Practice 2 (Limits and continuity)

Exercise 1. Find the average rate of change of the function over the given intervals

a) $f(x) = x^3 + 1$, $[2, 3]$;

b) $g(t) = 2 + cos(t)$, $[0, \pi]$, $[-\pi, \pi]$.

Exercise 2. Find the slope of the curve at the given point P as the limit of secant slopes

$$\frac{\triangle y}{\triangle x} = \frac{f(x+h) - f(x)}{h}$$

as h approaches 0 (don't use derivatives here). Find an equation of the tangent line at P.

a) $f(x) = x^2 - 3$, $P(2, 1)$;

b) $g(x) = x^2 - 4x$, $P(1, -3)$.

Exercise 3. For the function $f(t)$ graphed here, find the following limits or explain why they do not exist.

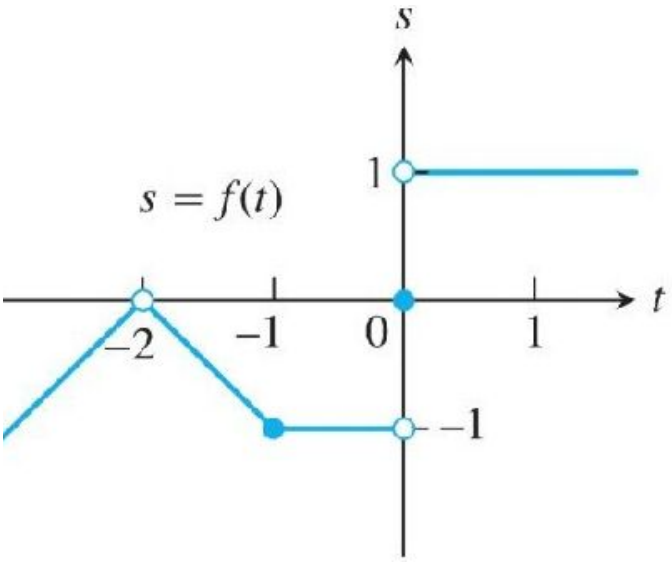
a) $\lim_{t \to -1} f(t)$

b) $\lim_{t \to -2} f(t)$

c) $\lim_{t \to 0.5} f(t)$

d) $\lim_{t \to 0} f(t)$

e) $\lim_{t \to 1} f(t)$



Exercise 4. Find the following limits or explain why they do not exist.

a) $\lim_{x\to -1} 3x+1$

b) $\lim_{x\to 3} \dfrac{x-1}{x+3}$

c) $\lim_{x\to 0} \dfrac{\frac{1}{x-1}}{x}$

d) $\lim_{x\to 2} \dfrac{x^2-7x+10}{x-2}$

e) $\lim_{x\to 0} \dfrac{x}{|x|}$

f) $\lim_{x\to -1} \dfrac{\sqrt{x^2+8}-3}{x+1}$

g) $\lim_{x\to 4} \dfrac{4-x}{5-\sqrt{x^2+9}}$

h) $\lim_{x\to 2^-} \dfrac{\sqrt{2x}(x-2)}{|x-2|}$

i) $\lim_{x\to 2^+} \dfrac{\sqrt{2x}(x-2)}{|x-2|}$

j) $\lim_{y\to 0} \dfrac{\sin 3y}{4y}$

k) $\lim_{\theta\to 0} \dfrac{\sin(\sin\theta)}{\sin\theta}$

l) $\lim_{x\to 2} \dfrac{2-x}{(x-2)^3}$

m) $\lim_{x\to \infty} \dfrac{3x-2}{9x+7}$

n) $\lim_{x\to \infty} \sqrt{x+9}-\sqrt{x+4}$

o) $\lim_{x\to \infty} 2x-\sqrt{4x^2+3x-2}$

Exercise 5. Let

$$f(x) = \begin{cases} 10 - x & \text{if} \quad x < 3 \\ \\ \dfrac{x}{3} - 2 & \text{if} \quad 3 < x \end{cases}$$

a) Find $\lim\limits_{x \to 3^+} f(x)$ and $\lim\limits_{x \to 3^-} f(x)$.

b) Does $\lim\limits_{x \to 3} f(x)$ exist? If so, what is it? If not, why not?

c) Find $\lim\limits_{x \to 9^+} f(x)$ and $\lim\limits_{x \to 9^-} f(x)$.

d) Does $\lim\limits_{x \to 9} f(x)$ exist? If so, what is it? If not, why not?

e) Does $\lim\limits_{x \to c} f(x)$ exist at every c in the open interval $(1, 3)$?

Exercise 6. Find a function that satisfies the given conditions at the same time and sketch its graph.

a) $\lim\limits_{x \to -\infty} h(x) = -1$.

b) $\lim\limits_{x \to \infty} h(x) = 1$.

c) $\lim\limits_{x \to 0^-} h(x) = -1$.

d) $\lim\limits_{x \to 0^+} h(x) = 1$.

Exercise 7. Let

$$f(x) = \begin{cases} \sqrt{1 - x^2} & \text{if} \quad 0 \leqslant x < 1 \\ 1 & \text{if} \quad 1 \leqslant x < 2 \\ 2 & \text{if} \quad x = 2 \end{cases}$$

a) Graph the function

b) Give the domain and range of $f(x)$

c) At what points c, if any, does $\lim\limits_{x \to c} f(x)$ exist? Justify your answer.

Exercise 8. At what points are the functions continuous? Justify your answer.

a) $f(x) = \dfrac{1}{|x|+1} - \dfrac{x^2}{2}$

b) $f(x) = \dfrac{x+2}{\cos x}$

c) $\sqrt{2x+3}$

d) $h(x) = \begin{cases} \dfrac{x^2-x}{x^2-1} & \text{if} \quad x \neq \pm 1 \\ 1 & \text{if} \quad x = 1 \end{cases}$

Exercise 9. For what value of c is $h(x) = \begin{cases} x & \text{if} \quad x < -2 \\ cx^2 & \text{if} \quad x \geqslant -2 \end{cases}$ continuous at every x?