MSU June 2019

Nuclear reactions in a three body model

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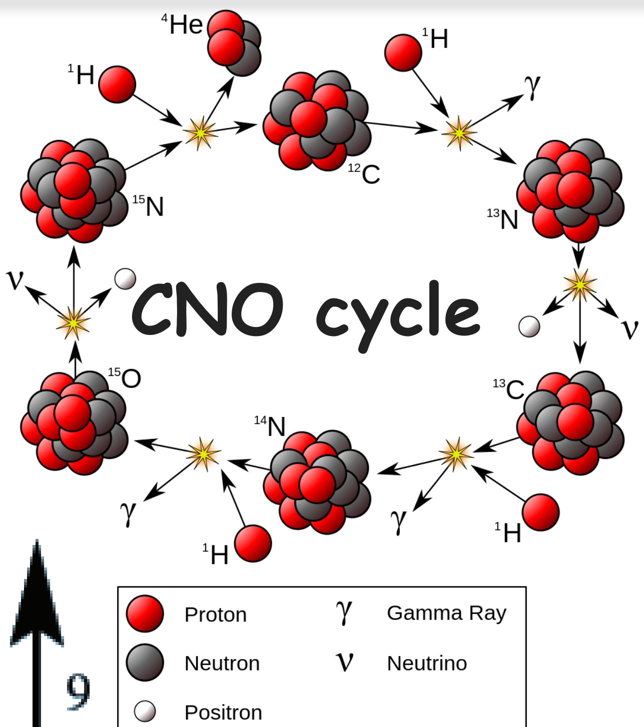
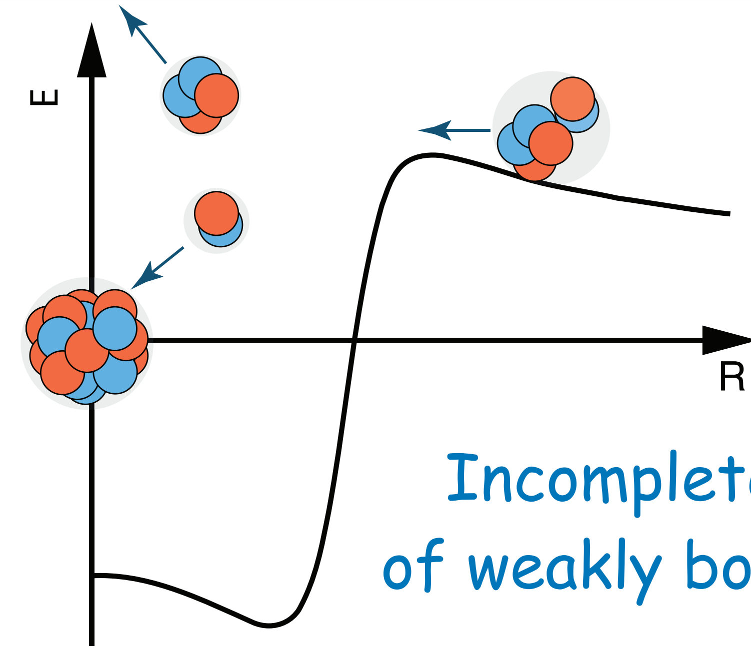
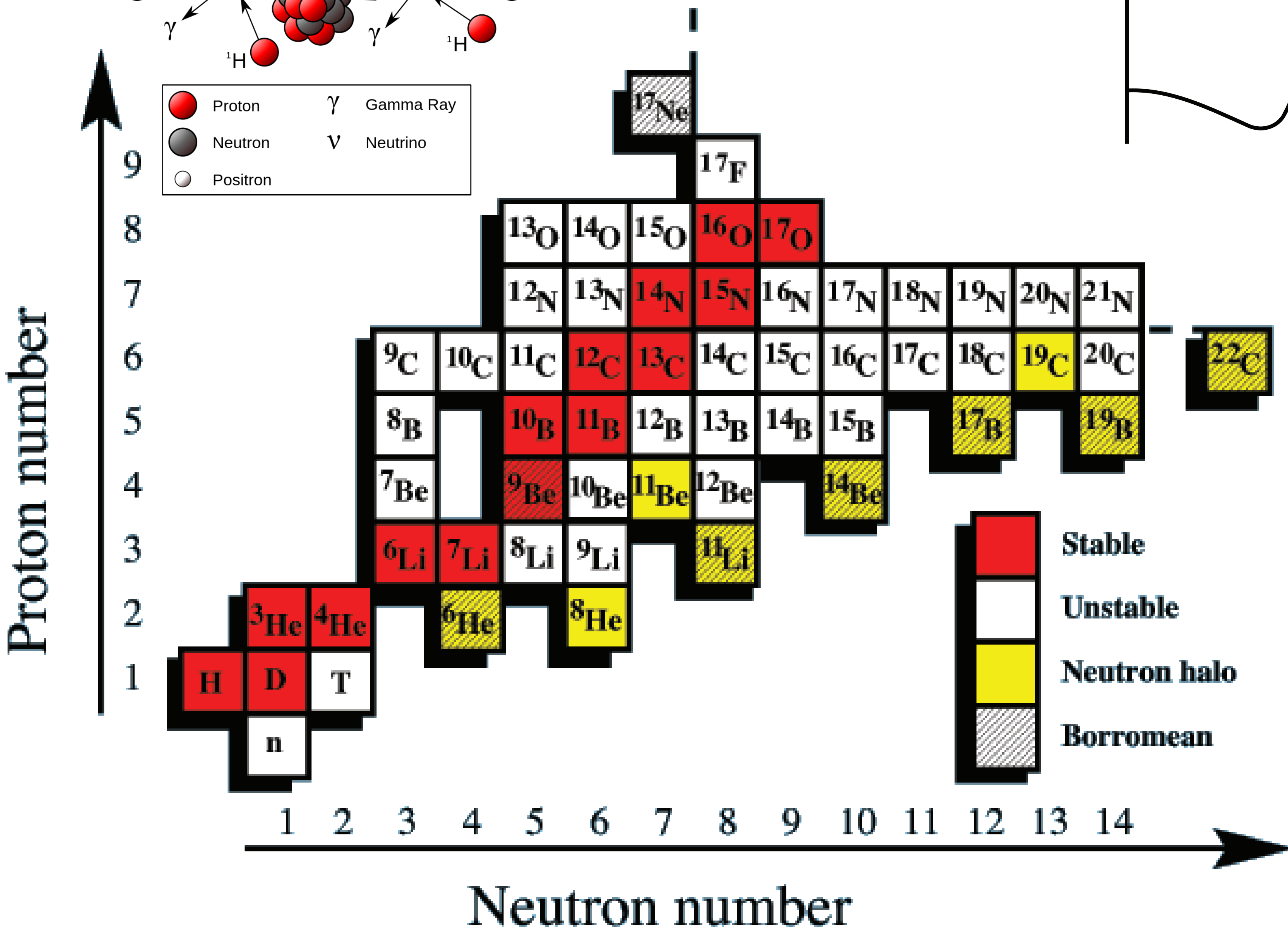
Ohio University

Nuclear reaction: use light nuclei as projectile

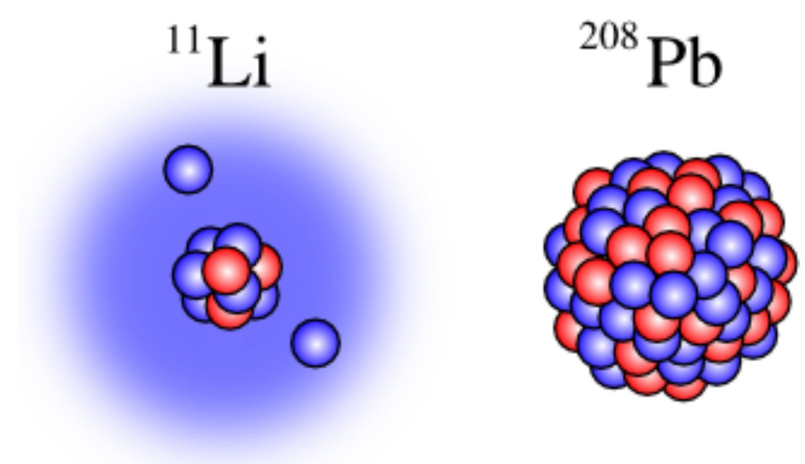




Incomplete fusion
of weakly bound nuclei

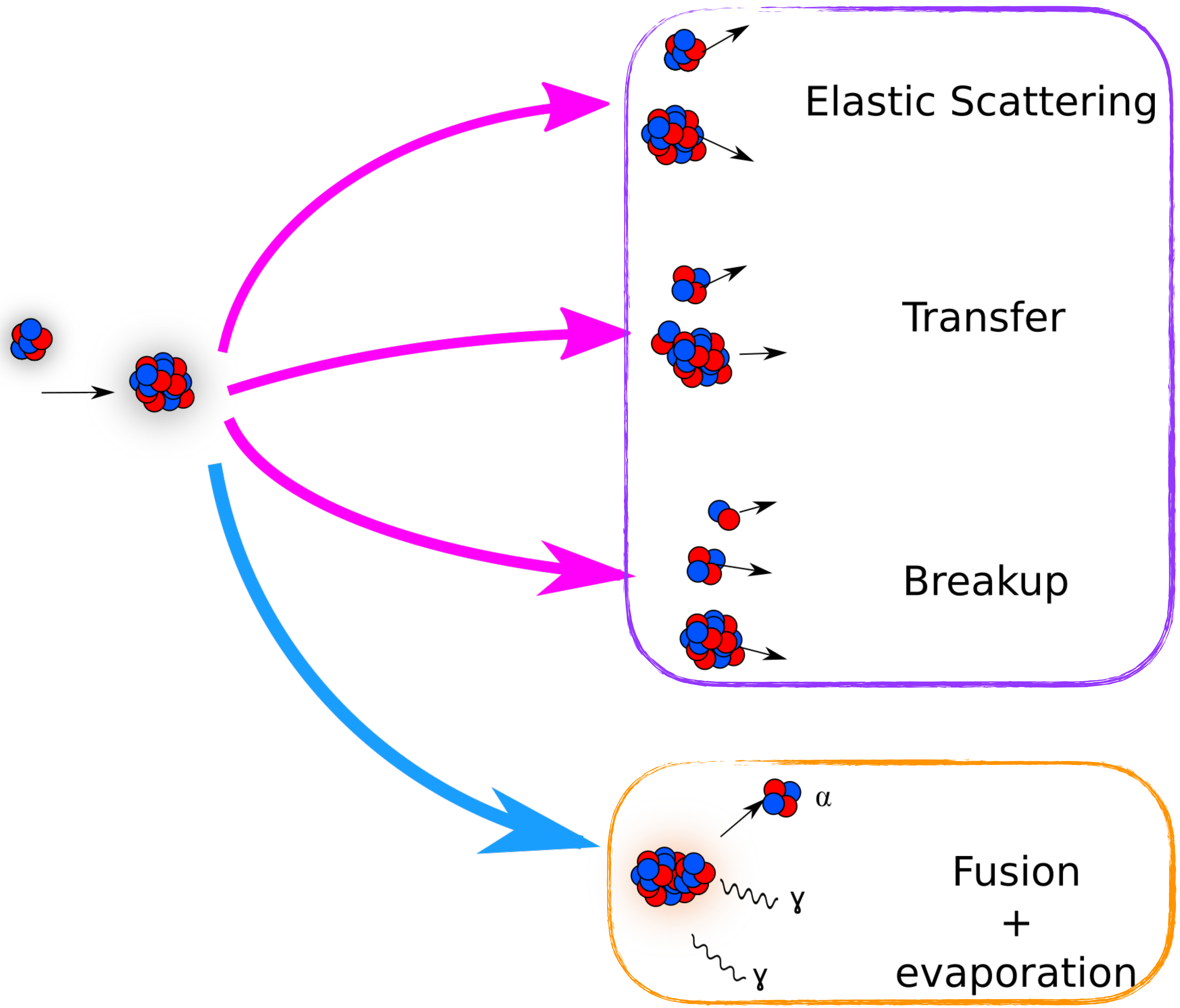


Properties
of halo nuclei



What is a nuclear reaction

- Nuclear reaction is the interaction between a projectile and a target



Extract optical potential, rms radius, density distributions

Extract spin, parity, spectroscopic factors example: $^{132}Sn(d,p)^{133}Sn$

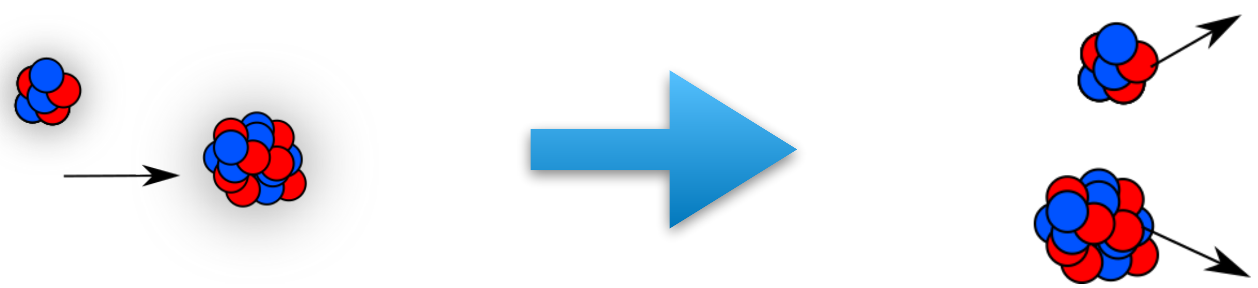
Study nuclear dynamics properties of halo nuclei

Provide energy for star burning synthesis of elements

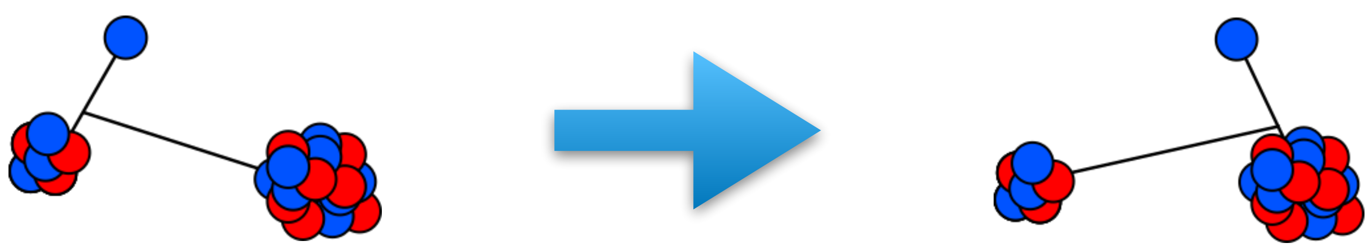
Why three body model

Key word: degree of freedom

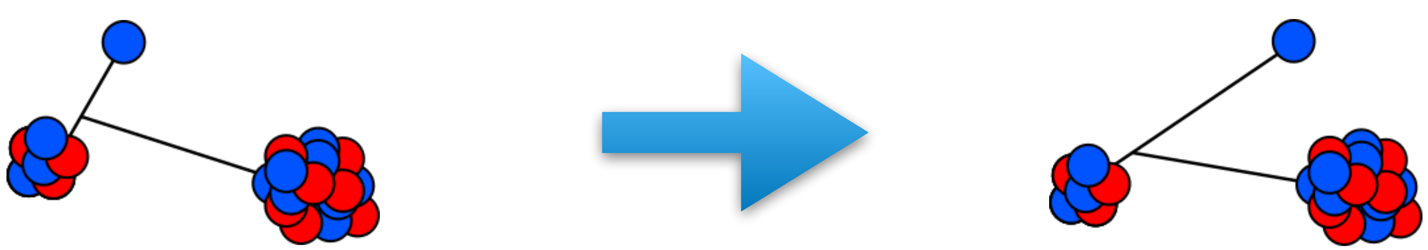
- Elastic scattering: only relative coordinate between projectile and target



- Transfer reaction: from one Jacobi coordinate set to another

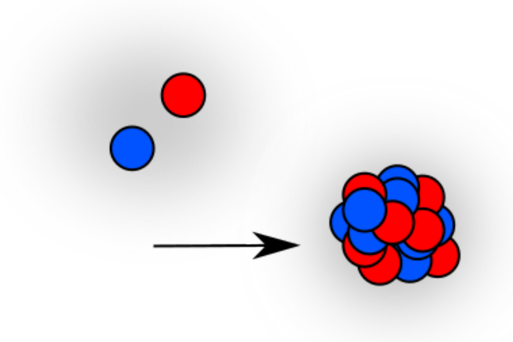


- Breakup reaction: stay in one Jacobi coordinate, but from bound state to continuum

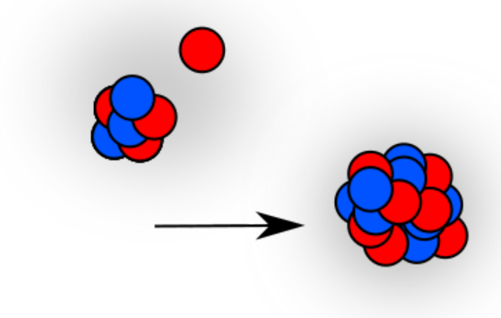


The cases can be considered as three body model

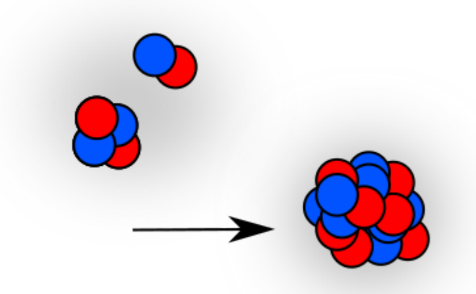
- d(n+p)+A reaction



- halo nuclei induced reaction: ^{11}Be(^{10}Be+n)+A; ^{8}B(^{7}Be+p)+A



- clustered structured projectile induced reaction: ^{6}Li(α+d)+A



Three body model

- Assume the projectile has a two body structure

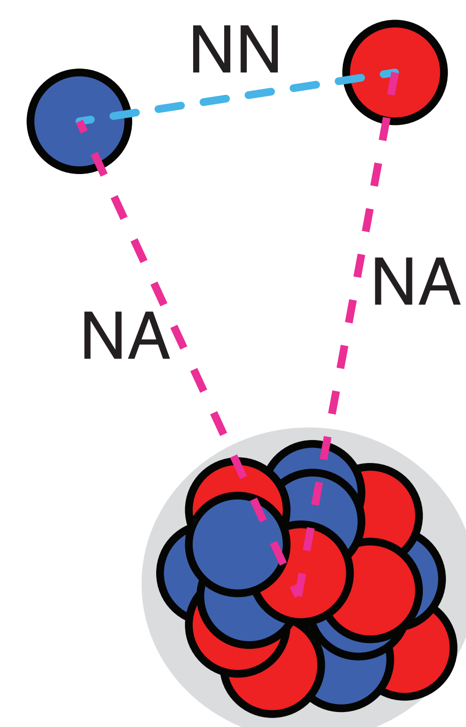
R. Kozack and F. S. Levin,
Phys. Rev. C 34 1511 (1986)

OR

- Hamiltonian for effective three body problem: take d+A reaction as an example

$$H = \underbrace{H_0}_{\text{kinetic energy}} + \underbrace{V_{np} + U_{pA} + U_{nA}}_{\text{two body interaction}}$$

- Nucleon-nucleon interaction "well" known
  - chiral interactions, 'high precision' potentials
- Effective proton (neutron) interactions
  - Phenomenological optical potentials fitted to data
  - Optical potentials with theoretical guidance
  - Microscopic optical potentials
  - Ab initio derivation of effective interaction



Solving the three body problem

Faddeev Equations

L. Hlophe, JL, et al., Phys. Rev. C 96, 064003 (2017)
JL, L. Hlophe, et al., Phys. Rev. C 98, 051001(R) (2018)

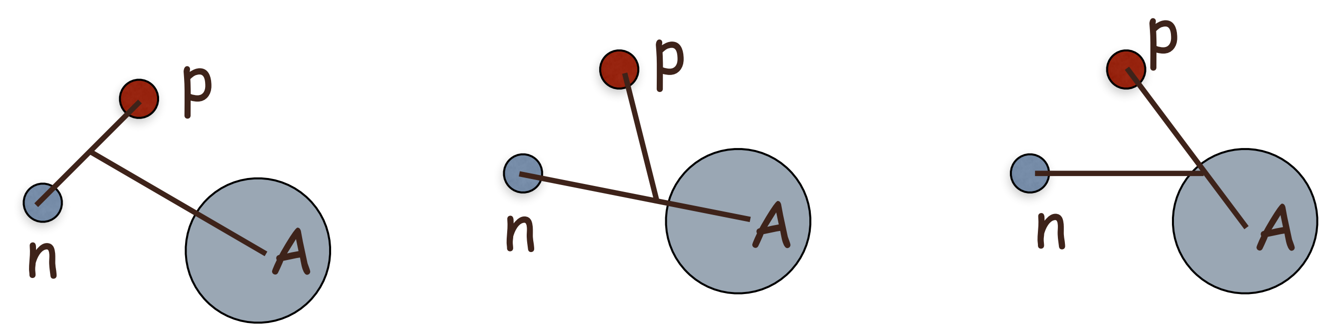
Solving the three body Schrodinger equation: two-body projectile with inert cores on inert target(take d+A system as an example)

$$E|\Psi\rangle = H|\Psi\rangle \longrightarrow H = H_0 + V_{np} + U_{nA} + U_{pA}$$

Faddeev equation: expand the three body wave function in three Jacobi systems

L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960).

$$|\Psi\rangle = |\psi_{np}\rangle + |\psi_{nA}\rangle + |\psi_{pA}\rangle$$



$$\left(E - H_0 - V_{np}\right)|\psi_{np}\rangle = V_{np}\left(|\psi_{nA}\rangle + |\psi_{pA}\rangle\right)$$

$$\left(E - H_0 - U_{nA}\right)|\psi_{nA}\rangle = U_{nA}\left(|\psi_{np}\rangle + |\psi_{pA}\rangle\right)$$

$$\left(E - H_0 - U_{pA}\right)|\psi_{pA}\rangle = U_{pA}\left(|\psi_{np}\rangle + \psi_{nA}\rangle\right)$$

Faddeev equations
(coordinate space)

E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B 2, 167(1967).

$$U^{ij} = \bar{\delta} G_0^{-1}(E) + \sum_{\sigma} \bar{\delta}_{i\sigma} t^{\sigma}(E) G_0(E) U^{\sigma j}$$

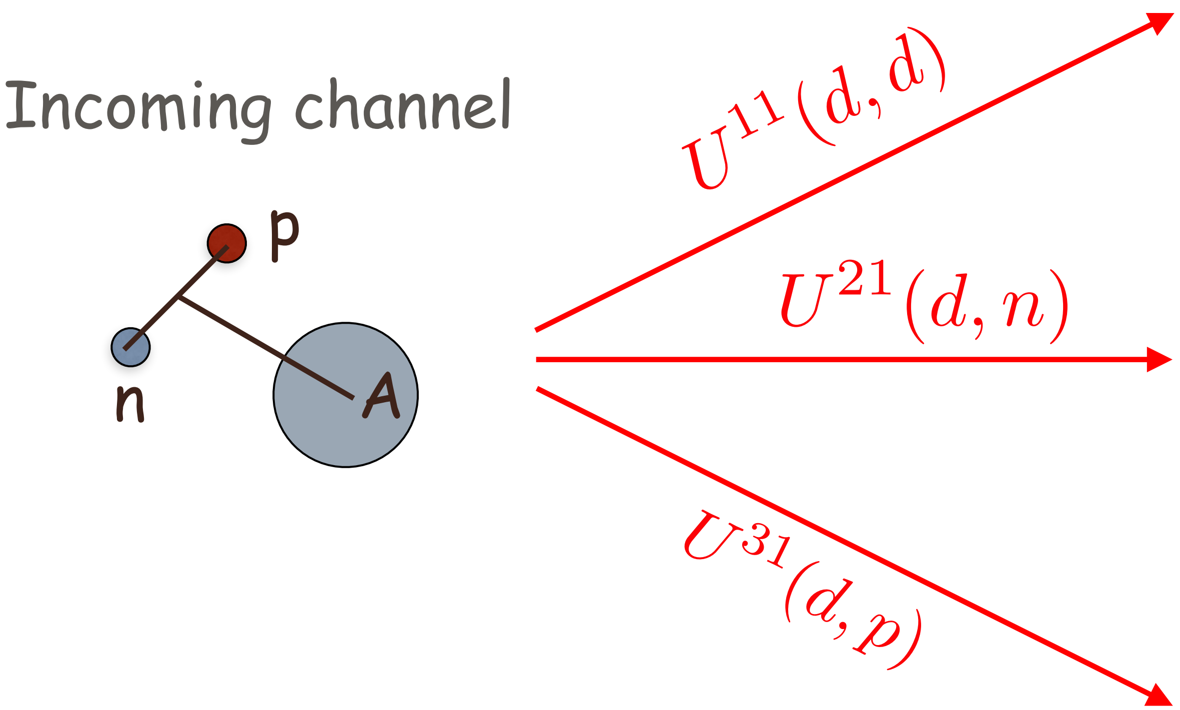
$$t^{\sigma} = v^{\sigma} + v^{\sigma} G_0 t^{\sigma}$$

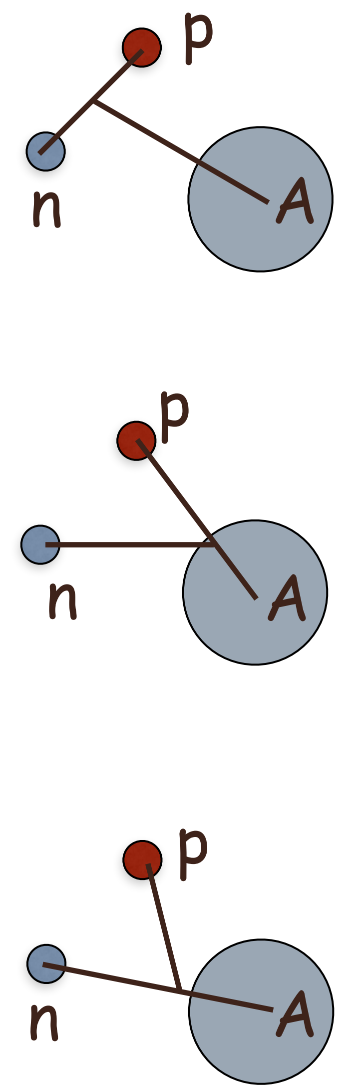
AGS-equations
(momentum space)

Solving the three body problem

Faddeev-AGS equations:

E. O. Alt, P. Grassberger, and W. Sandhas., Nucl.Phys. B2 (1967) 167





$$U^{ij} = \bar{\delta}_{ij} G_0^{-1}(E) + \sum_k \bar{\delta}_{ik} t_k G_0(E) U^{kj}$$

$$U^{0j} = G_0^{-1}(E) + \sum_k t_k G_0 U^{kj}$$

two body t-matrix: $t_k = v_k + v_k g_o t_k$

Observables: $\sigma_{i \leftarrow j} \propto |\langle \Phi_i | U^{ij} | \Phi_j \rangle|^2$

separable potential: $V(p,p') = h(p)\lambda h(p')$

Advantages:

- Elastic, transfer and breakup are treated on equal footing
- Easy boundary conditions

Disadvantages: Coulomb potential

- Current implementations (screening) unstable for high Z target

Remedy:

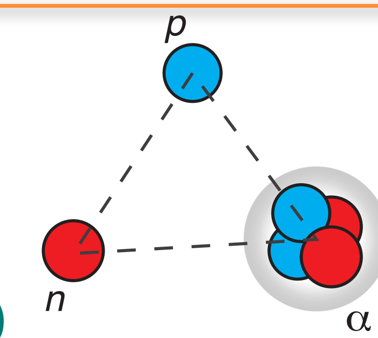
- Use separable two-body potentials

A. Mukhamedzhanov, et al. Phys.Rev. C86, 034001 (2012).

- Coupled equations in one variable allow full inclusion of Coulomb interaction via reformulation of AGS equation in Coulomb basis

Benchmark separable and non-separable Faddeev-AGS equations

- Take n+p+α three body system as an example
- n-p interaction: CD-Bonn — R. Machleidt, Phys. Rev. C 63, 024001 (2001)
- n/p-α interaction: J. Bang potential — J. Bang et al., Nucl. Phys. A 405, 126 (1983); I. J. Thompson et al, .Phys. Rev. C 61, 024318 (2000)
- Omit the Coulomb interaction
- EST scheme: $v^{EST} = VP(PVP)^{-1}PV$ — Ernst et al., Phys.Rev. C9, 1780 (1974)



^{6}Li three body bound state

L. Hlophe, JL, et al., Phys. Rev. C 96, 064003 (2017)

CD-Bonn np potential

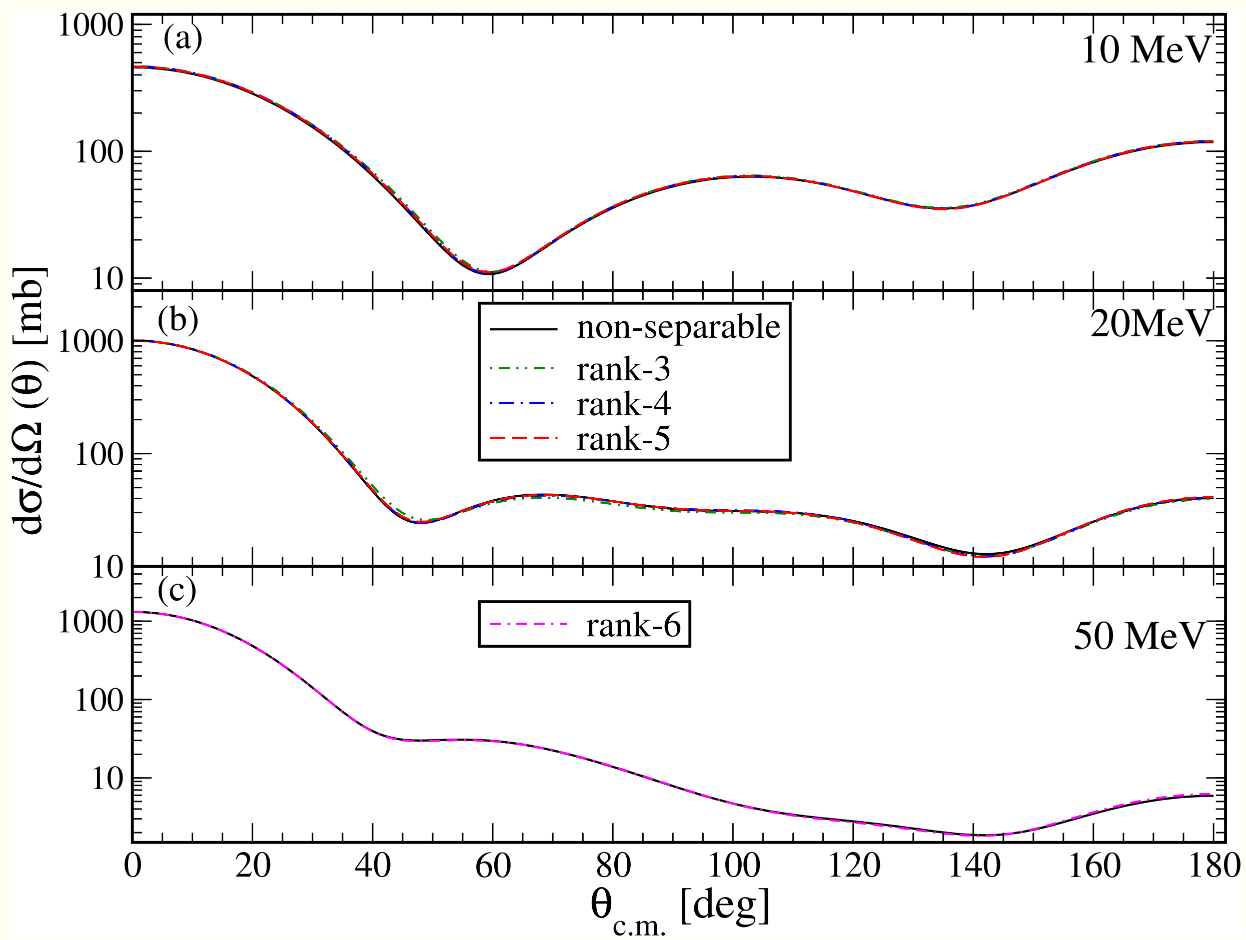
label	rank	E_3 [MeV]
EST5-1	5	-**3.78**47
EST5-2	5	-**3.78**48
EST5-3	5	-**3.78**55
EST6-1	6	-**3.78**67
EST6-2	6	-**3.78**68
EST6-3	6	-**3.78**71
EST7-1	7	-**3.7867**
EST7-2	7	-**3.7867**
EST7-3	7	-**3.7867**
non-separable:		-3.787

Bang $n\alpha$ potential

label	rank	E_{3b} [MeV]
EST6-1	6	-**3.785**6
EST6-2	6	-**3.785**2
EST6-3	6	-**3.785**2
EST7-1	7	-**3.786**8
EST7-2	7	-**3.786**4
EST7-3	7	-**3.786**7
EST8-1	8	-**3.787**0
EST8-2	8	-**3.787**0
EST8-3	8	-**3.786**6
non-separable:		-3.787

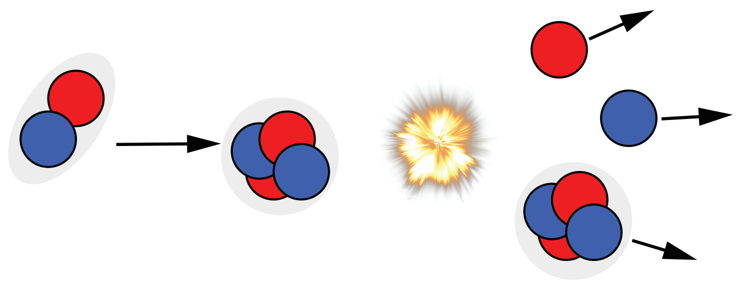
d+α scattering

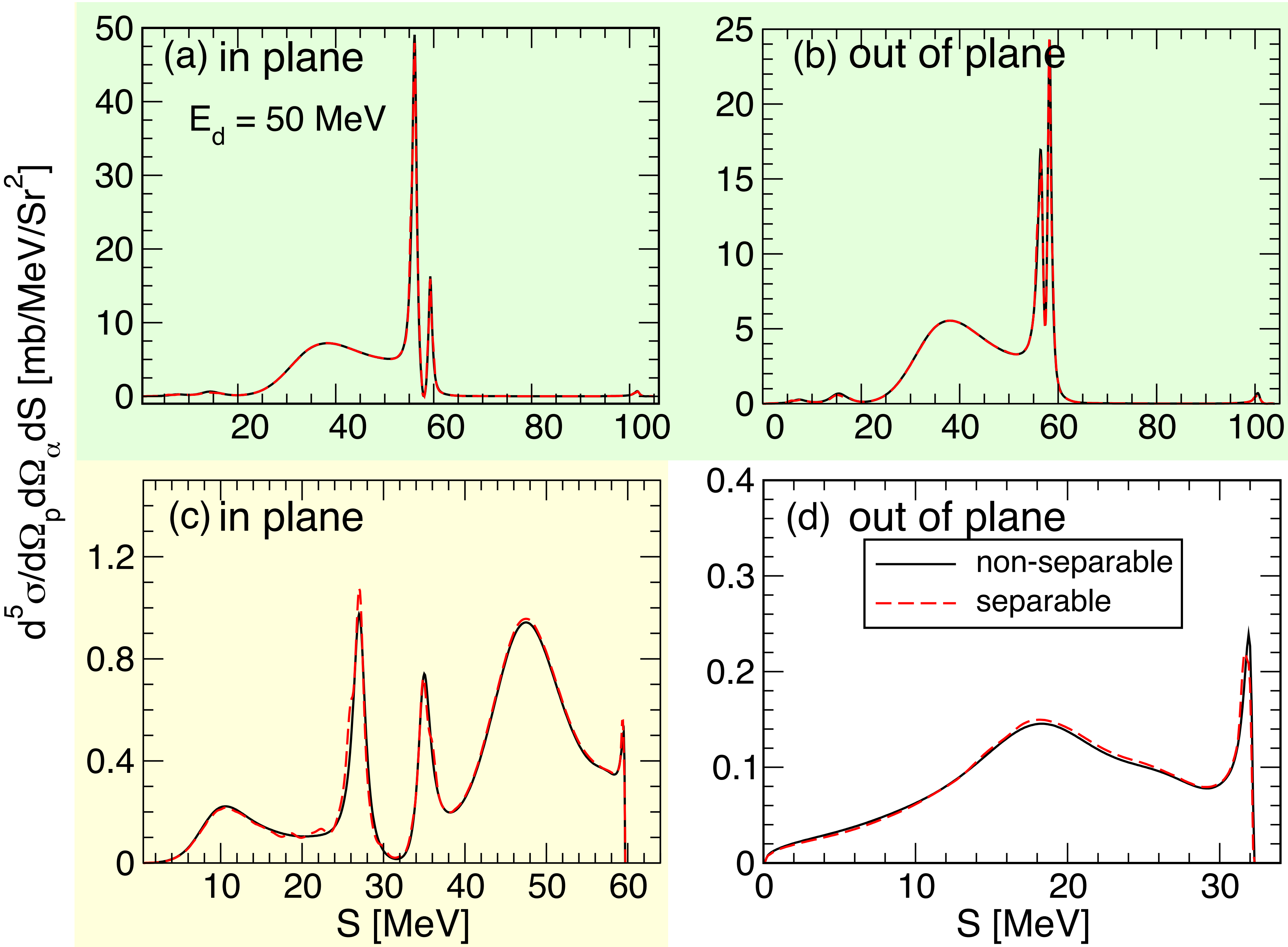
L. Hlophe, JL, et al., in preparation



Benchmark separable and non-separable Faddeev-AGS equations

- Breakup reaction: d+α -> n+p+α
- S-curve for selected angles $dS = \sqrt{dE_P^2 + dE_\alpha^2}$





(a) $\theta_\alpha = 14.0^\circ$ $\phi_\alpha = 0^\circ$ $\theta_p = 7.2^\circ$ $\phi_p = 180^\circ$

(b) $\theta_\alpha = 14.0^\circ$ $\phi_\alpha = 0^\circ$ $\theta_p = 7.2^\circ$ $\phi_p = 120^\circ$

(c) $\theta_\alpha = 22.0^\circ$ $\phi_\alpha = 0^\circ$ $\theta_p = 104.4^\circ$ $\phi_p = 180^\circ$

(d) $\theta_\alpha = 22.0^\circ$ $\phi_\alpha = 0^\circ$ $\theta_p = 104.4^\circ$ $\phi_p = 100^\circ$

Separable and non-separable calculations agree very well

Solving the three body problem

N. Austern, M. Yahiro, and M. Kawai Phys. Rev. Lett. 63, 2649 (1989)

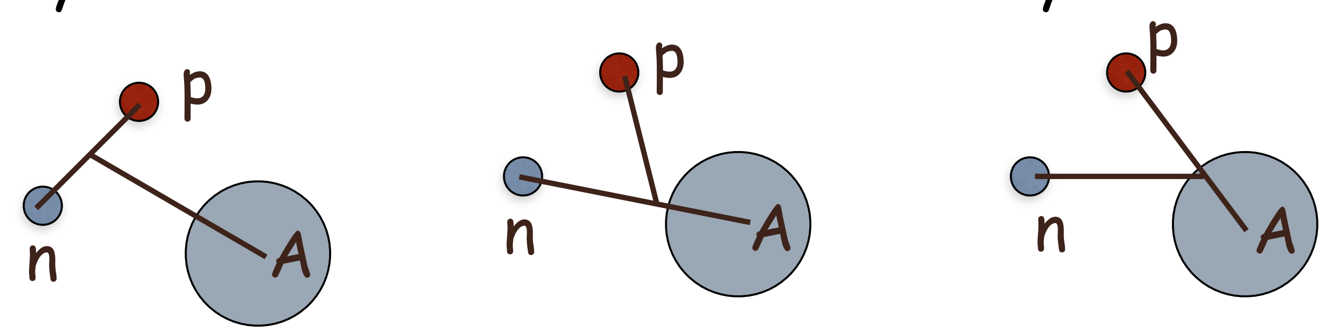
CDCC Equations: Distorted Faddeev Equations

Solving the three body Schrodinger equation: two-body projectile with inert cores on inert target(take d+A system as an example)

$$E|\Psi\rangle = H|\Psi\rangle \longrightarrow H = H_0 + V_{np} + U_{nA} + U_{pA}$$

Faddeev equation: expand the three body wave function in three Jacobi systems

$$|\Psi\rangle = |\psi_{np}\rangle + |\psi_{nA}\rangle + |\psi_{pA}\rangle$$



$$\big(E - H_0 - V_{np} - \mathscr{P}(U_{nA} + U_{pA})\mathscr{P}\big)|\psi_{np}\rangle = V_{np}\big(|\psi_{nA}\rangle + |\psi_{pA}\rangle\big)$$

$$\big(E - H_0 - U_{nA}\big)|\psi_{nA}\rangle = (U_{nA} - \mathscr{P}U_{nA}\mathscr{P})|\psi_{np}\rangle + U_{nA}|\psi_{pA}\rangle$$

$$\big(E - H_0 - U_{pA}\big)|\psi_{pA}\rangle = (U_{pA} - \mathscr{P}U_{pA}\mathscr{P})|\psi_{np}\rangle + U_{pA}|\psi_{nA}\rangle$$

$$(E - H_0 - U)(|\psi_{pA}\rangle + |\psi_{nA}\rangle) = (U - \mathscr{P}U\mathscr{P})|\psi_{np}\rangle$$ weak coupling to the first equation

$$\mathscr{P} = \sum_{\alpha b}\int R^2 dR\,|\phi^b_{bx}R\alpha\rangle\langle\phi^b_{bx}R\alpha| + \sum_{\alpha}\int R^2 dR\int dk\,|\phi^k_{bx}R\alpha\rangle\langle\phi^k_{bx}R\alpha|$$

Inclusion of the continuum in CC calculations: continuum discretization

Quantum Hamiltonian

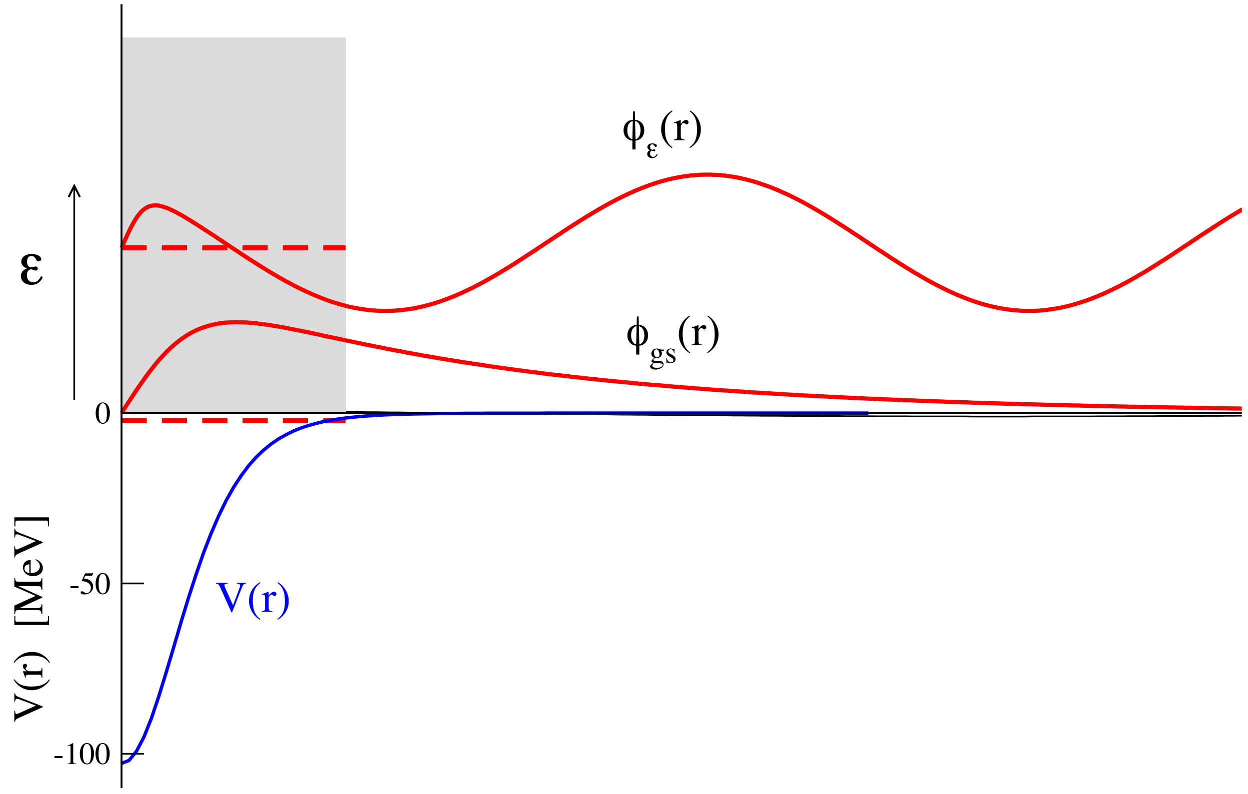
Bound states
- Discrete
- Finite
- Normalizable

Unbound states
- Continuous
- Infinite
- Non-normalizable

Continuum discretization: represent the continuum by a finite set of square-integrable states

***True* continuum**	$\rightarrow$	**Discretized continuum**
Non normalizable	$\rightarrow$	Normalizable
Continuous	$\rightarrow$	Discrete

Bound versus scattering states



Continuum state:

$$\phi^m_{k,\ell j}(\mathbf{r}) = \frac{u_{k,\ell j}(r)}{r}[Y_\ell(\hat{r}) \otimes \chi_s]_{jm}$$

Unbound states are not suitable for CC calculations:

- Continuous (infinite) distribution in energy.
- Non-normalizable: $\langle u_{k,\ell sj}(r)^* | u_{k',\ell sj}(r) \rangle \propto \delta(k-k')$

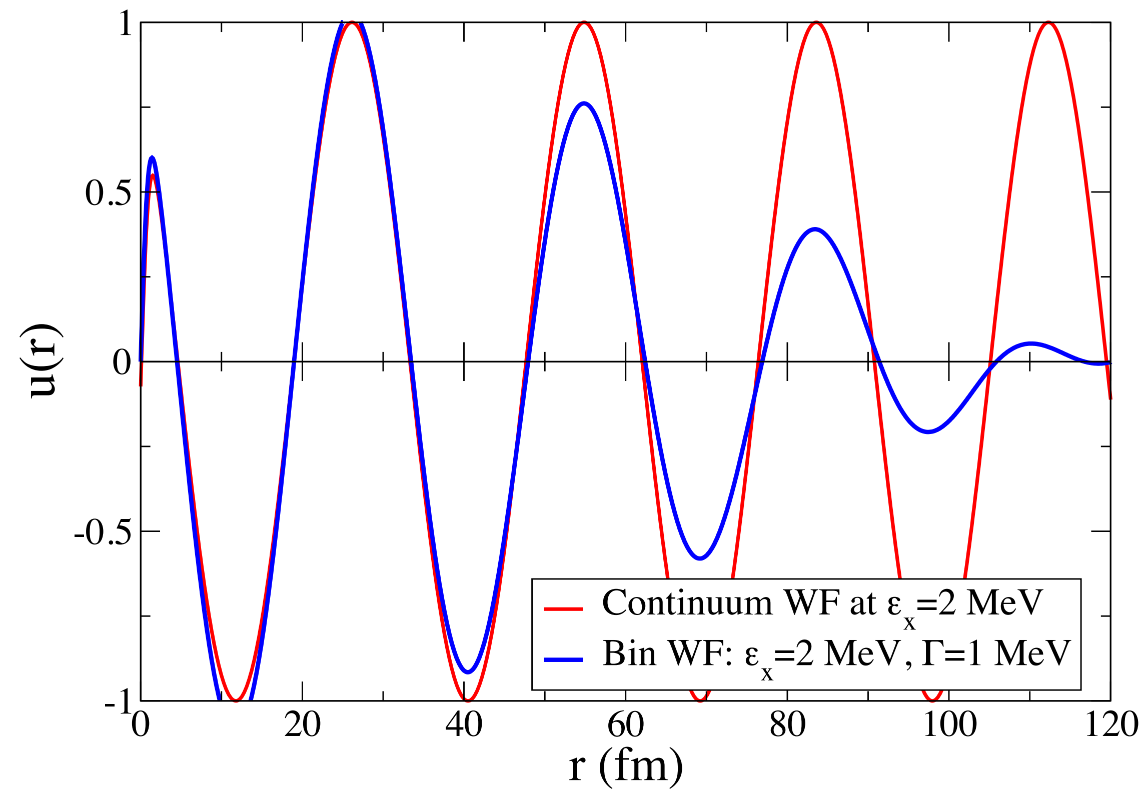
SOLUTION $\Rightarrow$ continuum discretization

CDCC formalism: construction of the bin wave functions

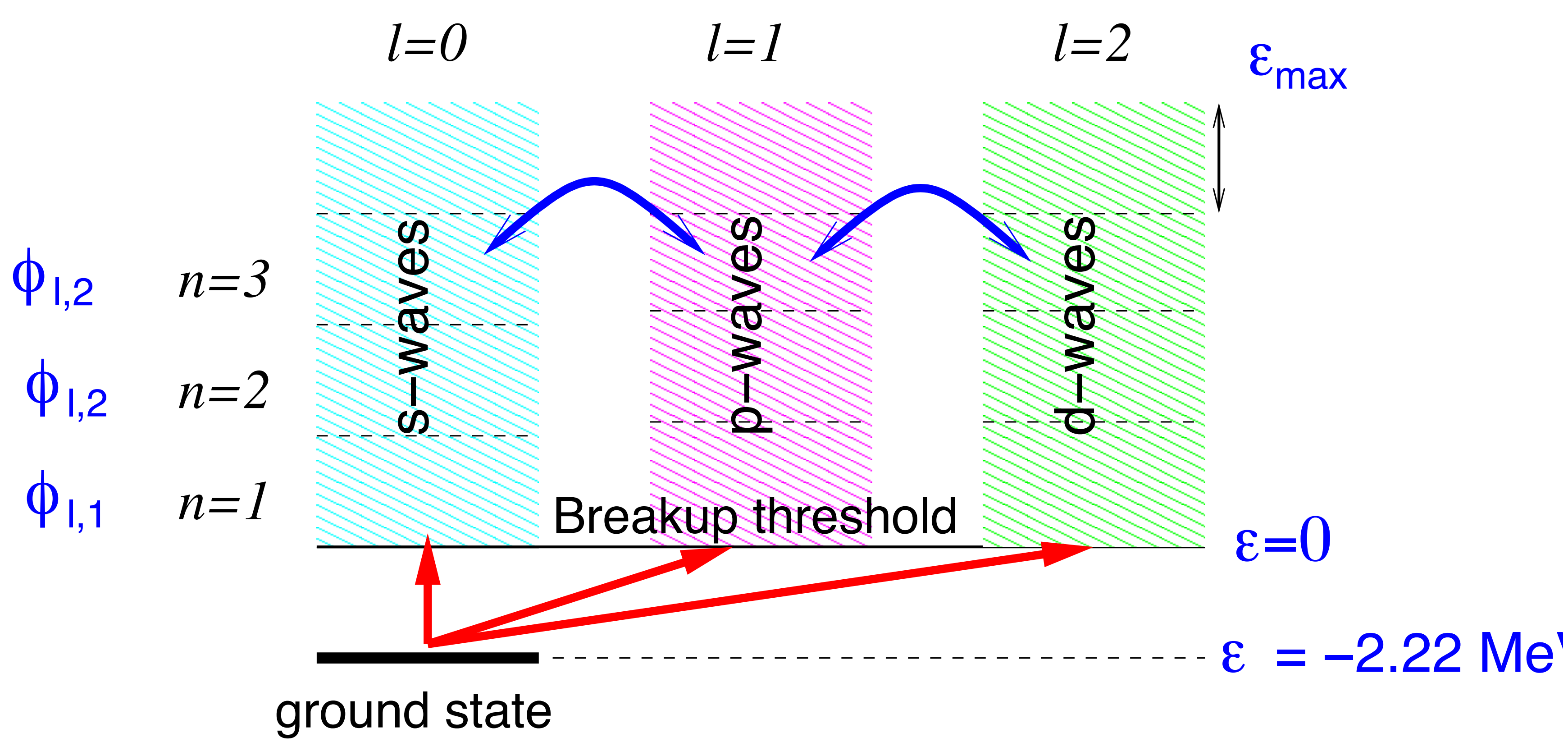
Bin wavefunction:

$$\boxed{u_{\ell s j,n}(r) = \sqrt{\frac{2}{\pi N}} \int_{k_1}^{k_2} w(k) u_{k,\ell s j}(r) dk}$$

- k: linear momentum
- $u_{k,\ell s j}(r)$: scattering states (radial part)
- $w(k)$: weight function

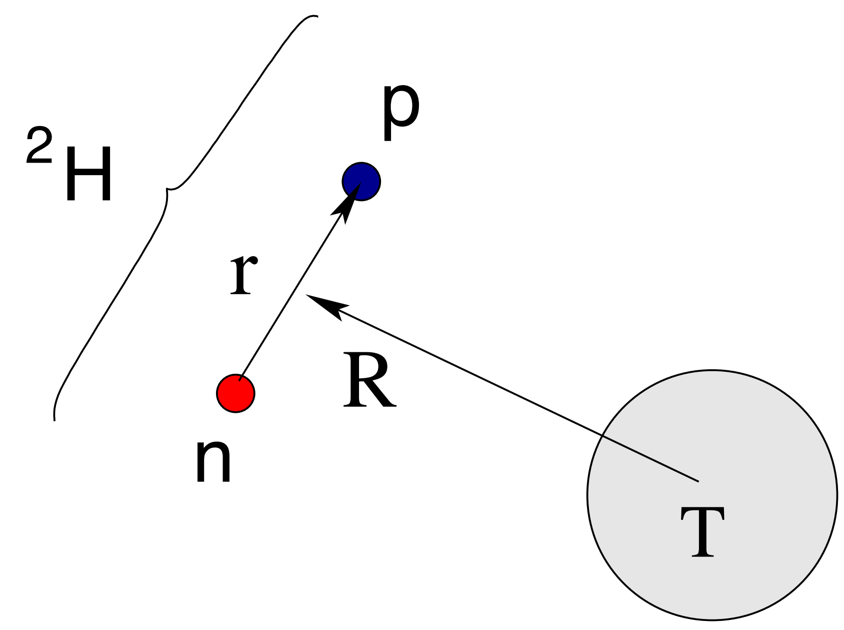


Continuum discretization for deuteron scattering



- Select a number of partial waves ($\ell = 0, \ldots, \ell_{\max}$).
- For each ℓ, set a maximum excitation energy $\varepsilon_{\max}$.
- Divide the interval $\varepsilon = 0 - \varepsilon_{\max}$ in a set of sub-intervals (*bins*).
- For each *bin*, calculate a representative wavefunction.

CDCC equations for deuteron scattering



- Hamiltonian:

  $$H = T_R + h_r(\mathbf{r}) + V_{pt}(\mathbf{r}_{pt}) + V_{nt}(\mathbf{r}_{nt})$$

- Model wavefunction:

$$\Psi(\mathbf{R},\mathbf{r}) = \phi_{gs}(\mathbf{r})\chi_0(\mathbf{R}) + \sum_{n>0}^{N} \phi_n(\mathbf{r})\chi_n(\mathbf{R})$$

- Coupled equations: $[H - E]\Psi(\mathbf{R},\mathbf{r}) = 0$

$$\boxed{[E - \varepsilon_n - T_R - V_{n,n}(\mathbf{R})]\,\chi_n(\mathbf{R}) = \sum_{n' \neq n} V_{n,n'}(\mathbf{R})\chi_{n'}(\mathbf{R})}$$

- Transition potentials:

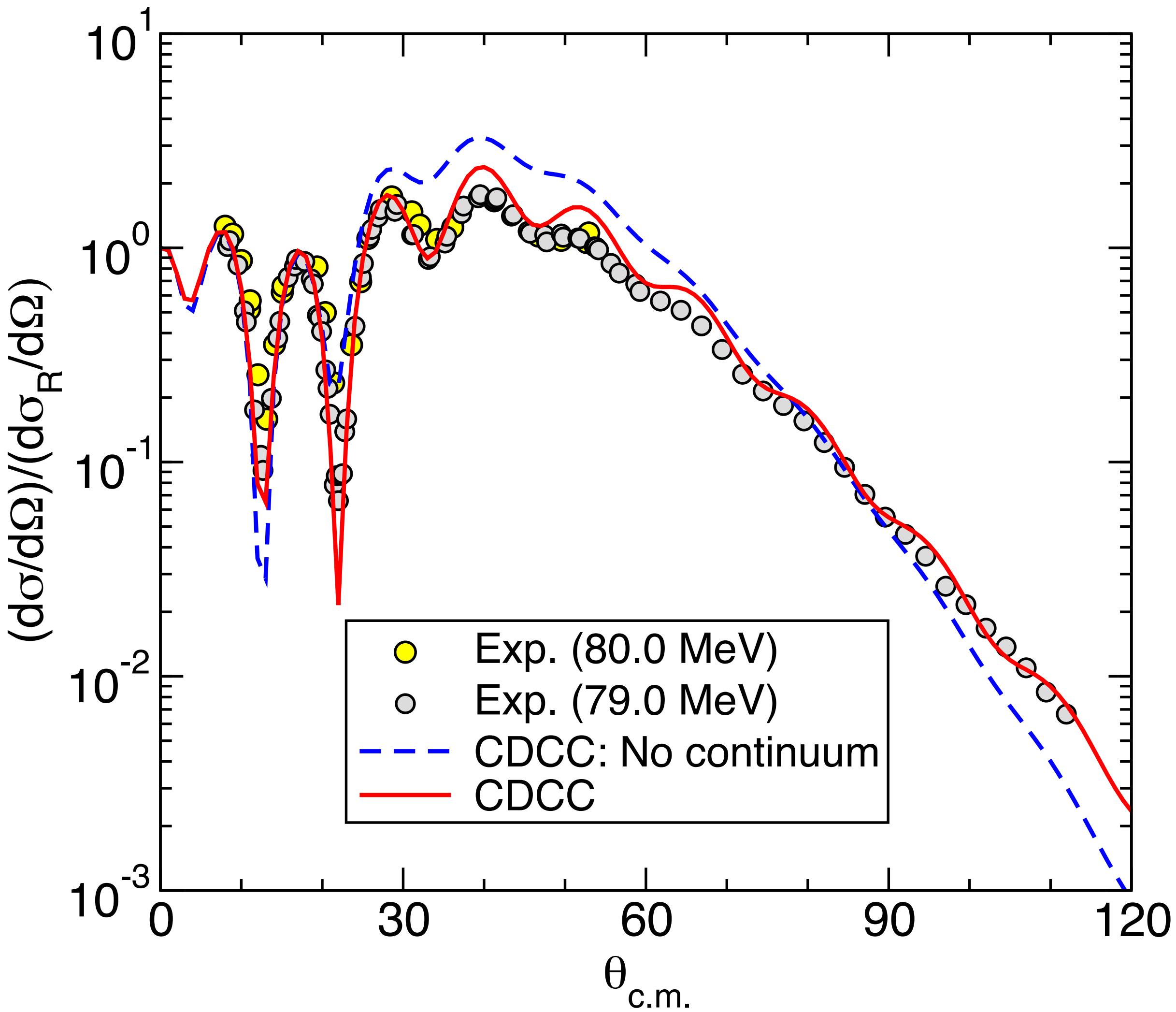
$$V_{n;n'}(\mathbf{R}) = \int d\mathbf{r}\phi_n(\mathbf{r})^* \left[V_{pt}(\mathbf{R} + \frac{\mathbf{r}}{2}) + V_{nt}(\mathbf{R} - \frac{\mathbf{r}}{2})\right] \phi_{n'}(\mathbf{r})$$

What observables can we study with CDCC

- ⇨ Elastic scattering
- ⇨ Breakup angular distribution, as a function of excitation energy:
- ⇨ Breakup energy distribution, as a function of c.m. angle:

☞ From the S-matrices, more complicated breakup observables can be obtained, such as angular/energy distribution of one of the fragments

Application of the CDCC formalism: d+ 58Ni



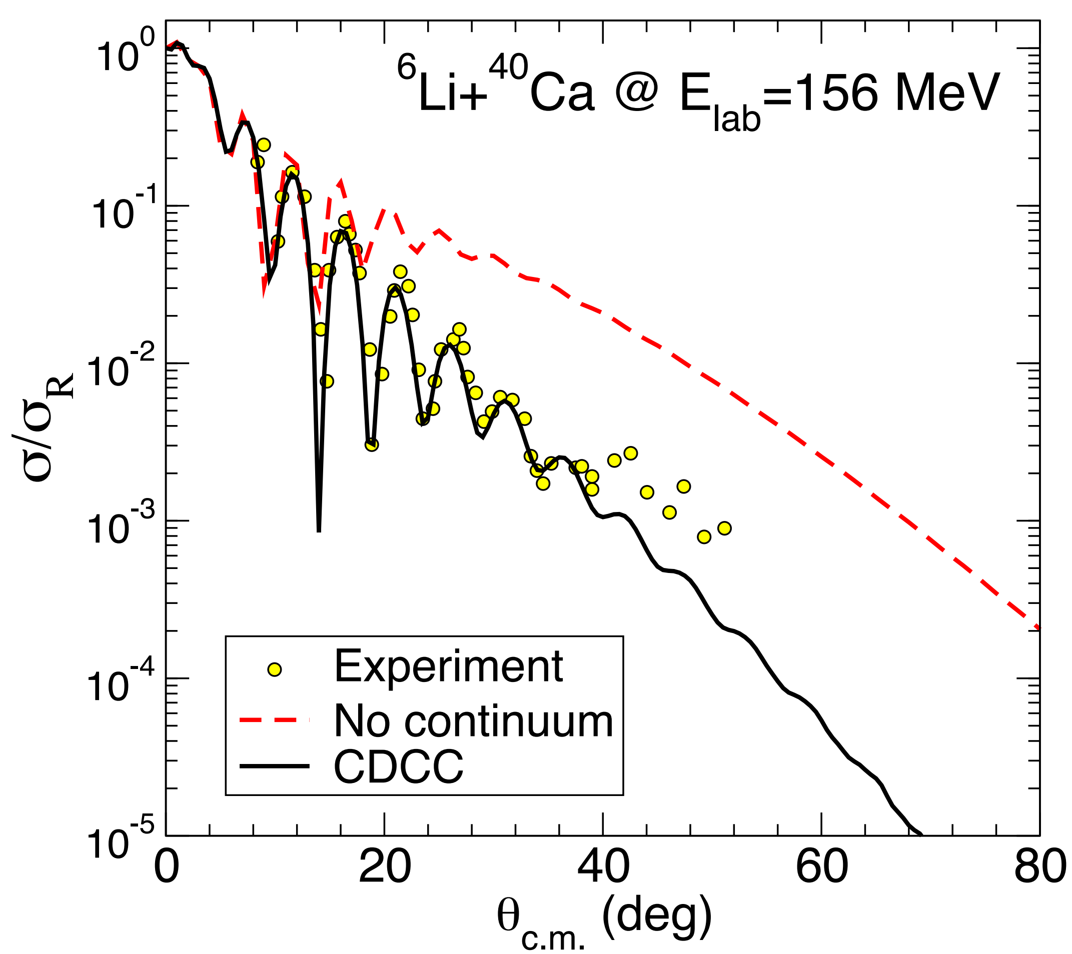
- $\ell = 0, 2$ continuum
- $p+^{58}$Ni and $n+^{58}$Ni from Koning-Delaroche OMP.
- $V_{pn}(r) = -72.15\ \exp[-(r/1.484)^2]$

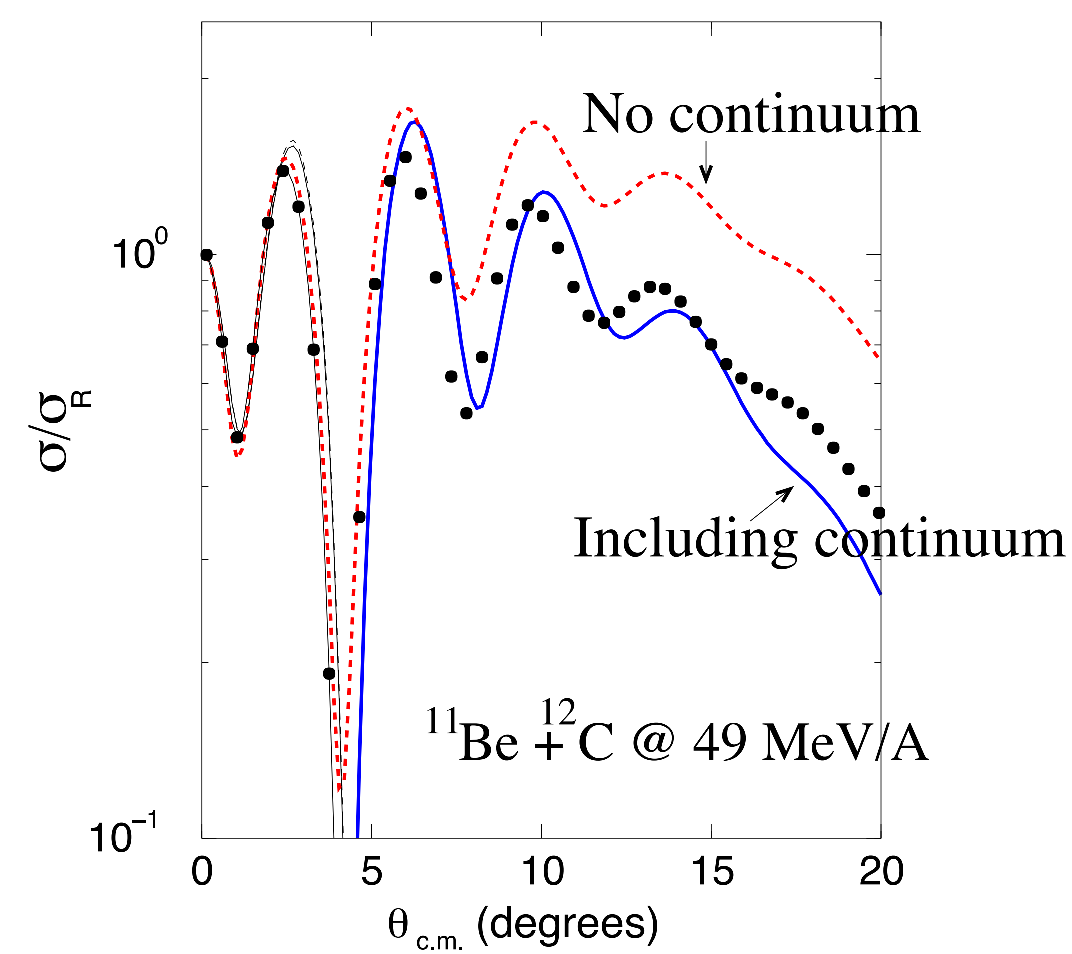
☞*Coupling to breakup channels has a important effect on the reaction dynamics*

Application of the CDCC method: 6Li and 6He scattering

☞ The CDCC has been also applied to nuclei with a cluster structure:

- ^{6}Li=α + d
- ^{11}Be=^{10}Be + n





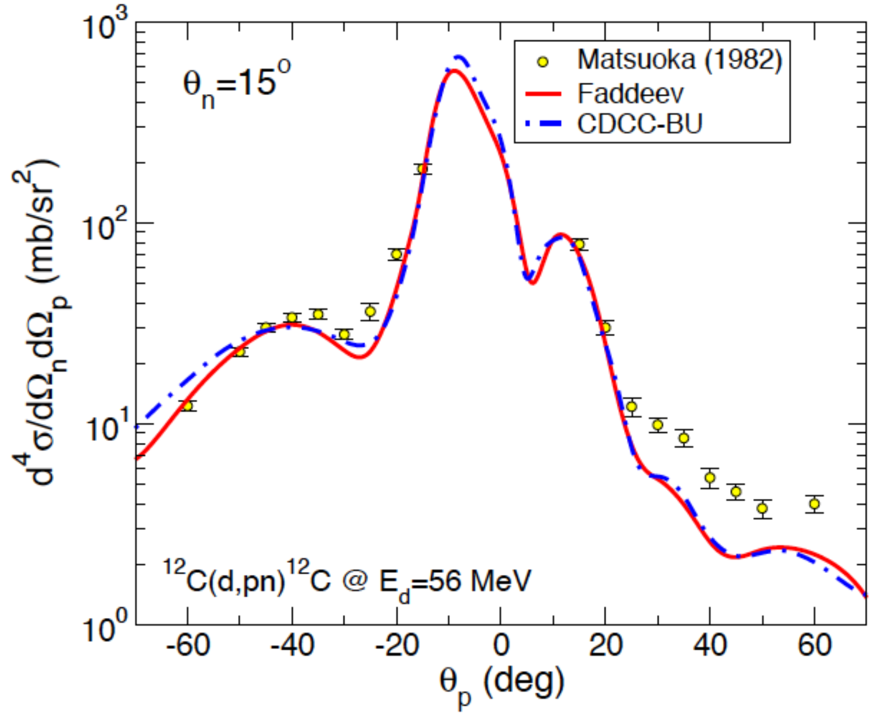
☞ In Fraunhofer scattering the presence of the continuum produces a reduction of the elastic cross section

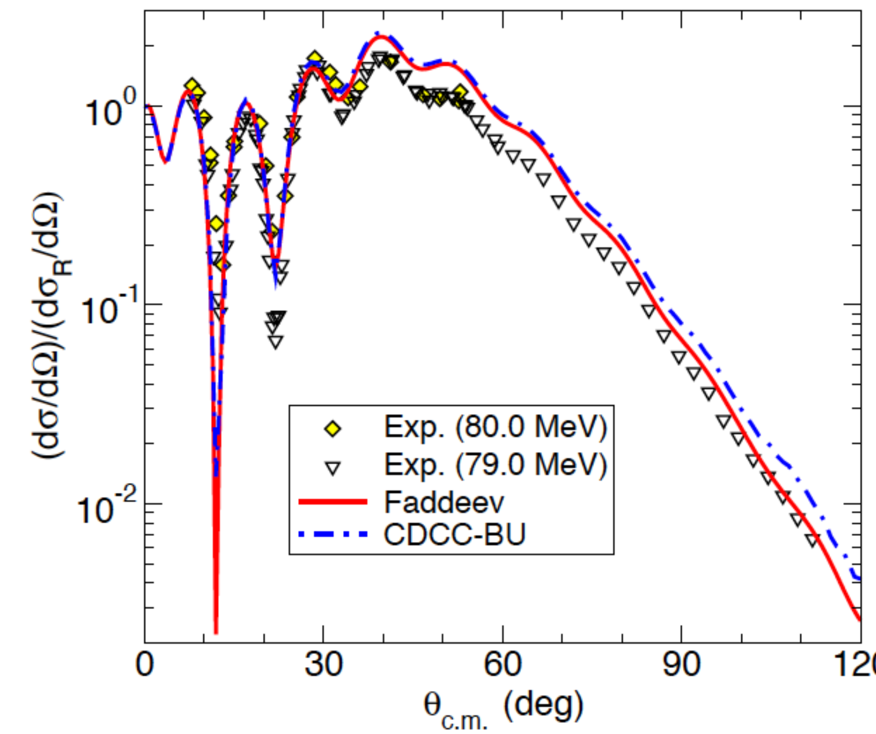
Compare Faddeev with CDCC

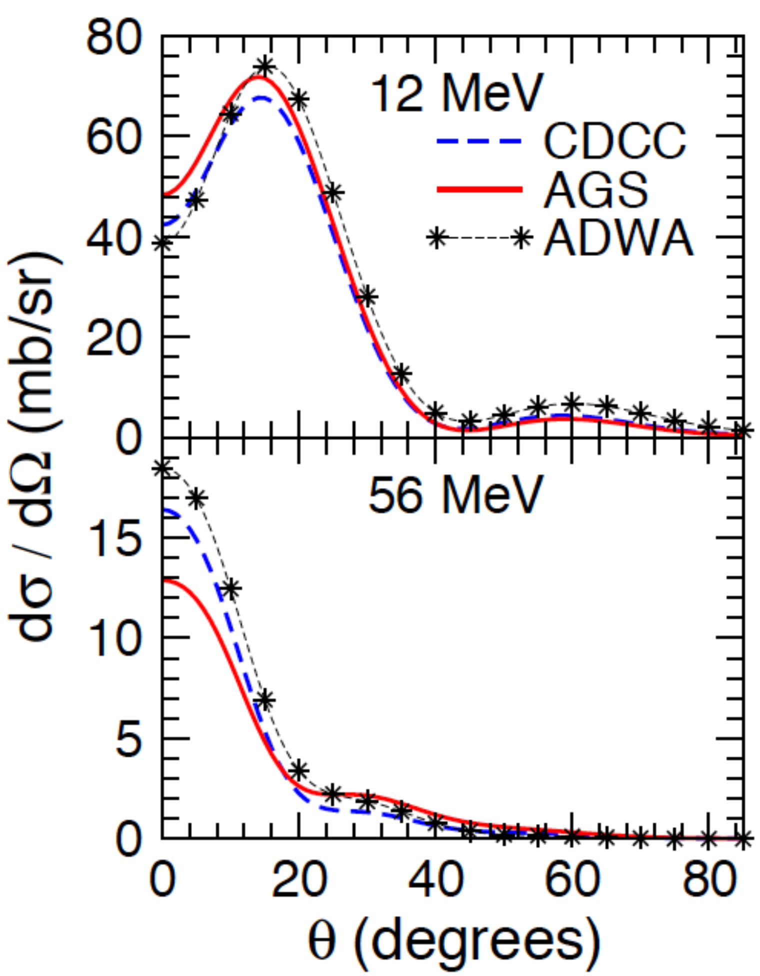
Test case
$^{12}C(d,pn)^{12}C$
$^{58}Ni(d,d)^{58}Ni$

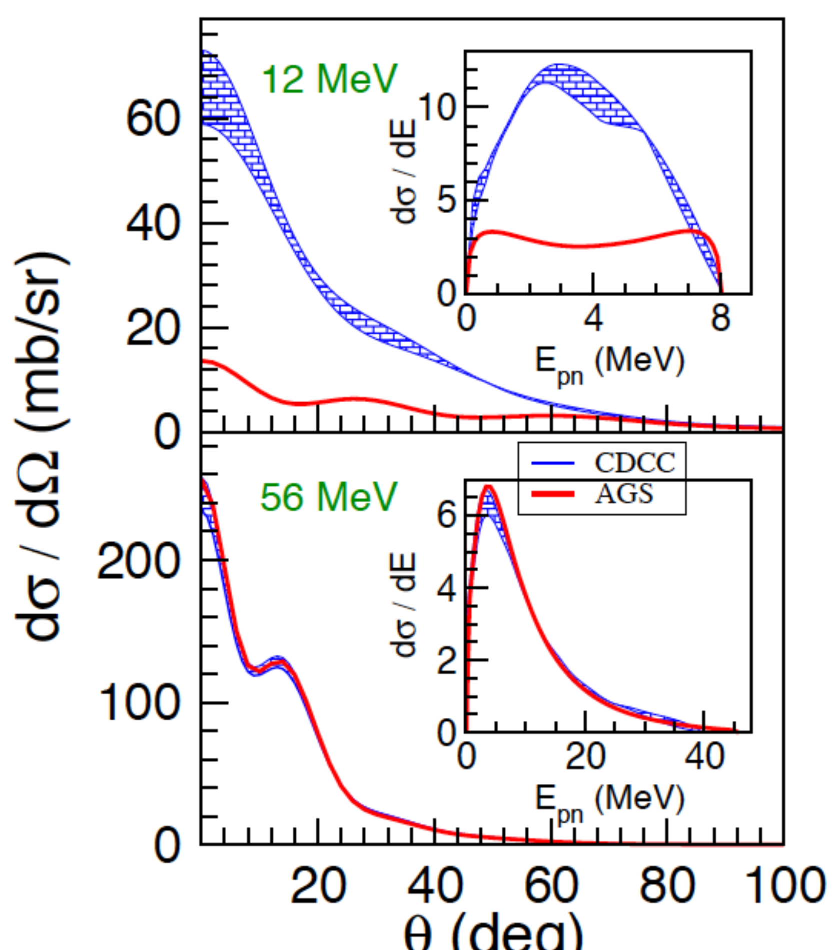
Good agreement in both elastic scattering and breakup

A. Deltuva et al.,
Phys. Rev. C 76, 064602 (2007)









Test case
$^{12}C(d,p)^{13}C$
$^{12}C(d,pn)^{12}C$

Disagreement in transfer and breakup

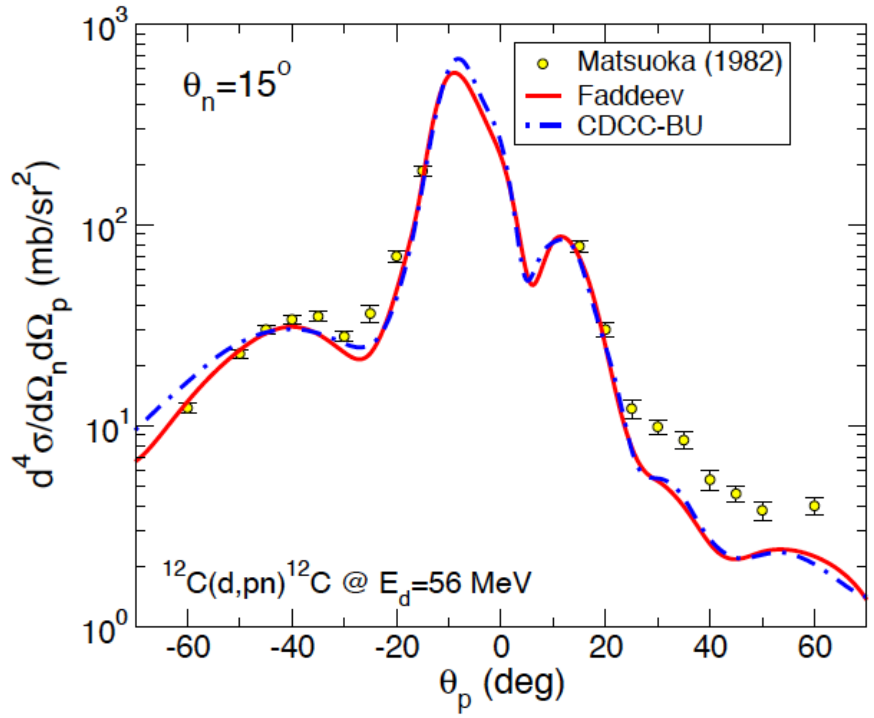
N. J. Upadhyay, A. Deltuva, and F. M. Nunes,
Phys. Rev. C 85, 054621 (2012)

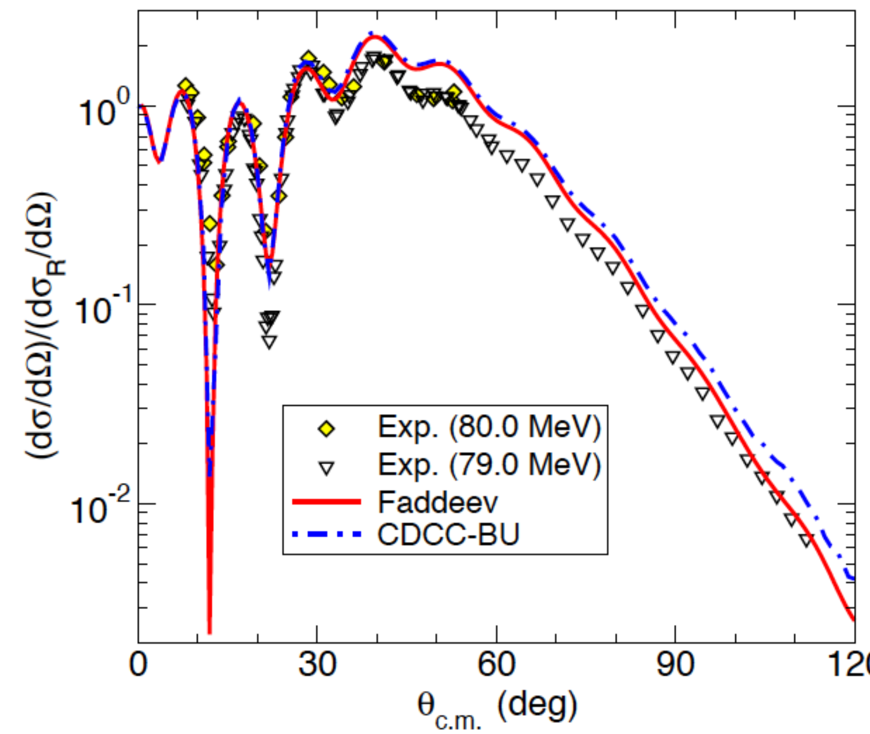
Compare Faddeev with CDCC

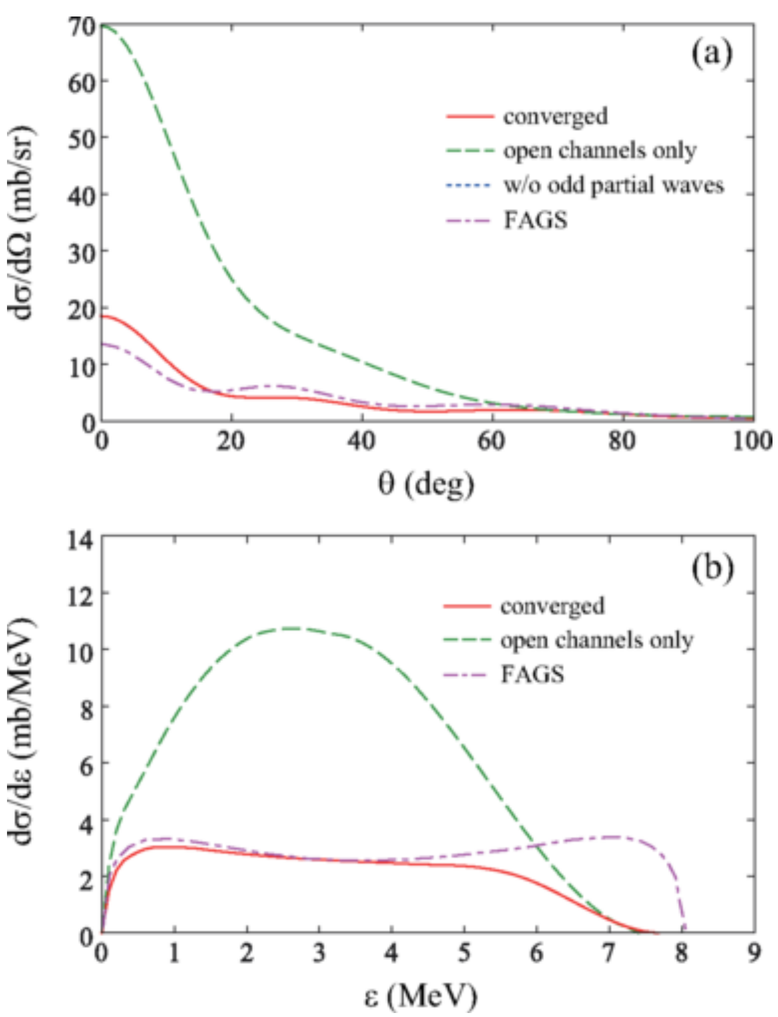
Test case
$^{12}C(d,pn)^{12}C$
$^{58}Ni(d,d)^{58}Ni$

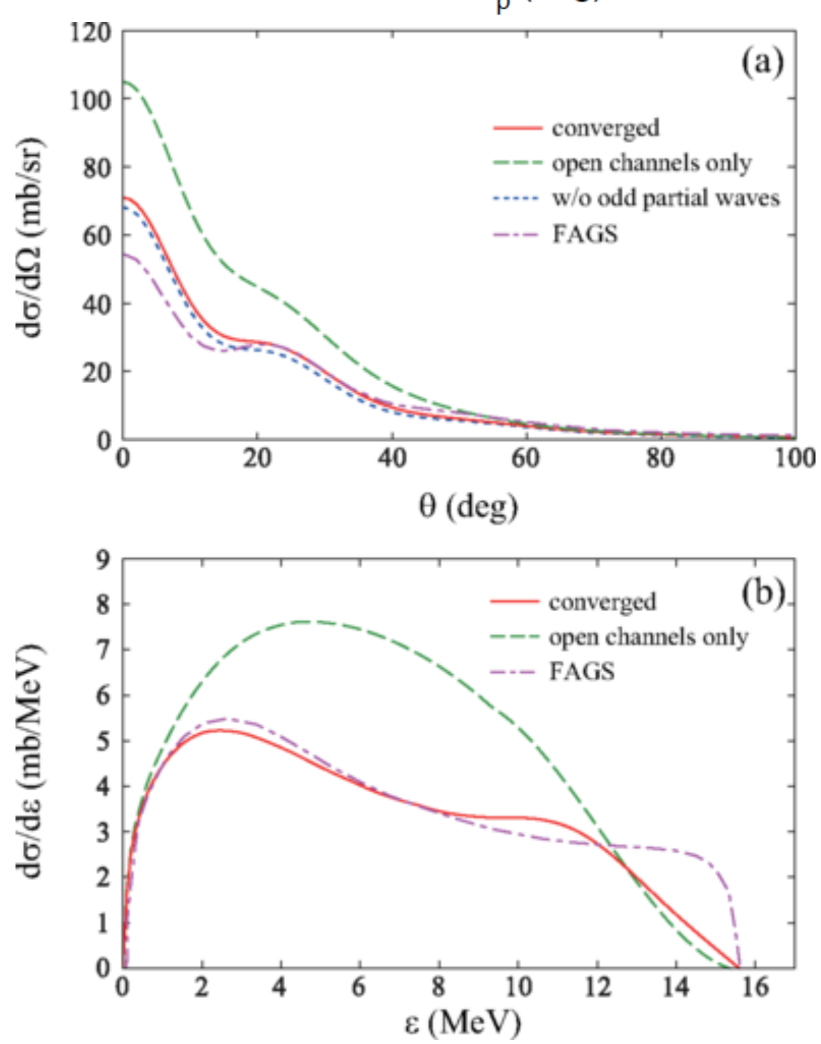
Good agreement in both elastic scattering and breakup

A. Deltuva et al.,
Phys. Rev. C 76, 064602 (2007)









Test case
$^{12}C(d,pn)^{12}C$@12MeV
$^{10}Be(d,p)^{10}Be$@21MeV

Agree with Faddeev when including the close channels

K. Ogata and K. Yoshida
Phys. Rev. C 94, 051603(R) (2016)