1. (a)

$$\begin{aligned}
E[Y_{(1)}] &= \int_0^\infty (y * \frac{d}{dy} F_{Y_{(1)}}(y))dy \\
&= \int_0^\infty (\frac{-ny}{\theta} e^{\frac{-ny}{\theta}})dy \\
&= \int_0^\infty (ye^{\frac{-ny}{\theta}})dy \\
&= 0 + 0 - 0 - \frac{-\theta}{n} \\
&= \frac{\theta}{n}
\end{aligned}$$

Unbiased after a sample size of one.

(b) Multiply by the sample size to receive an unbiased estimator for θ.

$$\begin{aligned}
E[Y_{(1)}] &= \int_0^\infty (n * y * \frac{d}{dy} F_{Y_{(1)}}(y))dy \\
&= \int_0^\infty (\frac{-n^2 y}{\theta} e^{\frac{-ny}{\theta}})dy \\
&= \int_0^\infty (yne^{\frac{-ny}{\theta}})dy \\
&= n(0 + 0 - 0 - \frac{-\theta}{n}) \\
&= \theta
\end{aligned}$$

2. If we show:

$$\lim_{n\to\infty} Pr(|\hat{p} - p| > \epsilon) = 0, \forall \epsilon > 0$$

then $\hat{p}$ is consistent for p.
$\bar{X}$ is consistent for μ, therefore we can write;

$$\lim_{n\to\infty} Pr(|n\hat{p} - np| > \epsilon) = 0, \forall \epsilon > 0$$
$$\lim_{n\to\infty} Pr(|\hat{p} - p|\,|n| > \epsilon) = 0, \forall \epsilon > 0$$

n is always a natural number, therefore;

$$\lim_{n\to\infty} Pr(|\hat{p} - p|\,n > \epsilon) = 0, \forall \epsilon > 0$$
$$\lim_{n\to\infty} Pr(|\hat{p} - p| > \frac{\epsilon}{n}) = 0, \forall \epsilon > 0$$

Then without loss of generality for ϵ;

$$\lim_{n\to\infty} Pr(|\hat{p} - p| > \epsilon) = 0, \forall \epsilon > 0$$

$\hat{p}$ is consistent for p, then.

3. (a)

$$\int_0^1 (y\theta y^{\theta-1})dy = E[Y]$$
$$\int_0^1 (\theta y^{\theta})dy = E[Y]$$
$$\int_0^1 (\theta y^{\theta})dy = E[Y]$$
$$\frac{\theta y^{\theta+1}}{\theta+1}]_0^1 = E[Y]$$
$$\frac{\theta}{\theta+1} = E[Y]$$

   (b) $\bar{X}$ is consistent for μ by WLoLN
   So $\bar{Y}$ is consistent for $\frac{\theta}{\theta+1}$

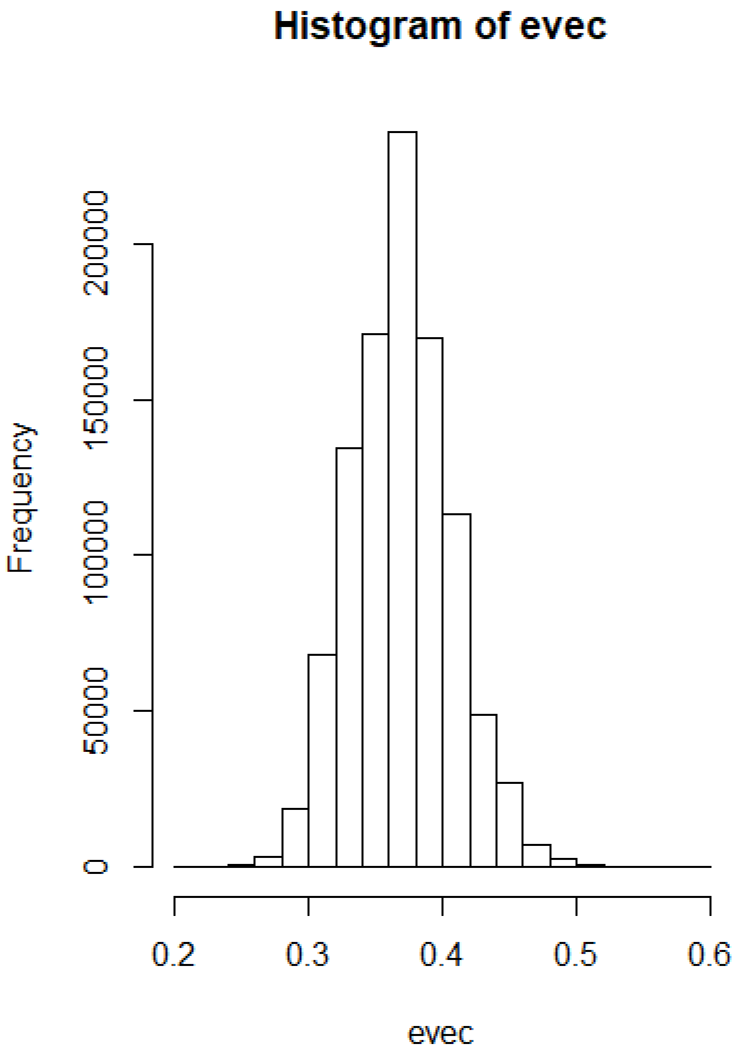
4. (a) It looks normal with the proper symmetry and the fifteen percent variance.

   (b)

$$\sqrt{n}(\bar{X} - \lambda) \to N(0, \sigma^2(x))$$
$$\sqrt{100}(\bar{X} - \lambda) \to N(1, 1)$$
$$(\bar{X} - \lambda) \to N(1, \frac{1}{100})$$

   (c) $\bar{X} = 1.00004195$
   $\bar{X} - X \approx 0$
   $Var[X] = .009978$
   These match.

5. (a) It sure looks normal to me. It's even more symmetric than the last one with a similar quartile range.

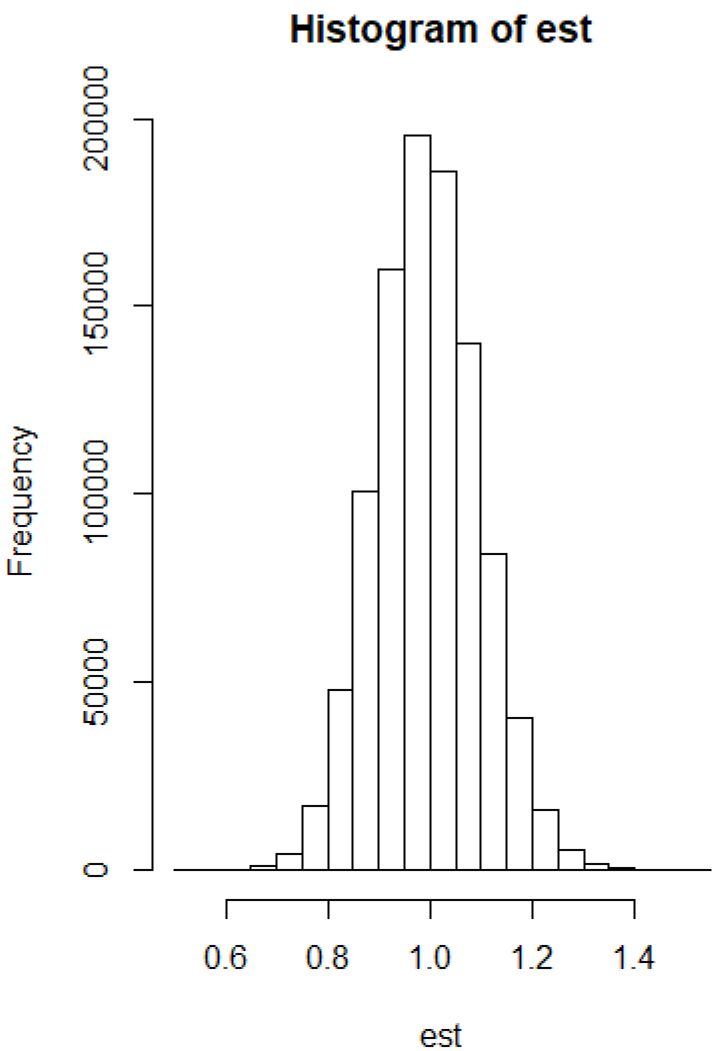


(b)

$$\sqrt{n}(\bar{X}-\lambda) \to N(e^{-1}, \sigma^2(x)\frac{d}{dx}(e^{-x})^2)$$
$$(\bar{X}-\lambda) \to N(e^{-1}, \frac{1}{100}(-e^{-1})^2)$$
$$(\bar{X}-\lambda) \to N(e^{-1}, \frac{e^{-2}}{100})$$

(c) $\bar{X} = .369698$
$\bar{X} - \lambda \approx 0$

$Var[X] = .001357471$
These match.


Histogram of est
Frequency
0
50000
100000
150000
200000
0.6
0.8
1.0
1.2
1.4
est