1. If H and K are subgroups of G, show that $H \cap K$ is a subgroup of G. (can you see that the same proof shows that the intersection of any number of subgroups of G, finite or infinite, is again a subgroup of G?)
   (a) Associativity. Since H and K are subgroups they are associative so $H \cap K$ inherits associativity.
   (b) Identity. As subgroups of G we can assume e, the identity of G, is $\in H$ and $\in K$.
   (c) Closure. $\forall a, b \in H$ we can say $ab \in H$ and $\forall a, b \in K$ we can say $ab \in K$. So $\forall a, b \in H \cap K$ we can say $ab \in H \cap K$.
   (d) Inverses. Since H and K are subgroups we can say $\exists c^{-1} \in H$ and K for $\forall c \in H$ and K. So $\exists c^{-1} \in H \cap K$ for $\forall c \in H \cap K$.
2. For any group elements a and x, prove that $|x^{-1}ax| = |a|$. For the finite case we have: $(xax^{-1})^n = xa^nx^{-1}$ so substituting $|a|$ in for n leaves us with $xa^{|a|}x^{-1} = xex^{-1} = e$. For the infinite case we have $(xax^{-1})^n = xa^nx^{-1}$ so if the order of a is infinite the order of xax^{-1} cannot be finite.
3. Prove that a group with two elements of order 2 that commute must have a subgroup of order 4. Consider $H = e, a, b, ab$ with $|a|, |b| = 2$ for $a, b \in G$. Then H is a valid subgroup of order 4 and is a subgroup of G.
4. Prove that $\langle a \rangle$ is the smallest subgroup of G containing a. Let H be the smallest subgroup of G containing a. Then by closure $\forall a^n$ for $\forall n \in \mathbb{Z}$ is $\in H$. Then $\langle a \rangle \subset H$ and since H is the smallest subgroup of G containing a then $\langle a \rangle$ is the smallest subset of G containing a.
5. (a) Suppose G is an abelian group. Prove that Z(G) = C(a) = G for any $a \in G$. Since G is abelian the centeralizer of the group is equal to G. Since G is abelian the center of an element a is equal to the group G for $\forall a \in G$.
   (b) Must the center of a group be abelian? Justify your answer. Yes since the center commutes with all other elements it is abelian.
   (c) Must the centralizer of a group element be abelian? Justify your answer. The centralizer of a group element need not be abelian since that element may not be abelian with all other elements in the centralizer of that element.