1. (a)

$$G(x,\bar{x}) = \begin{cases} \bar{x}(x-\pi), \bar{x} \in [0,x] \\ x(\bar{x}-\pi), \bar{x} \in [x,\pi] \end{cases}$$
$$u(x) = \int_0^\pi G(x,\bar{x})(-f(\bar{x}))d\bar{x}$$

(b)

$$\begin{aligned} u''(x) &= -sin(x) \\ u(x) &= \int_0^\pi G(x,\bar{x})sin(\bar{x})d\bar{x} \\ u(x) &= \int_0^x \bar{x}sin(\bar{x})d\bar{x}(x-\pi) + \int_x^\pi (\bar{x}-\pi)sin(\bar{x})d\bar{x}x \\ u(x) &= [-\bar{x}cos(\bar{x}) + sin(\bar{x})]_0^x(x-\pi) + [sin(\bar{x}) - (\bar{x}-\pi)cos(\bar{x})]_x^\pi x \\ u(x) &= [-xcos(x) + sin(x)][x-\pi] + x[sinx - (x-\pi)cos(x)] \\ u(x) &= -x^2cos(x) + xsin(x) + \pi xcos(x) - \pi sin(x) + xsin(x) - x^2cos(x) + x\pi cos(x) \\ u(x) &= -2x^2cos(x) + 2xsin(x) + 2\pi xcos(x) - \pi sin(x) \end{aligned}$$

(c)

$$\begin{aligned} u(x) &= csin(x) \\ u(0) &= u(\pi) = 0 \\ u''(x) + u(x) &= 0 \\ -csin(x) + csin(x) &= 0 \end{aligned}$$

(d) As an ode we have (c) as a solution but the greens function is also a valid solution starting with two different linear approaches which satisfy the boundary conditions. We have two y intercepts from integrating twice and getting $c_1 + c_2x = y$ with two different c_1 and c_2 values. Since the two y equations are valid solutions, by superposition, the combination of the two are valid so long as they satisfy the boundary conditions.

2. (a)

(b)

(c)

3. (a)

(b)

4. (a)

(b)