

1.

$$MSE[\hat{\theta}] = V[\hat{\theta}] + Bias[\hat{\theta}]^2 \quad (1)$$

$$Var[X] = E[X^2] - E[X]^2 \quad (2)$$

(a)

$$\hat{\theta} = \frac{Y_1 + 2Y_2}{3}$$

$$\hat{\theta} = \frac{Y_1}{3} + \frac{2Y_2}{3}$$

$$E[\hat{\theta}] = E\left[\frac{Y_1}{3} + \frac{2Y_2}{3}\right]$$

$$E[\hat{\theta}] = \frac{\theta}{3} + \frac{2\theta}{3}$$

$$E[\hat{\theta}] = \theta$$

$$Bias[\hat{\theta}] = 0$$

$$Var[\hat{\theta}] = Var\left[\frac{Y_1}{3} + \frac{2Y_2}{3}\right]$$

$$Var[\hat{\theta}] = \frac{1}{9}Var[Y_1 + 2Y_2]$$

$$Var[\hat{\theta}] = \frac{1}{9}(\theta^2 + 4\theta^2)$$

$$Var[\hat{\theta}] = \frac{5\theta^2}{9}$$

By (1):

$$MSE[\hat{\theta}] = \frac{5\theta^2}{9} + 0^2$$

$$MSE[\hat{\theta}] = \frac{5\theta^2}{9}$$

(b)

$$\hat{\theta} = \bar{Y}$$

$$E[\hat{\theta}] = \theta$$

$$Bias[\hat{\theta}] = 0$$

$$Var[\hat{\theta}] = \frac{\theta^2}{3}$$

By (1):

$$MSE[\hat{\theta}] = \frac{\theta^2}{3} + 0^2$$

$$MSE[\hat{\theta}] = \frac{\theta^2}{3}$$

(c)

$$\hat{\theta} = Y_1^2$$

$$E[\hat{\theta}] = E[Y_1^2]$$

By (2):

$$E[\hat{\theta}] = E[Y_1]^2 + Var[Y_1]$$

$$E[\hat{\theta}] = \theta^2 + \theta^2$$

$$E[\hat{\theta}] = 2\theta^2$$

Then for the Variance:

$$Var[\hat{\theta}] = (Y_1^2 - \theta)^2 P(Y = Y_1^2)$$

$$Var[\hat{\theta}] = (Y_1^2 - \theta)^2 e^{\frac{-Y_1^2}{\theta}}$$

2. (a) $\hat{\lambda} = \frac{\bar{X}}{8}$

It is unbiased since we should expect this to be poisson with the same mean every hour of every work day.

(b)

$$E[C] = E[50X + 2X^2]$$

$$E[C] = 50E[X] + 2E[X^2]$$

By (2):

$$E[C] = 50 * 8 * \lambda + 2(V[X] + E[X]^2)$$

$$E[C] = 400 * \lambda + 2(V[X] + E[X]^2)$$

$$E[C] = 400\lambda + 2(8 * \lambda + (8 * \lambda)^2)$$

$$E[C] = 416\lambda + 128\lambda^2$$

3. (a)

$$E[\bar{Y}] = a - \frac{1}{2}$$

$$Bias[\bar{Y}] = -\frac{1}{2}$$

(b)

$$\hat{a} = \bar{Y} + \frac{1}{2}$$

(c)

$$\begin{aligned}
 \text{Var}[\bar{Y}] &= \frac{1}{12} \\
 \text{MSE}[\bar{Y}] &= \frac{1}{12} + \left(-\frac{1}{2}\right)^2 \\
 \text{MSE}[\bar{Y}] &= \frac{1}{12} + \frac{1}{4} \\
 \text{MSE}[\bar{Y}] &= \frac{4}{12} \\
 \text{MSE}[\bar{Y}] &= \frac{1}{3}
 \end{aligned}$$

4.

$$\begin{aligned}
 \text{Bias}[\hat{p}_1] &= 0 \\
 \text{Var}[\hat{p}_1] &= \frac{p(1-p)}{n} \\
 \text{MSE}[\hat{p}_1] &= \frac{p(1-p)}{n} \\
 \text{Bias}[\hat{p}_2] &= \frac{np+1}{n+2} - p \\
 &= \frac{np+1}{n+2} - \frac{np+2p}{n+2} \\
 &= \frac{1-2p}{n+2} \\
 \text{Var}[\hat{p}_2] &= \frac{np(1-p)}{(n+2)^2} + \frac{1}{(n+2)^2} \\
 \text{MSE}[\hat{p}_2] &= \frac{np(1-p)+1}{(n+2)^2} + \left(\frac{1-2p}{n+2}\right)^2
 \end{aligned}$$

Simulation

1. .000292
.2547849
2. 1.5 and .255
Simulated vales are very close
1.500292 and .25478486
3. .05287404
.06467135
4. 1 and .05
1.05287404 and .0618756832