1. (a)

$$\begin{aligned}
f(\sigma;X) &= \frac{1}{\sigma^2\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{X_i-\mu}{\sigma})^2}\\
l(\sigma;X) &= -\frac{1}{2}log(\sigma^2) - \frac{1}{2}log(2\pi) - \frac{1}{2}(\frac{X_i-\mu}{\sigma})^2\\
l'(\sigma;X) &= -\frac{1}{\sigma} - \frac{-2}{2}\frac{(X_i-\mu)^2}{\sigma^3}\\
l''(\sigma;X) &= \frac{1}{(\sigma)^2} - 3\frac{(X_i-\mu)^2}{\sigma^4}\\
-E[l''(\sigma;X)] &= \frac{2}{\sigma^2}
\end{aligned}$$

(b)

$$\begin{aligned}
L(\sigma;X) &= \prod_{i=1}^{N}\frac{1}{\sigma^2\sqrt{2\pi}}e^{-\frac{1}{2}(\frac{X_i-\mu}{\sigma})^2}\\
l(\sigma;X) &= -\frac{N}{2}log(\sigma^2) - \frac{N}{2}log(2\pi) - \sum_{i=1}^{N}\frac{1}{2}(\frac{X_i-\mu}{\sigma})^2\\
l'(\sigma;X) &= \frac{-N}{\sigma} + \sum_{i=1}^{N}\frac{(X_i-\mu)^2}{\sigma^3}\\
l'(\sigma;X) &= 0\\
0 &= \frac{-N}{\sigma} + \sum_{i=1}^{N}\frac{(X_i-\mu)^2}{\sigma^3}\\
\sigma^2 &= \sum_{i=1}^{N}\frac{(X_i-\mu)^2}{N}\\
\hat{\sigma} &= \sum_{i=1}^{N}\frac{(X_i-\mu)}{\sqrt{N}}
\end{aligned}$$

(c)

$$\sqrt{n}(\hat{\sigma}-\sigma) \rightarrow N(0,\frac{\sigma^2}{2})$$

2. (a)

$$\sqrt{n}(\hat{\lambda}-\lambda) \rightarrow N(0,\lambda^2)$$

(b) .9992687

3. (a)

$$\sqrt{n}(\bar{Y}-\mu) \rightarrow N(0,\sigma^2)$$

(b)

$$(\bar{Y} - \mu) \rightarrow N(0, \frac{\sigma^2}{n})$$

Therefore it is asymptotically efficient.

(c)

$$\sqrt{n}(\bar{Y} - \lambda) \rightarrow N(0, \lambda)$$

(d)

$$(\bar{Y} - \lambda) \rightarrow N(0, \frac{\lambda}{n})$$

Therefore it is asymptotically efficient.

4. (a)

$$\begin{aligned}
L(\theta; X) &= \prod_{i=1}^{N} \frac{1}{2}\theta^3 x^2 e^{-\theta x} \\
l(\theta; X) &= \sum_{i=1}^{N} log(\frac{1}{2}) + 3log(\theta) + 2log(x_i) - \theta x_i \\
l'(\theta; X) &= 0 \\
0 &= 3N\frac{1}{\theta} - \sum_{i=1}^{N} x_i \\
\hat{\theta} &= \frac{3}{\bar{X}}
\end{aligned}$$

(b)

$$\begin{aligned}
L(\theta; X) &= \frac{1}{2}\theta^3 x^2 e^{-\theta x} \\
l(\theta; X) &= log(\frac{1}{2}) + 3log(\theta) + 2log(x_i) - \theta x_i \\
l''(\theta; X) &= -3\frac{1}{\theta^2} \\
-E(l''(\theta; X) &= 3\frac{1}{\theta^2} \\
\sqrt{n}(\hat{\theta} - \frac{3}{\theta}) &\rightarrow N(0, \frac{\theta^2}{3})
\end{aligned}$$

(c)

$$E[X] = \frac{3}{\theta}$$
$$V[X] = \frac{3}{\theta^2}$$
$$g(\mu) = \frac{3}{\mu}$$
$$g'(\mu) = \frac{3}{\mu^2}$$
$$g'(\frac{3}{\theta}) = \frac{\theta^2}{3}$$
$$\sqrt{n}(\hat{\theta} - \frac{3}{\theta}) \to N(0, \frac{3}{\theta^2}\frac{\theta^4}{9})$$
$$\sqrt{n}(\hat{\theta} - \frac{3}{\theta}) \to N(0, \frac{\theta^2}{3})$$

(d) Yes.

(e) When finding the standard deviation is hard and involves a lot of calculus.