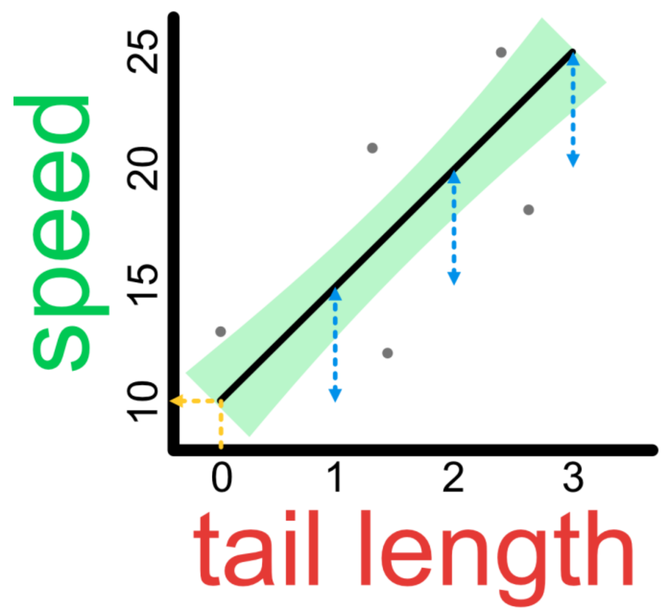
Modelling normal distribution

Pärt Prommik, PhD

Ülo Maiväli, PhD

Recap



Coefficients

Y — *Predicted output*, *Outcome variable*, *Response variable*, *Dependent variable*

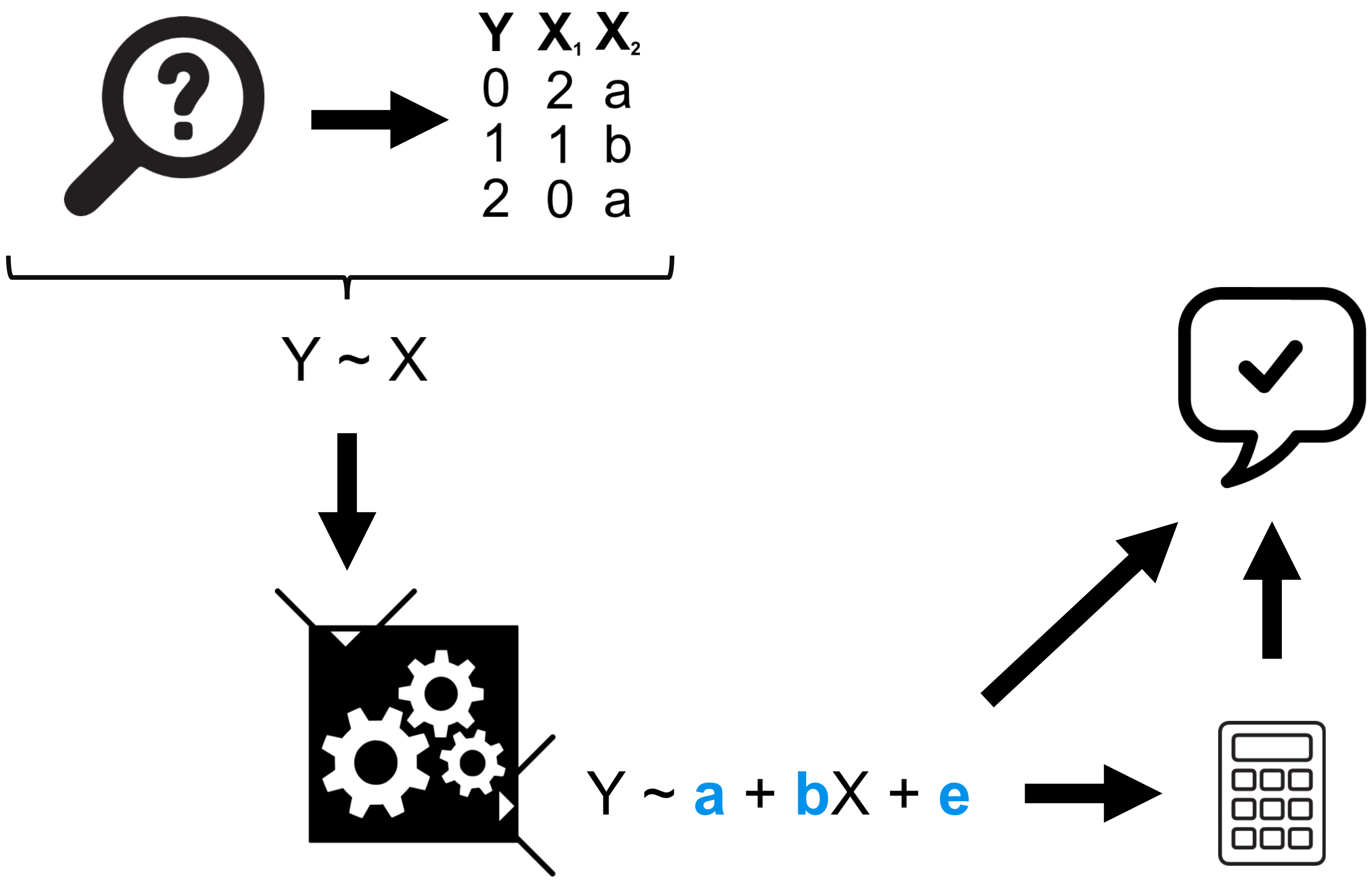
Intercept — *Constant*

Slope

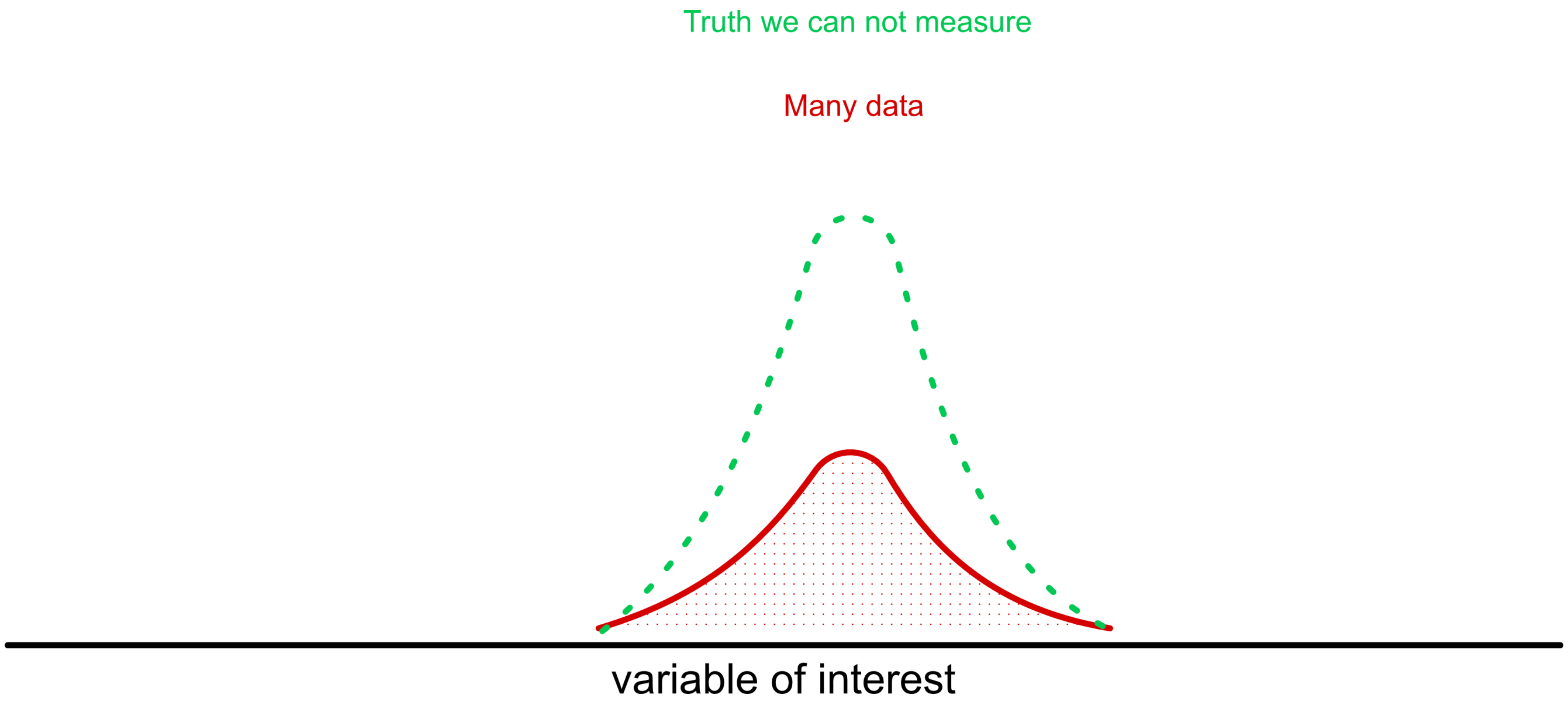
X — *Explanatory variable*, *Predictor variable*, *Independent variable*

Residual standard deviation — *Residual standard error*, *Error*, *Disturbance*

speed = a + b×tail_length + e



Classical regression is data-driven



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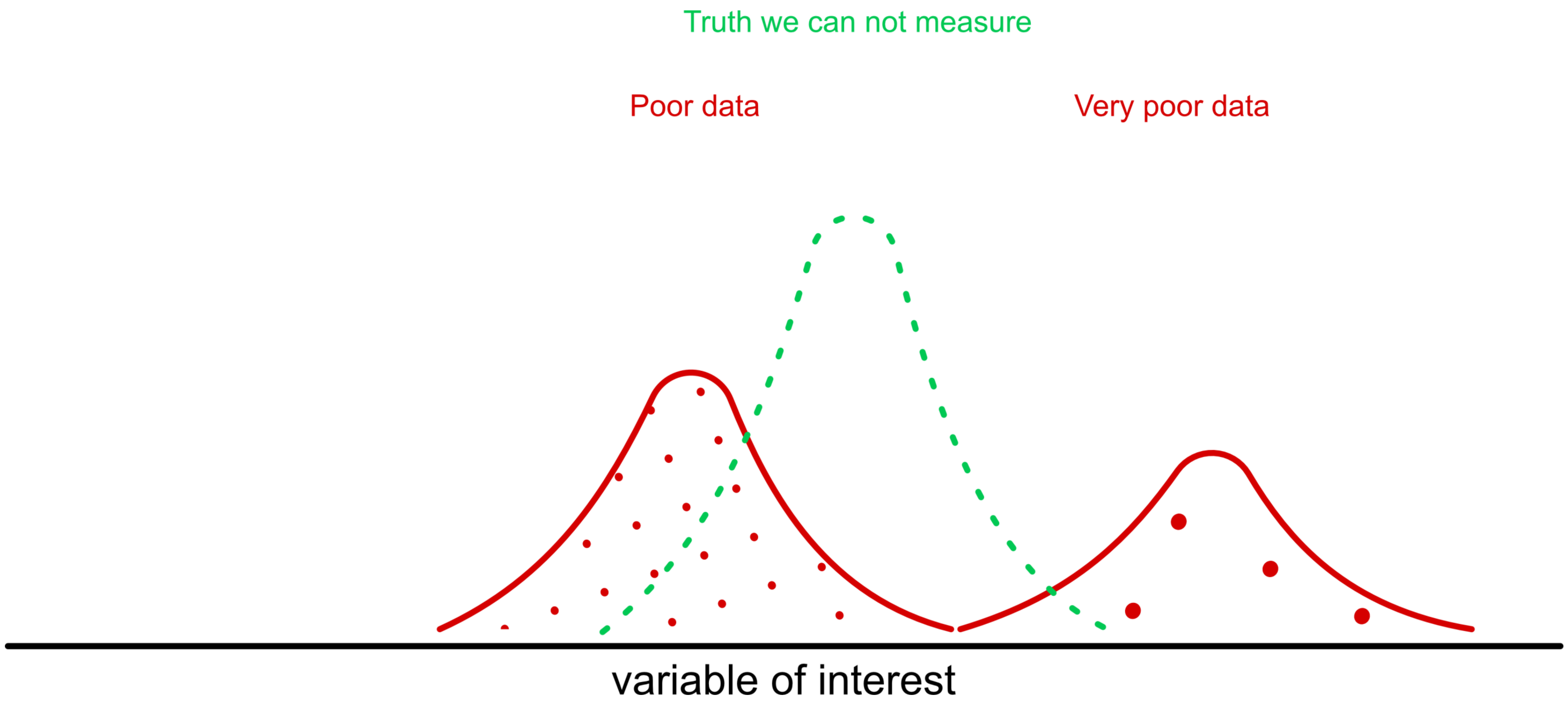


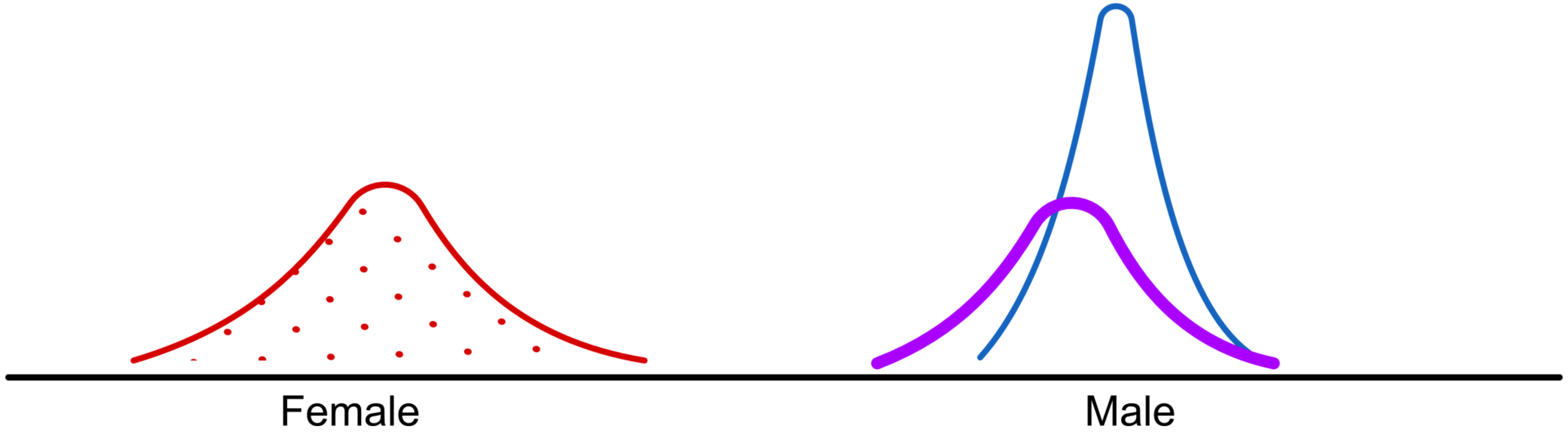
Bayesian thinking is different

Y	X_1	X_2
0	2	a
1	1	b
2	0	a

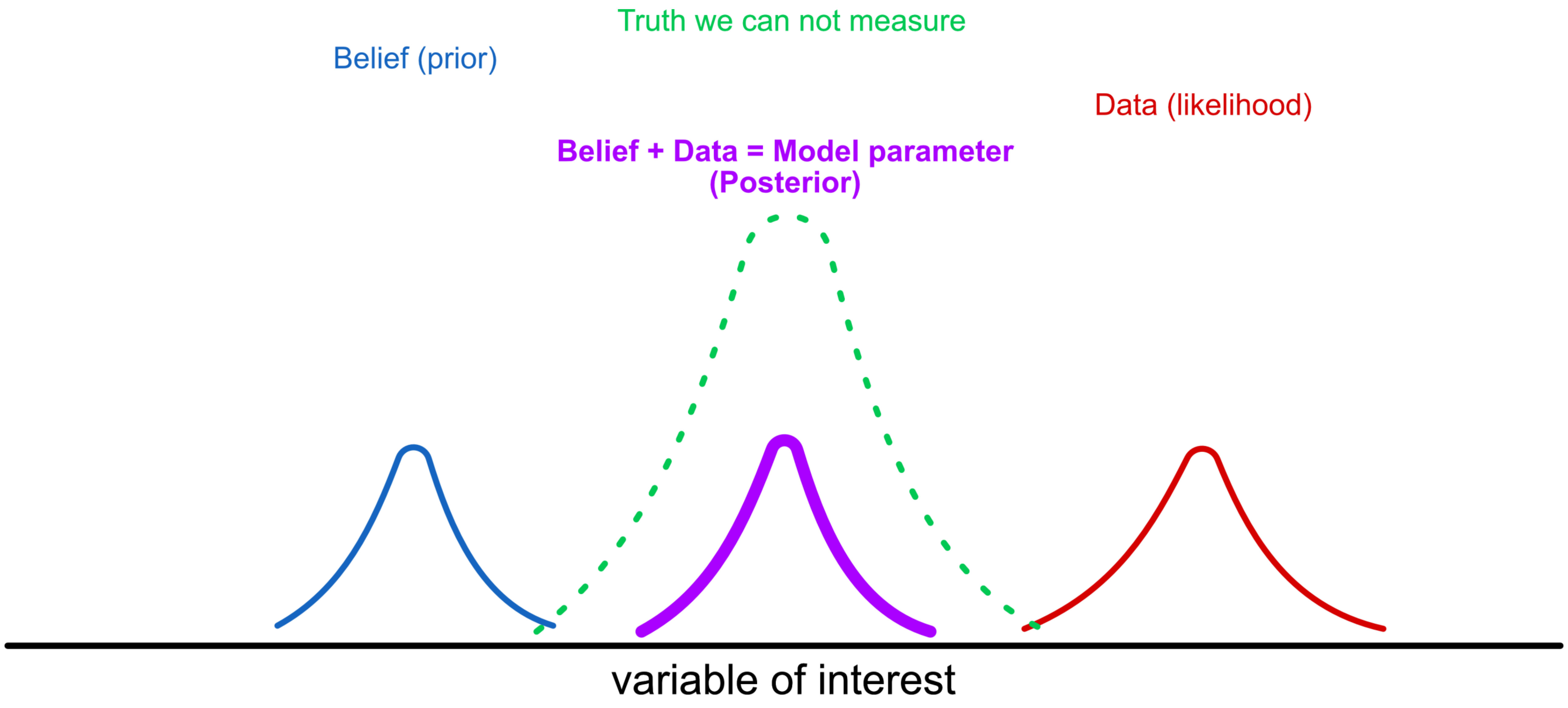
! New data solely do not determine our beliefs. New data updates them

Few

Mostly belief-driven posterior



Inputting domain knowledge to an analysis



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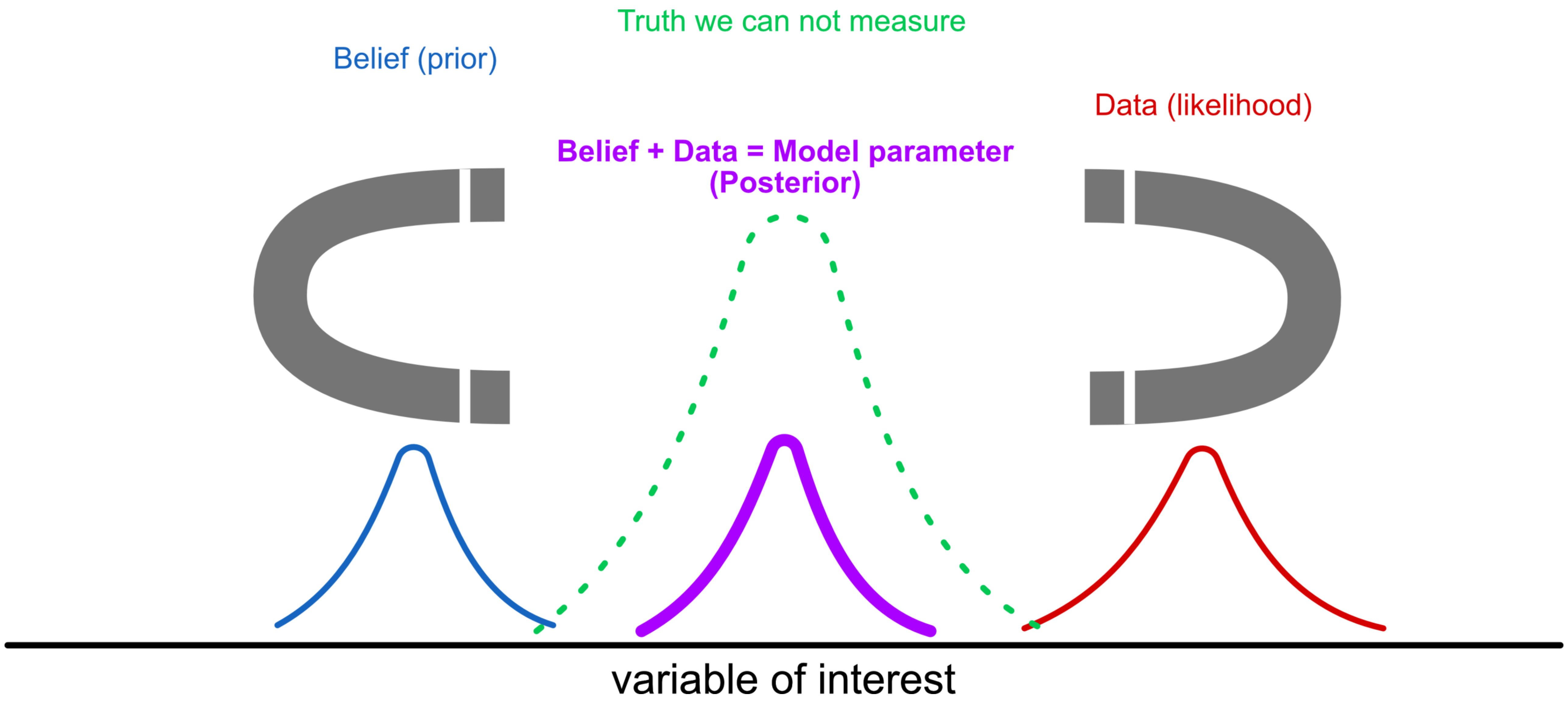
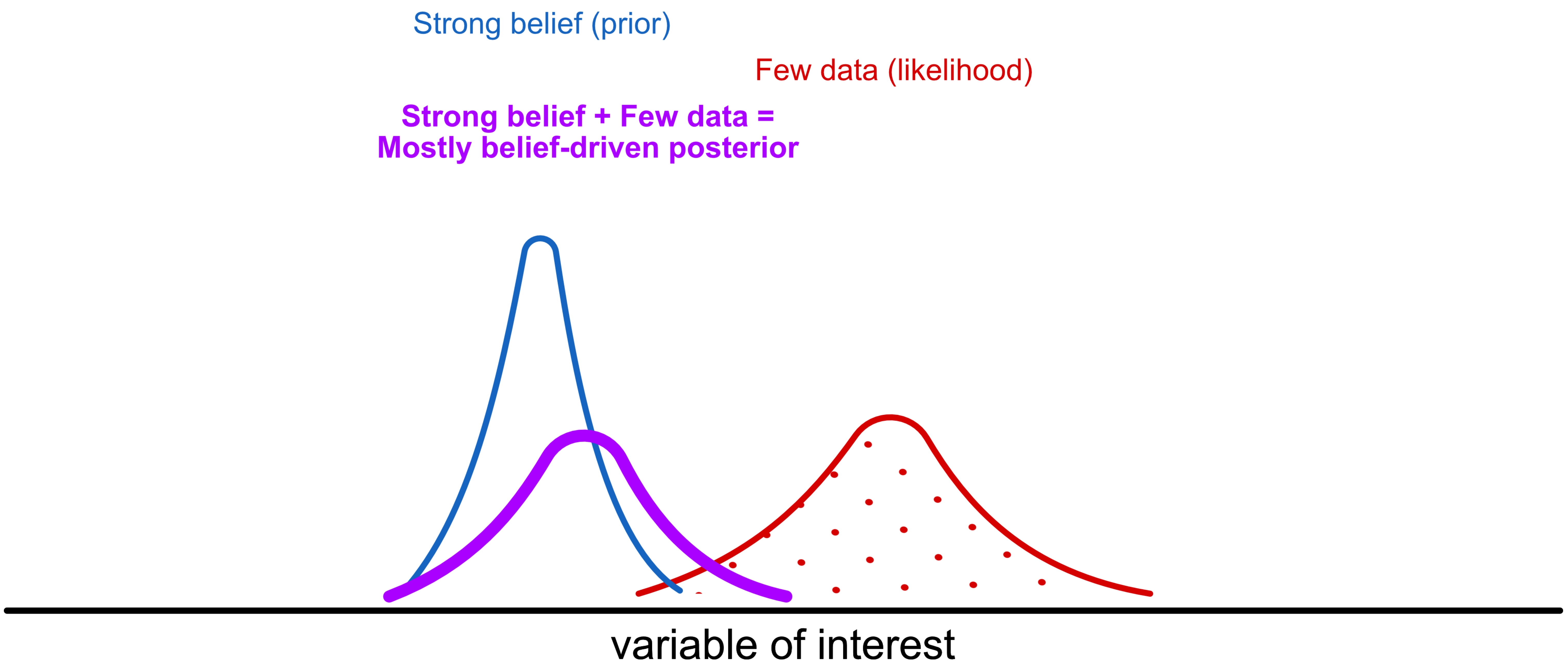


How **data** and **prior** influence **posterior**



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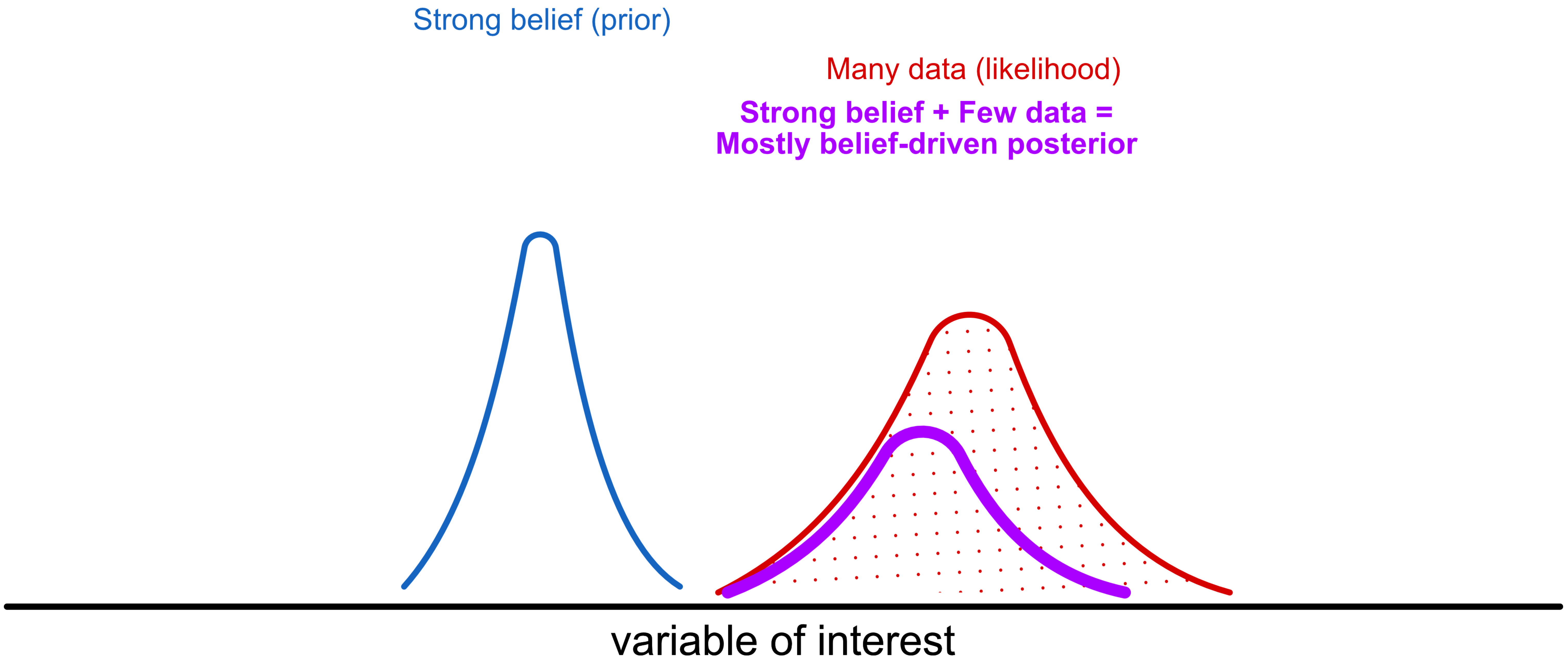

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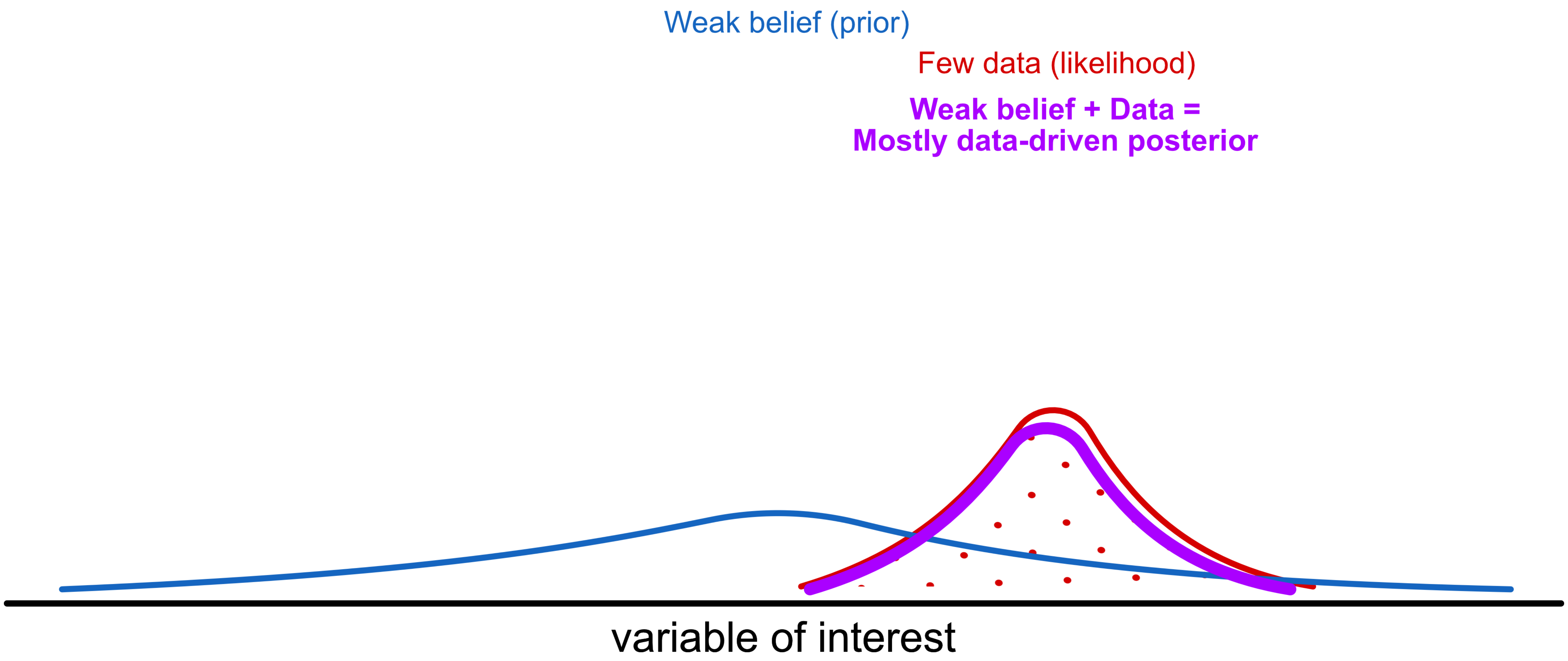

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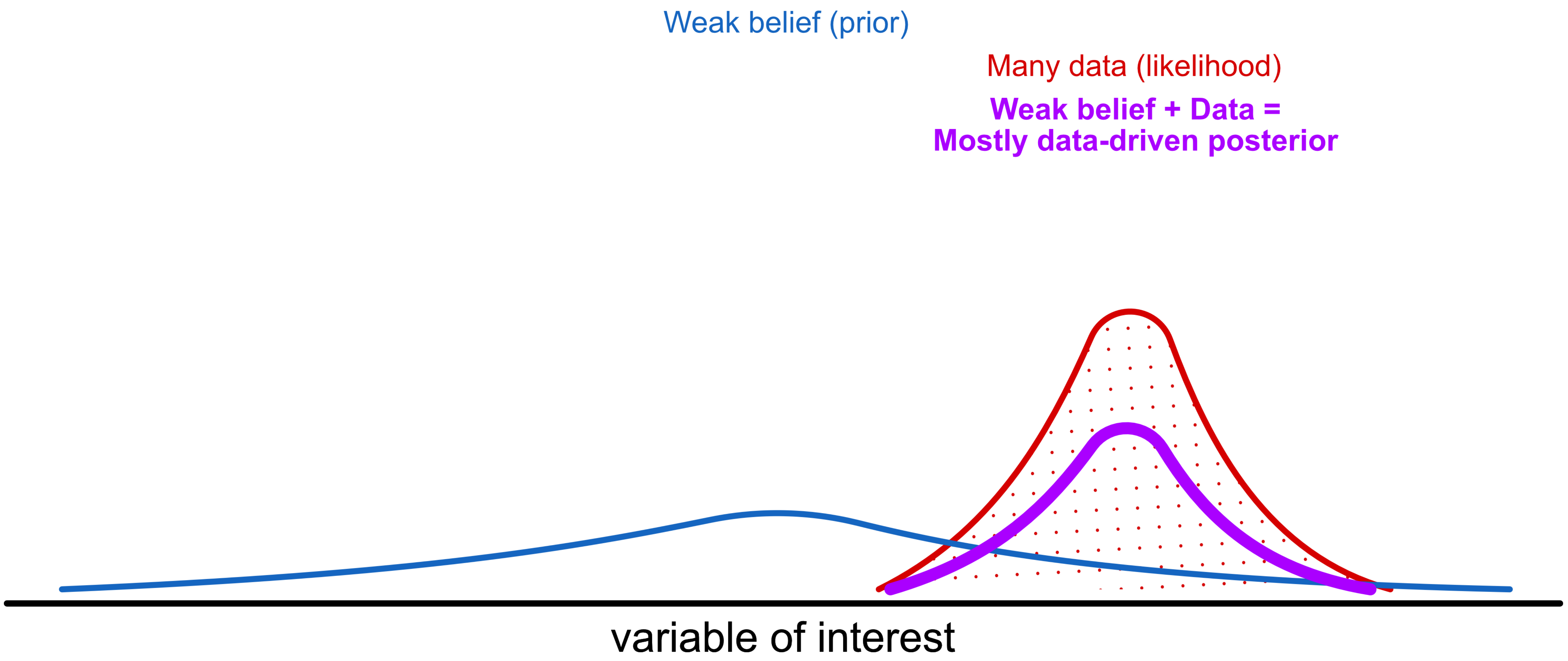
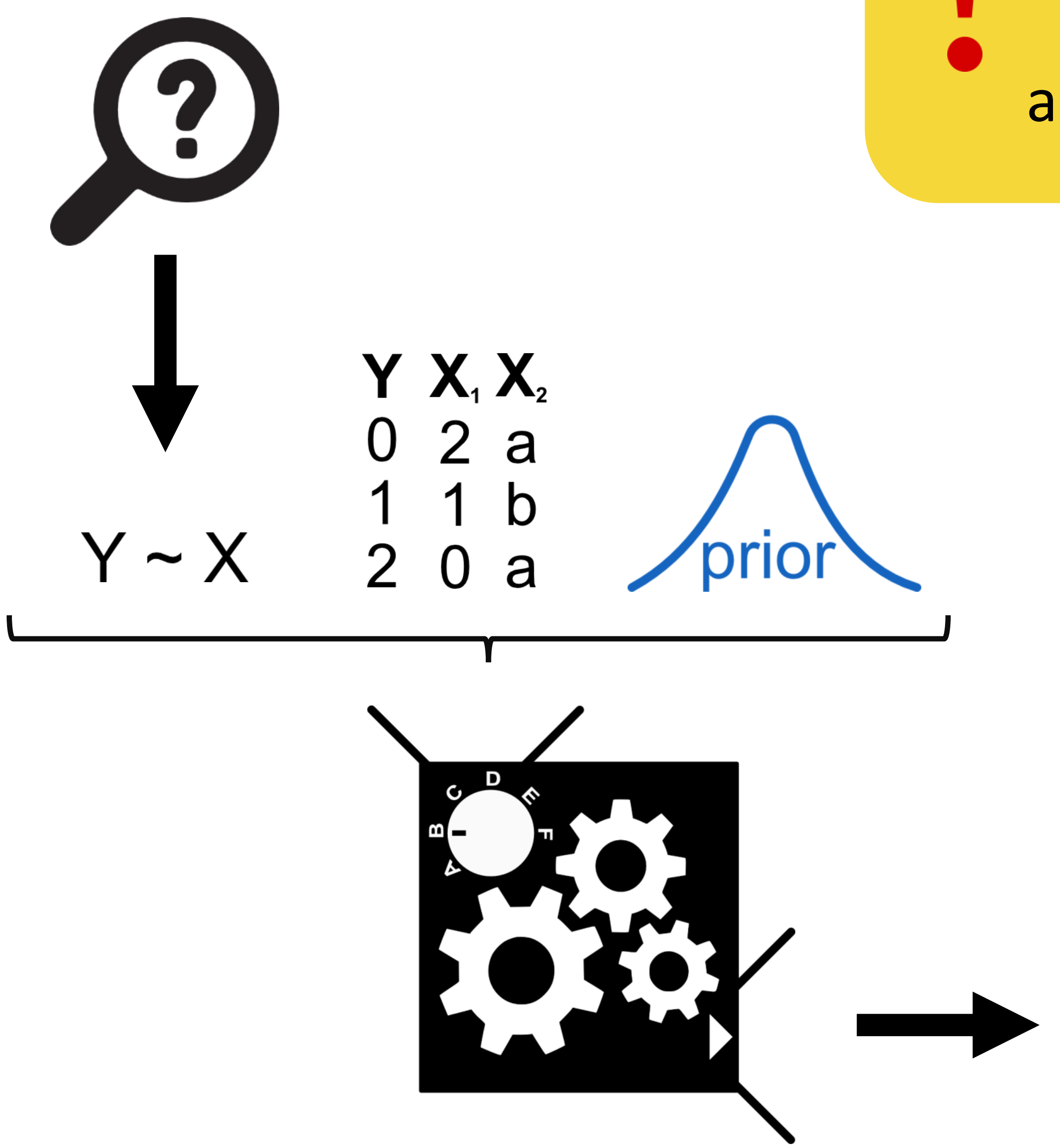
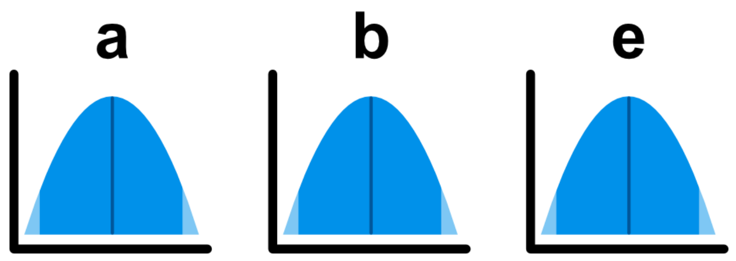


Bayesian regression = working with distributions

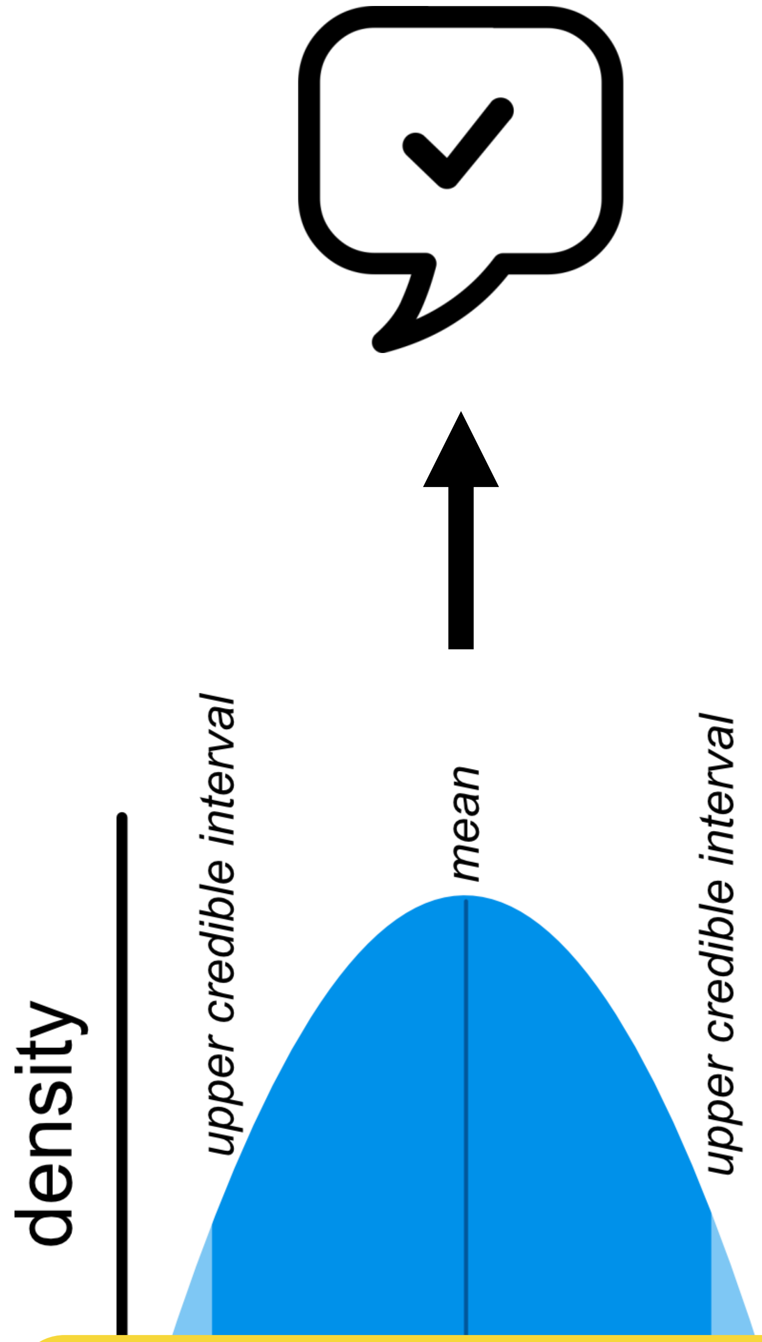
Posterior is a table containing
all possible parameter values.
It's each line is a draw, giving
a value for every model parameter.



a	b	e
1.1	3.2	0.2
1.0	3.1	0.1
1.3	3.4	0.4
0.9	2.9	0.3
...	...	...

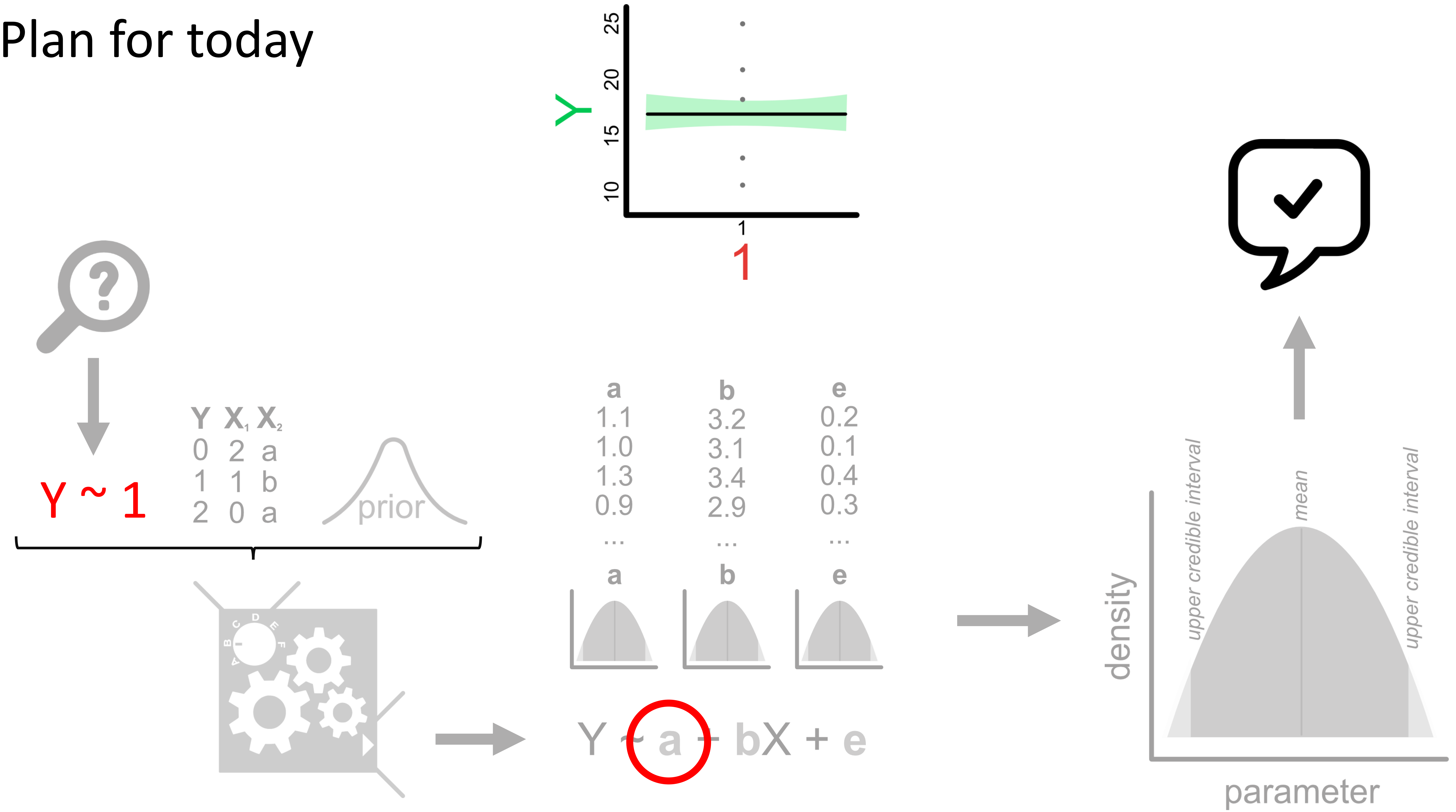


Y ~ **a** + **b**X + **e**

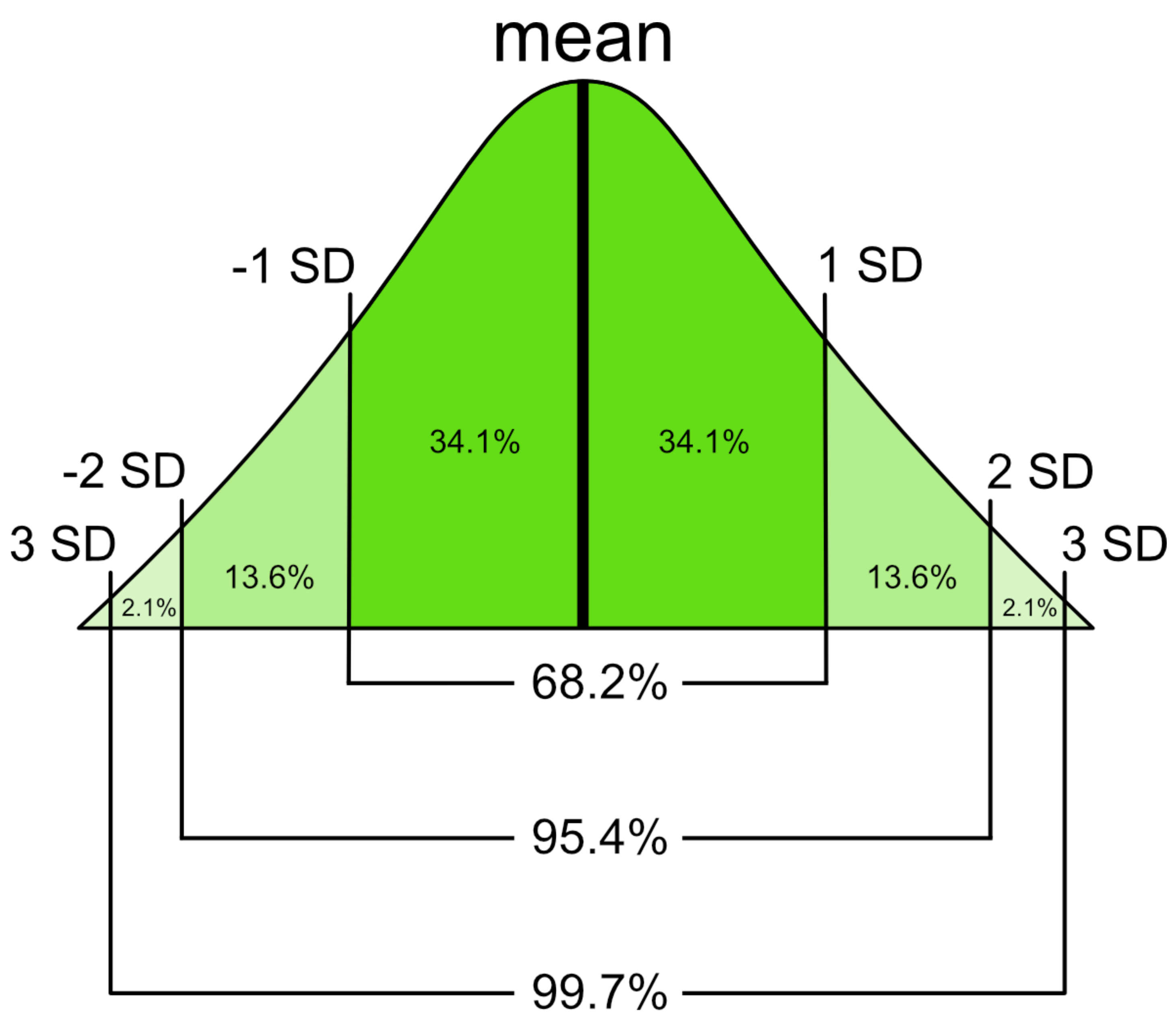


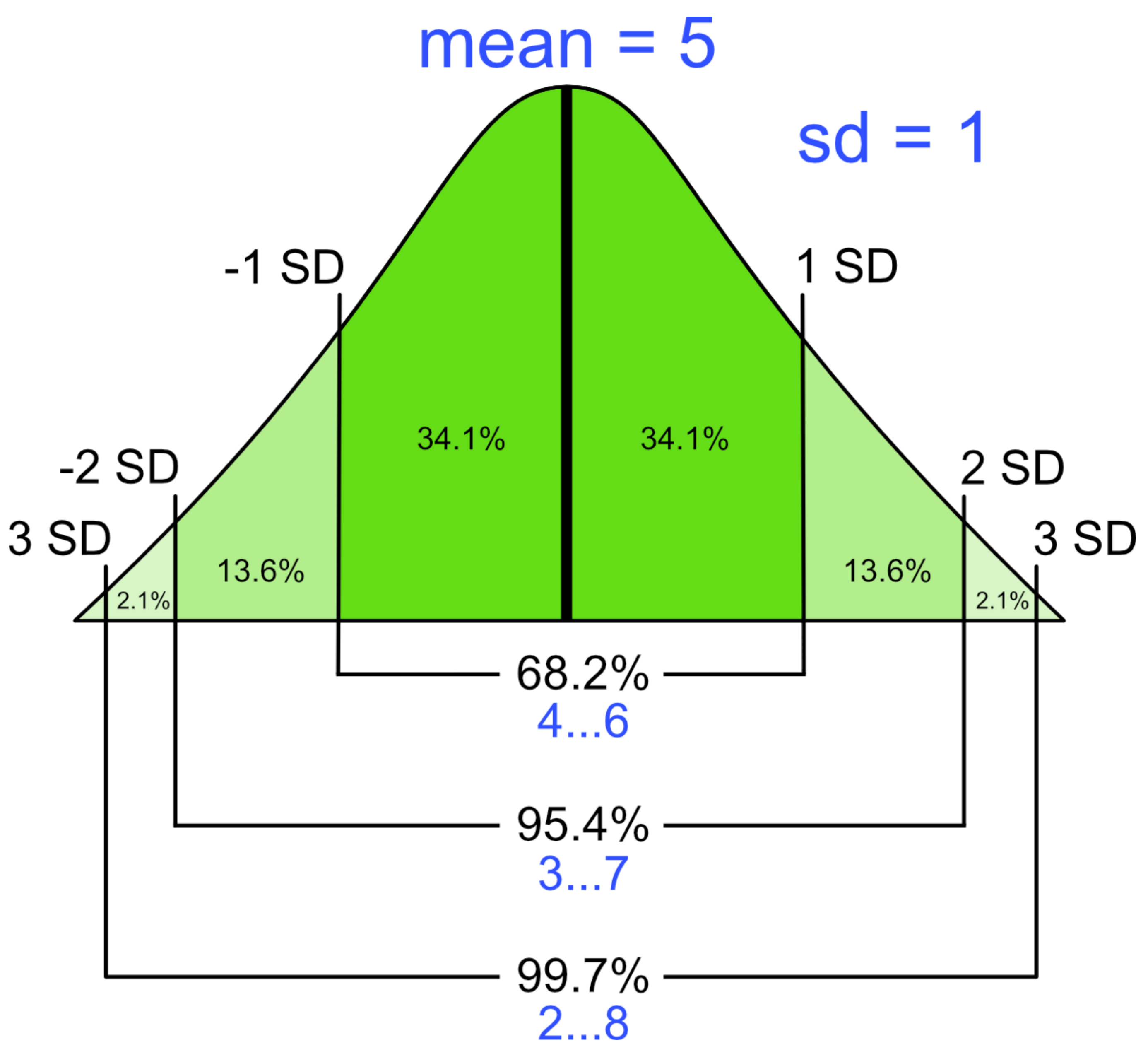
Statistical uncertainty
we need!

Plan for today



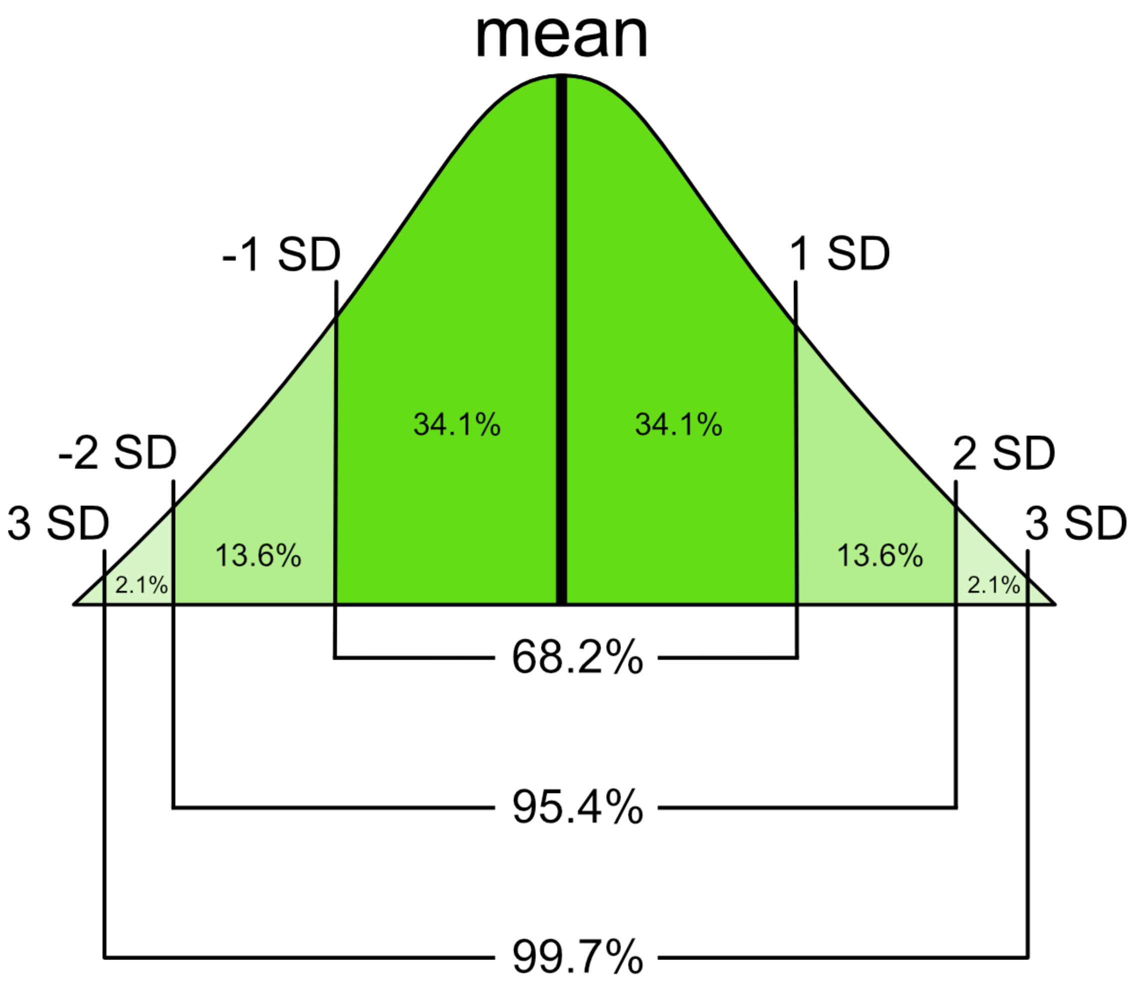
Prior is specified as a distribution





Define your first prior for writing speed in general population

1. Mean typed words per minute
2. How much the mean may vary (SD)?



Let’s do some modelling

brms model summary

```
 Family: gaussian
  Links: mu = identity; sigma = identity
Formula: words ~ 1
   Data: data (Number of observations: 17)
  Draws: 4 chains, each with iter = 2000; warmup = 1000; thin = 1;
         total post-warmup draws = 4000

Population-Level Effects:
          Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
Intercept    52.73      5.45    42.06    63.42 1.00     2509     2300

Family Specific Parameters:
      Estimate Est.Error l-95% CI u-95% CI Rhat Bulk_ESS Tail_ESS
sigma    22.24      3.26    16.54    29.34 1.00     2635     2411

Draws were sampled using sampling(NUTS). For each parameter, Bulk_ESS
and Tail_ESS are effective sample size measures, and Rhat is the potential
scale reduction factor on split chains (at convergence, Rhat = 1).
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1	F	0
2	T	1
3	F	0

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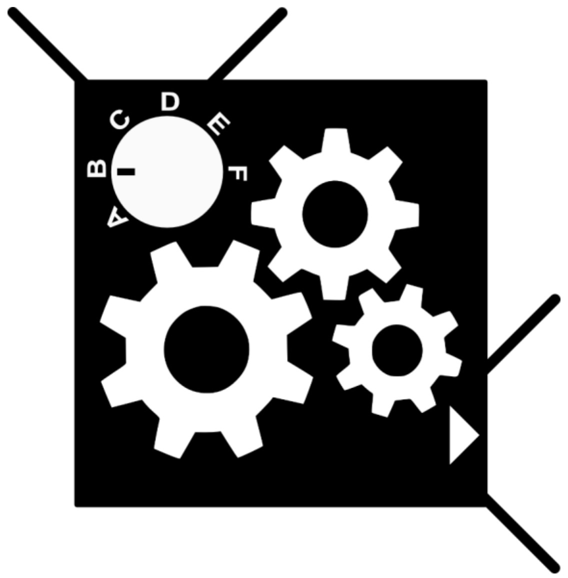


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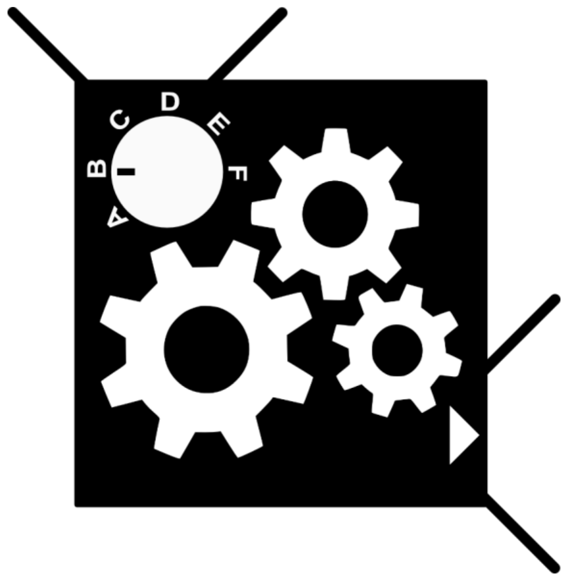
Y ~ X

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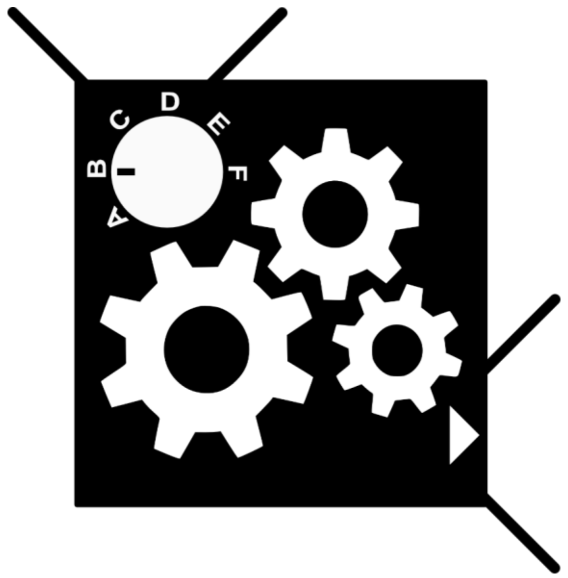

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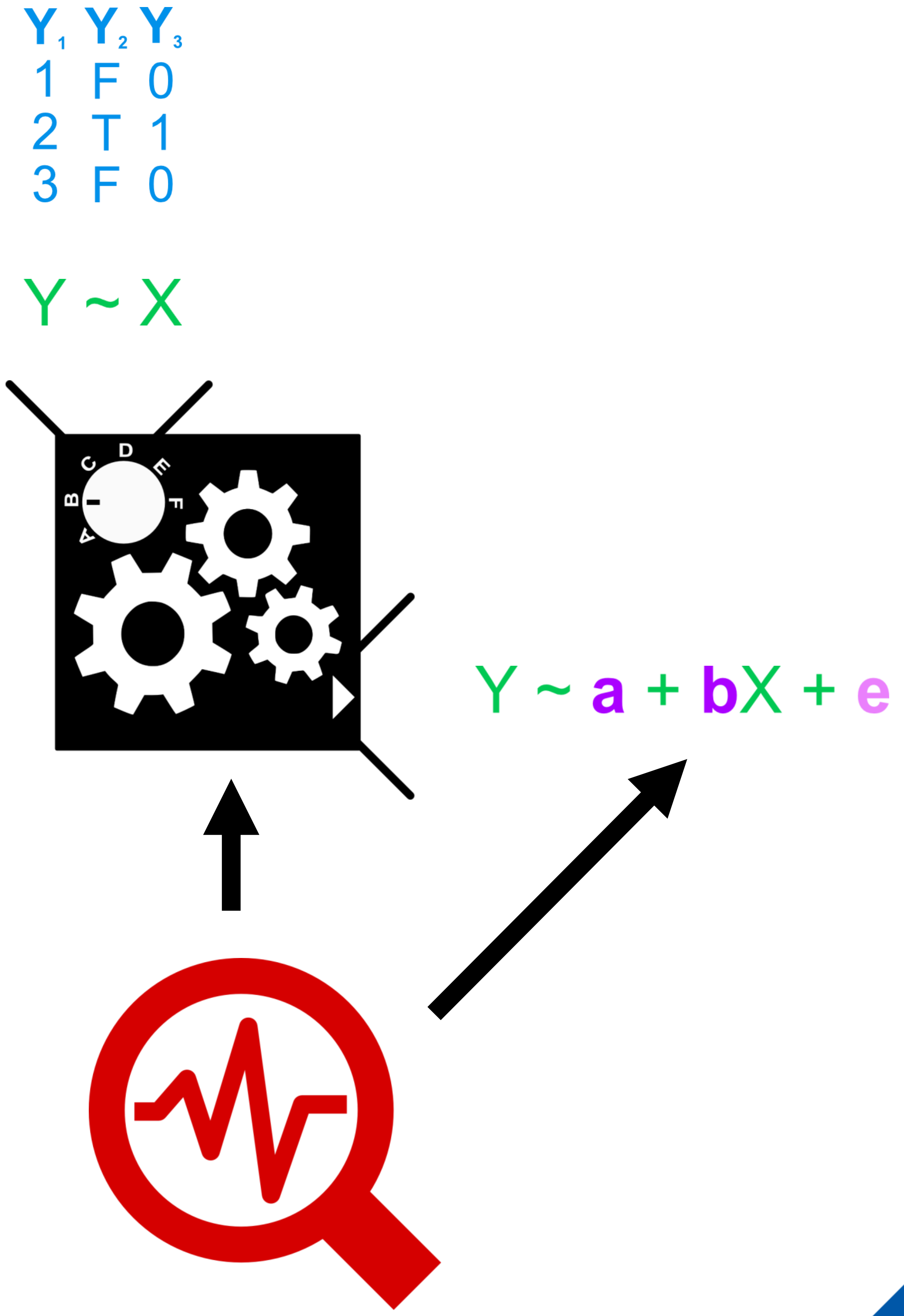
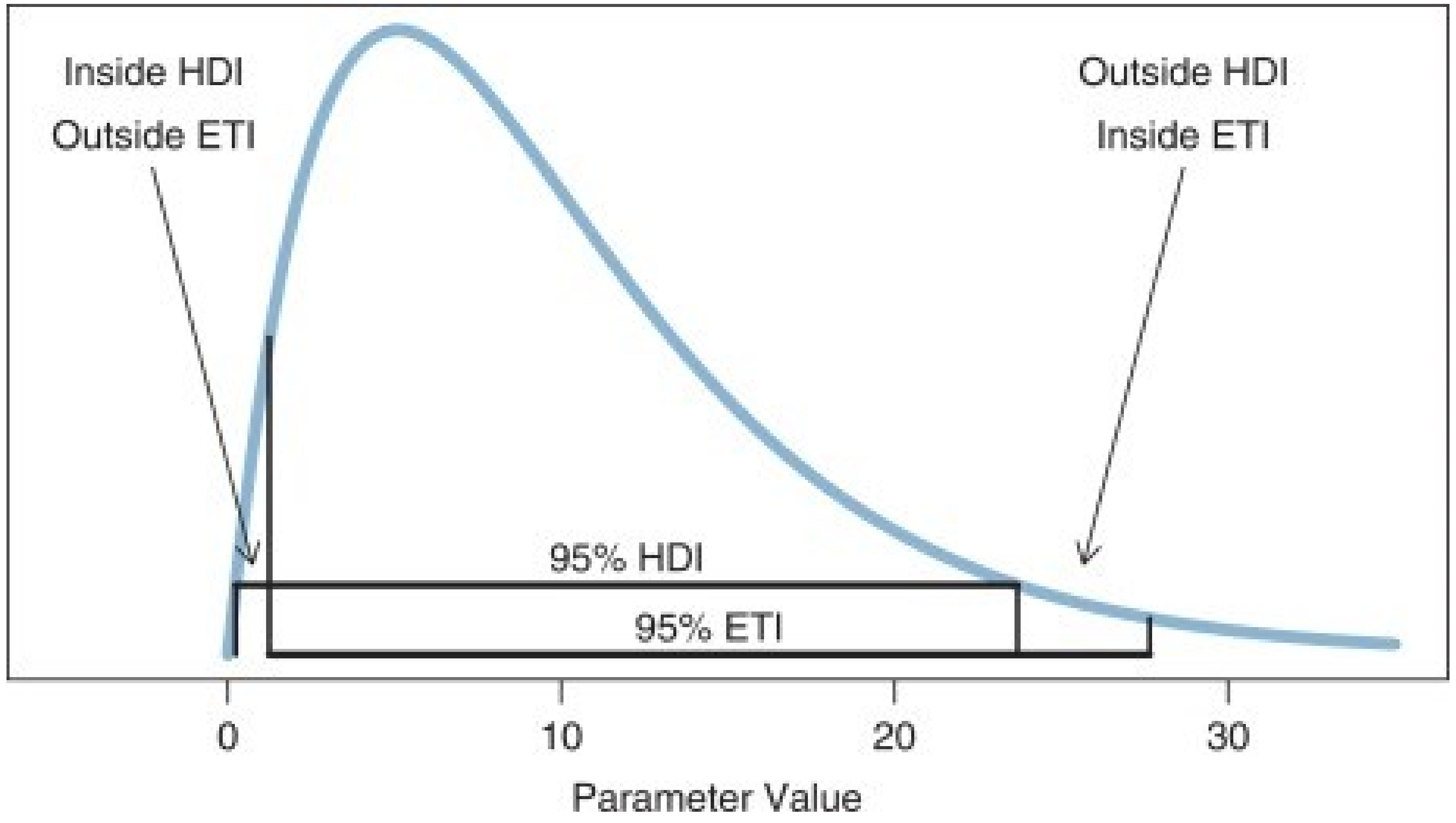
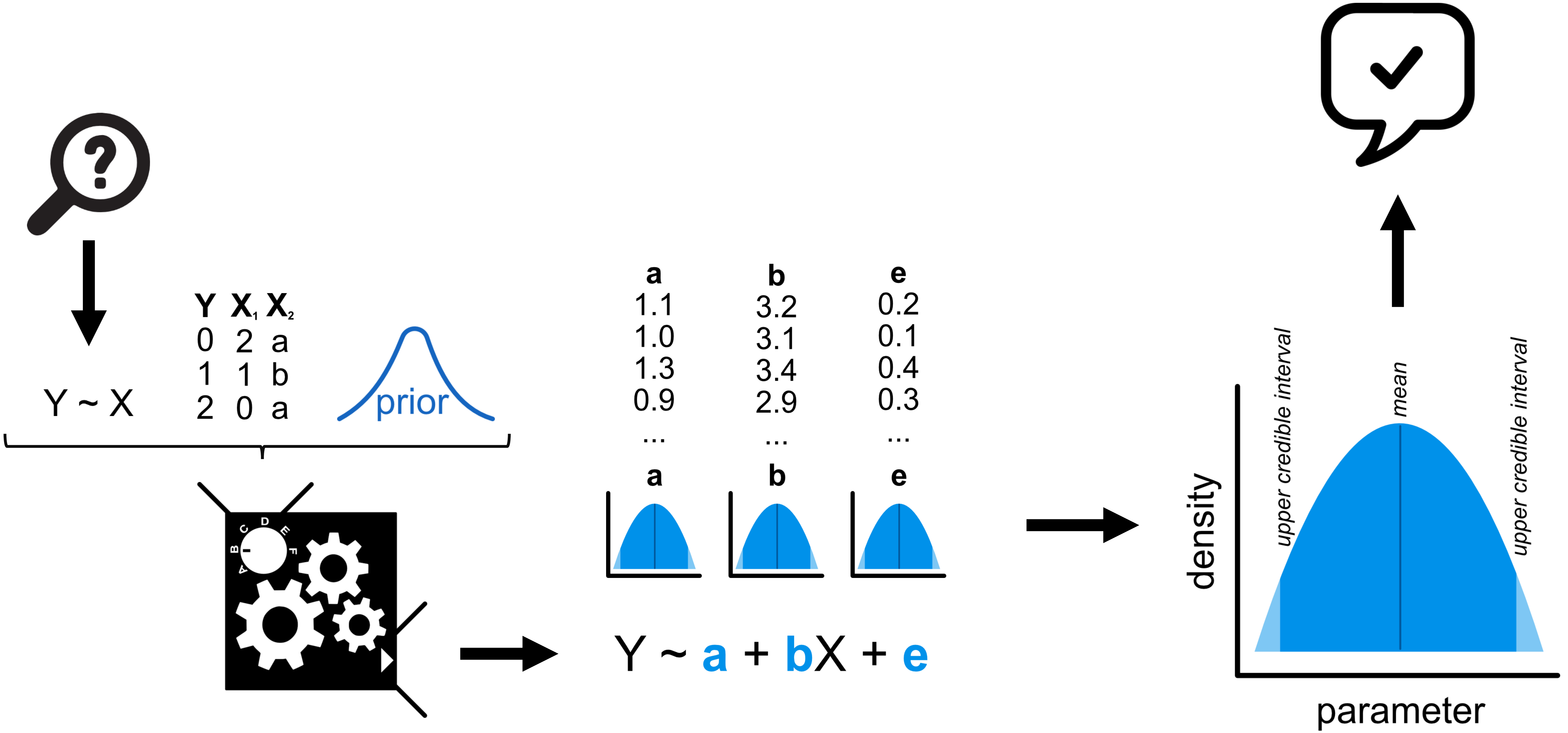


HDI vs ETI



Inside HDI
Outside ETI
Outside HDI
Inside ETI
95% HDI
95% ETI
0
10
20
30
Parameter Value

Summary



Y ~ X
Y X1 X2
0 2 a
1 1 b
2 0 a
prior
a b e
1.1 3.2 0.2
1.0 3.1 0.1
1.3 3.4 0.4
0.9 2.9 0.3
... ...
a b e
Y ~ a + bX + e
density
parameter
upper credible interval
mean
upper credible interval

Need help?

Google

(R OR tidyverse) AND "your problem"

stack overflow

discourse.mc-stan.org