Integral Equations Notes: [457]

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Definition 1. *An integral equation is any equyation in which the unknown function is inside the integral sign.*

Example 1. $\int_a^b k(x,y)u(y)dy = f(x), x \in (a,b)$ *Here k and f are given and u(x) is the unknown function.*

Example 2. $\int_0^\infty e^{-xt}u(t)dt = f(x), x \in (0,\infty)$ *f(x) is the laplace transform of u.*

Remark 1. *There is an inversion formula giving u from f but it involves an integhrral of f(x) over complex values of x. In practice we may only know f(x) for real values of x.*

Example 3. $\int_{-1}^1 u(t)dt = 1$ *One solution* $u(t) = \frac{1}{2}, t \in (-1,1)$ *Clearly there are lots of solutions, e.g.* $u(t) = \frac{1}{2}+$ *any odd function of t or* $u(t) = \frac{1}{2} + \sum_{n=1}^\infty \{a_n cos(nt\pi) + b_n sin(nt\pi)\}$ *For any reasonable* a_n *and* b_n*.*