

1. (a)

$$\begin{aligned}
 E[Y_{(1)}] &= \int_0^\infty (y * \frac{d}{dy} F_{Y_{(1)}}(y)) dy \\
 &= \int_0^\infty (\frac{-ny}{\theta} e^{\frac{-ny}{\theta}}) dy \\
 &= ye^{\frac{-ny}{\theta}} \Big|_0^\infty - \int_0^\infty (e^{\frac{-ny}{\theta}}) dy \\
 &= 0 - 0 + \frac{-\theta}{n} e^{\frac{-ny}{\theta}} \Big|_0^\infty \\
 &= 0 - \frac{-\theta}{n} \\
 &= \frac{\theta}{n}
 \end{aligned}$$

Unbiased after a sample size of one.

(b) Multiply by the sample size to receive an unbiased estimator for  $\theta$ .

$$\begin{aligned}
 E[Y_{(1)}] &= \int_0^\infty (n * y * \frac{d}{dy} F_{Y_{(1)}}(y)) dy \\
 &= \int_0^\infty (\frac{-n^2 y}{\theta} e^{\frac{-ny}{\theta}}) dy \\
 &= n(0 + 0 - 0 - \frac{-\theta}{n}) \\
 &= \theta
 \end{aligned}$$

2. If we show:

$$\lim_{n \rightarrow \infty} Pr(|\hat{p} - p| > \epsilon) = 0, \forall \epsilon > 0$$

then  $\hat{p}$  is consistent for  $p$ .

$\bar{X}$  is consistent for  $\mu$ , therefore we can write;

$$\begin{aligned}
 \lim_{n \rightarrow \infty} Pr(|n\hat{p} - np| > \epsilon) &= 0, \forall \epsilon > 0 \\
 \lim_{n \rightarrow \infty} Pr(|\hat{p} - p| |n| > \epsilon) &= 0, \forall \epsilon > 0
 \end{aligned}$$

$n$  is always a natural number, therefore;

$$\begin{aligned}
 \lim_{n \rightarrow \infty} Pr(|\hat{p} - p| n > \epsilon) &= 0, \forall \epsilon > 0 \\
 \lim_{n \rightarrow \infty} Pr(|\hat{p} - p| > \frac{\epsilon}{n}) &= 0, \forall \epsilon > 0
 \end{aligned}$$

Then without loss of generality for  $\epsilon$ ;

$$\lim_{n \rightarrow \infty} Pr(|\hat{p} - p| > \epsilon) = 0, \forall \epsilon > 0$$

$\hat{p}$  is consistent for  $p$ , then.

3. (a)

$$\int_0^1 (y\theta y^{\theta-1})dy = E[Y]$$

$$\int_0^1 (\theta y^{\theta})dy = E[Y]$$

$$\int_0^1 (\theta y^{\theta})dy = E[Y]$$

$$\frac{\theta y^{\theta+1}}{\theta+1} \Big|_0^1 = E[Y]$$

$$\frac{\theta}{\theta+1} = E[Y]$$

(b)  $\bar{X}$  is consistent for  $\mu$  by WLoLN  
 So  $\bar{Y}$  is consistent for  $\frac{\theta}{\theta+1}$

4. (a) It looks normal with the proper symmetry and the fifteen percent variance.

(b)

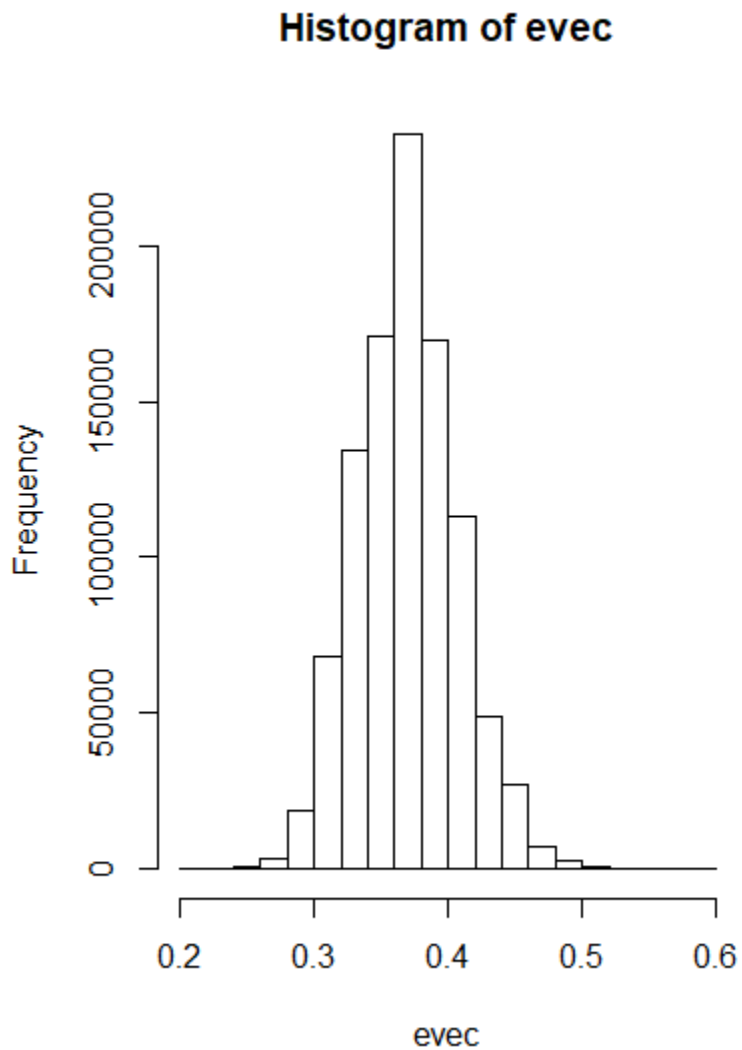
$$\sqrt{n}(\bar{X} - \lambda) \rightarrow N(0, \sigma^2(x))$$

$$\sqrt{100}(\bar{X} - \lambda) \rightarrow N(1, 1)$$

$$(\bar{X} - \lambda) \rightarrow N(1, \frac{1}{100})$$

(c)  $\bar{X} = 1.00004195$   
 $\bar{X} - X \approx 0$   
 $Var[X] = .009978$   
 These match.

5. (a) It sure looks normal to me. It's even more symmetric than the last one with a similar quartile range.



(b)

$$\sqrt{n}(\bar{X} - \lambda) \rightarrow N(e^{-1}, \sigma^2(x) \frac{d}{dx}(e^{-x})^2)$$

$$(\bar{X} - \lambda) \rightarrow N(e^{-1}, \frac{1}{100}(-e^{-1})^2)$$

$$(\bar{X} - \lambda) \rightarrow N(e^{-1}, \frac{e^{-2}}{100})$$

(c)  $\bar{X} = .369698$   
 $\bar{X} - \lambda \approx 0$

$Var[X] = .001357471$   
These match.

