Towards a Theory of Confidence in Market-Based Predictions

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ABSTRACT

Prediction markets are a way to yield probabilistic predictions about future events, theoretically incorporating all available information. In this paper, we focus on the *confidence* that we should place in the prediction of a market. When should we believe that the market probability meaningfully reflects underlying uncertainty, and when should we not? We discuss two notions of confidence. The first is based on the expected profit that a trader could make from correcting the market if it were wrong, and the second is based on expected market volatility in the future. Our paper is a stepping stone to future work in this area, and we conclude by discussing some key challenges.

1 INTRODUCTION

Making probabilistic judgments about the future is fundamental to sound decision making. An agent may base its predictions on its own direct evidence or on forecasts crowdsourced from others. Market-based predictions and other wisdom-of-crowds forecasts, formed by taking an aggregate (e.g., an average, median, or equilibrium) of many individual predictions, tend to be more reliable than domain experts [23].

All financial markets yield predictions indirectly. A *prediction market* is a financial market directly designed to crowdsource predictions. Bettors trade securities that eventually realize some value based on the outcome of the event in question. The price of the securities at any point in time can be interpreted as a forecast probability that the event occurs.

We have researched the design and use of prediction markets for two decades. We have been regular contributors to [redacted for anonymity]. The first question many journalists and readers ask is: what is the *margin of error* in a prediction market? What are its confidence bounds? We don't have good answers. No one does.

A poll has a formula for margin of error.[1] Although polls are *not* predictions, pundits and consumers regularly compare them, especially during elections, and wonder how to think about the accuracy of, and confidence in, both polls and predictions. A prediction market has no concept of margin of error, even in theory. If two markets selling the exact same contract disagree, which should we trust more? How confident can we be in the accuracy of the

(aggregated or separate) predictions? In this paper, we attempt to paint a way forward to answering these questions.

By way of motivation, consider three scenarios predicting the winner of a football match:

- (Game 1) a normal professional regular-season game
- (Game 2) a normal professional regular-season game where, 24 hours before the game, there will be a rapid COVID-19 test to determine the eligibility of the star player
- (Game 3) a pick-up game

Now imagine that the wisdom of the crowd (via whatever method) gives Team A a 70% chance of winning 48 hours before the game in all three scenarios. Intuitively, that crowd knows that Game 1 is very precisely defined due to years of detailed statistics and precise models. In Game 2, we will treat it as known that the star player's COVID-19 test is essentially a fair coin flip and that Team A has a 50% chance of winning without their star and 90% with them. That gives a multi-modal future probability [19]: 0.5*90% + 0.5*50% = 70%. Finally, the Game 3 estimate is based on scant information; who knows who may show up to play or what may happen? For all three games, 70% for Team A is our probability, but that 70% number is distilling intuitively very different states of uncertainty about the outcomes.

We can model our three games as being faced with a (binary) random number generator that outputs 1 with probability p and 0 with probability $1 - p$. In Game 1 we are able to observe 10,000 outputs of this random number generator: 7,000 are 1s and 3,000 are 0s. In Game 2 we are able to observe 10,000 outputs of this random number generator under each of the two conditions: we observe 5,000 1s when the test is positive and 9,000 when the test is negative. Finally in Game 3 we are only able to observe 10 outputs: 7 1s and 3 0s. We assume an agent with a non-informative prior who concludes 70% as the probability of 1 in each of the games. But what would this agent say when pressed about its confidence, both relative and absolute, for these three games?

In some ways, these three situations are similar. A fan of Team A who gets $100 worth of utility from attending the game if Team A wins, and zero otherwise should pay no more than $70 for a ticket to the game. However, in other ways they are very different. If we are offered a bet on the next output of the random number generator at odds slightly favorable to our 70% estimate by an opponent who we know independently witnessed 5000 observations of their own, we would accept the bet in the first case but not the third. We're at a clear informational disadvantage [14] in that case. If we could pay $1 to see an additional output to help us refine our estimate of the probability, we would be more inclined to pay for outputs of the second or third generator than the first. In the second case, we may want to wait until after the player's test result to buy a ticket, for example. These are ways that we are able to express feeling *more confident* that $p_1 = 70\%$ than we are that p_2 or $p_3 = 70\%$.

In this paper, we ask whether it is possible to formalize that feeling of confidence: to ascertain and explain which, if any, of the

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[1] In practice, based on the random-sample error, the stated margin of error ignores all of the other error in both data collection and analytics, accounting for roughly half of the total empirically derived error [21]. Regardless, polling error is a well-defined concept.

three random number generators is most reminiscent of a given real-world probabilistic prediction. That is, we seek to develop a theory of confidence in predictions. While we feel that this is an intriguing topic in any setting where predictions are made, we will focus our attention on prediction markets, given their widespread use and influence in popular forecasting domains such as sports and politics.

Related Work. A large body of literature focuses on the relative accuracy of prediction markets and other wisdom-of-crowds forecasts compared to polls and traditional expert forecasting methods [6, 9, 20, 25, 26]. However, the majority of these papers do not formally examine the notion of confidence, or consider ex-ante forecast accuracy. Closest to the notion of market confidence, Berg et al. [3, 4] empirically examine prediction market accuracy using data from the Iowa Electronic Markets. Berg et al. [3] estimate a model for a market's absolute average prediction error based on election-eve data, finding that trading volume (in terms of number of contracts and dollar amount) and differences in bid and ask queues are the most important factors in market accuracy. Berg et al. [4] explicitly construct forecast standard errors, however they focus on vote share markets as opposed to probabilistic predictions.

Previous work has considered the meaning and necessity of a notion of second-order probabilities to express uncertainty over a probability [2, 10, 11, 19]. Related, imprecise probabilities are defined by an upper and lower bound rather than a single value. Work on imprecise probabilities [1] has studied the problem of aggregating multiple imprecise probabilities [15, 18, 22], generally from an axiomatic perspective. These are certainly relevant literatures for the research direction we propose, although they do not directly shed light on market confidence. Frongillo et al. [7] study the problem of aggregating probabilistic beliefs when forecasters may have differing levels of information, but assumes that information is provided directly to the principal and not via a market.

2 A CONFIDENCE MEASURE FROM THE EFFICIENT MARKET HYPOTHESIS

Why do we trust that (prediction) market prices reflect the underlying value of an asset? The *efficient market hypothesis* states that markets incorporate all available information. The idea behind the hypothesis is that if some relevant piece of information is not reflected in the market price, then someone with that piece of information could profit from buying or selling the asset until its price reaches the appropriate level. Since the existence of such profit opportunities is not a stable state of a market, in equilibrium we would expect that all information is incorporated.

Unfortunately, real-world markets may not be fully efficient, with information often failing to flow into the market. In these cases, opportunities to profit can persist — indeed, many people make careers out of finding and exploiting these opportunities. But we can at least expect small profit opportunities to persist longer and more reliably than large ones. We expect a $20 bill to go unnoticed on the sidewalk longer than a pot of gold!

In this section we exploit the ideas behind the efficient market hypothesis to propose a general method for expressing confidence in a market prediction. To set the stage more formally, suppose that we observe a prediction market for a random bit of uncertainty to be realized at some known time in the future. How can we estimate the probability p that the random bit takes value 1, and how can we express our confidence in that estimate?

Consider a logarithmic market scoring rule (LMSR) prediction market with current price q.[2] In our football match example, we have $q = 70\%$ as the market estimate. Now imagine that q is not actually representative of p, the underlying randomness in the event. Say $q = p + \epsilon$ for some $\epsilon > 0$. Then an omniscient being with knowledge of p would be able to make some expected profit $\$x > 0$ by moving the market price to p, exploiting all available trade in the market by doing so. The existence of this undiscovered profit opportunity would be at least mildly surprising, certainly more surprising than $q = p$ and all profit opportunities having been exploited. In this sense we can quantify exactly how surprised we would be if $p = q + \epsilon$: we would be $\$x$ *surprised*. The greater the ϵ, the greater the market inefficiency, and the greater the surprise.

Real-world prediction markets incorporate trading fees, may have low liquidity, or cap the amount that a single trader can invest. Tying up capital also entails opportunity cost. All of this distorts our ability to infer a probability from the market. For example, suppose that a market platform charges a flat 5% fee on the amount paid in any trade. Then, to make a profit, a trader must be able to buy or sell securities at a price at least 5% removed from what she truly estimates to be their value. In such a market, we could only expect the discovery of an accurate probability up to a 5% margin of error. Put another way, if the true probability p was anywhere within 5% of the market price q, the omniscient profit would be $0, just the same as if $p = q$ exactly.

This discussion therefore yields a natural way to discuss uncertainty in probabilities arising from prediction markets. For any market, and any probability p, we can provide an exact amount of money that quantifies how surprised we would be if the true probability took value p. This method is very general. For any probability p, we simply put ourselves in the shoes of a (budget unconstrained) omniscient trader who knows that p is the true probability that the event occurs. Then we imagine participating in the market, exhausting all possible trades that yield a positive expected profit, net of fees. The omniscient's expected profit $\$x$ is our level of surprise, should the true probability indeed be p.

Conversely, for an amount of money $\$x$, we can consider the set of all possible true probabilities that would yield an omniscient profit of at most $\$x$. In this way we can define an "$\$x$ margin of error" for any prediction market, as the set of probabilities that are consistent with the current state of the market up to an $\$x$ level of surprise. Figure 1 provides a graphical representation.

Note that many features of a market that we intuitively associate with increased accuracy will also be associated with narrow margins of error: high liquidity, no investment caps, and low fees. All of these features increase the (expected) profit that an omniscient trader could extract from participating in the market, shrinking the space of true probabilities that would result in a profit less than $\$x$.

Consider (hypothetical) prediction markets for the professional and pickup football matches described in Section 1. Due to the high

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[2]In an LMSR market, traders can always buy or sell securities from a market maker if their subjective probability is higher or lower than the market price, respectively. As traders buy (resp. sell) securities, the price increases (resp. decreases) continuously according to a predefined formula.
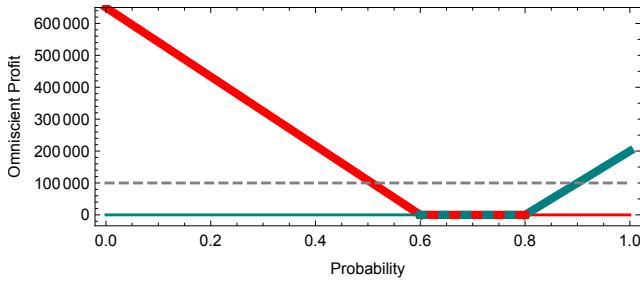
Figure 1: The thick line depicts the expected profit for a budget-unconstrained, omniscient bettor as a function of the true probability of Team A winning, p, in a market with current estimate $q = 0.7$. This is the maximum of the expected profit betting that Team A loses (in red) and betting that Team A wins (in green). The gray dashed line at $100k omniscient profit implies a $100k margin of error between its intercepts with the red and green lines. This is the set of values of p for which the omniscient bettor expects at most $100k profit: $0.5 \leq p \leq 0.9$ in this example. The flat segment where red and green overlap ($0.6 \leq p \leq 0.8$) indicates the set of probabilities consistent with a $0 omniscient profit. The bid-ask spread is necessarily contained in this range.

information flow and level of interest in the professional match, we would expect a thick market with many opportunities for trade. The margin of error in this market will be narrow, because a trader with knowledge that the market was wrong could make a large profit. On the other hand, we would expect the market in the pickup game to be thinly trafficked. Even a trader who had precise knowledge of the capabilities of the teams would be able to make very little money. The margin of error in this market would be wide, reflecting a low confidence in the market estimate.

In the case of continuous double auction (CDA) markets,[3] the $0 margin of error is always a superset of the bid-ask spread, since a trader whose belief lies within the spread has no buying or selling opportunities. In the case of zero fees and no opportunity cost, the $0 margin of error would coincide with the bid-ask spread exactly.

One can imagine prediction market estimates being reported in this way. For example, a data journalist could report that the probability of Joe Biden being elected president is 65%, with the $1000 margin of error being between 57% and 70%. A market with a narrower margin of error at the x level should be more trustworthy than a market with a wider margin of error.

This method can even extend to settings with multiple markets for the same event, or situations with very general betting structures that need not take the form of conventional prediction markets. When there are multiple markets (say, two different platforms each have a market for the same event), we can imagine an informed trader who is able to invest in all markets, profiting from each one. Since a trader can make higher expected profit from participating in two markets than just one, the margin of error that results from considering multiple markets will be no wider than the margin of error from considering a subset of those markets. Once again, this

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[3]In a CDA market, traders place bid offers to buy securities, and ask offers to sell, with the market matching all mutually beneficial offers. The gap between the highest bid and the lowest ask is known as the bid-ask spread.

matches our intuition; additional information should not make us *less* confident in our prediction. Similarly, in any betting situation, we can ask how much profit a bettor with infinite budget and precise knowledge of the underlying true probability p could make in expectation, given the bets laid down by the other parties.

3 CONFIDENCE AS VOLATILITY

Let us return to Game 2 from the introduction, in which a 50/50 random event will occur *before* the football match in question, the outcome of which will inform us whether Team A has a 50% or a 90% chance of winning the match.

Can we say we're confident in today's estimate of 70%? By the measure we suggest in Section 2, yes. The information structure is public so we expect plenty of traders willing to bet if the implied odds deviate from 70% in either direction. But in another sense, no. We're expecting new information, 24 hours before the game, that will render the 70% estimate wrong in hindsight. A layperson told that a probabilistic prediction has high confidence would be taken aback to see the prediction change drastically the following day. We can capture that notion of confidence (or lack thereof) by measuring the expected *volatility*. A prediction with high expected volatility indicates, in our toy example, additional layers of uncertainty that will be resolved before the event itself resolves.

In financial markets, options and other derivatives reveal the distribution and variance of their underlying instruments. For example, a *butterfly option* pays the absolute difference between the market price at time t_1 and the price at time t_2: $|p_{t_1} - p_{t_2}|$. Such contracts could be traded in a market secondary to any prediction, allowing us to estimate the volatility of the primary prediction.

In other words, a *second-order* market can predict volatility in the original market. To be clear, second-order probabilities can always be collapsed into first-order probabilities [19]. Nonetheless, the concept of second-order probabilities are well-defined and capture the expectation of new information that will change our probability estimate. For the example of Game 2, the price of the butterfly option would reveal that the current prediction, a 70% chance of Team A winning, is expected to change by ±20% exactly when the COVID-19 test occurs. This would reflect a lack of confidence in the prediction, at least relative to that point in the future.

We believe that this "volatility" notion of confidence is complementary to the "x margin of error" notion of confidence. One can imagine scenarios, such as the Game 2 example, where the x margin of error is very small while the volatility is large. The converse appears possible, but less likely: a high x margin of error implies high uncertainty about the current prediction, which suggests volatility of the price in the future. In any case, both measures together may give a clearer picture of the market's confidence.

4 DISCUSSION AND CHALLENGES

4.1 Evaluating a Measure of Confidence

So far, we have avoided defining confidence formally, instead appealing to intuition about the nature of a confident forecast. However, before these notions can be used, they need to be validated empirically and/or theoretically, which necessitates a definition. For the purpose of illustration, let us define a probabilistic prediction q to be *accurate* if it is close to the true probability p. We can then define

a confident forecast as one that we believe to be accurate, that is, to properly reflect the inherent randomness.

If we accept this definition, we would hope that a confident prediction would, on average, be more accurate than a less confident one. Measuring accuracy is challenging in itself, since we never observe true probabilities p, but instead only the event outcomes. While it is possible to score predictions in such a way that more accurate predictions receive higher scores in expectation than less accurate ones [5, 8], it is impossible to know if a single prediction is accurate or not.

If two forecasters each predict the same set of events and each forecast is scored, the more accurate forecaster will achieve the higher score over time. One may hope that this technique would allow us to validate a measure of confidence, in the sense that high-confidence forecasts should achieve a higher score on average than low-confidence forecasts if confidence is indeed reflecting accuracy.

Unfortunately, to do this would require a set of high-confidence predictions and low-confidence predictions *for the same set of events*. Otherwise, an accurate prediction of $q = p = 0.5$ may receive a lower score, even in expectation, than an erroneous prediction $q = 0.8$ of a different event with $p = 0.9$. Due to the intertwined nature of real-world markets, finding such a pair of markets for any event is a difficult, perhaps impossible, proposition. To pursue this direction, markets may need to be created in laboratory conditions, or a more clever evaluation method designed.

Additionally, we may want a way to interpret market confidence in an absolute sense, not relative to another market, in the same way that polling margin of errors have a precise interpretation as confidence intervals. This would seem to be a very challenging question to resolve empirically, since we never observe the underlying probabilities. Perhaps it is a question better suited to a theoretical modeling approach, in which (hypothetical) traders observe some information, trade is simulated, and the features of the corresponding market observed.

4.2   Model variants

Much of our discussion so far has been predicated on the idea that every event has some inherent and unknowable, yet quantifiable and well-defined, uncertainty occurring at a precise moment in time. Alternative models are of course possible, and may affect how we define and think about uncertainty. This gets much more complicated as the speed of information shifts: our examples take place before a football game, when new information is relatively slow, versus during a football game when new information comes rapidly. While these periods are relatively well defined for sports, they are more ambiguous for politics: did the 2024 presidential election start before or after the 2020 presidential election ended? Announcements and debates happen periodically, shaking probabilities. During times of particularly intense new information (like during a football game or political debate), uncertainty can shift rapidly, and market volume generally increases dramatically. Our key indicators of bidding and price tend to behave very differently during these periods than in times of low information flow, possibly shifting their relationship to confidence.

4.3   Value of Information

The notions of confidence in Sections 2 and 3 can both be interpreted in terms of the value of information acquisition. In Section 2, an equivalent interpretation of the $x margin of error is that it contains all the predictions that could result if a forecaster spent $x acquiring new information (otherwise, a forecaster could make greater than $x profit from the market by spending only $x on information acquisition, violating the efficient market hypothesis[4]). In Section 3, the expected market volatility expresses how much we expect the prediction to change as new information comes to light. High expected volatility says that the new information will be highly valuable compared to old information. Further exploring the relationship between value of information and market confidence may be fruitful.

4.4   Communicating probabilistic uncertainty

Regardless of the precise definition of confidence in a prediction, we're interested in how to convey it to a lay audience. For example, one way to conceptualize a 20% probability is to imagine 20 red balls and 80 blue balls. The probability of the event occurring is the same as a randomly chosen ball being red. Could we use the same analogy to communicate an imprecise probability, where the number of red balls may be anywhere between 16 and 24? What about if some numbers of red balls are more likely than others? Of course, the exercise could be repeated for a host of probability explanation and visualization tools.

4.5   Deriving a single probability from a market

Our discussion has been phrased in terms of second-order uncertainty: how confident can we be that a probabilistic prediction accurately reflects the inherent underlying randomness. However, even in the presence of second-order uncertainty, a Bayesian agent must be able to represent their belief by a single probability [19]. How best to derive a probability from a real-world prediction market remains an open problem. In a hypothetical high-liquidity LMSR with no fees and no opportunity cost for tying up funds, the market price should be our best estimate of the underlying probability. For a real-world continuous double auction market with a bid-ask spread, how do we infer a probability from what the market is telling us? Plausibly the midpoint of the bid-ask spread is the best we can do but that's an empirical question worth answering. Other hypotheses include (a) the edge of the bid-ask spread closest to or furthest from 50%, due to known distortion effects in markets at extreme probabilities, (b) something outside the bid-ask spread that accounts for fees and opportunity cost, (c) a more elaborate, perhaps machine-learned, function of the order book.

The right way to derive a single probability is, in principal, solved by Bayesian theory: (1) start with a joint prior over the events and the markets, (2) treat all attributes of the markets as evidence, and (3) compute the posterior. This is called the *supra-Bayesian* approach to aggregating probabilities [12, 13, 16, 17, 24]. Although this approach is correct in theory, it sidesteps all the hard practical problems: how does the agent generate and represent a joint prior encoding every detail about how evidence from markets correlates with events, and compute the posterior tractably?

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[4]In reality, we don't expect markets to be so efficient that this holds exactly.

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