Homework 1

1. Give two reasons why the set of odd integers under addition does not form a group.

2. Show that the set of 2×2 matrices with determinant 1 and with entries from $\mathbb{R}$ forms a group under matrix multiplication.

$$SL(2, \mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \middle| \; a, b, c, d \in \mathbb{R}, \; ad - bc = 1 \right\}$$

   (In general, when F is any of $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, or $\mathbb{Z}_p$, we let $SL(n, F)$ be the group of $n \times n$ matrices with determinant equal to 1 under matrix multiplication. We call the group $SL(n, F)$ the *special linear group of $n \times n$ matrices over F*.)

3. Suppose $(G, *)$ is a group with elements $\{e, a, b, c\}$ where e is the identity element. Construct all possible Cayley tables for this group. (Hint: There are more than 1.)

4. Let G be a group with the property that $xy = zx$ implies $y = z$ for all $x, y, z \in G$. Prove that G is abelian.

5. For any elements a and b from a group and any integer n, prove that $(a^{-1}ba)^n = a^{-1}b^n a$. (Hint: use induction on n.)