1. (a)

$$Z = \frac{6-2}{2\sqrt{2}}$$
$$Z = \sqrt{2}$$
$$P(x > Z) = .07927$$

(b) Normal with:

$$\mu = \frac{8X + 4 * 2}{4+8}$$
$$\mu = \frac{2X+2}{3}$$
$$\sigma^2 = \frac{4*8}{8+4}$$
$$\sigma^2 = \frac{32}{12}$$
$$\sigma^2 = \frac{8}{3}$$

(c)

$$\mu = \frac{2(2)+2}{3}$$
$$\mu = 2$$
$$Z = \frac{6-2}{\sqrt{\frac{8}{3}}}$$
$$Z = 2.449$$
$$P(x > 2.449) = .00714$$

(d)

$$\hat{\theta} = \frac{2X+2}{3}$$

(e) Yes. Gaussian likelihood with Gaussian Prior and Gaussian Posterior.

2. (a)

$$\mu = \frac{1}{1+3}$$
$$\mu = \frac{1}{4}$$

(b)

$$\alpha = 1 + X$$
$$\beta = 3 + 20 - X$$
$$\beta = 23 - X$$

(c)

$$\hat{\theta} = \frac{\bar{X}}{20}$$
$$\hat{\theta} = \frac{X}{20}$$

(d)

$$\begin{aligned}
\hat{\theta}_{Bays'} &= \frac{1+X}{24} \\
\hat{\theta}_{Bays'} &= \hat{\theta}_{MLE} \\
\frac{X}{20} &= \frac{1+X}{24} \\
24X &= 20 + 20X \\
X &= 5 \\
\hat{\theta}_{Bays'} &< \hat{\theta}_{MLE}, X \in (5, \infty) \\
\hat{\theta}_{Bays'} &> \hat{\theta}_{MLE}, X \in (-\infty, 5)
\end{aligned}$$

(e)

$$\hat{\theta}_{Bays'} = \frac{1+X}{24}$$

3.

$$\begin{aligned}
\Gamma(1) &= 1 \\
\Gamma(2) &= 1 \\
B(1,1) &= \frac{x^{1-1}(1-x)^{1-1}}{\frac{\Gamma(1)\Gamma(1)}{\Gamma(1+1)}} I(x)_{0,1} \\
B(1,1) &= \frac{(1)(1)}{\frac{(1)(1)}{(1)}} I(x)_{0,1} \\
B(1,1) &= 1I(x)_{0,1} \\
B(1,1) &= U(0,1)
\end{aligned}$$

4. (a) X $\sim$ Bin(50,θ), X = 3, U(0,1) = B(1,1)

$$\begin{aligned}
B(\alpha + X, \beta + n - x) &= B(1+3, 1+50-3) \\
&= B(4, 48)
\end{aligned}$$

(b)

$$\hat{\theta}_{Bays'} = \frac{4}{52}$$