1. The function:

$$f(t,u) = u^2 e^{-t^2} sin(t)$$

is Lipschitz continous in u on:

$$\mathcal{D} = \{(t,u) : t \in \mathbb{R}, u \in [0,2]\}$$

if

$$\begin{aligned} \exists L \in \mathbb{R}_+ : |f(t,u) - f(t,u^*)| &\leq L\,|u - u^*| \\ \left|u^2 e^{-t^2} sin(t) - u^{*2} e^{-t^2} sin(t)\right| &\leq L\,|u - u^*| \\ \left|u^2 - u^{*2}\right| \left|e^{-t^2} sin(t)\right| &\leq L\,|u - u^*| \\ |u - u^*|\,|u + u^*| \left|e^{-t^2} sin(t)\right| &\leq L\,|u - u^*| \end{aligned}$$

Since $f(t,u)$ is differentiable with respect to u and bounded in $\mathcal{D}$ then we can set the Lipschitz constant:

$$\begin{aligned} L &= \max_{(t,u)\in\mathcal{D}} |f_u(t,u)| \\ &= \max_{(t,u)\in\mathcal{D}} \left|2ue^{-t^2} sin(t)\right| \\ &\approx 4 * .397 \\ &\approx 1.588 \end{aligned}$$

2. (a) Since $f(u)$ is differentible at all points $u \in \mathbb{R}$, $f(u)$ is Lipshitz at every point $u \in \mathbb{R}$. With L $\leq$ 2.

   (b) Take Dirchlet's 'jagged' discontinuous function which is continuous nowhere and is differentiable nowhere but is nonetheless bounded by 1.

3.

$$U = \begin{bmatrix} u_1 & 0 & 0 & 0 \\ 0 & u_2 & 0 & 0 \\ 0 & 0 & u_3 & 0 \\ 0 & 0 & 0 & u_4 \end{bmatrix}$$

$$U' = \begin{bmatrix} u_2 & 0 & 0 & 0 \\ 0 & -\frac{u_1}{(u_1^2+u_3^2)^{3/2}} & 0 & 0 \\ 0 & 0 & u_4 & 0 \\ 0 & 0 & 0 & -\frac{u_3}{(u_1^2+u_3^2)^{3/2}} \end{bmatrix}$$