1. The function:

$$f(t,u) = u^2 e^{-t^2} sin(t)$$

is Lipschitz continous in u on:

$$\mathcal{D} = \{(t,u) : t \in \mathbb{R}, u \in [0,2]\}$$

if

$$\begin{aligned}
\exists L \in \mathbb{R}_+ : |f(t,u) - f(t,u^*)| &\leq L\,|u-u^*| \\
\left|u^2 e^{-t^2} sin(t) - u^{*2} e^{-t^2} sin(t)\right| &\leq L\,|u-u^*| \\
\left|u^2 - u^{*2}\right| \left|e^{-t^2} sin(t)\right| &\leq L\,|u-u^*| \\
|u-u^*|\,|u+u^*| \left|e^{-t^2} sin(t)\right| &\leq L\,|u-u^*|
\end{aligned}$$

Since $f(t,u)$ is differentiable with respect to u then we can set the Lipschitz constant:

$$\begin{aligned}
L &= \max_{(t,u)\in\mathcal{D}} |f_u(t,u)| \\
&= \max_{(t,u)\in\mathcal{D}} \left|2ue^{-t^2} sin(t)\right| \\
&\approx 4 * .397 \\
&\approx 1.588
\end{aligned}$$

2. (a) Since $f(u)$ is differentible at all points $u \in \mathbb{R}$ $f(u)$ is Lipshitz at every point $u \in \mathbb{R}$. With L $\leq$ 2.

   (b) Take Dirchlet's 'jaggged' discontinuous function which is continuous nowhere and is differentiable nowhere but is nonetheless bounded by 1.