PH 223 Week 5

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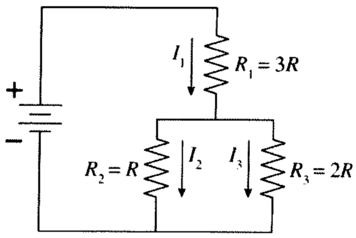
These problems are borrowed/adapted from Chapter 28 of the *Student Workbook* for *Physics for Scientists and Engineers.*

Activity 1

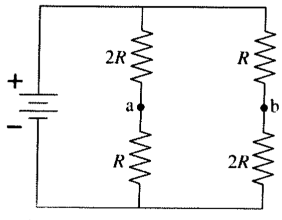
Rank in order, from largest to smallest, the three currents I_1 to I_3.



Order: $I_1 > I_2 > I_3$
Explanation: By Kirchhoff's junction rule, $I_1 = I_2 + I_3$, so I_1 must be larger than both of the other two currents. The voltage drop across R_2 and R_3 is the same, but $R_2 < R_3$, so $I = \frac{\Delta V}{R}$ suggests that $I_2 > I_3$.

Activity 2

(a) Consider the points a and b. Is the potential difference $\Delta V_{ab} = 0$? If so, why? If not, which point is more positive?



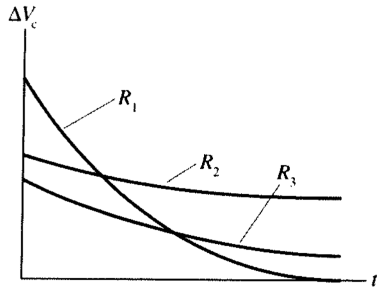
No, the potential difference is not zero. Each branch of the circuit has the same effective resistance and the same voltage drop, so by $\Delta V = IR$, we know $I_a = I_b$. However, this also suggests that the voltage drop across the $2R$ resistor is greater than the voltage drop across the smaller resistor. As such, point b is more positive.

(b) If a wire is connected between points a and b, does a current flow through it? If so, in which direction—to the right or to the left? Explain.

Current flows to the left from b to a. The connecting wire will cause a and b to be at equal potentials. Now, the current across the first pair of parallel resistors will favor the path of least resistance. Since ΔV from the top of the circuit to the a and b connection is the same for both resistors, but one has twice the resistance, the $2R$ resistor will have half as much current as the R resistor. The same goes for the lower pair of resistors. Half of the current leaving the top R resistor will cross from b to a to enter the bottom R resistor.

Activity 3

The graph shows the voltage as a function of time on a capacitor as it is discharged (separately) through three different resistors. Rank in order, from largest to smallest, the values of the resistances R_1 to R_3.



Order: $R_2 > R_3 > R_1$
Explanation: The decay time of an RC circuit is $\tau = RC$. The decay time is longest for the R_2 curve and shortest for the R_1 curve. Since they all share the same capacitor, this directly corresponds to their relative resistances.

For a discharging capacitor, the loop law is

$$0 = -V_C - V_R = -\frac{Q}{C} - IR = -\frac{Q}{C} - \frac{dQ}{dt}R,$$

which may seem strange, since it assumes that current goes into the positive end of the capacitor (thus allowing both voltage changes to be negative as we go with the current through the resistor and then from the positive terminal of the capacitor to its negative terminal). After all, we know the current should be going the other way. However, if we make that change, then we must redefine the current to be $I = -\frac{dQ}{dt}$, since a positive current flowing away from the capacitor would reduce the charge on it. Either way, the differential equation is

$$\frac{dQ}{dt} = -\frac{Q}{RC},$$

and the solution is

$$Q(t) = Q_0 e^{-t/RC}.$$

When we take the derivative, we find that

$$I(t) = -\frac{Q_0}{RC} e^{-t/RC},$$

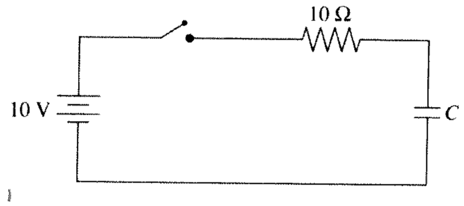
and we see that the current is going the opposite direction from what we assumed! That said, this problem deals with the voltage:

$$V_C(t) = \frac{Q(t)}{C} = \frac{Q_0}{C} e^{-t/RC}.$$

The solution for a charging capacitor (see the next activity) is $Q(t) = C\Delta V_{bat}\left(1 - e^{-t/RC}\right)$, which differentiates to $I(t) = \frac{\Delta V_{bat}}{R} e^{-t/RC}$.

Activity 4

The charge on the capacitor is zero when the switch closes at $t = 0$ s.



(a) What will be the current in the circuit after the switch has been closed for a long time? Explain.

After the switch has been closed a long time, the capacitor will be fully charged and the current will approach zero.

(b) Immediately after the switch closes, before the capacitor has had time to charge, the potential difference across the capacitor is zero. What must be the potential difference across the resistor in order to satisfy Kirchhoff's loop law? Explain.

Going clockwise, Kirchhoff's loop law gives

$$0 = \Delta V_{bat} + \Delta V_R + \Delta V_C.$$

We know that $\Delta V_{bat} = \mathcal{E} = 10$ V. $\Delta V_C = -Q/C$, but the capacitor hasn't charged, so this is zero. As such

$$0 = \mathcal{E} + \Delta V_R,$$

so $\Delta V_R = -10$ V.

(c) Based on your answer to part (b), what is the current in the circuit immediately after the switch closes?

The current in the circuit is the same as the current through the resistor, and by Ohm's law (using $\Delta V_R = 10$ V, the magnitude of the potential difference, to remove the sign issue), we know that

$$I = \frac{\Delta V_R}{R} = \frac{10 \text{ V}}{10\ \Omega} = 1 \text{ A}.$$

(d) Sketch a graph of the current versus time, starting from just before $t = 0$ s and continuing until the switch has been closed for a long time. There are no numerical values for the horizontal axis, so you should think about the *shape* of the graph.

The current decays as charge accumulates on the capacitor and its voltage drop ΔV_C increases. ΔV_{bat} is constant, so ΔV_R must decrease as ΔV_C increases.