MATH 437 HW2

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1.a. Singular values are $\sigma_1 = 4$,$\sigma_2 = 2$,$\sigma_3 = 1$, and $\sigma_4 = 5$. 1.b. Singular values are $\sigma_1 = 1$,$\sigma_2 = 1$,$\sigma_3 = 1$, and $\sigma_4 = 1$. 1.c. $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ A is different from the identity and is orthogonal. 2.a. $\hat{\beta} = (A^T\Sigma^{-1}A)^{-1}A^T\Sigma^{-1}y$ 2.b. mean=$\hat{\beta} = (A^T\Sigma^{-1}A)^{-1}A^T\Sigma^{-1}A\beta$ $E(\hat{\beta}) = \beta$ $Cov(\hat{\beta}) = (A^T\Sigma^{-1}A)^{-1}$ 2.c. $Cov(r) = I - A(A^T\Sigma^{-1}A)^{-1}A^T\Sigma^{-1}\Sigma(I - A(A^T\Sigma^{-1}A)^{-1}A^T\Sigma^{-1})$ 3.a. $= \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \\ a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$ 3.b. $\begin{bmatrix} a_{1,1} & a_{1,1} & a_{1,1} & a_{1,2} & a_{1,2} & a_{1,2} \\ a_{2,1} & a_{2,1} & a_{2,1} & a_{2,2} & a_{2,2} & a_{2,2} \end{bmatrix}$ 3.c. $\begin{bmatrix} a_{1,1} & a_{1,2} & 0 & 0 \\ a_{2,1} & a_{2,2} & 0 & 0 \\ 0 & 0 & a_{1,1} & a_{1,2} \\ 0 & 0 & a_{2,1} & a_{2,2} \end{bmatrix}$ 3.d. $\begin{bmatrix} a_{1,1} & 0 & a_{1,2} & 0 \\ 0 & a_{1,1} & 0 & a_{1,2} \\ a_{2,1} & 0 & a_{2,2} & 0 \\ 0 & a_{2,1} & 0 & a_{2,2} \end{bmatrix}$ 3.e. $\begin{bmatrix} a_{1,1}a_{1,1} & a_{1,1}a_{1,2} & a_{1,2}a_{1,1} & a_{1,2}a_{1,2} \\ a_{1,1}a_{2,1} & a_{1,1}a_{2,2} & a_{1,2}a_{2,1} & a_{1,2}a_{2,2} \\ a_{1,1}a_{2,1} & a_{2,1}a_{1,2} & a_{2,2}a_{1,1} & a_{1,2}a_{2,2} \\ a_{2,1}a_{2,1} & a_{2,1}a_{2,2} & a_{2,2}a_{2,1} & a_{2,2}a_{2,2} \end{bmatrix}$ 3.f. let A=$\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}$ then $(A \otimes A)(I \otimes A) = \begin{bmatrix} 7 & 18 & 21 & 54 \\ 12 & 31 & 36 & 93 \\ 14 & 36 & 35 & 90 \\ 24 & 62 & 60 & 155 \end{bmatrix}$ 4.a. \$E(Y) = (0,0,0,0)^{T} \$ 4.b. $Var(Y) = (2, 2, 1, 1)^T$ 4.c. $E(U) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $Var(U) = \begin{bmatrix} 2 & 2 \\ 1 & 1 \end{bmatrix}$ 5.a. x1=N(1,1) x2=N(1,5) 5.b.

```
sigma <- matrix(c(1,-2,-2,5),nrow=2)
mu=c(1,1)
x = matrix(c(1,0,1,-1),nrow=2)
x%*%mu
```

```
##      [,1]
## [1,]    2
## [2,]   -1
```

```
t(x)%*%sigma%*%x
```

```
##      [,1] [,2]
## [1,]    1    3
## [2,]    3   10
```

5.c.

```
x = matrix(c(1,1,1,-1),nrow=2)
x%*%mu
```

```
##      [,1]
## [1,]    2
## [2,]    0
```

```
t(x)%*%sigma%*%x
```

```
##      [,1] [,2]
## [1,]    2   -4
## [2,]   -4   10
```

5.d. new $\mu = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} \mu$ new $\sigma = (\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix})^T \Sigma \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix}$ 5.e. new $\mu = \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} b\mu$ new $\sigma = (\begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} b)^T \Sigma \begin{bmatrix} a_{1,1} & a_{1,2} \\ a_{2,1} & a_{2,2} \end{bmatrix} b$ 5.f. $a = 1, b = -1$ 6.a. Since the vector is positive and the sigma matrix is always semi definite the matrix $a^T \Sigma a$ is always positive definite.