Practice (Integrals (second part))

Exercise 1. Find the derivative of the functions:

a) $h(x) = \displaystyle\int_1^{e^x} \ln t \ dt$

b) $g(x) = \displaystyle\int_1^{\sqrt{x}} \frac{z^2}{z^4+1} \ dz$

c) $F(x) = \displaystyle\int_{1-2x}^{1+2x} t \sin t \ dt, \ 0 < x.$

Exercise 2. Evaluate the integrals using integration by parts

a) $\displaystyle\int x^2 e^x \ dx$

b) $\displaystyle\int e^x \cos x \ dx$

c) $\displaystyle\int x \sin \frac{x}{2} \ dx$

d) $\displaystyle\int_1^2 x \ln x \ dx$

e) $\displaystyle\int_1^e x^3 \ln x \ dx$

f) $\displaystyle\int x e^{3x} \ dx$

g) $\displaystyle\int (x^2 - 2x + 1) e^{2x} \ dx$

h) $\displaystyle\int_0^{\frac{\pi}{2}} \sin(2x) \cos(4x) \ dx$

Exercise 3. Evaluate the integrals:

a) $\displaystyle\int_0^1 \sqrt[3]{1+7x} \ dx$

b) $\displaystyle\int_0^{\frac{\pi}{6}} \frac{\sin t}{\cos^2 t}\ dt$

c) $\displaystyle\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}\ dx$

d) $\displaystyle\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\ dx$

e) $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

f) $\displaystyle\int_0^1 \frac{e^z + 1}{e^z + z}\ dz$

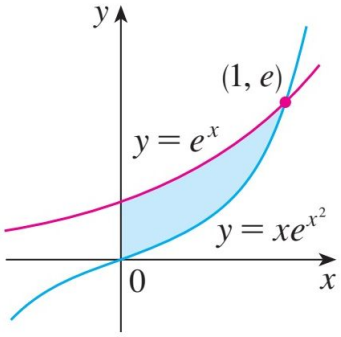
Exercise 4. You and a companion are about to drive a twisty stretch of dirt road in a car whose speedometer works but whose odometer (kilometer counter) is broken.To find out how long this particular stretch of road is, you record the car's velocity at 10-sec intervals, with the results shown in the accompaning table. Estimate the length of the road using a) left-endpoint values; b) right-endpoint values.

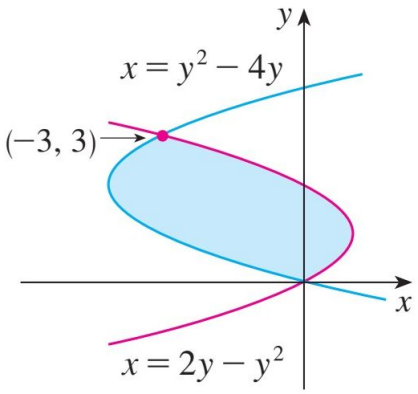
Time (sec)	0	10	20	30	40	50	60	70	80	90	100	110	120
Velocity (converted to m/sec)	0	13	5	11	9	13	11	5	7	11	13	9	11

Exercise 5. The speed of a runner increased steadily during the first three seconds of a race. Her speed at half-second intervals is given in the table. Find lower and upper estimates for the distance that she traveled during these three seconds.

t (s)	0	0.5	1.0	1.5	2.0	2.5	3.0
v (ft/s)	0	6.2	10.8	14.9	18.1	19.4	20.2

Exercise 6. Find the area of the shaded regions.





Exercise 7. Sketch the region enclosed by the given curves and find its area.

a) $y = 12 - x^2, \quad y = x^2 - 6$

b) $y = \cos x, \quad y = 2 - \cos x, \ \ 0 \leqslant x \leqslant 2\pi$

c) $y = \sqrt{x-1}, \quad x - y = 1$

d) $y = \dfrac{1}{x}, \quad y = x, \quad y = \dfrac{1}{4}x, \ \ x > 0$

e) $x = y^2, \quad x = y + 2$

Exercise 8. Find the area of the region between the curve $y = 3 - x^2$ and the line $y = -1$ by integrating with respect to a) x and b) y.