# Objectives and motivation of the project

In the aeronautical industry, it is of vital importance the study of the fluid’s behaviour in different conditions as the air behaves as a fluid and in order to design an aircraft (airfoil, sustaining surfaces, etc ) it is necessary these study with the aim of being able to perform their functions at all times.

In fluid dynamics, the study of the fluid behaviour has a great complexity; several manipulations of complex equations are required and some simplifications are needed to simplify them and attempt achieving an analytical solution.

However, apart from these simplifications, nowadays we have help of computers and computational fluids dynamics to facilitate even more the process of obtaining the solutions of these problems related with the behaviour study of fluids, giving more visual results and using numerical resolution techniques instead of complex analytical solutions.

This project is orientated to the simulation of one of the process that occurs inside the supersonic air flux, in concrete, the study of an expansive wave, also known as Prandtl-Meyer expansion.

# Description of the physic problem and its relevance



Figure 1. Prandtl-Meyer expansive wave

The Prandtl-Meyer expansion or expansive wave is the process that occurs inside a supersonic flux when it expands over a convex corner (that forms an angle θ with the horizontal), creating infinity of Mach waves.

This expansion is caused by the drastic change in the direction of the geometry of flux, as it is a convex corner. This phenomenon affects the fluid properties in a gradual and continuous way, making the temperature, pressure and density decrease at the same time that velocity increases.

It has to be empathized that the increase in Mach number and velocity is soft and, as the variations of temperature, pressure and density are also infinitesimal, we can consider the flux as isentropic.

This flux has a leading edge with which creates an angle and a trailing edge with which the final flux forms an angle . These angles are the Mach angles and, as their name already indicates, they are related with the Mach numbers at the beginning () and final () with the following expressions:

This problem is of great interest to us in the aeronautical industry for the design of wing profiles, since practically all aircraft move within supersonic flows (due to the high speeds they reach) where the wing of an aircraft in question acts in a similar to the convex corner of the Prandtl-Meyer model. The air flow will maintain its direction parallel to the wing surface at the leading edge and it will be when, upon reaching the trailing edge, the flow direction will change producing an expansion wave and accelerating the air behind the wing (at the same time temperature, pressure and density decrease).

Moreover, a boundary condition must be applied to the system: tangent flow to the wall. Following the Abbett’s boundary condition treatment, we first calculate the u and v values at the wall and compute the direction of the resulting velocity (Ф) as tan-1(v/u) and the Mach number.

With the formulas previously seen, Pcal, Tcal, ρcal can be obtained. Once we have the calculated values Pcal, Tcal, ρcal we must compute the actual values. They can be computed as:

However the Mact is not defined yet and it must be obtained by trial and error. As Mcal is known it can be computed the fcal with the formula 1.2 and as fact=fcal + Ф it can be solved the value of fcal. With this value it is proposed a trial and error resolution yet we are using a numerical method (secant).

With the secant method two initial random but realistic values of Mach must be defined and the tangent line is computed. Then, it is evaluated the value when the tangent line is 0 and this new point, and the last of the previous Mach points defined, is used to compute the next tangent line to find the new value of 0. This is done iteratively until a value is found, defining some tolerances.