1.

$$\begin{aligned}
f(y) &= (\theta+1)y^{\theta} \\
\mu = \int_{\mathcal{D}} f(y)dy &= \int_0^1 (\theta+1)y^{\theta} y dy \\
&= \int_0^1 (\theta+1)y^{\theta+1} dy \\
&= \frac{(\theta+1)}{(\theta+2} y^{\theta+2}]_0^1 \\
&= \frac{(\theta+1)}{(\theta+2} \\
\bar{Y} &= \frac{(\theta+1)}{(\theta+2} \\
\bar{Y}(\theta+2) &= \theta+1 \\
\bar{Y}\theta + 2\bar{Y} &= \theta+1 \\
\bar{Y}\theta - \theta &= 1 - 2\bar{Y} \\
\theta(\bar{Y}-1) &= 1 - 2\bar(Y) \\
\hat{\theta} &= \frac{1-2\bar{Y}}{\bar{Y}-1}
\end{aligned}$$

2. (a)

$$l(\theta; X) = \binom{100}{33}\theta^{33}(1-\theta)^{100-33}$$

(b)

$$\begin{aligned}
\mu &= n\theta \\
\frac{\bar{X}}{100} &= \hat{\theta}
\end{aligned}$$

3. (a)

$$\begin{aligned}
\mu &= \frac{1}{\lambda} \\
\bar{X} &= \frac{1}{\hat{\lambda}} \\
\frac{1}{\bar{X}} &= \hat{\lambda}
\end{aligned}$$

(b)

$$l(\lambda; X) = \prod_{n=1}^{N} \lambda e^{-\lambda X_n}$$
$$ln(l(\lambda; X) = ln(\prod_{n=1}^{N} \lambda e^{-\lambda X_n})$$
$$ln(l(\lambda; X) = N ln(\lambda) - \lambda \sum_{n=1}^{N} X_n$$
$$\frac{d}{d\lambda} ln(l(\lambda; X) = \frac{d}{d\lambda}[N ln(\lambda) - \lambda \sum_{n=1}^{N} X_n]$$
$$\frac{d}{d\lambda} ln(l(\lambda; X) = \frac{N}{\lambda} - \sum_{n=1}^{N} X_n$$
$$\frac{d}{d\lambda} ln(l(\lambda; X) = 0$$
$$0 = \frac{N}{\lambda} - \sum_{n=1}^{N} X_n$$
$$\sum_{n=1}^{N} X_n = \frac{N}{\lambda}$$
$$\frac{\sum_{n=1}^{N} X_n}{N} = \frac{1}{\lambda}$$
$$\frac{1}{\bar{X}} = \hat{\lambda}$$

(c)

$$\sqrt{n}(g(\hat{\lambda}) - g(\mu)) \to N(0, g'(\mu)^2 \sigma^2)$$
$$g'(\mu) = -\mu^{-2}$$
$$g'(\frac{1}{\lambda}) = -\lambda^2$$
$$\sqrt{n}(\frac{1}{\bar{X}} - \lambda) \to N(0, \lambda^2)$$

4. (a)

$$P(X = 1) = \lambda e^{-\lambda}$$

(b)

$$\hat{\lambda} = \frac{1}{n}$$

(c)

$$\sqrt{n}(\frac{1}{n} - 1) \to N(0, n)$$

(d) $n > 0$

(e)

(f)

(g)