

1. The group is not closed. The group contains no identity.
2. (a) Closure. Let $A, B \in SL(2, \mathbb{R})$ and $A \circ B = C$. Then $\det(C) = \det(AB) = \det(A)\det(B) = 1$ thus $C \in SL(2, \mathbb{R})$
- (b) Associativity. Suppose $A, B, C \in SL(2, \mathbb{R})$ then $A \circ (B \circ C) = (A \circ B) \circ C$ and then $\det(A)\det(BC) = \det(AB)\det(C) = \det(A)\det(B)\det(C) = 1$ so closure holds.
- (c) Identity. Let the identity be the 2x2 unit matrix (e). $\det(e) = 1$ so $e \in SL(2, \mathbb{R})$.
- (d) Inverse. Suppose $\exists A^{-1}, A \in SL(2, \mathbb{R})$ such that $A \circ A^{-1} = A^{-1} \circ A = e$. Then we have $\det(A \circ A^{-1}) = \det(A^{-1}) = \det(e) = 1$. So $A^{-1} \in SL(2, \mathbb{R})$.

3.

	e	a	b	c
e	e	a	b	c
a	a	b	c	e
b	b	c	e	a
c	c	e	a	b

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	a	e
c	c	b	e	a

	e	a	b	c
e	e	a	b	c
a	a	c	e	b
b	b	e	c	a
c	c	b	a	e

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

4. If $xy = zx$ implies $y = z$ then $xz = zx$ and $xy = yx$ for $\forall x, y, z \in G$ therefore the group G is abelian.

5. For $n=0$ the case is trivial and becomes the identity. Consider $n+1$ case. Then we have:

$$\begin{aligned}
 (a^{-1}ba)^{n+1} &= a^{-1}b^{n+1}a \\
 a^{-n-1}b^{n+1}a^{n+1} &= a^{-1}b^{n+1}a \\
 a^{-1}b^{n+1}a^1 &= a^{-1}b^{n+1}a \\
 a^{-1}b^{n+1}a &= a^{-1}b^{n+1}a
 \end{aligned}$$

Then consider the $n-1$ case. Then we have:

$$\begin{aligned}(a^{-1}ba)^{n-1} &= a^{-1}b^{n-1}a \\ a^{-n+1}b^{n-1}a^{n-1} &= a^{-1}b^{n-1}a \\ a^1b^{n-1}a^{-1} &= a^{-1}b^{n-1}a \\ a^{-1}b^{n+1}a &= a^{-1}b^{n+1}a\end{aligned}$$