Harmonic functions. Uncertainty principle, wave-particle duality.

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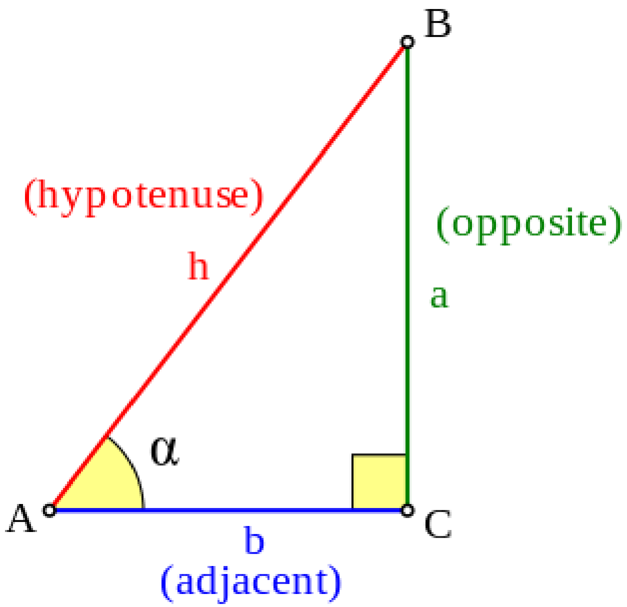
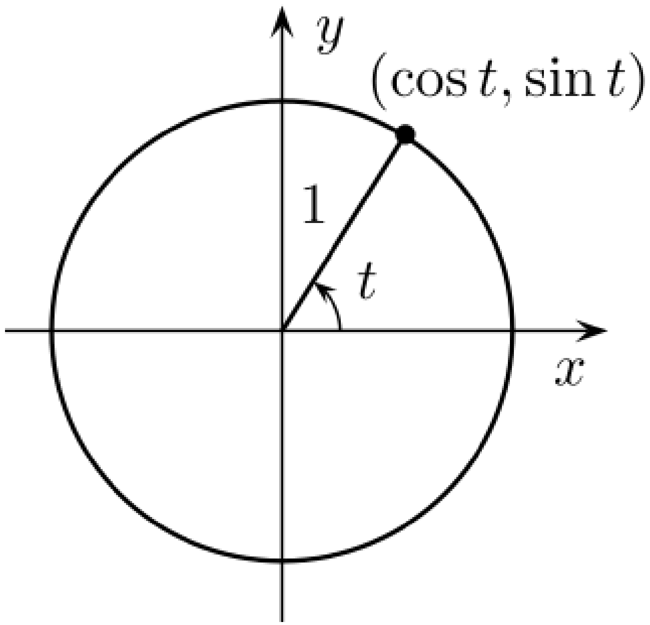
Sine and Cosine functions



$$\sin\alpha = \frac{opposite}{hypotenuse}$$

$$\cos\alpha = \frac{adjastent}{hypotenuse}$$



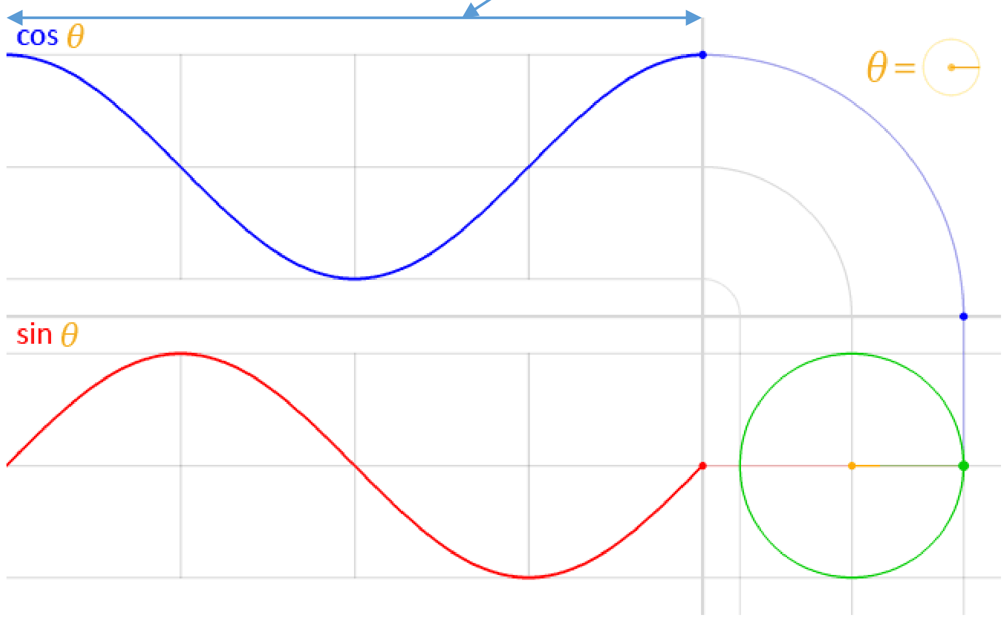
Sin and Cos functions fluctuate between -1 and +1 thus creating a circle on X,Y plane.

Sine and Cosine functions

- Defined from -∞ to + ∞
- Period 2π (radians)
- 2 π = 2*3.14 =
  1 full circle =360 degrees

What is a
radian ?

Wavelength

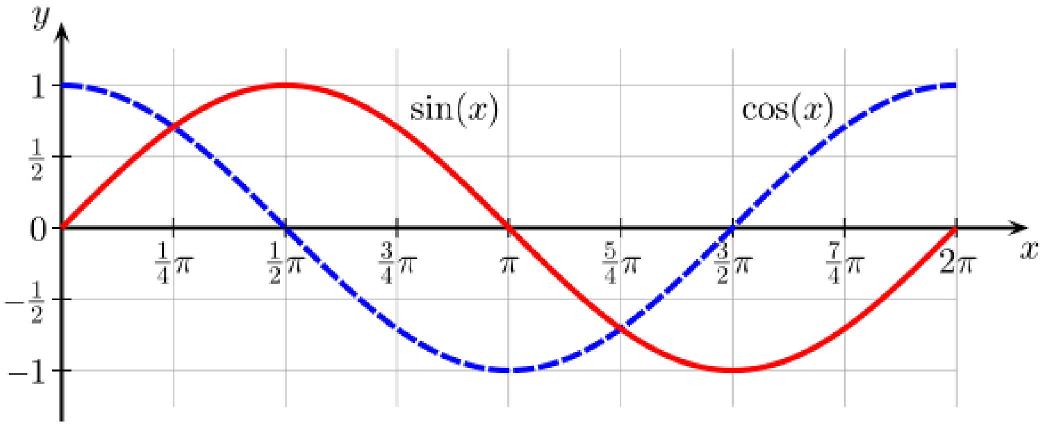


$$\cos\alpha = \sin(a + \frac{\pi}{2})$$

This is called
phase shift !

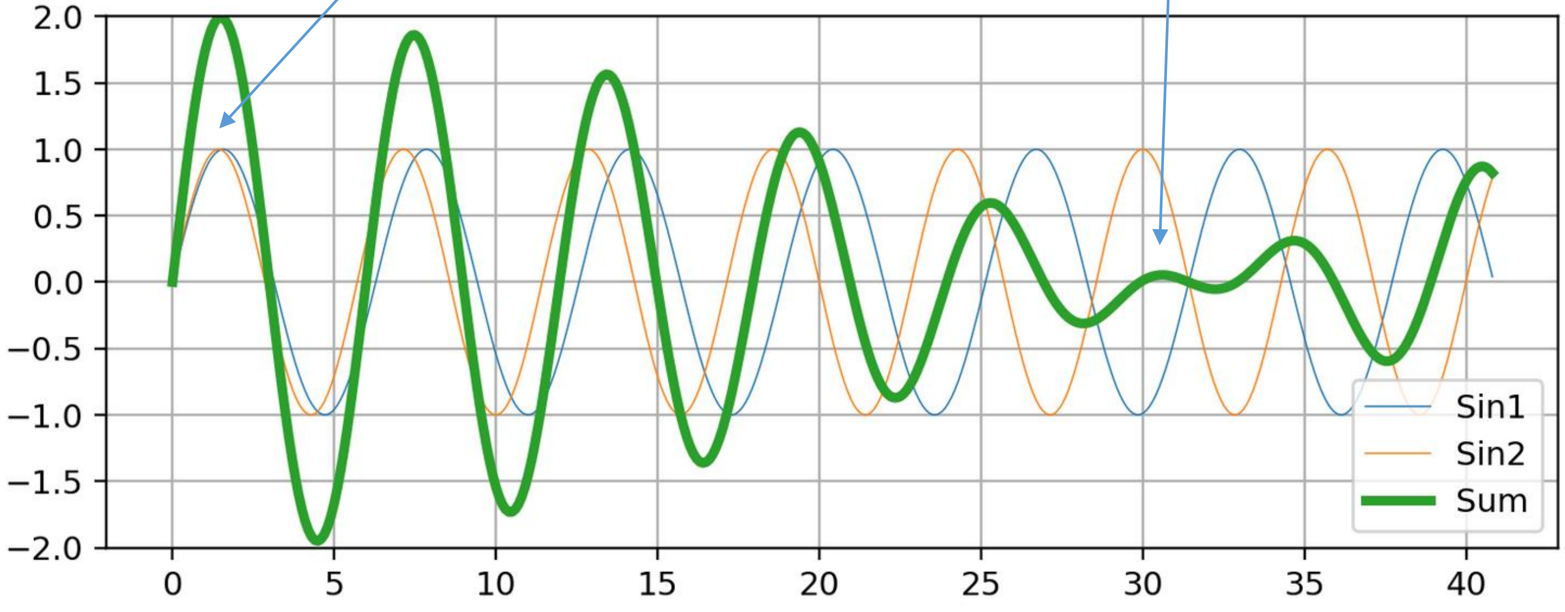
$$f = \frac{v}{\lambda} = \frac{c}{\lambda} \cdot \left(\frac{m}{s \cdot m}\right)$$

How we add waves together ?

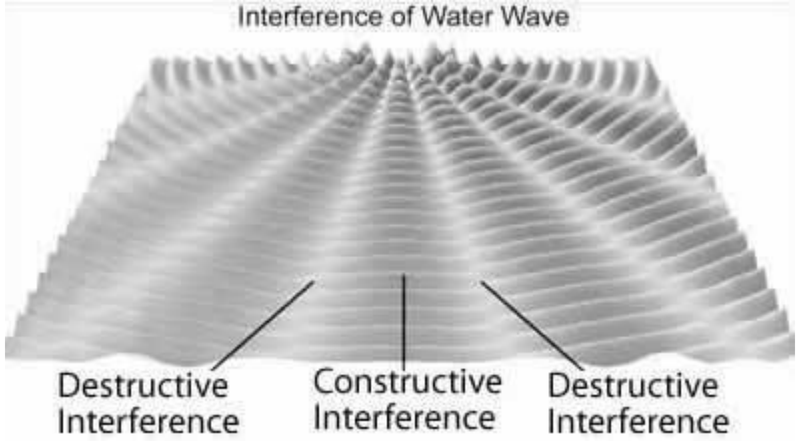


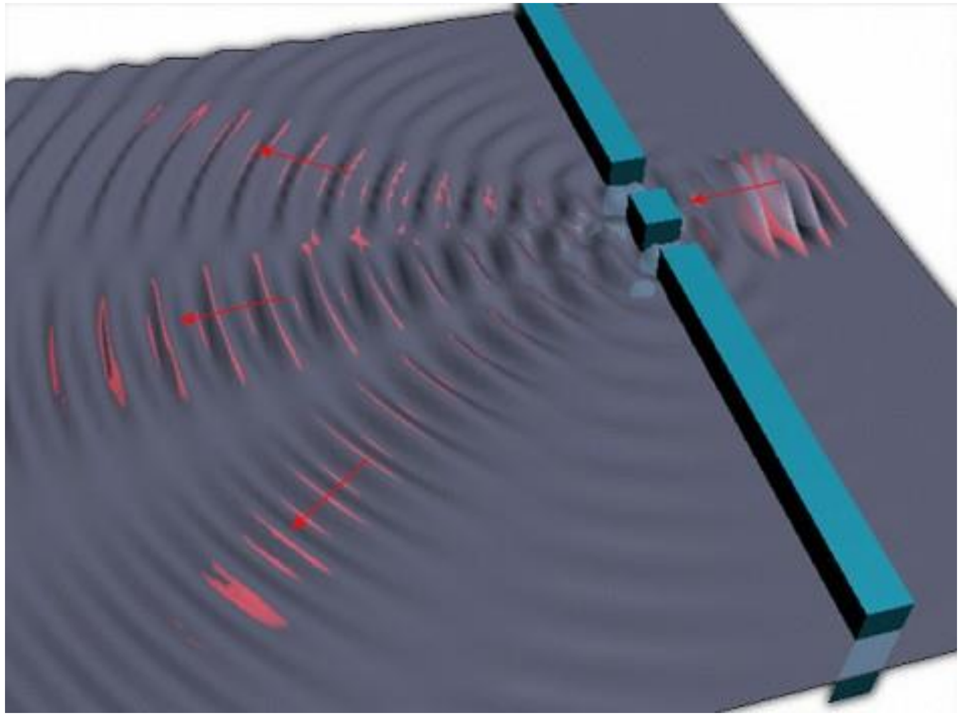
Constructive interference

Destructive interference

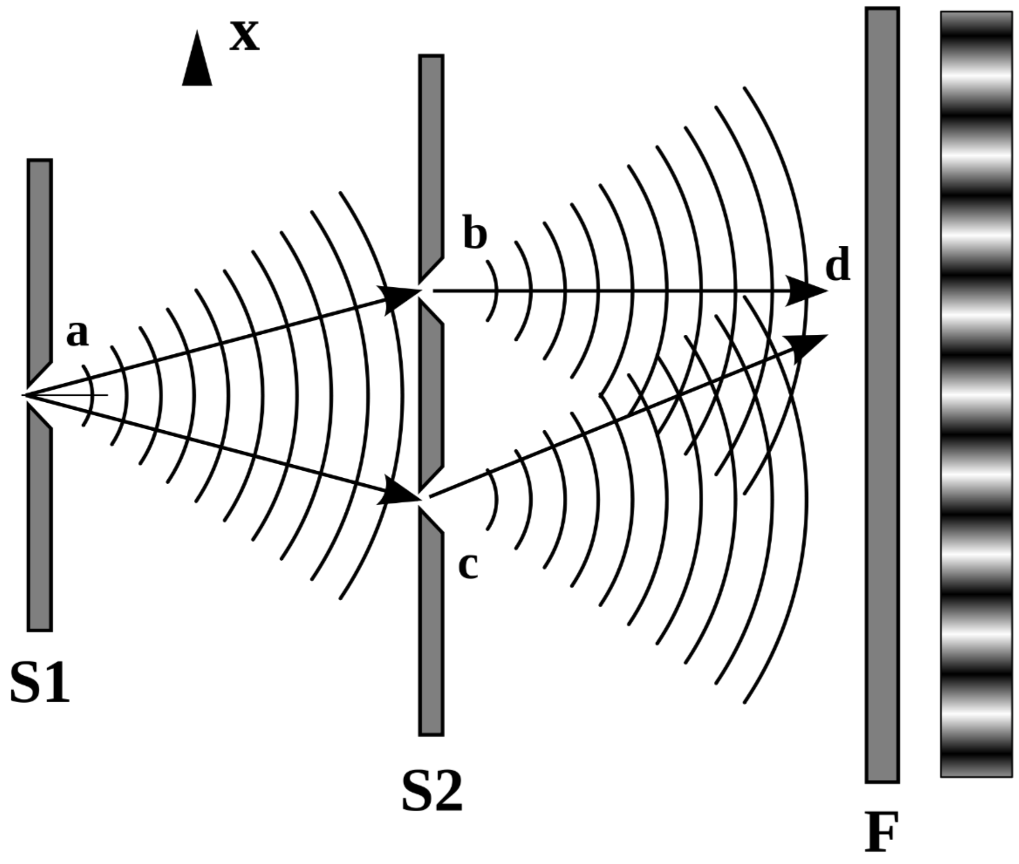


Interference

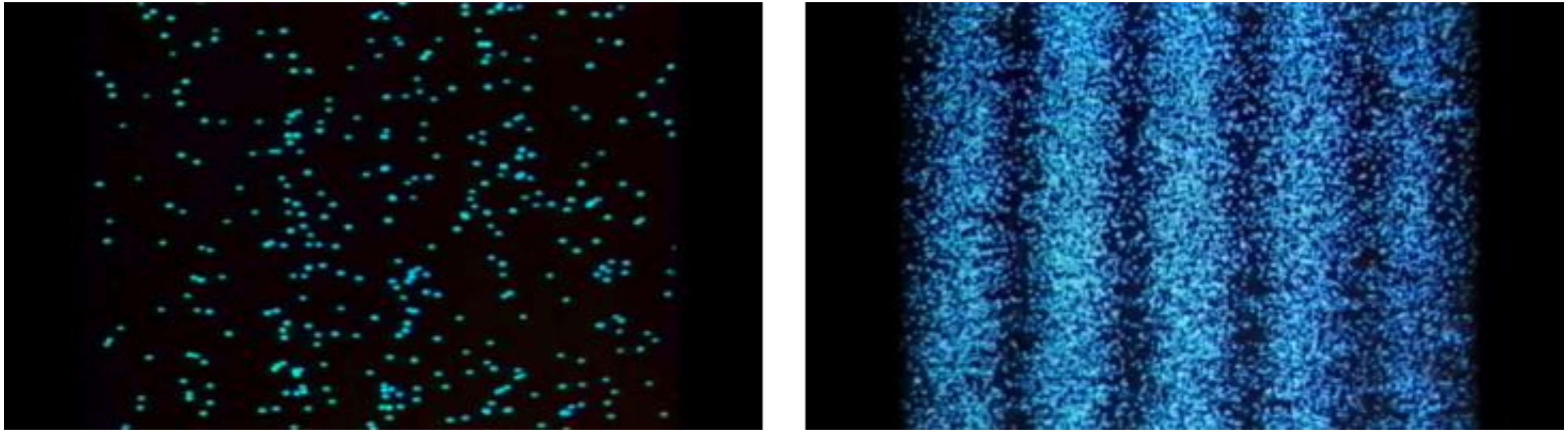




Interference of light



From particle to wave…



Young’s Interference Experiment with a single photon
(Hamamatsu Photonics, 1982)

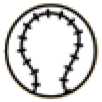
Wave particle duality

- Light functions as both a particle and a wave, depending on how the experiment is conducted and when observations are made.
- All matter exhibits both wave and particle properties under the right circumstances. Obviously, massive objects exhibit very small wavelengths, so small in fact that it's rather pointless to think of them in a wave fashion. But for small objects, the wavelength can be observable and significant, as attested to by the double slit experiment with electrons.

All matter as waves
DeBroglie Hypothesis

$$\lambda = \frac{h}{p} = \frac{h}{mv} \qquad h = 6.626 \cdot 10^{-34} \cdot J \cdot s$$

Does this relationship apply to all particles? Consider a pitched baseball:



$$v = 40m/s \approx 90mi/hr$$

$$m = 0.15kg \qquad \lambda = \frac{h}{mv} = \frac{6.626x10^{-34}\,J\cdot s}{(0.15kg)(40m/s)} = 1.1x10^{-34}m$$

For an electron accelerated through 100 volts: v= $5.9x10^{6}m/s$

$$\lambda = \frac{6.626x10^{-34}\,J\cdot s}{(9.11x10^{-31}kg)(5.9x10^{6}m/s)} = 1.2x10^{-10}m = 0.12nm$$

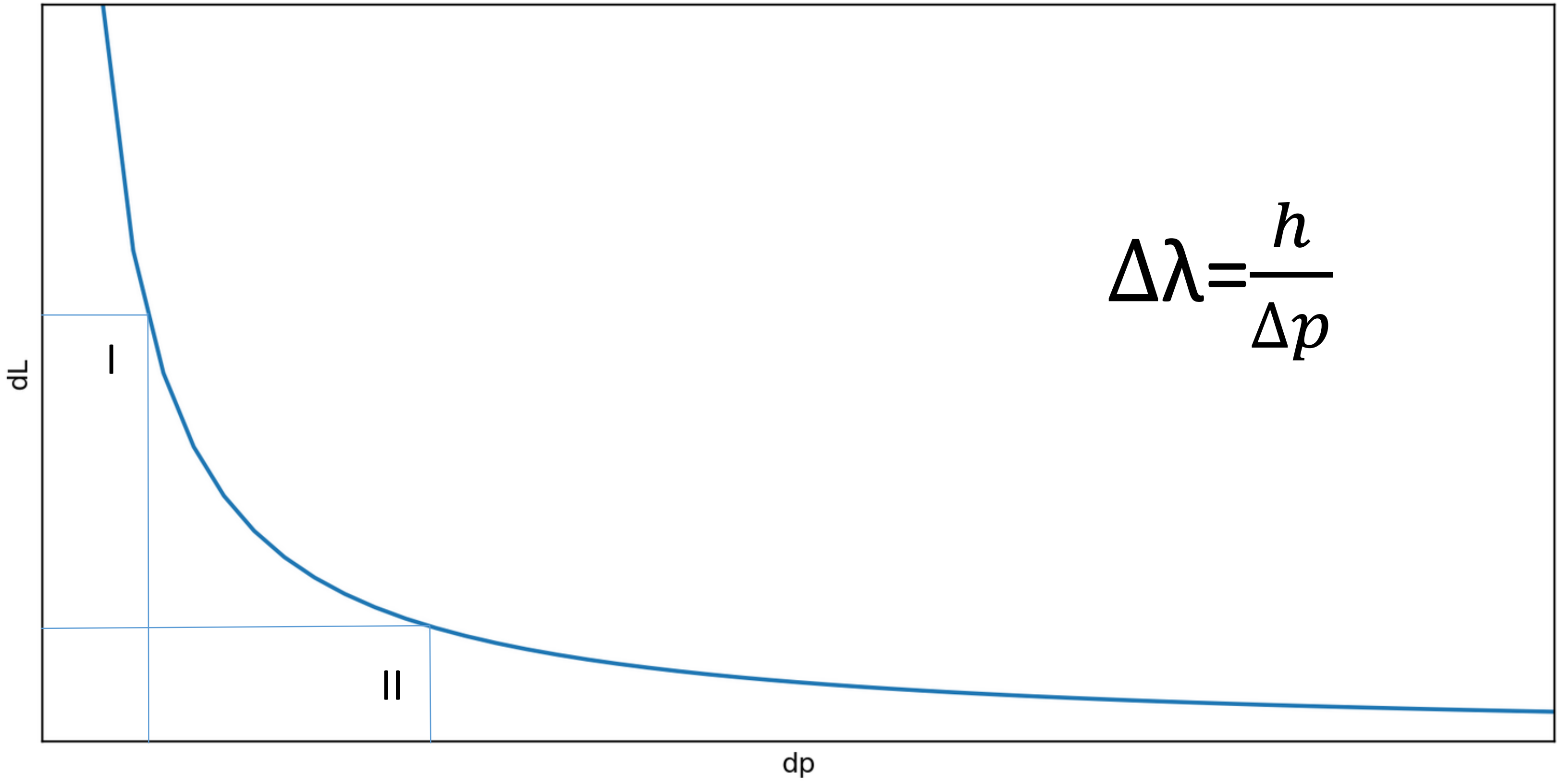
This is on the order of atomic dimensions and is much shorter than the shortest visible light wavelength of about 390 nm.

Atomic diameter
$10^{-10}m$
$10^{-14}m$
Nuclear diameter

All matter is presented as ideal periodic, sinusoidal waves of infinite length.

What happens if particles have dimensions less than infinity ?

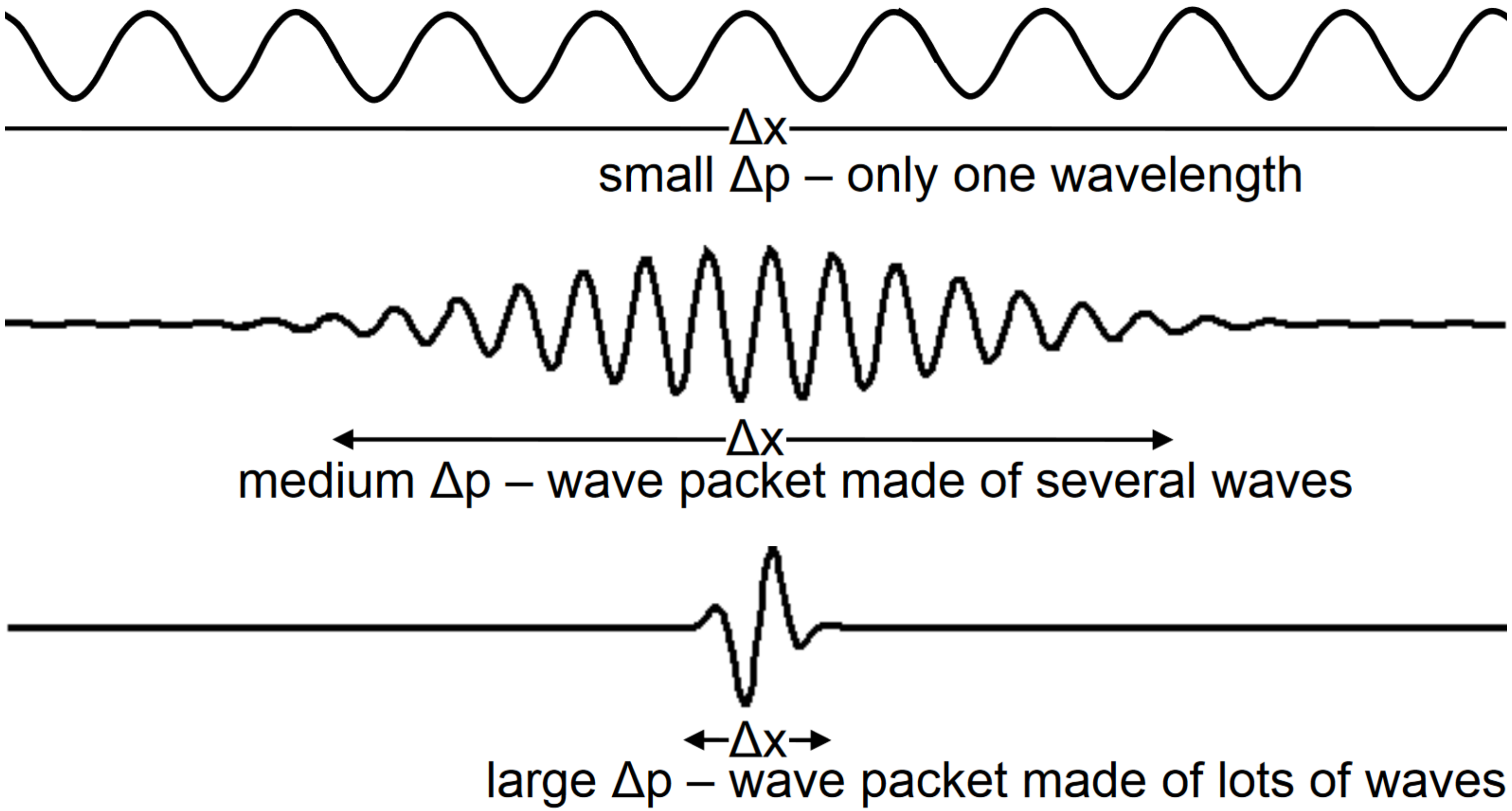
Spread of wave momentum as wave is „compressed“ in space



$$\Delta\lambda = \frac{h}{\Delta p}$$

Reciprocial function

Heisenberg uncertainty principle



Heisenberg uncertainty principle

$$\Delta \mathrm{p} \Delta x > \frac{h}{2\pi}$$

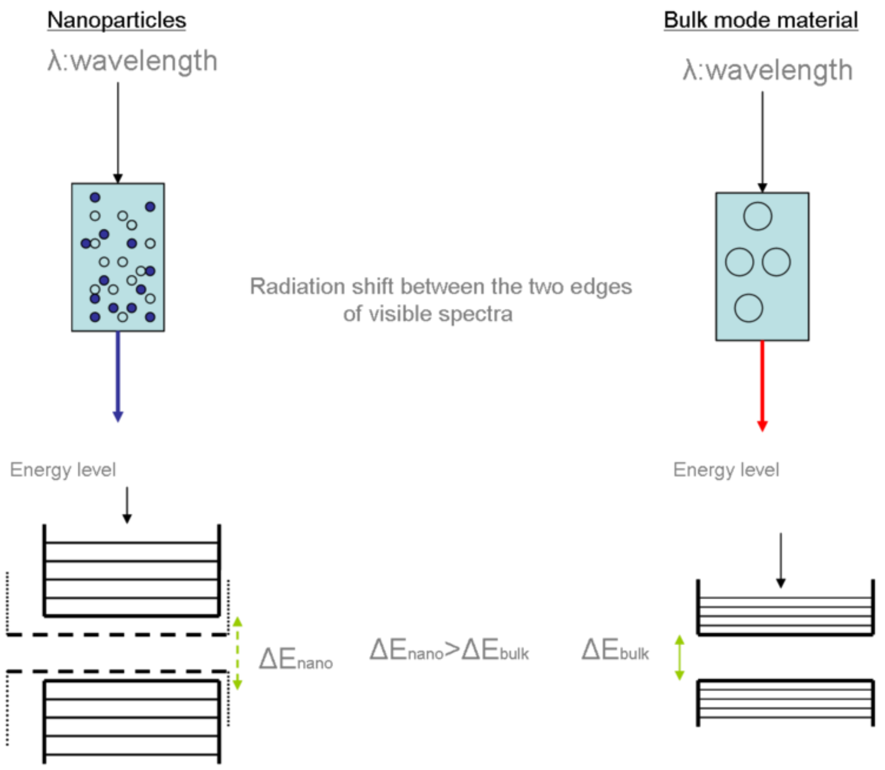
$$\Delta p = m \cdot \Delta v$$

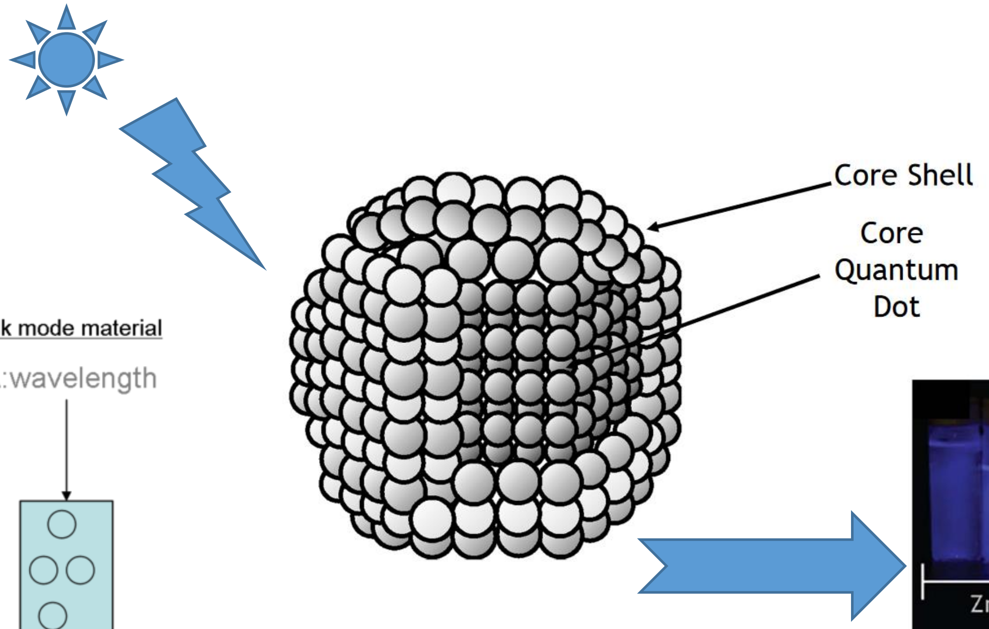
Whats the speed of electron around atomic nucleus ?

R(atom) = 10^{-10} m

Practical example – quantum dot emission

$$\Delta E = h \cdot f = \frac{h \cdot c}{\Delta\lambda} = \Delta p \cdot c$$





Red: bigger dots!

Blue: smaller dots!

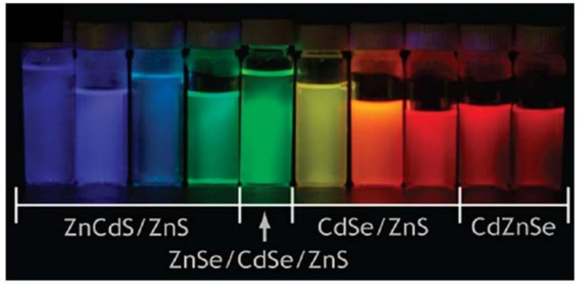


Photo by J. Halpert, Courtesy of M. Bawendi Group, EECS, MIT.

Quantum Dot TV from Samsung