

Practice (Integrals)

Exercise 1. Using rectangles whose height is given by the value of the function at the midpoint of the rectangle's base, estimate the area under the graph of the function using 1) two rectangles of equal width, 2) four rectangles of equal width.

a) $f(x) = x^3$ between $x = 0$ and $x = 1$;

b) $f(x) = 4 - x^2$ between $x = -2$ and $x = 2$.

Exercise 2. Write the sums without sigma notation. Then evaluate them

a) $\sum_{k=1}^3 \frac{k-1}{k}$

b) $\sum_{k=1}^5 \sin k\pi$

c) $\sum_{k=1}^4 (-1)^k \cos k\pi$

Exercise 3. Express the sums in sigma notation.

a) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5}$;

b) $-\frac{1}{5} + \frac{2}{5} - \frac{3}{5} + \frac{4}{5} - \frac{5}{5}$

Exercise 4. Evaluate the sums

a) $\sum_{k=1}^6 (3 - k^2)$

b) $\sum_{k=1}^n 4$

c) $\sum_{k=18}^{71} k(k-1)$

Exercise 5. Find an antiderivative for each function.

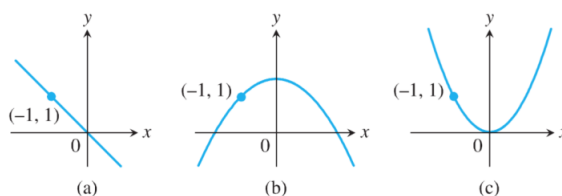
a) $f(x) = 2 - \frac{5}{x^2}$

b) $f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}$

c) $f(x) = -\pi \sin \pi x$

d) $f(x) = \sin(\pi x) - 3 \cos(3x)$

Exercise 6. Which of the following graphs shows the solution of the initial value problem $\frac{dy}{dx} = -x, y = 1$ when $x = -1$?



Exercise 7. Suppose that $\int_1^9 f(x) \, dx = -1$, $\int_7^9 f(x) \, dx = 5$ and $\int_7^9 h(x) \, dx = 4$. Find

a) $\int_7^9 [2f(x) - 3h(x)] \, dx$

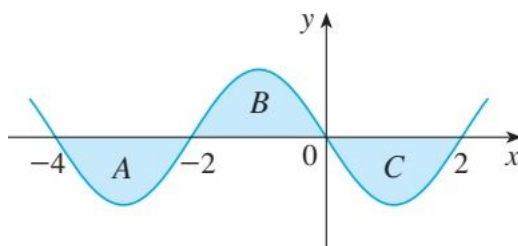
b) $\int_9^1 f(x) \, dx$

c) $\int_1^7 f(x) \, dx$

d) $\int_7^9 [h(x) - f(x)] \, dx$

Exercise 8. Each of the regions A , B , and C bounded by the graph of f and the x -axis has area 3. Find the value of

$$\int_{-4}^2 [f(x) + 2x + 5] \, dx$$



Exercise 9. Graph the integrands and use known areas to evaluate the integrals.

a) $\int_{-2}^4 \left(\frac{x}{2} + 3 \right) \, dx$

b) $\int_{-4}^0 \sqrt{16 - x^2} \, dx$

c) $\int_{-1}^1 (1 - |x|) \, dx$

Exercise 10. Find the derivative of the functions:

a) $g(x) = \int_1^x \ln(1 + t^2) \, dt$

b) $h(u) = \int_0^u \frac{\sqrt{t}}{t + 1} \, dt$

c) $F(x) = \int_x^0 \sqrt{1 + \sec t} \, dt$

Exercise 11. Evaluate the integrals:

a) $\int_0^\pi (1 + \cos x) \, dx$

b) $\int_{-4}^4 |x| \, dx$

c) $\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4) \, dt$

d) $\int_0^{\pi} \frac{1}{2} (\cos x - |\cos x|) \, dx$

Exercise 12. Find the total area between the region and the x -axis.

a) $y = -x^2 - 2x, \quad -3 \leq x \leq 2$

b) $y = x^3 - 3x^2 + 2x, \quad 0 \leq x \leq 2$

Exercise 13. Evaluate the integrals using the substitution property

a) $\int \frac{1}{\sqrt{5s+4}} \, ds$

b) $\int 3y\sqrt{7-3y^2} \, dy$

c) $\int \left(1 - \cos \frac{t}{2}\right)^2 \sin \frac{t}{2} \, dt$

d) $\int \frac{\sin(2t+1)}{\cos^2(2t+1)} \, dt$