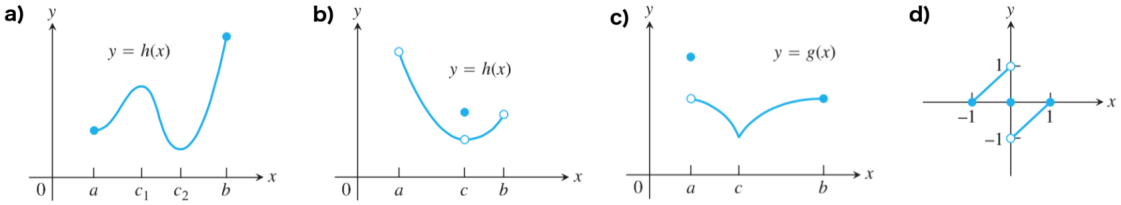
Practice (Applications of differentiations)

Exercise 1. Determine from the graph whether the function has any absolute extreme values on [a, b] and where they occur.



Exercise 2. Determine all critical points of each function

a) $6x^2 - x^3$

b) $(x-1)^2(x-3)^2$

c) $\dfrac{x^2}{x-2}$

d) $\sqrt{2x - x^2}$

Exercise 3. Find the extreme values (absolute and local) of the function and where they occur.

a) $f(x) = x^2\sqrt{3-x}$;

b) $g(t) = \begin{cases} 3 - t & \text{if } \ t < 0 \\ \\ 3 + 2t - t^2 & \text{if } \ t \geqslant 0 \end{cases}$

Exercise 4. Find the value or values of c that satisfy the equation

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

in the conclusion of the Mean Value Theorem for the function $f(x) = x^{\frac{2}{3}}$ and the interval $[0, 1]$.

Exercise 5. A marathoner ran the 42.16-km New York City Marathon in 2.2 hours. Show that at least twice the marathoner was running at exactly 19 km/hr, assuming the initial and final speeds are zero.

Exercise 6. Find the open intervals on which the function is increasing and decreasing. Identify the function's local and absolute extreme values, if any, saying where they occur.

a) $f(x) = 4\sqrt{x} - x^2 + 3$

b) $f(t) = -3t^2 + 9t + 5$

c) $f(x) = \dfrac{x^3}{3x^2 + 1}$

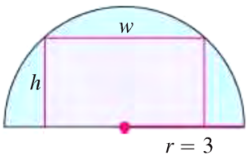
Exercise 7. Graph the functions. Include the coordinates of any local and absolute extreme points and inflection points.

a) $f(x) = -x^4 + 6x^2 - 4.$

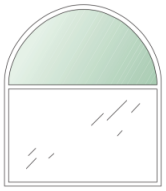
b) $f(x) = x - sinx,\ 0 \leqslant x \leqslant 2\pi.$

c) $f(x) = -\dfrac{x^2 - x + 1}{x - 1}$

Exercise 8. Determine the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 3.



Exercise 9. A window is in the form of a rectangle surmounted by a semicircle. The rectangle is of clear glass, whereas the semicircle is of tinted glass that transmits only half as much light per unit area as clear glass does. The total perimeter is 6 meters. Find the proportions of the window that will admit the most light. Neglect the thickness of the frame.



Exercise 10. Use l'Hospital's rule to find the limits.

a) $\lim_{x\to 2} \dfrac{x^3 - 7x^2 + 10x}{x^2 + x - 6}$

b) $\lim_{w\to -4} \dfrac{\sin(\pi w)}{w^2 - 16}$

c) $\lim_{x\to \infty} \dfrac{\ln(3x)}{x^2}$

d) $\lim_{z\to 0} \dfrac{\sin(2z) + 7z^2 - 2z}{z^2(z+1)^2}$

e) $\lim_{x\to \infty} x \sin \dfrac{5}{x}$