MATH 437 HW3

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##1.i.
$T^2(X,\mu) = 1(\bar{X}-\mu)^T s(X)(\bar{X}-\mu)$ T squared ##1.ii.
$\frac{n-1}{n}(\bar{X}-\mu)^T\hat{\Sigma}^{-1}(X)(\bar{X}-\mu)$ ##1.iii.
##1.iii.a.
$A\bar{X}+a$ $A^T\Sigma(X)A$ ##1.iii.b.
$A^TS(X)A$ $n(A\bar{X}+a-\mu_y)^TA^TS^{-1}(X)A(A\bar{X}+a-\mu_y)$ ##1.iii.c.
##2.
$(1-\alpha)*100\%$
$X \pm Z_{1-\alpha/2} * S/\sqrt{n}$
$X \pm KS/\sqrt{n}$
$K = Z_{1-\alpha/2}$
$\phi(k) = 1-\alpha/2$
$\alpha = 2-2\phi(k)$
##3.
For n = 25 p = 3: $a^TX - 1.809F^{\alpha}_{3,22}\frac{\sqrt{a^T\Sigma a}}{25} \leq a^T\mu \leq a^TX + 1.809F^{\alpha}_{3,22}\frac{\sqrt{a^T\Sigma a}}{25}$
For n = 25 p = 2: $a^TX - 1.445F^{\alpha}_{2,23}\frac{\sqrt{a^T\Sigma a}}{25} \leq a^T\mu \leq a^TX + 1.445F^{\alpha}_{2,23}\frac{\sqrt{a^T\Sigma a}}{25}$
##4.a.

```
lizard <- read.csv("~/School/Math437/HW3/lizard.dat", sep="")
xbar = c(mean(lizard$mass),mean(lizard$svl),mean(lizard$hls))
xbar
```

```
## [1]   8.6786  68.4000 129.3200
```

```
S=cov(lizard[-1])
S
```

```
##           mass       svl       hls
## mass  7.292144  20.87785  33.80618
## svl  20.877854  63.77083 102.08542
## hls  33.806175 102.08542 185.83083
```

##b.
$n(\bar{X}-\mu)^TS^{-1}(X)(\bar{X}-\mu)$ ##c.

```
mu = c(10,70,140)
t((xbar-(mu)))%*%solve(S)%*%(xbar-(mu))
```

```
##          [,1]
## [1,] 3.943406
```

##d.

```r
library(DescTools)
```

```
## Warning: package 'DescTools' was built under R version 4.1.3
```

```r
HotellingsT2Test(lizard[-1],mu=mu)
```

```
##
##  Hotelling's one sample T2-test
##
## data:  lizard[-1]
## T.2 = 30.123, df1 = 3, df2 = 22, p-value = 5.676e-08
## alternative hypothesis: true location is not equal to c(10,70,140)
```

##e.

```r
S= cov(lizard[,3:4])
S
```

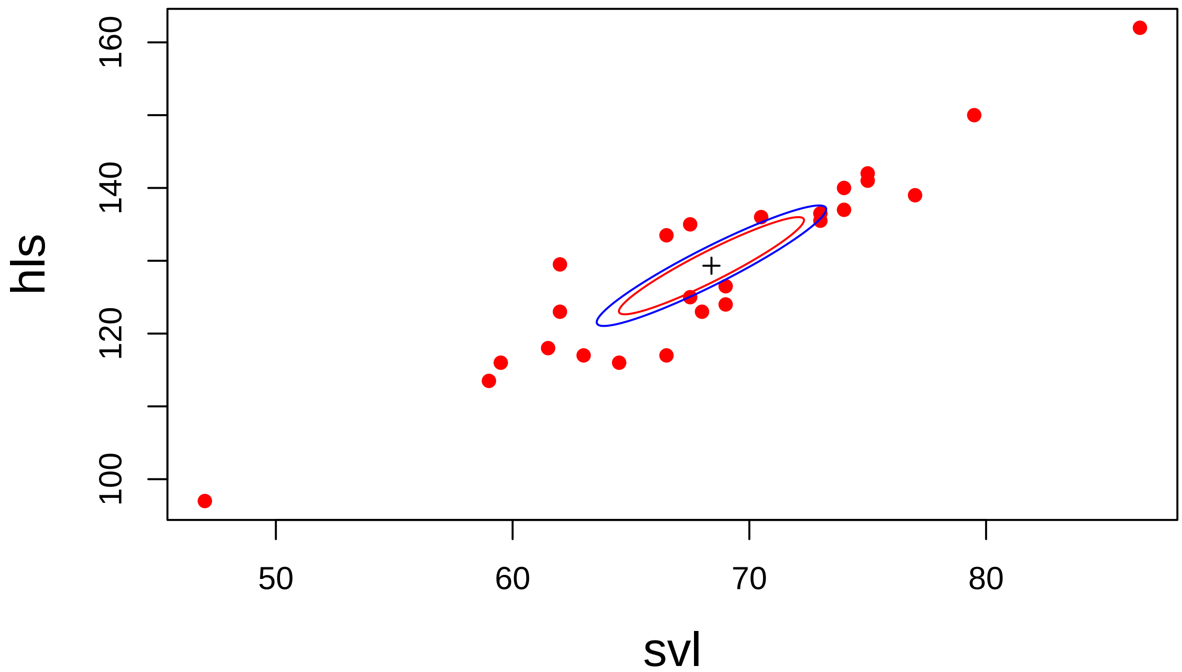
```
##           svl      hls
## svl  63.77083 102.0854
## hls 102.08542 185.8308
```

##f.

```r
bivCI <- function(s, xbar, n, alpha, m)
# returns m (x,y) coordinates of 1-alpha joint confidence ellipse of mean
{
x <- sin(2 * pi * (0 : (m - 1)) / (m - 1)) # m points on a unit circle
y <- cos(2 * pi * (0 : (m - 1)) / (m - 1))
cv <- qchisq(1 - alpha, 2) # chi-squared critical value
cv <- cv / n # value of quadratic form
for (i in 1 : m)
{
pair <- c(x[i], y[i]) # ith (x,y) pair
q <- pair %*% solve(s,pair) # quadratic form
x[i] <- x[i] * sqrt(cv / q) + xbar[1]
y[i] <- y[i] * sqrt(cv / q) + xbar[2]
}
cbind(x, y)
}


biv = lizard[,3:4]
plot(biv, col = "red", pch = 16, cex.lab = 1.5)
lines(bivCI(s = var(biv), xbar = colMeans(biv), n = dim(biv)[1],alpha = .01,
m = 1000),type = "l", col = "blue")
```

```r
lines(bivCI(s = var(biv), xbar = colMeans(biv), n = dim(biv)[1],alpha = .05,
m = 1000),type = "l", col = "red", lwd = 1)
# Add "+" sign
lines(colMeans(biv)[1], colMeans(biv)[2], pch = 3, cex = .8,type = "p", lwd = 1)
```



##g.

```r
conf=bivCI(s = var(biv), xbar = colMeans(biv), n = dim(biv)[1],alpha = .025,
m = 1000)
t.test(lizard[,3])
```

```
##
##  One Sample t-test
##
## data:  lizard[, 3]
## t = 42.827, df = 24, p-value < 2.2e-16
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  65.10368 71.69632
## sample estimates:
## mean of x
##      68.4
```

```r
t.test(lizard[,4])
```

```
##
##  One Sample t-test
##
## data:  lizard[, 4]
## t = 47.433, df = 24, p-value < 2.2e-16
## alternative hypothesis: true mean is not equal to 0
## 95 percent confidence interval:
##  123.693 134.947
## sample estimates:
## mean of x
##    129.32
```

```r
min(conf[,1])
```

```
## [1] 64.0621
```

```r
max(conf[,1])
```

```
## [1] 72.73799
```

```r
min(conf[,2])
```

```
## [1] 121.9154
```

```r
max(conf[,2])
```

```
## [1] 136.7253
```