $g(x)$ is called an annihilator.
- A low degree annihilator can be used for the attack.
- Necessary condition: Neither f nor $(1 \oplus f)$ should have a low degree annihilator.

Boolean Function Properties

Let $f(x_1, \ldots, x_n)$ be an n-variable Boolean function.

Balancedness: $wt(f) = 2^{n-1}$.

k**-CI:** $\mathsf{Prob}[f(x_1, \ldots, x_n) = x_{i_1} \oplus \cdots \oplus x_{i_k}] = 1/2$ for any choice of $i_1, \ldots, i_k$ from $\{1, \ldots, n\}$.

Nonlinearity: A function l is affine if $l(x_1, \ldots, x_n) = a_0 \oplus a_1 x_1 \oplus \cdots \oplus a_n x_n$ for some bit constants $a_0, a_1, \ldots, a_n$.

$$\mathsf{nl}(f) = \min_{\text{affine } l} d(f, l).$$

Algebraic Degree: Largest degree monomial in the algebraic normal form of f.

Boolean Function Properties (contd.)

Let n = the number of input bits; d = degree; t = the order of resiliency; $\mathcal{N}$ = nonlinearity.

Known Trade-Offs:

- $d \leq n - t - 1$.
- $\mathcal{N} \leq 2^{n-1} - 2^{t+1+\lfloor (n-t-2)/d \rfloor}$.
- Recent research has shown that it is possible to construct Boolean functions with optimal trade-offs.
- The question of algebraic resistance offered by such Boolean functions is still under investigation.

Exchange-Shuffle Paradigm

- Let A be an array of length N containing some permutation of 0 to $N-1$.
- Repeat the following steps N times.
  - Choose two random locations of A.
  - Exchange the values in these locations.
- At the end of N steps, A is expected to have a random permutation of 0 to $N-1$.
- **RC4:** Uses the above principle.
  - Key set-up phase: Starting from a secret key, ensures a "uniform" distribution.
  - Byte generation phase: generates bytes at each step.

RC4 – Key Setup

```
for i from 0 to 255
S[i] := i
j := 0
for i from 0 to 255
j := (j + S[i] + key[i mod keylength]) mod 256
swap(S[i],S[j])
```

RC4 – Byte Generation

Initialization:

$i = 0$;

$j = 0$;

Generation loop:

$i = i + 1$;

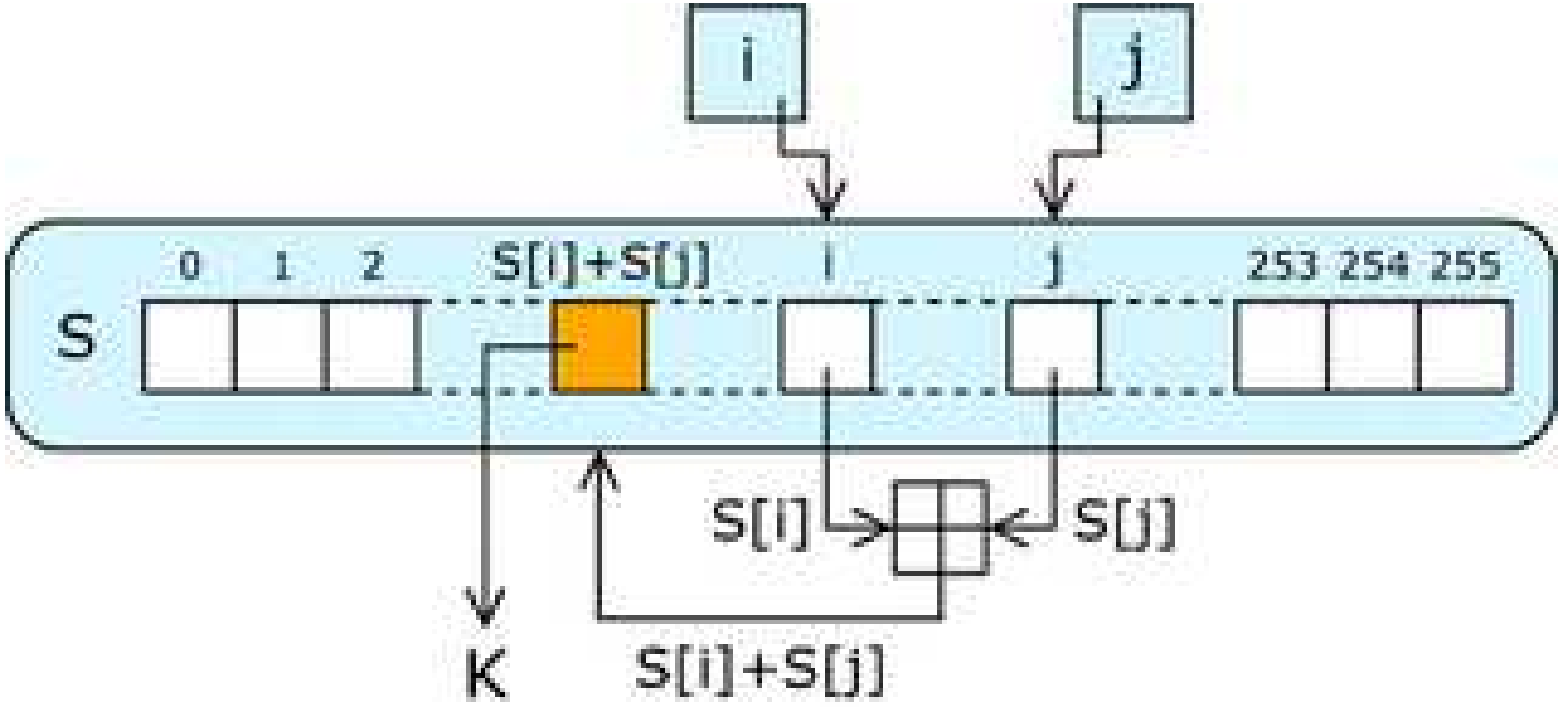
$j = j + A[i]$;

Swap($A[i]$,$A[j]$);

Ouput $z = A[A[i] + A[j]]$.

Note: $A[]$ is a byte-array of length 256.

RC4: Byte Generation



RC4: Weaknesses and Extensions

- Very efficient in software. But cannot take advantage of 32-bit architecture.
- Weaknesses pointed out by Shamir, Mantin, Golić, Paul-Preneel and others.
- Attempts to extend the design to 32-bit architecture. Recent study of weaknesses by Paul-Preneel (Asiacrypt 2006).

Initialization Vector

- In a stream cipher, the same secret key cannot be used more than once.
- Regular key change is a serious problem.
- Modern day stream ciphers use an initialization vector (IV).
- IV can be known to the adversary. Strength of stream cipher does not depend on IV.
- IV need not be random or unpredictable.
- Constraint: The same (key,IV) pair cannot be used twice.
- With this constraint, for the same key, several IVs can be used. Eases the problem of supplying fresh keys.

Security Goals

Confidentiality: This is the basic goal.

Authentication: If the message has been tampered with, then it should be possible to detect it. Usually achieved by computing a tag of the message.

Authenticated Encryption: The combined goal of confidentiality plus authentication.

Self-Synchronization:

- In a noisy channel, there may be bit flips, bit slips or bit inserts.
- Suppose a contiguous sequence of bits are received correctly.
- From this point, the decryption algorithm can correctly decrypt.

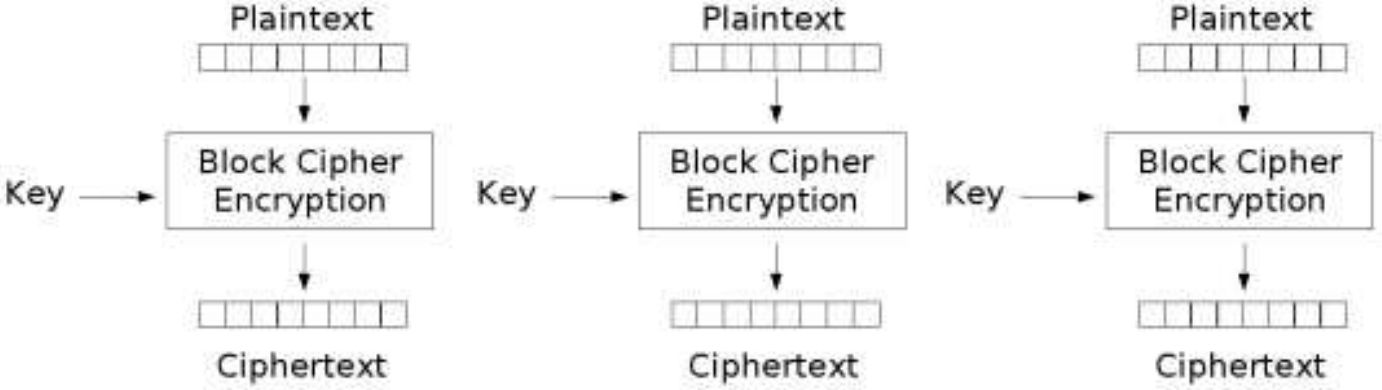
BC Modes of Operations

message: $M_1, M_2, M_3, \ldots$ (n-bit blocks);
initialization vector: n-bit IV (used as nonce).

Electronic codebook (ECB) mode:

$$C_i = E_K(M_i), i \geq 1.$$

ECB Mode



Electronic Codebook (ECB) mode encryption

Insecurity of ECB Mode

Source: Wikipedia, http://en.wikipedia.org/wiki/.

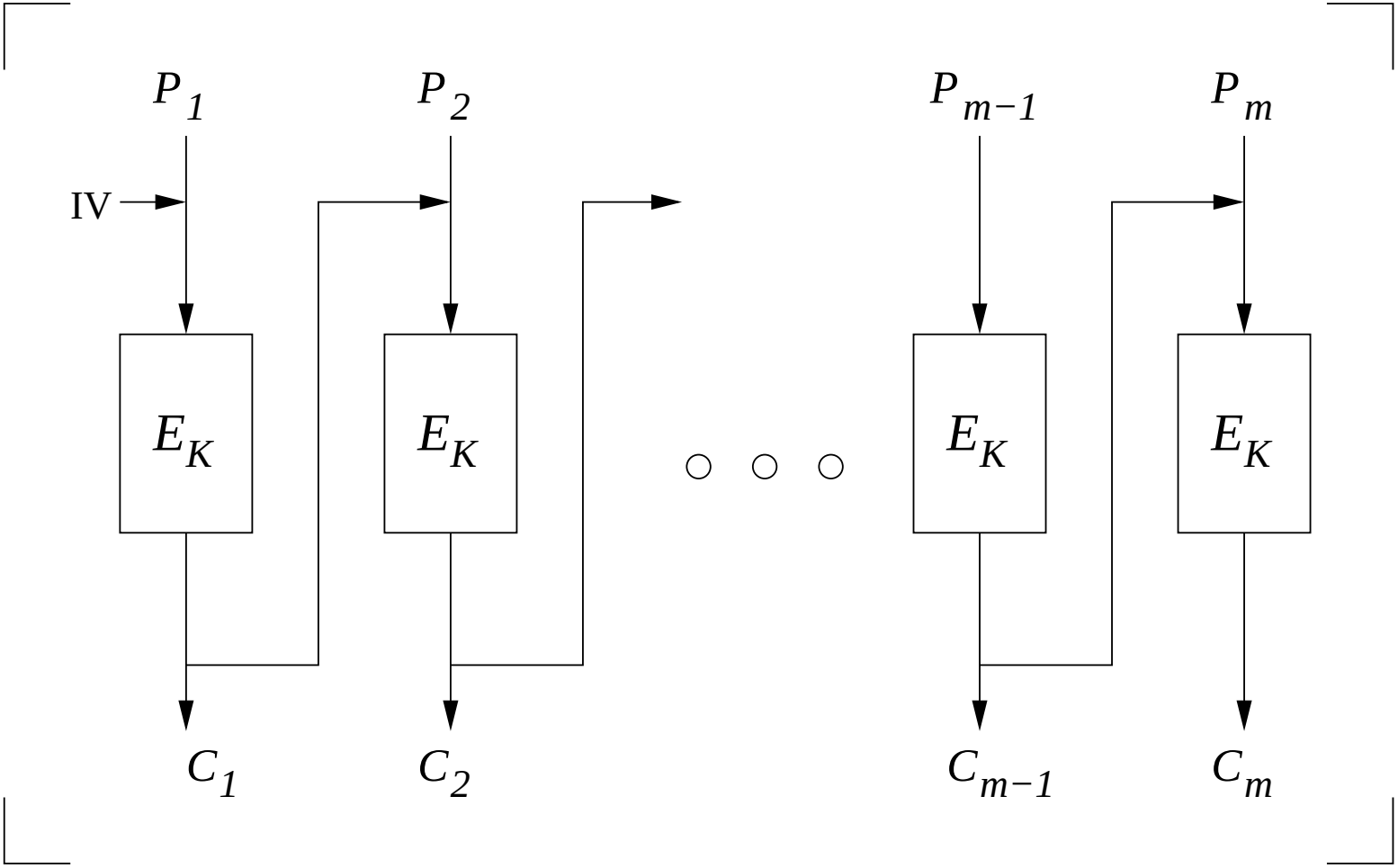
CBC Mode

Cipher Block Chaining (CBC) Mode:

$$C_1 = E_K(M_1 \oplus \mathrm{IV});$$

$$C_i = E_K(M_i \oplus C_{i-1}),\ i \geq 2.$$

CBC Mode



Modes of Operations (contd.)

Output feedback (OFB) mode:

$$Z_1 = E_K(\mathrm{IV});\ Z_i = E_K(Z_{i-1}),\ i \geq 2;$$
$$C_i = M_i \oplus Z_i,\ i \geq 1.$$

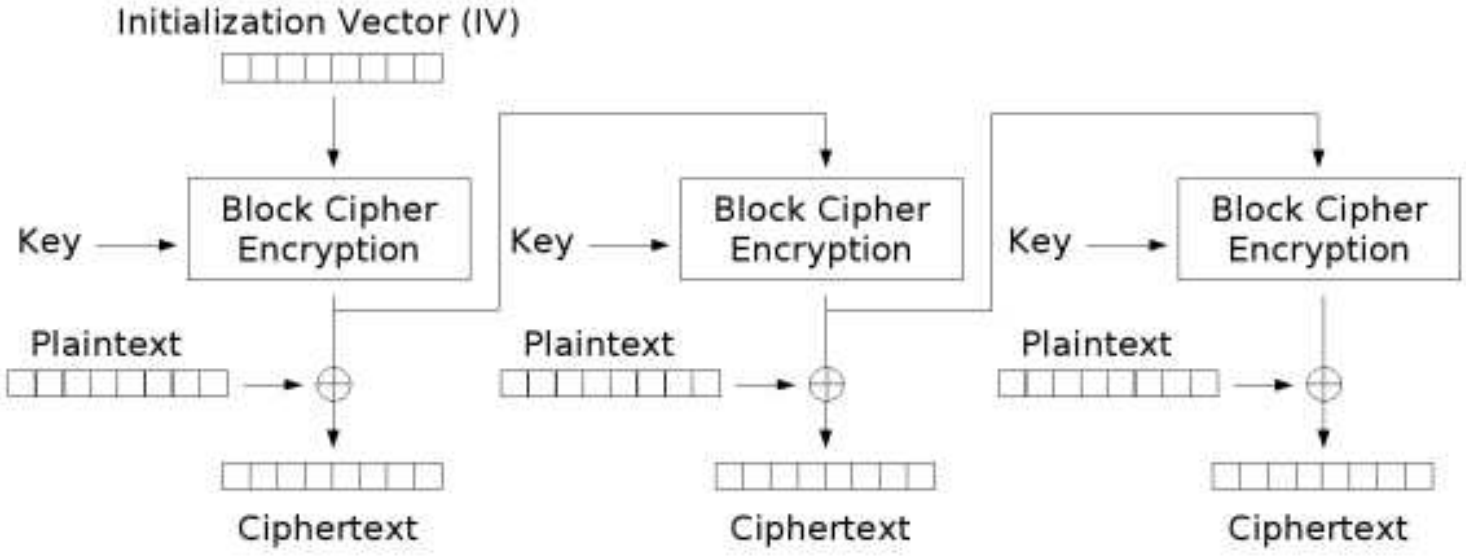
- This is essentially an additive stream cipher.

Cipher feedback (CFB) mode:

$$C_1 = M_1 \oplus E_K(\mathrm{IV});$$
$$C_i = M_i \oplus E_K(C_{i-1}),\ i \geq 2.$$

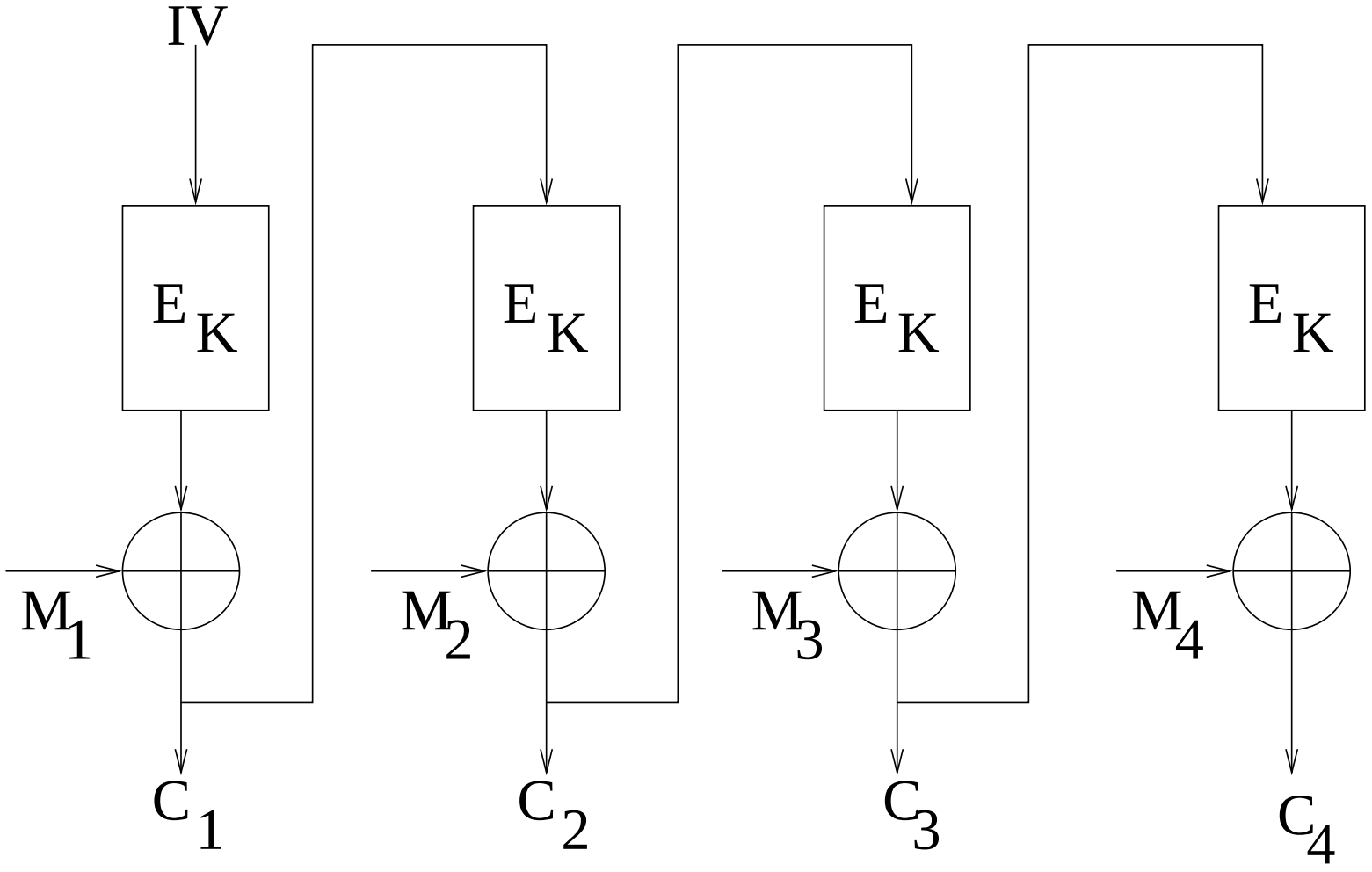
- Can be used as a self-synchronizing stream cipher in a 1-bit feedback mode.

OFB Mode



Output Feedback (OFB) mode encryption

CFB Mode



Modes of Operations (contd.)

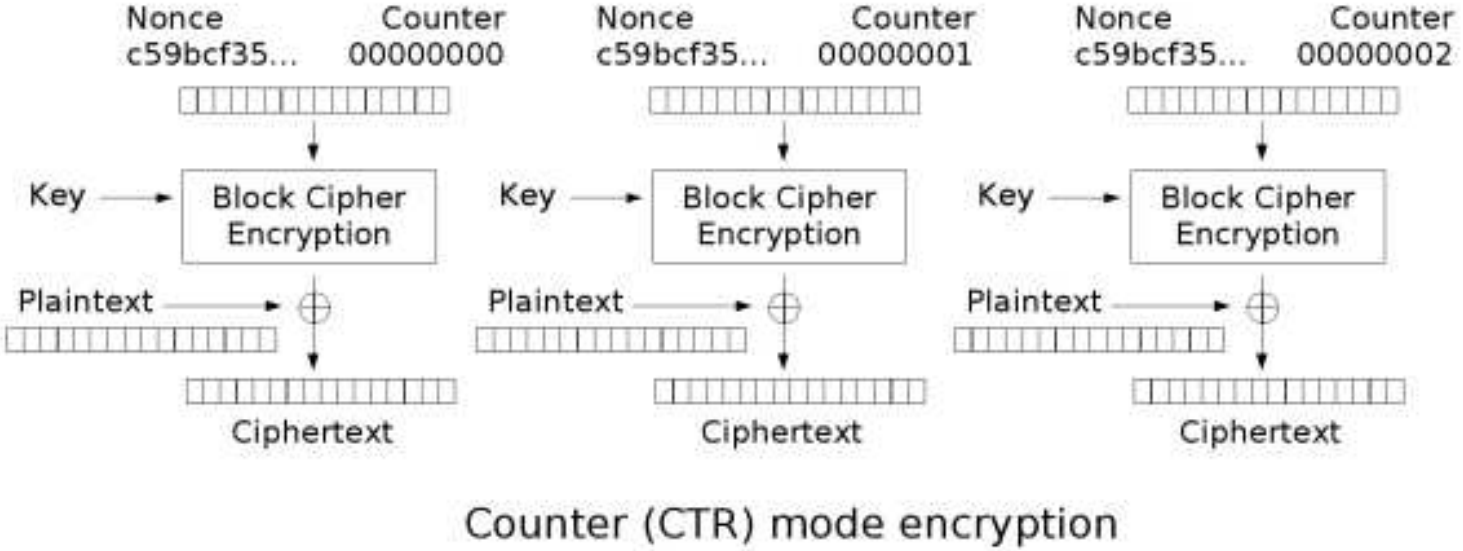
Counter (CTR) mode:

$$C_i = M_i \oplus E_K(\text{nonce}||\mathsf{bin}(i)),$$

$i \geq 1$.

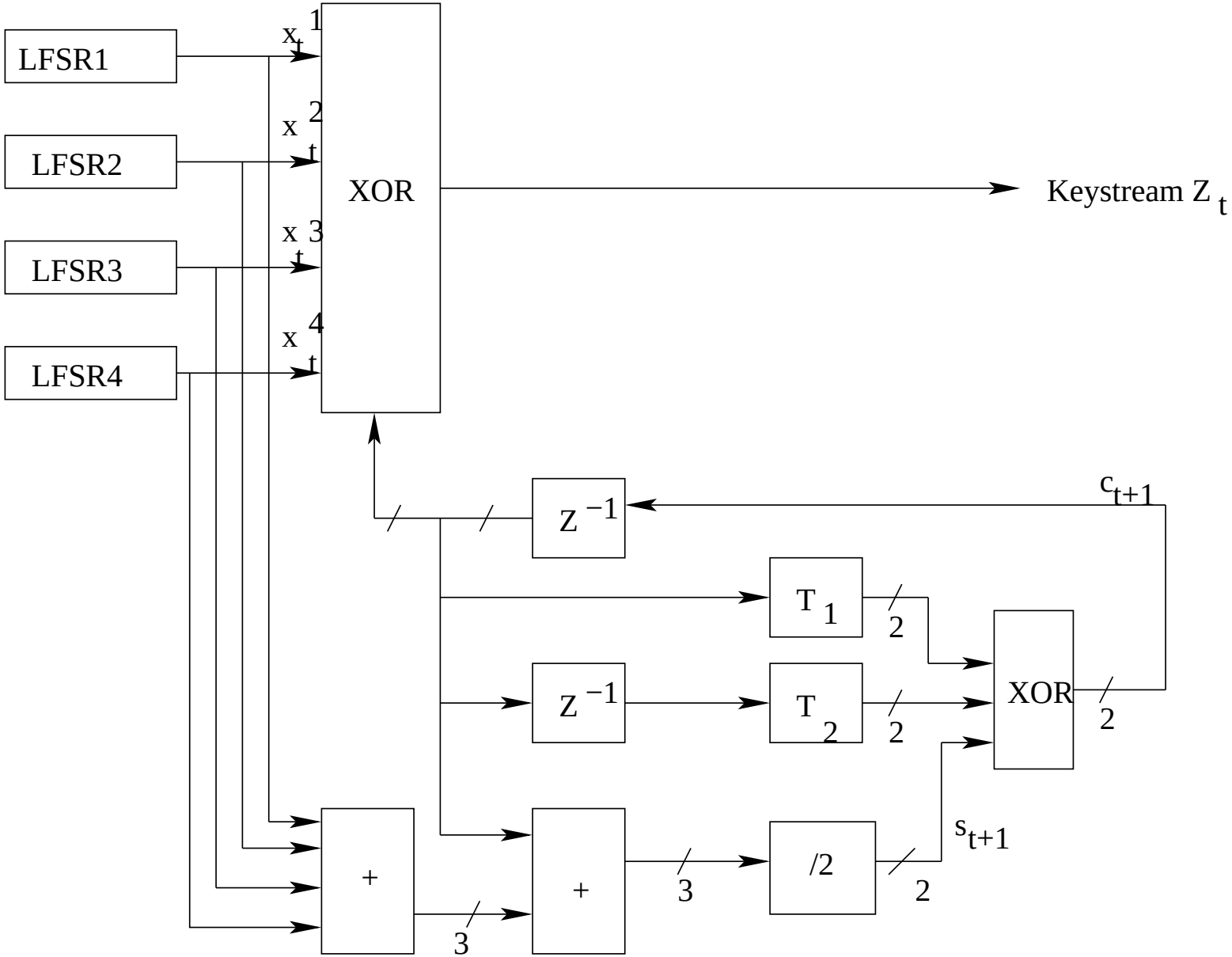
- Recent proposal – Salsa20 – due to Bernstein is based on this idea.
- nonce is an $n/2$-bit string.
- $\mathsf{bin}(i)$ is an $n/2$-bit binary representation of i.
- Requires an adder.
- Use of parallel generation of LFSR sequences in hardware (Mukhopadhyay-Sarkar, 2006).

Counter Mode



Counter (CTR) mode encryption

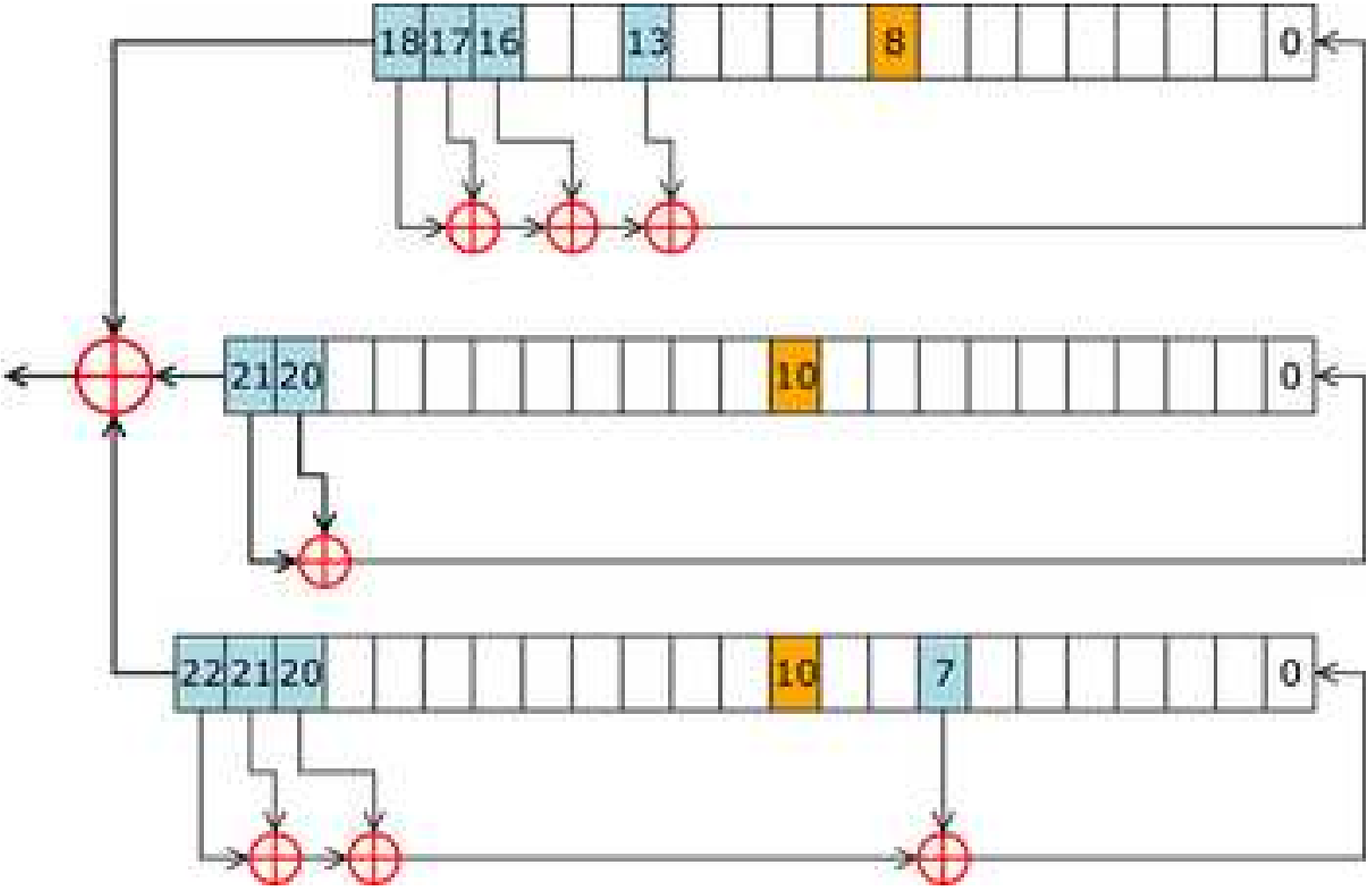
Bluetooth: E0



Bluetooth: E0

- The key length may vary; usually 128 bits.
- At each step, E0 generates a bit using four shift registers (lengths 25, 31, 33, 39 bits) and two 2-bit internal states.
- At each step, the registers are shifted and the two states are updated with the current state, the previous state and the values in the shift registers.
- Four bits are then extracted from the shift registers and added together.
- The algorithm XORs that sum with the value in the 2-bit register.
- The first bit of the result is output for the encoding.

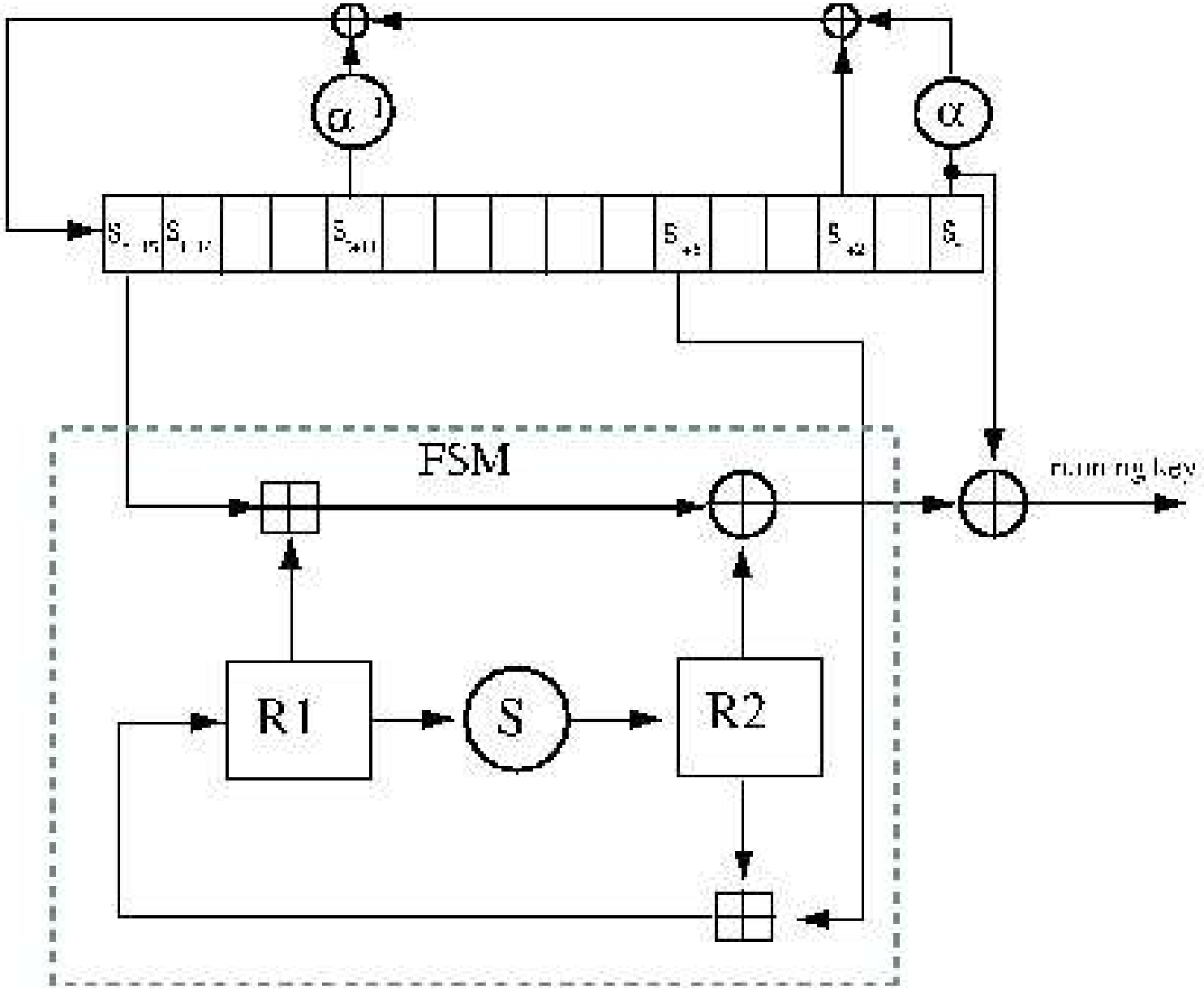
GSM: A5/1



GSM: A5/1

- The registers are clocked in a stop/go fashion using a majority rule.
- Each register has an associated clocking bit.
- At each cycle, the clocking bit of all three registers is examined and the majority bit is determined.
- A register is clocked if the clocking bit agrees with the majority bit.
- Hence at each step two or three registers are clocked, and each register steps with probability 3/4.

SNOW 2.0 Stream Cipher



Phelix

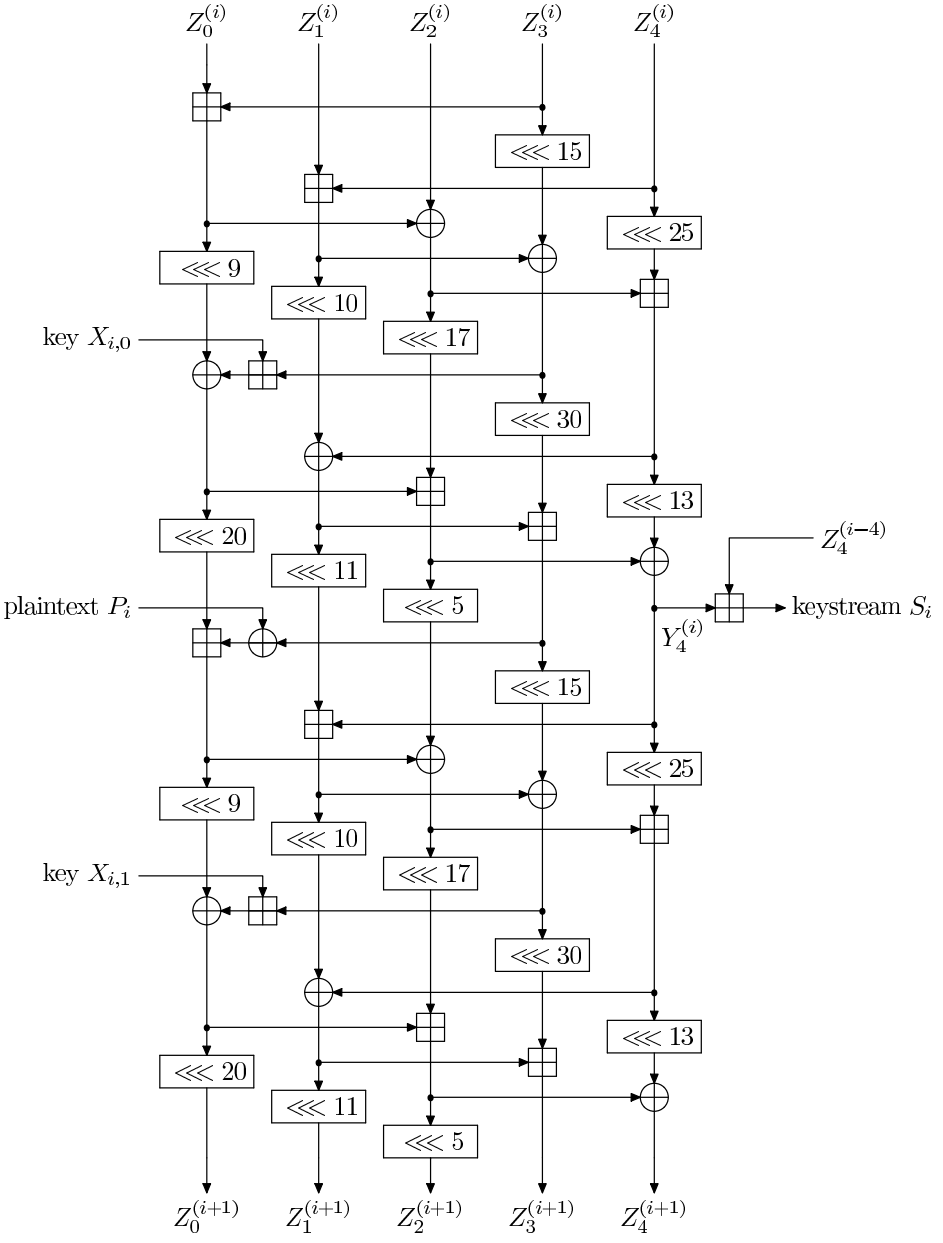


Fig. 2. One block of Phelix encryption

Ecrypt Stream Cipher Proposals

- Phelix.
- HC-256.
- Salsa20.
- Rabbit.
- Many others – generating a lot of interest and discussion on various aspects of stream cipher design and analysis.
- http://www.ecrypt.eu.org/stream/

Thank you for your attention!