1.

$$MSE[\hat{\theta}] = V[\hat{\theta}] + Bias[\hat{\theta}]^2 \tag{1}$$
$$Var[X] = E[X^2] - E[X]^2 \tag{2}$$

(a)

$$\hat{\theta} = \frac{Y_1 + 2Y_2}{3}$$
$$\hat{\theta} = \frac{Y_1}{3} + \frac{2Y_2}{3}$$
$$E[\hat{\theta}] = E[\frac{Y_1}{3} + \frac{2Y_2}{3}]$$
$$E[\hat{\theta}] = \frac{\theta}{3} + \frac{2\theta}{3}]$$
$$E[\hat{\theta}] = \theta$$
$$Bias[\hat{\theta}] = 0$$
$$Var[\hat{\theta}] = Var[\frac{Y_1}{3} + \frac{2Y_2}{3}]$$
$$Var[\hat{\theta}] = \frac{1}{9}Var[Y_1 + 2Y_2]$$
$$Var[\hat{\theta}] = \frac{1}{9}(\theta^2 + 4\theta^2]$$
$$Var[\hat{\theta}] = \frac{5\theta^2}{9}$$

By (1):

$$MSE[\hat{\theta}] = \frac{5\theta^2}{9} + 0^2$$
$$MSE[\hat{\theta}] = \frac{5\theta^2}{9}$$

(b)

$$\hat{\theta} = \bar{Y}$$
$$E[\hat{\theta}] = \theta$$
$$Bias[\hat{\theta}] = 0$$
$$Var[\hat{\theta}] = \frac{\theta^2}{3}$$

By (1):

$$MSE[\hat{\theta}] = \frac{\theta^2}{3} + 0^2$$
$$MSE[\hat{\theta}] = \frac{\theta^2}{3}$$

(c)

$$\hat{\theta} = Y_1^2$$
$$E[\hat{\theta}] = E[Y_1^2]$$

By (2):

$$E[\hat{\theta}] = E[Y_1]^2 + Var[Y_1]$$
$$E[\hat{\theta}] = \theta^2 + \theta^2$$
$$E[\hat{\theta}] = 2\theta^2$$
$$Bias[\hat{\theta}] = 2\theta^2 - \theta$$

Then for the Variance by (2):

$$\begin{aligned} Var[\hat{\theta}] &= E[Y_1^4] - E[Y_1^2]^2 \\ &= 24\theta^4 - 4\theta^4 \\ &= 20\theta^4 \end{aligned}$$

$$\begin{aligned} MSE[\hat{\theta}] &= 20\theta^4 + 4\theta^4 - 4\theta^3 + \theta^2 \\ &= 24\theta^4 - 4\theta^3 + \theta^2 \end{aligned}$$

2. (a) $\hat{\lambda} = \frac{\bar{X}}{8}$
It is unbiased since we should expect this to be poisson with the same mean every hour of every work day.

(b)

$$E[C] = E[50X + 2X^2]$$
$$E[C] = 50E[X] + 2E[X^2]$$

By (2):

$$E[C] = 50 * 8 * \lambda + 2(V[X] + E[X]^2)$$
$$E[C] = 400 * \lambda + 2(V[X] + E[X]^2)$$
$$E[C] = 400\lambda + 2(8 * \lambda + (8 * \lambda)^2)$$
$$E[C] = 416\lambda + 128\lambda^2$$

3. (a)

$$E[\bar{Y}] = a - \frac{1}{2}$$
$$Bias[\bar{Y}] = -\frac{1}{2}$$

(b)

$$\hat{a} = \bar{Y} + \frac{1}{2}$$

(c)

$$
\begin{aligned}
Var[\bar{Y}] &= \frac{1}{12} \\
MSE[\bar{Y}] &= \frac{1}{12} + (-\frac{1}{2})^2 \\
MSE[\bar{Y}] &= \frac{1}{12} + \frac{1}{4} \\
MSE[\bar{Y}] &= \frac{4}{12} \\
MSE[\bar{Y}] &= \frac{1}{3}
\end{aligned}
$$

4.

$$
\begin{aligned}
Bias[\hat{p_1}] &= 0 \\
Var[\hat{p_1}] &= \frac{p(1-p)}{n} \\
MSE[\hat{p_1}] &= \frac{p(1-p)}{n}
\end{aligned}
$$

$$
\begin{aligned}
Bias[\hat{p_2}] &= \frac{np+1}{n+2} - p \\
&= \frac{np+1}{n+2} - \frac{np+2p}{n+2} \\
&= \frac{1-2p}{n+2} \\
Var[\hat{p_2} &= \frac{np(1-p)}{(n+2)^2} + \frac{1}{(n+2)^2} \\
MSE[\hat{p_2} &= \frac{np(1-p)+1}{(n+2)^2} + (\frac{1-2p}{n+2})^2
\end{aligned}
$$

Simulation

1. .000292
   .2547849
2. 1.5 and .255
   Simulated vales are very close
   1.500292 and .25478486
3. .05287404
   .06467135
4. 1 and .05
   1.05287404 and .0618756832