Scalars and vectors. Inear, power and exponential functions. Half-life and time constant.

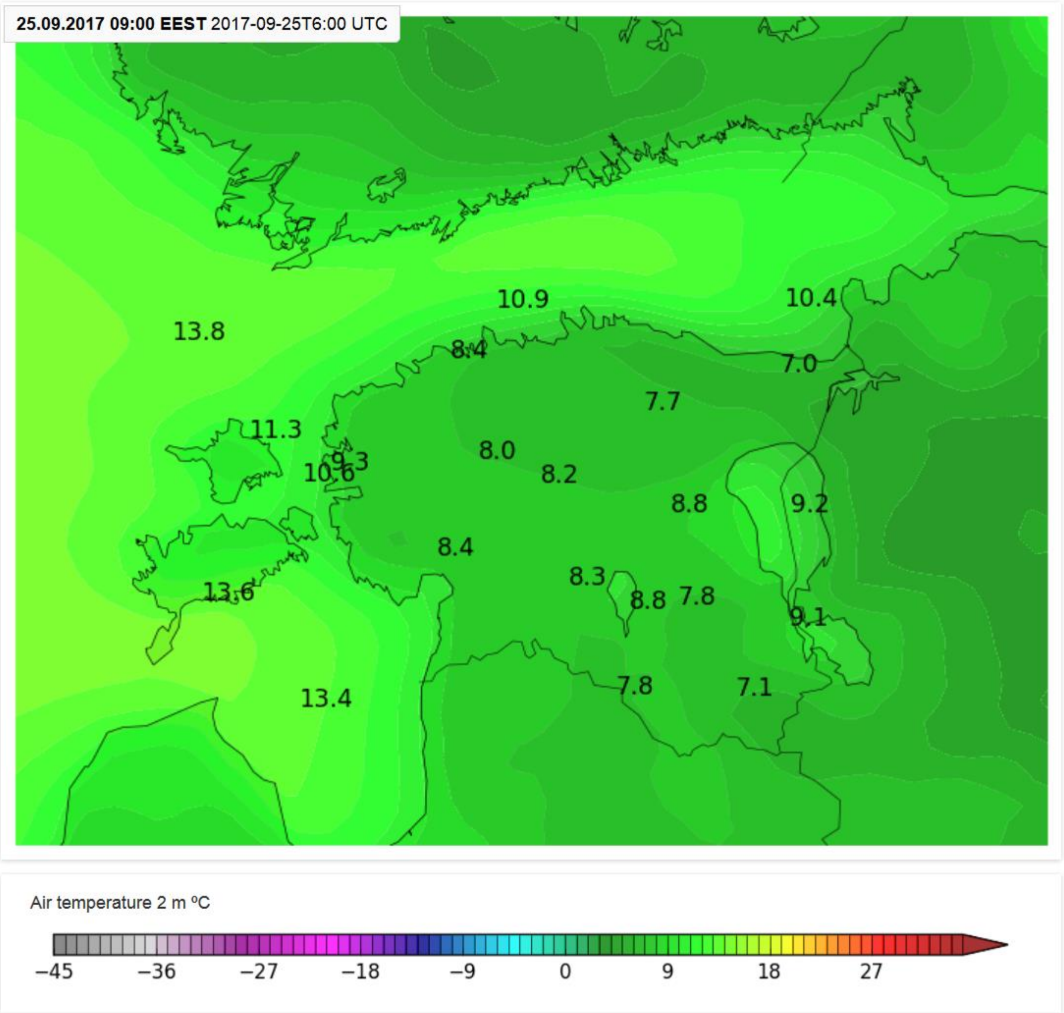
18.09.2020

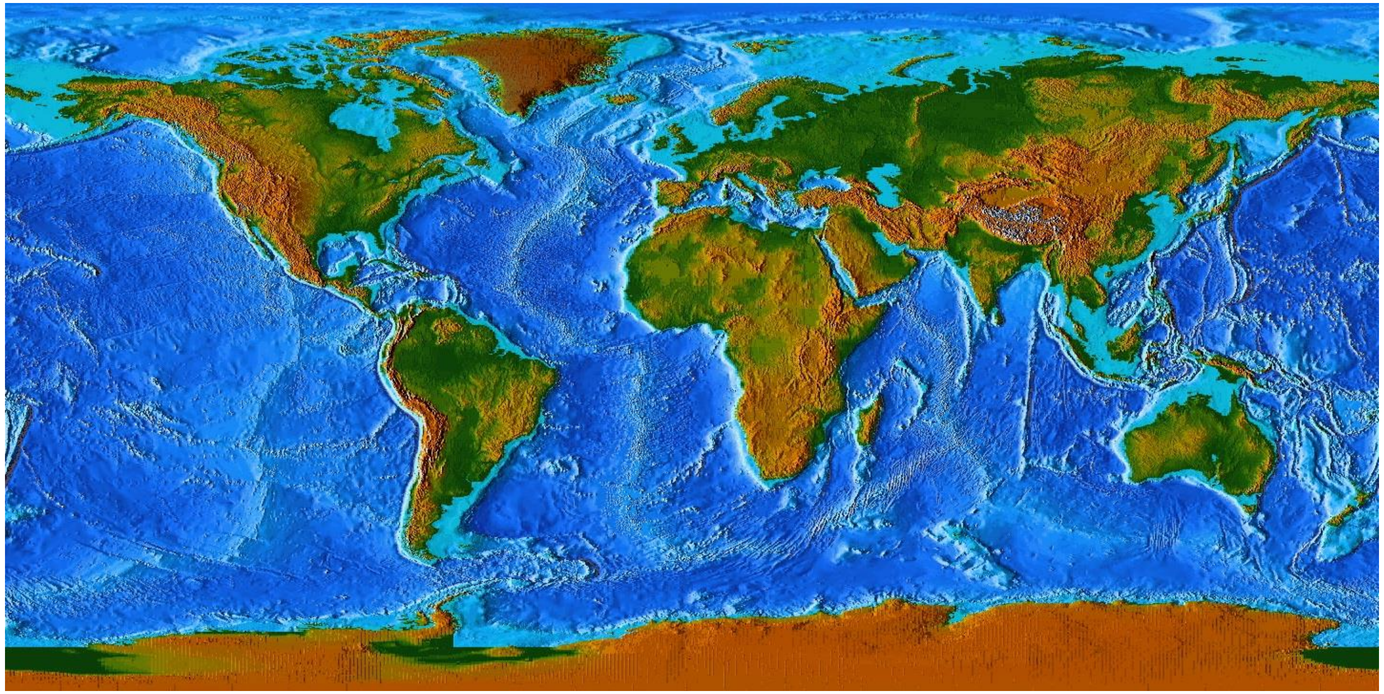
Scalar values

- Scalars are quantities that are fully described by a magnitude (or numerical value) alone.
- Examples:

  Weight=80 kg, Power = 100W, Height = 10 m

Examples of scalar fields



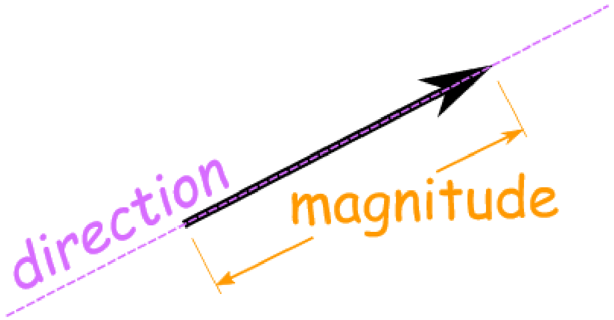


Adding scalar fields together

- Mathematics operations (add, subtract, divide, multiply) on scalar values produce scalar values.
- C=a+b, if a,b are scalar, C is scalar value
- Example: rising temperatuure of a cup of water by 10 degrees will increase the temperatuure at every point within the cup by 10 degrees (ignoring convection).

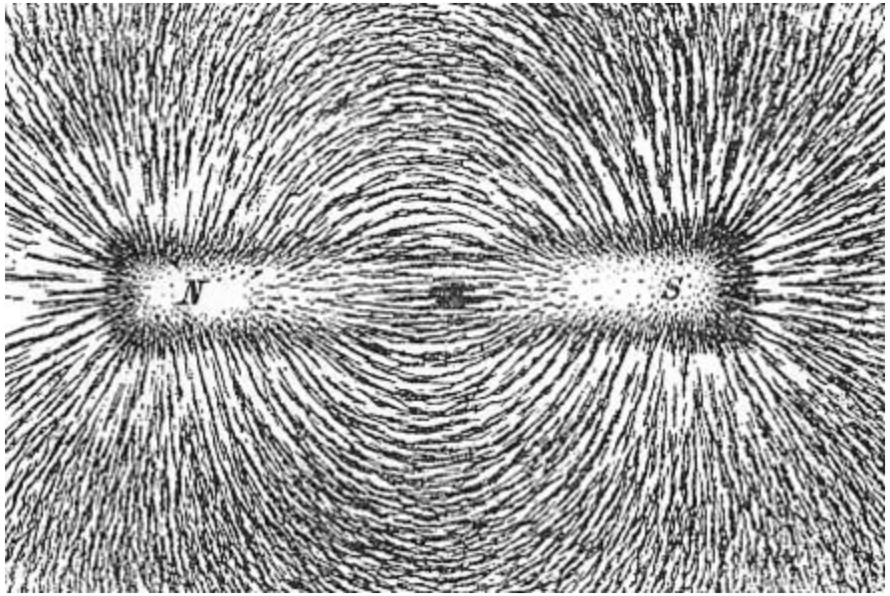
Vector values

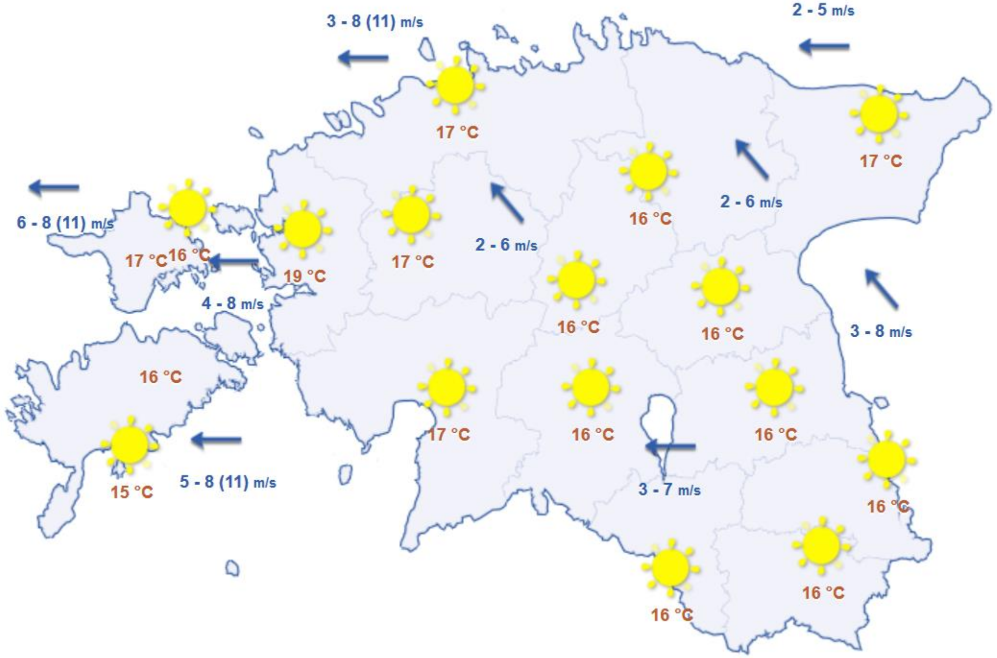
- Vectors are quantities that are fully described by both a magnitude and a direction.
- Examples: Wind speed: 10 m/s, SW,



|a| = magnitude

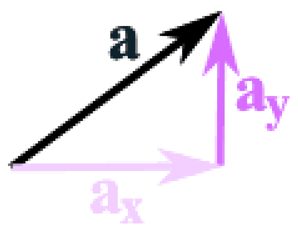
Vector field examples





Magnitude, coordinates

coordinates

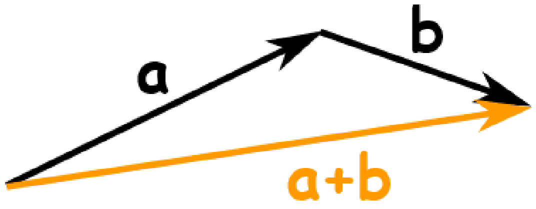


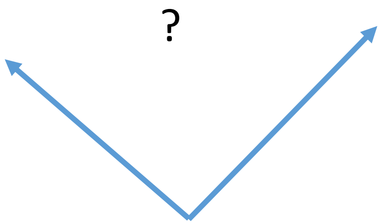
magnitude

$$|a| = \sqrt{a_x{}^2 + a_y{}^2}$$

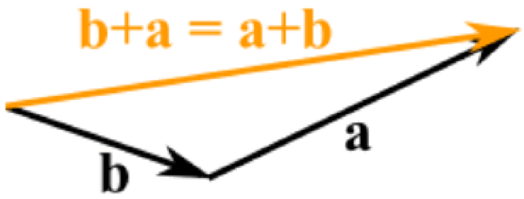
Adding vectors

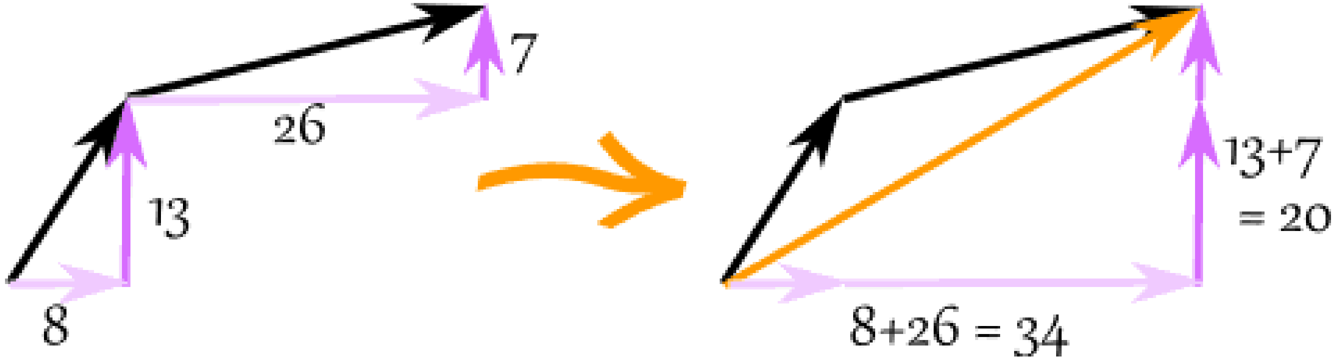
We can add two vectors by joining them head-to-tail:





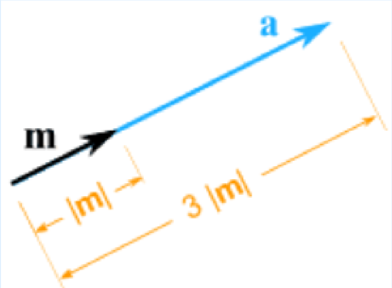
And it doesn't matter which order we add them, we get the same result:





Scalar product of a vector

Example: multiply the vector **m** = (7,3) by the scalar 3



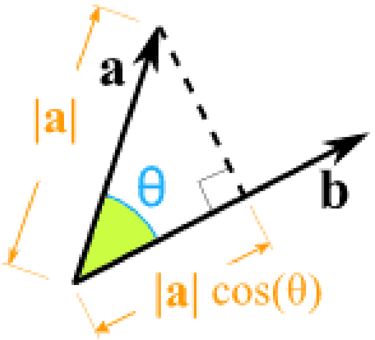
$\mathbf{a} = 3\mathbf{m} = (3\times7, 3\times3) = (21,9)$

It still points in the same direction, but is 3 times longer

See more: https://www.mathsisfun.com/algebra/vectors.html

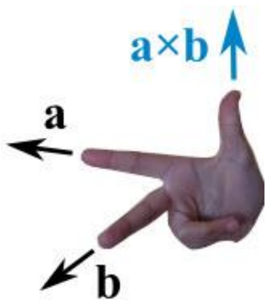
Dot product

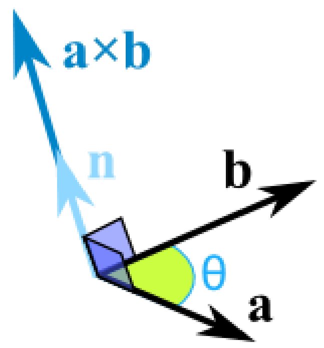
C=|a|*|b|*Cos(α), α = angle between a and b vectors



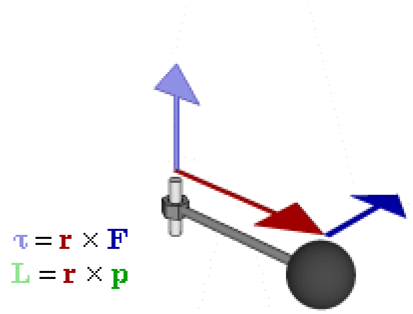
Practical example: Solar cell light capturing efficiency depending on suns position. How is it calculated ?

Vector cross product





Example: torgue vector in rotating bodies



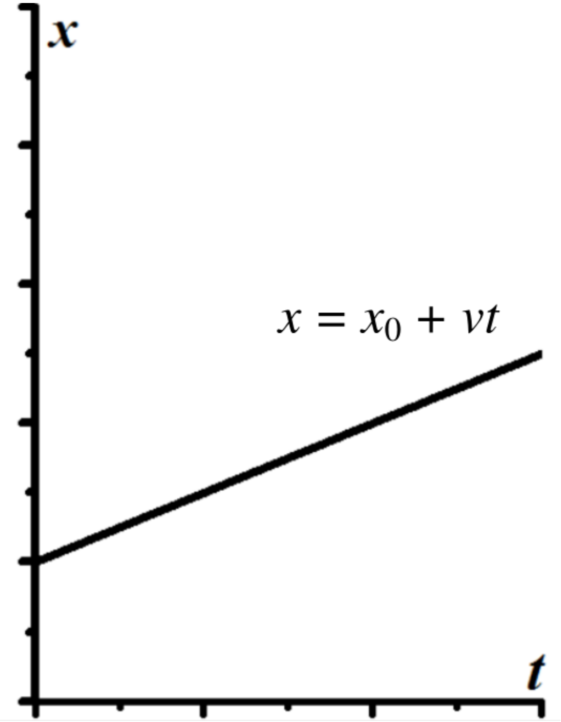
$$\mathbf{a} \times \mathbf{b} = |\mathbf{a}|\ |\mathbf{b}| \sin(\theta)\ \mathbf{n}$$

- $|\mathbf{a}|$ is the magnitude (length) of vector **a**
- $|\mathbf{b}|$ is the magnitude (length) of vector **b**
- θ is the angle between **a** and **b**
- **n** is the unit vector at right angles to both **a** and **b**

Why do good cars need high torgue motor ? Do you need one ?

Linear functions

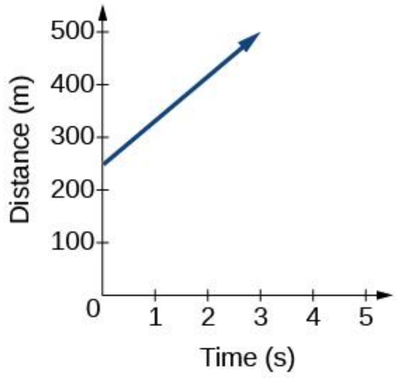
- F(x)=ax+c

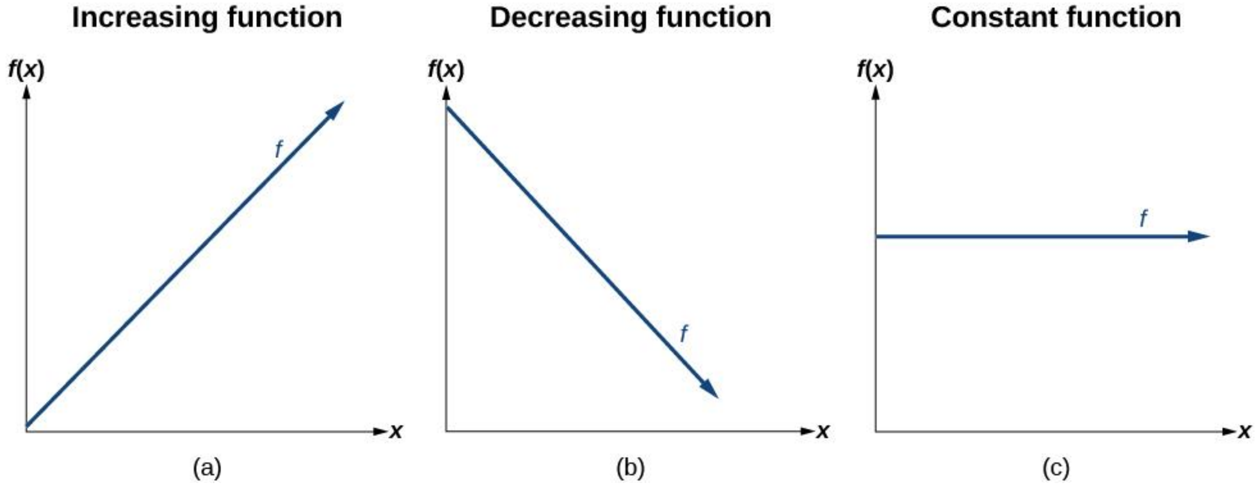


1 second 1 second 1 second

t	0	1	2	3
$D(t)$	250	333	416	499

83 meters 83 meters 83 meters



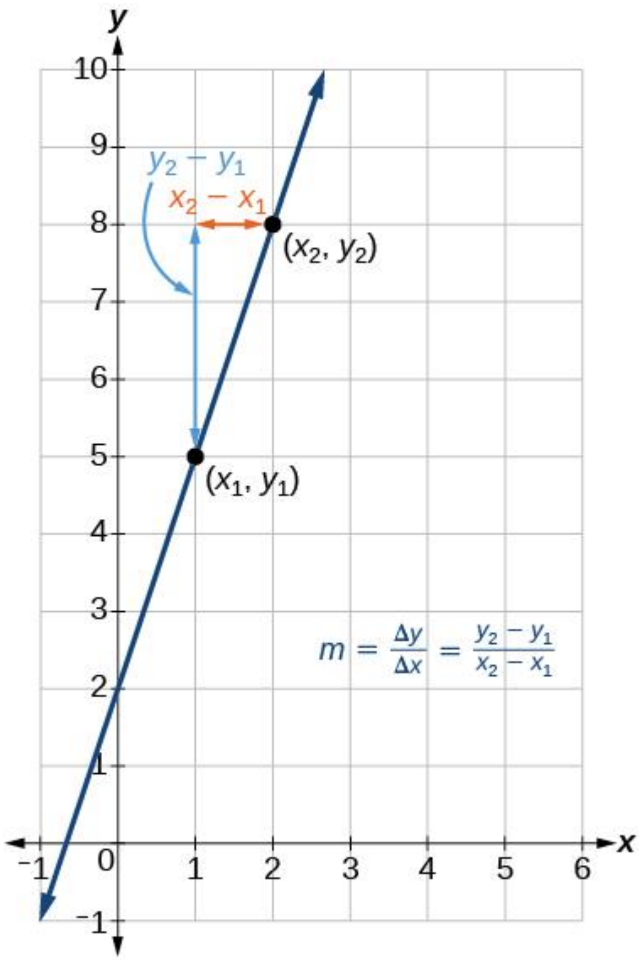


Slope of a function

- f(x)=mx+b is an increasing function if m>0.
- f(x)=mx+b is an decreasing function if m<0.
- f(x)=mx+b is a constant function if m=0.

$$m = \frac{\text{change in output (rise)}}{\text{change in input (run)}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

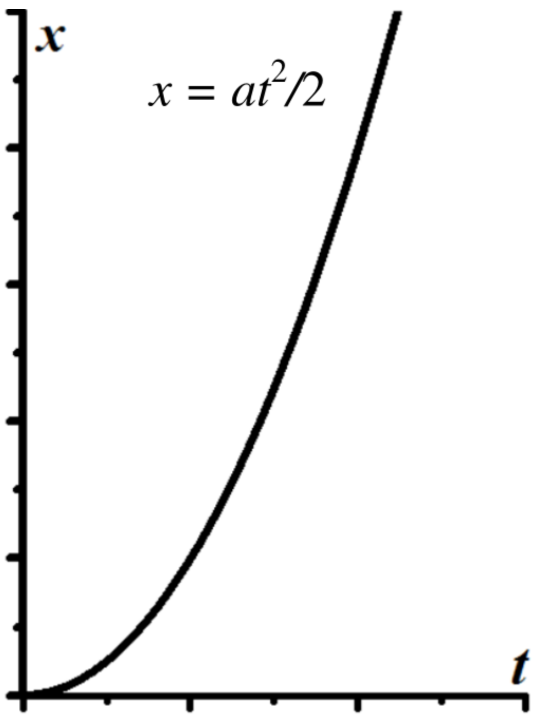
slope



Power functions

- $F(x)=x^n$

What is the slope of this function ?



Polynomial functions

An oil pipeline bursts in the Gulf of Mexico, causing an oil slick in a roughly circular shape. The slick is currently 24 miles in radius, but that radius is increasing by 8 miles each week. We want to write a formula for the area covered by the oil slick. The radius r of the spill depends on the number of weeks w that have passed.

$r(w)=24+8w$

We can combine this with the formula for the area A of a circle.

$A(r)=\pi r^2$

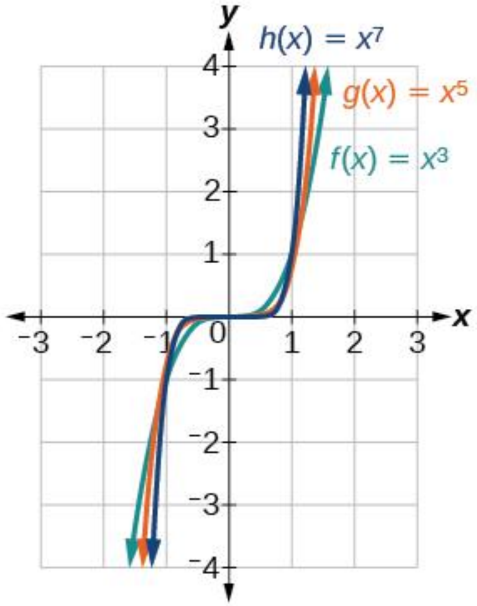
Composing these functions gives a formula for the area in terms of weeks.

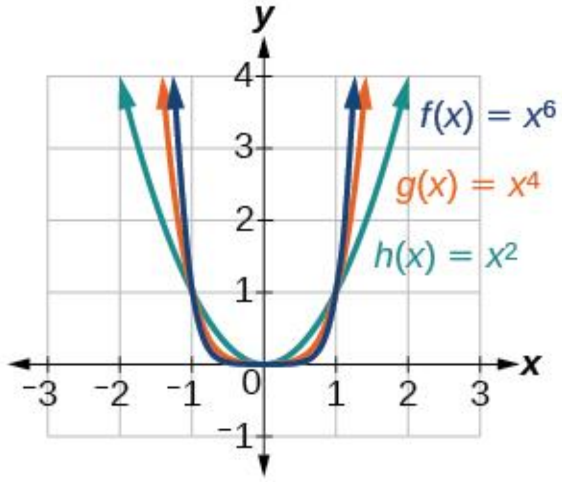
$A(w)=A(r(w))=A(24+8w)=\pi\,(24+8w)^2$

Multiplying gives the formula.

$$A(w)=576\pi+384\pi w+64\pi w^2$$

Constant or zero order first order second order

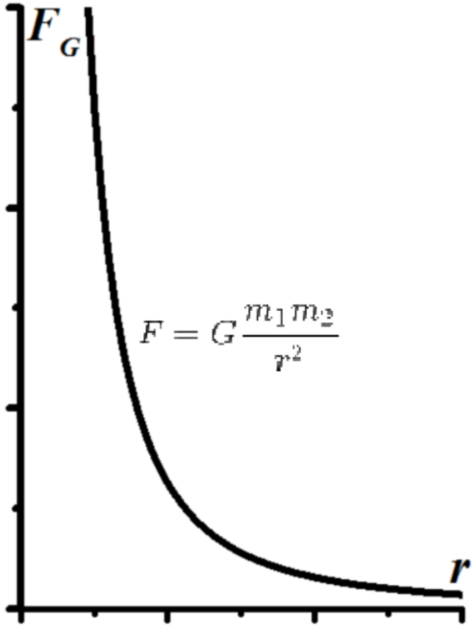




Inverse value

F(x)=$\frac{1}{x}$

Force of gravity

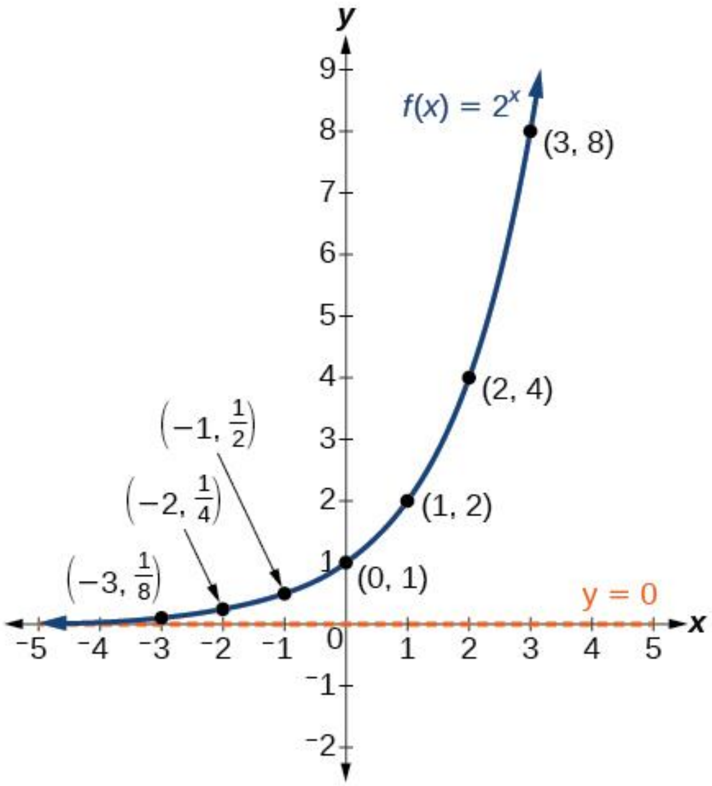


Electric field strength:

$$E = k\frac{Q}{r^2}$$

Exponential functions

- $F(x) = n^x$



Example: If Facebook decided that friends of all friends in your friends list will suddenly become also your friends, through how many such connections you are a freind with everybody on earth/internet ?

Exponential decay, process lifetime and time constant

$$F(t) = a_0 e^{-kt}$$

Growth when k < 0
Decay when k > 0
k=decay rate

Example: Decay of luminesence (cold light like LED lapms). Different types can give rise to range of values froma few ns to several days

$$F(t) = a_0 e^{-\frac{t}{\tau}}$$

$\tau\ - time\ constant\ (lifetime)$

τ is the time at which the population of the assembly is reduced to 1/*e* ≈ 0.367879441 times its initial value. e~2.718

Half-life

Half-life (symbol $\boldsymbol{t_{1/2}}$) is the time required for a quantity to reduce to half its initial value.

$$t_{1/2} = \frac{\ln(2)}{\lambda} = \tau \ln(2) \qquad N(t) = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}} = N_0 2^{-t/t_{1/2}}$$
$$= N_0 e^{-t \ln(2)/t_{1/2}}$$

Example: Decay of radioactive isotopes ,biological processes
For example, the biological half-life of water in a human being is about 9 to 10 days,
The biological half-life of cesium in human beings is between one and four months.

The natural logarithm of x is the power to which e would have to be raised to equal x. For example, ln(7.5) is 2.0149..., because $e^{2.0149...} = 7.5$. The natural log of e itself, ln(e), is 1, because $e^1 = e$, while the natural logarithm of 1, ln(1), is 0, since $e^0 = 1$.

Number of half-lives elapsed	Fraction remaining		Percentage remaining
0	$1/1$	100	
1	$1/2$	50	
2	$1/4$	25	
3	$1/8$	12	
4	$1/16$	6	
5	$1/32$	3	
6	$1/64$	1	
7	$1/128$	0	
...	...		
n	$1/2^n$		$100/(2^n)$