Practice 3 (Derivatives)

Exercise 1. Use the definition of a derivative to find:

a) $f'(2)$, where $f(x) = x^3 - 2x$

b) $f'(-3)$, $f'(0)$, where $f(x) = 4 - x^2$.

Exercise 2. Find a function f and a number a such that

$$\lim_{h \to 0} \frac{(2+h)^6 - 64}{h} = f'(a).$$

Exercise 3. Find the equation for the tangent to the curve at the given point. Then sketch the curve an tangent together.

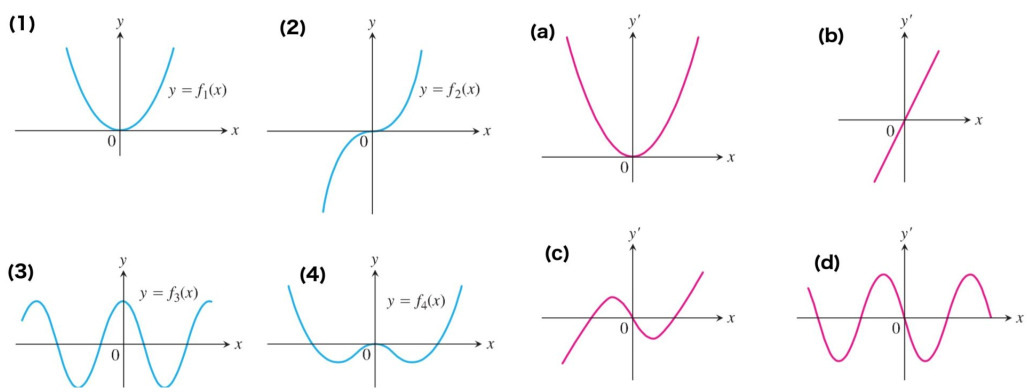
a) $f(x) = 4 - x^2$, $(-1, 3)$;

b) $g(t) = \dfrac{1}{t^2}$, $(-1, 1)$;

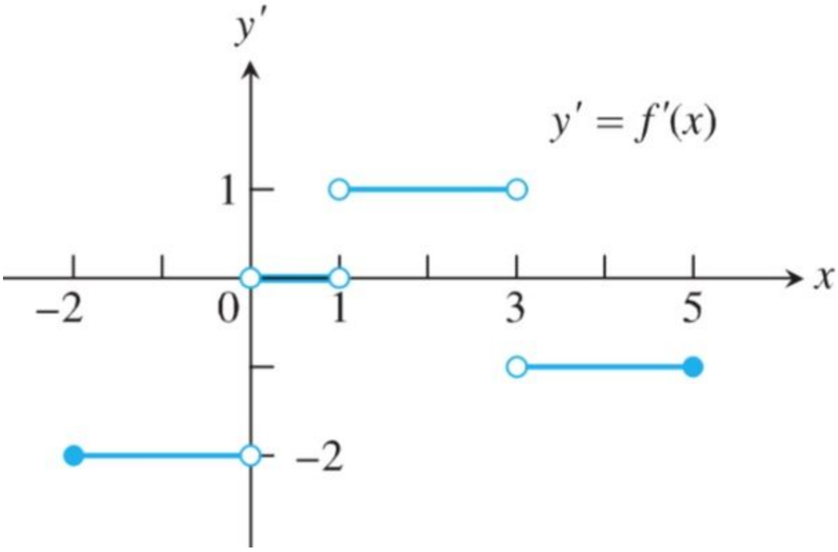
c) $h(x) = x^3$, $(-2, -8)$.

Exercise 4. At what points do the graph of the function $f(x) = x^2 + 4x - 1$ have horizontal tangents?

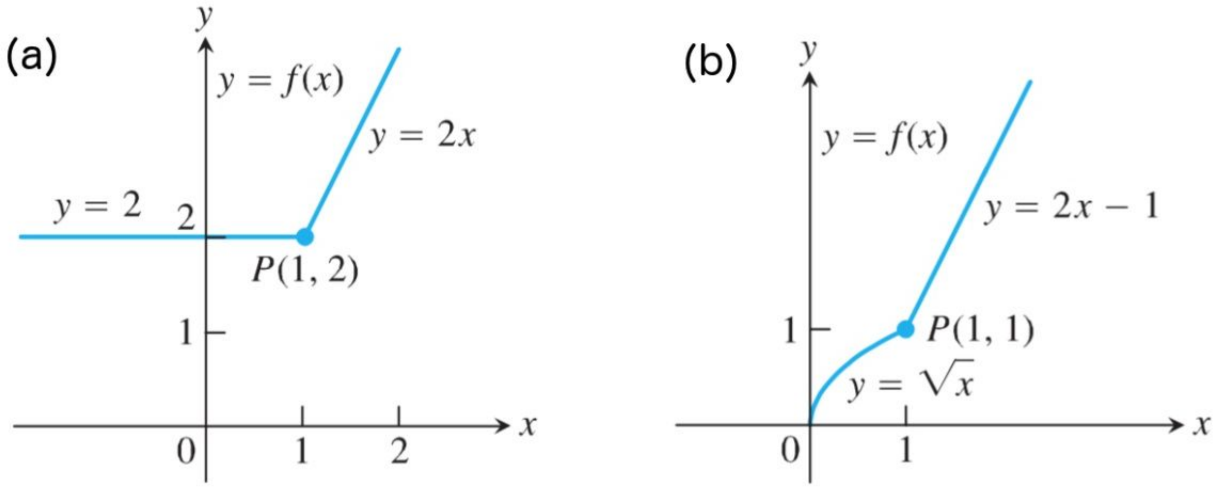
Exercise 5. Match the functions graphed (1)-(4) with the derivatives graphed (a)-(d).



Exercise 6. Use the following information to graph the function f over the closed interval $[-2, 5]$.
- The graph of f is made of closed line segments joined end to end;
- The graph starts at the point $(-2, 3)$;
- The derivative of f is the step function in the figure shown here.



Exercise 7. Compute the right-hand and left-hand derivatives as limits to show that the functions are not differentiable at the point P.



Exercise 8. Find the first and second derivative

a) $5t^3 - 3t^5$.

b) $\dfrac{1}{3s^2} - 3s^2$.

Exercise 9. Find the derivatives of the functions

a) $(3 - x^2)(x^3 - x + 1)$;

b) $\dfrac{4 - 3x}{3x^2 + x}$;

c) $\dfrac{\cos x}{1 + \sin x}$;

d) $x^3 \sin x \cos x$;

e) $(4 - 3x)^9$;

f) $e^{x^2 - x}$

g) $2^{3^{4^x}}$

h) $\sqrt[3]{2r - r^2}$;

i) $\dfrac{1}{x} \sin^{-5} x + \dfrac{x}{3} cos^3 x$;

j) $cos \sqrt{\sin(\tan \pi x)}$;

k) $\sqrt{x + \sqrt{x + \sqrt{x}}}$

Exercise 10. Function $s(t) = 6t - t^2$ gives the position of a body moving on a coordinate line, with s in meters and t in seconds.

a) Find the body's displacement and average velocity for $0 \leqslant t \leqslant 6$.

b) Find the body's speed and acceleration at the endpoints of the interval.

c) When, if ever, during the interval does the body change direction?

Exercise 11. A 13 ft ladder is leaning against a house when its base starts to slide away (see figure). By the time the base is 12 ft from the house, the base is moving at the rate of 5 ft/sec.

a. How fast is the top of the ladder sliding down the wall then?

b. At what rate is the area of the triangle formed by the ladder, wall, and ground changing then?

c. At what rate is the angle θ between the ladder and the ground changing then?