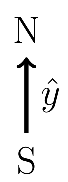
Lecture 22: Inelastic and Superelastic Collisions

Announcements

- End-of-Term Individual Meetings Open
  - 30 minute time slots to discuss anything before the final ungrading.
    * Wednesday: 10:00 – 11:30 a.m., 3:30 – 5:00 p.m.
    * Thursday: 10:00 a.m. – 2:00 p.m., 4:00 p.m. – 5:30 p.m.
    * Friday: 11:00 – 11:30 a.m., 3:30 – 5:00 p.m.
  - Must be booked at least 5 hours in advance (recommended to book far sooner).
  - Only 21 slots; ask me if you need to meet and nothing is available.

Case 1

To set up a momentum vector diagram for case 1, I began with entering the initial momenta for the two cars, which told me the initial momentum for the whole system. I also assumed that momentum was conserved (by the reasoning listed in L20-1), which allowed me to fill in the 1 & 2 column. Since the cars get tangled at the end, I know they have the same speed, and I know Car 2 is twice as big as Car 1, so it should have twice the momentum (which means it will have two thirds of the momentum of the whole system). This allowed me to fill in the rest of the final momentum row. The last two entries in the change in momentum row are now determined by the other entries in their columns.



	Car 1	Car 2	1 & 2
$\vec{p}_i$	↑	↓	↓
$\Delta\vec{p}$	↓	↑	·
$\vec{p}_f$	↓	↓	↓

Already, I know that my cars should go south after the collision. Let us calculate the velocity.

$$\begin{aligned} m_1 &= 100\ \text{kg} & m_2 &= 200\ \text{kg} & m_f &= m_1 + m_2 \\ \vec{v}_1 &= (4\ \text{m/s})\hat{y} & \vec{v}_2 &= -(3\ \text{m/s})\hat{y} & \vec{v}_f &= v_f\hat{y} = ? \end{aligned}$$

$$\begin{aligned} \vec{p}_i &= m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1v_1 - m_2v_2)\hat{y} \\ \vec{p}_f &= (m_1+m_2)\vec{v}_f = (m_1+m_2)v_f\hat{y} \end{aligned}$$

$$\begin{aligned} \vec{p}_i &= \vec{p}_f & v_f &= \frac{m_1v_1 - m_2v_2}{m_1+m_2} \\ m_1v_1 - m_2v_2 &= (m_1+m_2)v_f & &= \frac{(100\ \text{kg})(4\ \text{m/s}) - (200\ \text{kg})(3\ \text{m/s})}{100\ \text{kg} + 200\ \text{kg}} \\ & & &= \frac{400-600}{300}\ \text{m/s} = -\frac{2}{3}\ \text{m/s} \end{aligned}$$

$$\begin{aligned} K_i &= \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \\ &= \frac{1}{2}(100\ \text{kg})(4\ \text{m/s})^2 + \frac{1}{2}(200\ \text{kg})(3\ \text{m/s})^2 = 800\ \text{J} + 900\ \text{J} = 1700\ \text{J} \\ K_f &= \frac{1}{2}(m_1+m_2)v_f^2 = \frac{1}{2}(300\ \text{kg})\left(\frac{2}{3}\ \text{m/s}\right)^2 = \frac{200}{3}\ \text{J} \approx 66.7\ \text{J} \end{aligned}$$

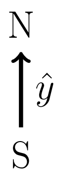
A lot of energy is lost to heat and deformation of metal.

L22-1: Bumper Cars

- Two bumper cars collide with each other and get tangled together.
- Car 1 (m_1) moves north at v_1. Car 2 (m_2) moves south at v_2.
- Case 1
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south at 3 m/s.
  - Find the final velocity of the cars.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.
- Case 2
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south.
  - Find the initial velocity of Car 2 assuming they both end at rest.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.

Case 2

To set up a momentum vector diagram for case 2, I began with entering the initial momentum for Car 1, along with the entire final momentum row (which is all zero). for the two cars, which told me the initial momentum for the whole system. I also assumed that momentum was conserved (by the reasoning listed in L20-1), which allowed me to fill in the 1 & 2 column with more zeros. The zero final momentum in the Car 1 column let me fill in the first car's change in momentum, then the zeros in the 1 & 2 column helped me to fill in the momenta for Car 2.



	Car 1	Car 2	1 & 2
$\vec{p}_i$	$\uparrow$	$\downarrow$	$\cdot$
$\Delta\vec{p}$	$\downarrow$	$\uparrow$	$\cdot$
$\vec{p}_f$	$\cdot$	$\cdot$	$\cdot$

Already, I know that the second car must have equal and opposite initial momentum to the first car.

$$m_1 = 100 \text{ kg} \qquad m_2 = 200 \text{ kg} \qquad m_f = m_1 + m_2$$
$$\vec{v}_1 = (4 \text{ m/s})\hat{y} \qquad \vec{v}_2 = -v_2\hat{y} = ? \qquad \vec{v}_f = \vec{0}$$

$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 v_1 - m_2 v_2)\hat{y}$$
$$\vec{p}_f = (m_1 + m_2)\vec{v}_f = \vec{0}$$

$$\vec{p}_i = \vec{p}_f \qquad v_2 = \frac{m_1}{m_2}v_1$$
$$m_1 v_1 - m_2 v_2 = 0 \qquad = \frac{1}{2}v_1$$
$$= 2 \text{ m/s}$$

Car 2 was going 2 m/s (to the south). They both end at rest, so $K_f = 0$ J.

$$\begin{aligned} K_i &= \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 \\ &= \frac{1}{2}(100 \text{ kg})(4 \text{ m/s})^2 + \frac{1}{2}(200 \text{ kg})(2 \text{ m/s})^2 = 800 \text{ J} + 400 \text{ J} = 1200 \text{ J} \end{aligned}$$

All of the energy was lost to heat and deformation of metal.

L22-1: Bumper Cars

- Two bumper cars collide with each other and get tangled together.
- Car 1 (m_1) moves north at v_1. Car 2 (m_2) moves south at v_2.
- Case 1
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south at 3 m/s.
  - Find the final velocity of the cars.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.
- Case 2
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south.
  - Find the initial velocity of Car 2 assuming they both end at rest.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.

When setting up a motion vector diagram for this problem, I know the initial momentum of the combined sled system, and I know both parts of the sled have half of this momentum. I also know that the final momentum of the back half is reversed, and the total momentum of the system is unchanged, so I can infer the rest of the table from there.

	Back	Front	Both
$\vec{p}_i$	$\rightarrow$	$\rightarrow$	$\longrightarrow$
$\Delta\vec{p}$	$\longleftarrow$	$\longrightarrow$	$\bullet$
$\vec{p}_f$	$\leftarrow$	$\longrightarrow$	$\longrightarrow$

The initial momentum of the system is $mv\hat{x}$, and the final momentum is $(\frac{m}{2}v_{fr} - \frac{m}{2}v)\hat{x}$. Since the momentum is conserved (no friction, and the normal force and gravitational force are in balance, so no other net impulse), we know that

$$\begin{aligned} mv\hat{x} &= \left(\frac{m}{2}v_{fr} - \frac{m}{2}v\right)\hat{x} \\ mv &= \frac{m}{2}v_{fr} - \frac{m}{2}v \\ 2v &= v_{fr} - v \\ v_{fr} &= 3v. \end{aligned}$$

The front half of the sled gets launched forward at triple its original speed!

As for kinetic energy, the system started with $K_i = \frac{1}{2}mv^2$, and now it has

$$K_f = \frac{1}{2}\frac{m}{2}v^2 + \frac{1}{2}\frac{m}{2}(3v)^2 = \frac{5}{2}mv^2,$$

therefore the change in kinetic energy is

$$\Delta K = K_f - K_i = 2mv^2.$$

This came from the spring. If the spring constant is k and the spring was compressed by a length Δx, then we have

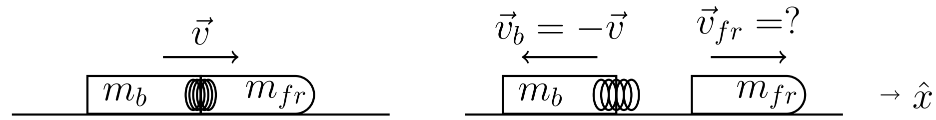
$$\begin{aligned} \frac{1}{2}k\Delta x^2 &= 2mv^2 \\ k\Delta x^2 &= 4mv^2. \end{aligned}$$

This has some interesting design implications. For example, say each half of our sled is 100 kg (say that accounts for the machinery and the load of a single passenger on each half) and its initial speed was a lazy 1 m/s. That would mean the spring has to store 400 J of energy. If $\Delta x = 0.5$ m (which may be too much of a compression for a reasonable use of Hooke's law), then $k = 3200$ N/m (or 32 N/cm), which is a pretty stiff spring. If we cannot get a spring this stiff, then we need more compression, but if we cannot obtain a spring that compresses far enough without permanently deforming, then we need it stiffer. The key will be finding the perfect middle ground. It is also worth considering whether having only a single spring is the best option.

L22-2: Springloaded Sled

You are designing a sled with a compressed spring inside, which can be released to separate the sled into two pieces of equal mass $(m/2)$. You are racing the sled across level snow at speed v when you trigger the separation.

Right after the two halves push apart, the back end of the sled is moving backward with speed v. What is the velocity of the other piece? How much kinetic energy did the system gain?



Main Ideas

- When kinetic energy is lost in a collision, the collision is *inelastic*.
  - A collision in which the objects stick together and move with the same velocity is *perfectly inelastic*.
- When kinetic energy increases in a collision, the collision is *superelastic*.