1 Context

NPZ model: The NPZ model is the simplest meaningful model for the phytoplankton growth. It is a ODE model that links phytoplankton concentrations (P) to preying zooplankton (Z) and available nutrient (N) concentrations. In our case we augment the NPZ model by including a nutrient flux term (Φ) that models an input of nutrient from the deep sea. The equations of the NPZ model are shown below:

$$\frac{dP}{dt} = \mu \frac{N}{k+N} P - gPZ - \varepsilon_P P - \varepsilon_P^* P^2, \tag{1}$$

$$\frac{dZ}{dt} = \gamma g P Z - \varepsilon_Z Z - \varepsilon_Z^* Z^2, \tag{2}$$

$$\frac{dN}{dt} = -\mu \frac{N}{k+N} P + (1-\gamma) gPZ + \varepsilon_P P + \varepsilon_Z Z + \Phi_N(t). \tag{3}$$

2 Statistical modeling

Denote $Y_1, ..., Y_T$ the observations of chlorophyll concentration, $\mathbf{X}_1, ..., \mathbf{X}_T$ the state variables, with $\mathbf{X}_t = [N_t, P_t, Z_t, \Phi_t]^T$, the concentration of each of the ODE state variables at time t. For now on, we actually considere the log of these concentrations.

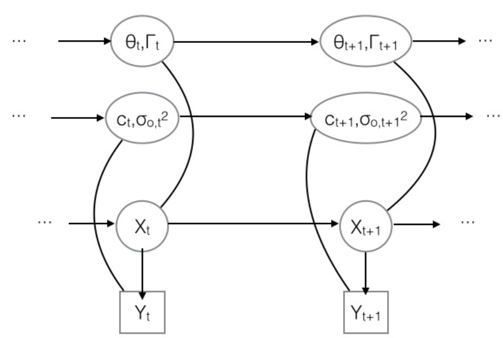

Figure 1: BHM second attempt

Data model: The observations are independent conditionally on the state of the system.

$$Y_t = c_t + P_t + w_t^o, \tag{4}$$

with $w_t^o \sim \mathcal{N}(0, (\sigma_t^o)^2)$ iid and c_t in the log-conversion rate between the chlorophyll and the phytoplankton concentrations.

Process model: The state variables are assumed Markovian:

$$X_t = \begin{bmatrix} N_t \\ P_t \\ Z_t \\ \Phi_t \end{bmatrix} = \begin{bmatrix} f(X_{t-1}; \theta_t) \\ \Phi_{t-1} \end{bmatrix} + \mathbf{w}_t, \mathbf{w}_t \sim \mathcal{N}(0, diag(\Gamma_t)) \tag{5}$$

f is the result of the integration of the ODE system between times $t-1$ and t with initial conditions $[N_{t-1}, P_{t-1}, Z_{t-1}]$ and a fixed nutrient flux Φ_{t-1}. $\Gamma_t = \left[(\sigma_t^N)^2, (\sigma_t^P)^2, (\sigma_t^Z)^2, (\sigma_t^\Phi)^2\right]$. Φ_t is a random walk process.

Transition model: We denote: $\lambda_t = [\theta_t^T, \Gamma_t^T, c_t, \sigma_t^o]^T$

We have to specify a transition model for the time-varying parameters. We choose a random walk process of the form:

$$\text{logit}(\lambda_{t+1}^{(i)}) = \text{logit}(\lambda_t^{(i)}) + v_t^{(i)}, \text{ if } \lambda_t^{(i)} = \gamma_t, \tag{6}$$

$$\log(\lambda_{t+1}^{(i)}) = \log(\lambda_t^{(i)}) + v_t^{(i)}, \text{ otherwise}, \tag{7}$$

where v_t^λ is a iid Gaussian random error of fix variance.