Homework 1.3: [530]

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Exercise 2.28 (a)

$$1 = c\frac{1}{2}^{x}, x = 1, 2, 3, 4 \tag{1}$$

$$1 = c[\frac{1}{2} + \frac{1}{2}2 + \frac{1}{2}^{3} + \frac{1}{2}^{4}] \tag{2}$$

$$1 = c[\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16}] \tag{3}$$

$$1 = c[\frac{8}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16}] \tag{4}$$

$$1 = c[\frac{15}{16}] \tag{5}$$

$$c = \frac{16}{15} \tag{6}$$

$$\tag{7}$$

Exercise 2.28 (b)

$$F(x) = \sum_{n=1}^{x} f(n), x = 1, 2, 3, 4 \tag{8}$$

$$F(x) = \sum_{n=1}^{x} \frac{16}{15} \frac{1}{2}^{n}, x = 1, 2, 3, 4 \tag{9}$$

Exercise 2.32 (a)

$$
\begin{aligned}
1 &= \int_0^1 .5dx + \int_1^3 .5 + c(x-1)dx \\
1 &= .5x]_0^1 + .5x - cx + \frac{1}{2}cx^2]_1^3 \\
1 &= .5 + .5(3) - 3c + \frac{9}{2}c - .5(1) + c(1) - c\frac{1}{2}(1)^2 \\
1 &= 1.5 + 2c \\
c &= -\frac{1}{4}
\end{aligned}
$$

Exercise 2.32 (b)

$$F(x) = \begin{cases} .5x, 0 < x < 1 \\ .5x + \frac{-1}{8}x^2 + \frac{1}{4}x, 1 \leq x < 3 \\ 0, otherwise \end{cases}$$

Exercise 2.34 (a) Discrete

Exercise 2.34 (b) .4

Exercise 2.34 (c) .6

Exercise 2.35 (a)

$$f(x) = \begin{cases} \frac{1}{N}, x = 1, 2, 3, ..., N \\ 0, otherwise \end{cases}$$

Exercise 2.35 (b)

$$E[X] = \sum_{x=1}^{N} x * f(x) \tag{10}$$

$$E[X] = \frac{1}{N}[1 + 2 + ... + N] \tag{11}$$

$$E[x] = \frac{1}{N}\frac{N(N+1)}{2} \tag{12}$$

$$E[x] = \frac{N+1}{2} \tag{13}$$

$$E[x^2] = \frac{1}{N}\sum_{x=1}^{x} x^2 \tag{14}$$

$$= \frac{1}{N}\frac{N(N+1)(2N+1)}{6} \tag{15}$$

$$= \frac{(N+1)(2N+2)}{6} \tag{16}$$

$$var[x] = E[x^2] - E^2[x] \tag{17}$$

$$= \frac{N^2-1}{12} \tag{18}$$

$$\tag{19}$$

Exercise 2.35 (c)

$$E[x] = 3.5 \tag{20}$$

$$var[x] = \frac{35}{12} \tag{21}$$

Exercise 2.42 (a)

$$1 = \int_0^1 cx(1-x)dx \tag{22}$$

$$1 = \frac{cx^2}{2} - \frac{cx^3}{3}]_0^1 \tag{23}$$

$$1 = c[\frac{1}{2} - \frac{1}{3}] \tag{24}$$

$$c = 6 \tag{25}$$

Exercise 2.42 (b)

$$F(x) = \begin{cases} 3x^2 - 2x^3, x \in [0,1) \\ 0, otherwise \end{cases}$$

Exercise 2.42 (c)

$$
\begin{aligned}
E[x] &= \int_0^1 x f(x) dx \\
E[x] &= \frac{1}{2} \\
var[x] &= E[x^2] - E^2[x] \\
&= 6[\frac{1}{4} - \frac{1}{5}] - \frac{1}{2}^2 \\
&= \frac{1}{20}
\end{aligned}
$$

Exercise 2.52 (a) 0, .3 1, .33 2, .26 3, .09 4, .02 Exercise 2.52 (b)

$$f(x) = \begin{cases} .3, x = 0 \\ .33, x = 1 \\ .26, x = 2 \\ .09, x = 3 \\ .02, x = 4 \end{cases}$$

Exercise 2.52 (c) E[x] = 1.2 var[x] = 1.06

Exercise 2.52 (d) It would cause E[x] and var[x] to decrease.

Exercise 2.54 (a)

$$f(x) = \begin{cases} .5, x = 100 \\ .5, x = 250 \end{cases}$$

$$f(y) = \begin{cases} .25, x = 0 \\ .25, x = 100 \\ .5, x = 200 \end{cases}$$

X and ty are not independent.

Exercise 2.54 (b) 0 100 200 p(y/x = 100) =.4 =.2 =.4 p(y/x =200) =.1 =.3 =.6

Exercise 2.54 (c) cov(x,y) = 1875

Exercise 2.58
$p(15 < x < 25) > .2256$