1. (a)

$$\begin{aligned} E[Y_{(1)}] &= \int_0^\infty (1 - F_{Y_{(1)}}(y))dy \\ &= \int_0^\infty (1 - 1 + e^{\frac{-ny}{\theta}})dy \\ &= \int_0^\infty (e^{\frac{-ny}{\theta}})dy \\ &= 0 - \frac{-\theta}{n} \\ &= \frac{\theta}{n} \end{aligned}$$

Unbiased after a sample size of one.

(b) Multiply by the sample size to receive an unbiased estimator for θ.

2. If we show:

$$\lim_{n\to\infty} Pr(|\hat{p} - p| > \epsilon) = 0, \forall\epsilon > 0$$

then $\hat{p}$ is consistent for p.
$\bar{X}$ is consistent for μ, therefore we can write;

$$\lim_{n\to\infty} Pr(|n\hat{p} - np| > \epsilon) = 0, \forall\epsilon > 0$$
$$\lim_{n\to\infty} Pr(|\hat{p} - p|\,|n| > \epsilon) = 0, \forall\epsilon > 0$$

n is always a natural number, therefore;

$$\lim_{n\to\infty} Pr(|\hat{p} - p|\,n > \epsilon) = 0, \forall\epsilon > 0$$
$$\lim_{n\to\infty} Pr(|\hat{p} - p| > \frac{\epsilon}{n}) = 0, \forall\epsilon > 0$$

Then without loss of generality for ϵ;

$$\lim_{n\to\infty} Pr(|\hat{p} - p| > \epsilon) = 0, \forall\epsilon > 0$$

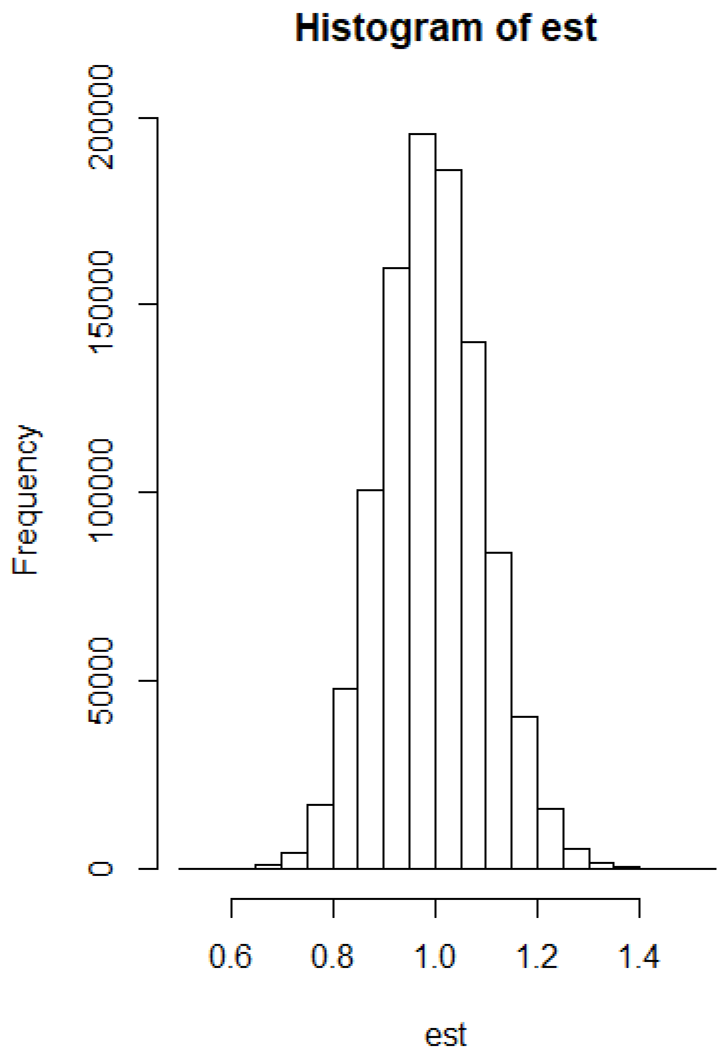
$\hat{p}$ is consistent for p, then.

3. (a)

$$\begin{aligned} \int_0^1 (y\theta y^{\theta-1})dy &= E[Y] \\ \int_0^1 (\theta y^{\theta})dy &= E[Y] \\ \int_0^1 (\theta y^{\theta})dy &= E[Y] \\ \frac{\theta y^{\theta+1}}{\theta+1}]_0^1 &= E[Y] \\ \frac{\theta}{\theta+1} &= E[Y] \end{aligned}$$

(b) $\bar{X}$ is consistent for μ by WLoLN
So $\bar{Y}$ is consistent for $\frac{\theta}{\theta+1}$

4. (a) It looks normal with the proper symmetry and the fifteen percent variance.

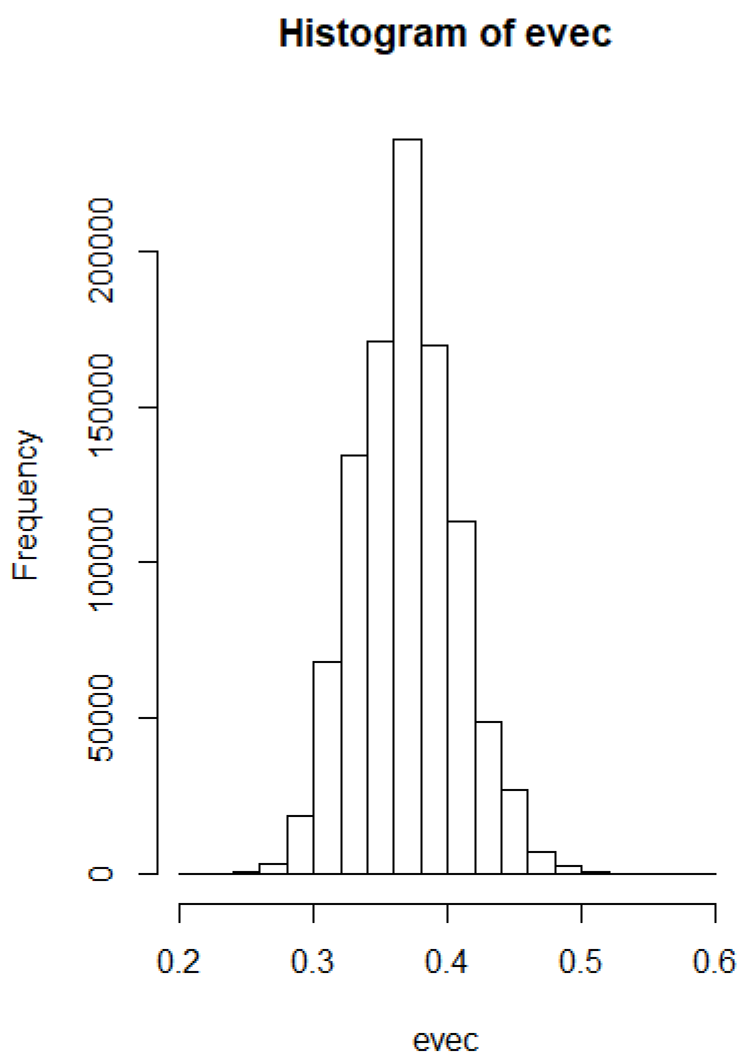


(b)

$$\sqrt{n}(\bar{X} - X) \to N(0, \sigma^2(x))$$
$$\sqrt{100}(\bar{X} - X) \to N(0, 1)$$
$$(\bar{X} - X) \to N(0, \frac{1}{100})$$

(c) $\bar{X} = 1.00004195$
$\bar{X} - X \approx 0$
$Var[X] = .009978$
These match.

5. (a) It sure looks normal to me. It's even more symmetric than the last one with a similar quartile range.



(b)

$$\sqrt{n}(\bar{X} - \lambda) \to N(0, \sigma^2(x))$$
$$(\bar{X} - \lambda) \to N(0, 1)$$
$$(\bar{X} - \lambda) \to N(\frac{\sqrt{10}}{10}, \frac{1}{100})$$

(c) $\bar{X} = .369698$
$\bar{X} - \lambda \approx 0$
$Var[X] = .001357471$