1.

$$
\begin{aligned}
f(y) &= (\theta+1)y^{\theta} \\
\mu = \int_{\mathcal{D}} f(y)dy &= \int_0^1 (\theta+1)y^{\theta} y dy \\
&= \int_0^1 (\theta+1)y^{\theta+1} dy \\
&= \frac{(\theta+1)}{(\theta+2} y^{\theta+2}]_0^1 \\
&= \frac{(\theta+1)}{(\theta+2} \\
\bar{Y} &= \frac{(\theta+1)}{(\theta+2} \\
\bar{Y}(\theta+2) &= \theta+1 \\
\bar{Y}\theta + 2\bar{Y} &= \theta+1 \\
\bar{Y}\theta - \theta &= 1 - 2\bar{Y} \\
\theta(\bar{Y}-1) &= 1 - 2\bar(Y) \\
\hat{\theta} &= \frac{1-2\bar{Y}}{\bar{Y}-1}
\end{aligned}
$$

2. (a)

$$
l(\theta; X) = \binom{100}{33}\theta^{33}(1-\theta)^{100-33}
$$

(b)

$$
\begin{aligned}
\mu &= n\theta \\
\frac{\bar{X}}{100} &= \hat{\theta}
\end{aligned}
$$

3. (a)

$$
\begin{aligned}
\mu &= \frac{1}{\lambda} \\
\bar{X} &= \frac{1}{\hat{\lambda}} \\
\frac{1}{\bar{X}} &= \hat{\lambda}
\end{aligned}
$$

(b)

$$l(\lambda;X)=\prod_{n=1}^{N}\lambda e^{-\lambda X_n}$$
$$ln(l(\lambda;X)=ln(\prod_{n=1}^{N}\lambda e^{-\lambda X_n})$$
$$ln(l(\lambda;X)=Nln(\lambda)-\lambda\sum_{n=1}^{N}X_n$$
$$\frac{d}{d\lambda}ln(l(\lambda;X)=\frac{d}{d\lambda}[Nln(\lambda)-\lambda\sum_{n=1}^{N}X_n]$$
$$\frac{d}{d\lambda}ln(l(\lambda;X)=\frac{N}{\lambda}-\sum_{n=1}^{N}X_n$$
$$\frac{d}{d\lambda}ln(l(\lambda;X)=0$$
$$0=\frac{N}{\lambda}-\sum_{n=1}^{N}X_n$$
$$\sum_{n=1}^{N}X_n=\frac{N}{\lambda}$$
$$\frac{\sum_{n=1}^{N}X_n}{N}=\frac{1}{\lambda}$$
$$\frac{1}{\bar{X}}=\hat{\lambda}$$

(c)

$$\sqrt{n}(g(\hat{\lambda})-g(\mu))\to N(0,g'(\mu)^2\sigma^2)$$
$$g'(\mu)=-\mu^{-2}$$
$$g'(\frac{1}{\lambda})=-\lambda^2$$
$$\sqrt{n}(\frac{1}{\bar{X}}-\lambda)\to N(0,\lambda^2)$$

4. (a)

$$P(X=1)=\lambda e^{-\lambda}$$

(b)

$$M\hat{L}E=\bar{X}e^{-\bar{X}}$$

(c)

$$\sqrt{n}(\bar{X}e^{-\bar{X}}-\lambda e^{-\lambda})\to N(0,\lambda(-\lambda e^{-\lambda}+e^{-\lambda})^2)$$

(d) $\lambda \neq 1$

(e)

$$M\hat{L}E = \bar{X}^2$$

(f)

$$\sqrt{n}(\bar{X}^2 - \lambda^2) \to N(0, \lambda 4\lambda^2)$$

(g) $\lambda > 0$