Homework 2

1. If H and K are subgroups of G, show that $H \cap K$ is a subgroup of G. (can you see that the same proof shows that the intersection of any number of subgroups of G, finite or infinite, is again a subgroup of G?)

2. For any group elements a and x, prove that $|x^{-1}ax| = |a|$.

3. Prove that a group with two elements of order 2 that commute must have a subgroup of order 4.

4. Prove that $\langle a \rangle$ is the smallest subgroup of G containing a.

5. (a) Suppose G is an abelian group. Prove that $Z(G) = C(a) = G$ for any $a \in G$.
   (b) Must the center of a group be abelian? Justify your answer.
   (c) Must the centralizer of a group element be abelian? Justify your answer.