

Homework 3

1. Let G be an abelian group. (Do not assume that G is finite.)
 - (a) Prove that $H = \{x \in G \mid |x| \text{ is odd}\}$ is a subgroup of G .
 - (b) Give an example to show that $K = \{x \in G \mid |x| \text{ is 1 or even}\}$ need not be subgroup of G .
2. Show that $U(n)$ is a group under multiplication modulo n .
3. Find a noncyclic subgroup of order 4 in $U(36)$. (*Hint: Use Homework 2 Problem 3 for inspiration.*)
4. Determine the subgroup lattice of \mathbb{Z}_{16} . Generalizer to \mathbb{Z}_{p^n} where p is prime and n is some positive integer. (No justification required.)
5. Suppose that G is a group with more than one element. If the only subgroups of G are $\{e\}$ and G , prove that G is cyclic and has prime order. (Do not assume from the start that G is finite.)