Driving to Portland

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XX-1: Driving to Portland

Discuss "sensemaking" with your group. Identify several ways of making sense of answers or contexts you have used in math or science courses.

(a) Identify several ways of making sense of answers or contexts you have used in math or science courses.

- **Numerical Sensemaking:** Compare a numerical result against a reference number.
- **Unit Check:** Check the units of your expression to make sure they are what you expect.
  Example: The units of β must be $[\beta] = \frac{\mathrm{gal}}{\mathrm{h}^2}$ for βt^2 to have units of gallons.
- **Covariation:** See how changing the variables changes the output of your expression. See what the signs in your expression tell you.
  Example: The minus sign in $G_0 - \beta t^2$ tells us that the tank is getting emptier as time progresses (assuming $\beta > 0$), which we expect of a vehicle consuming gas as an energy source.
- **Special Case Analysis:** Choose values for variables which correspond to "special cases," where the physical expectation is obvious and the math is simpler.
  Example: We should have the most gas in the tank when we start, and if we plug in an initial time of $t = 0$, we get $G(0) = G_0$. This tells us that G_0 is the initial amount of gas in the tank.

IMPORTANT! Do not just comment on the behavior of an expression without comparing to physical expectations. For example, do NOT just say "$G(t)$ is decreasing, which makes sense." Say **why**: "$G(t)$ is decreasing, which makes sense, as gas is consumed during travel."

You are driving from Corvallis to Portland, and you measure how full your gas tank is (in gallons) as a function of time (in hours):

$$G(t) = G_0 - \beta t^2$$

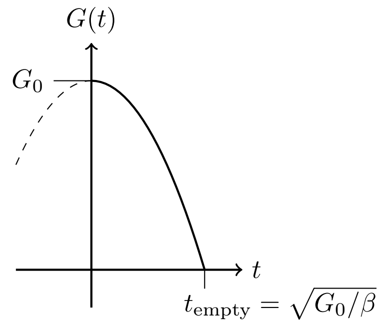
(b) Make sense of this expression with your group in as many different ways as you can, making use of as many different representations as you can.

Multiple sensemaking examples are given in part (a). Additionally, we can learn a lot by looking at a visual representation. Let us graph the function:

There is a really important lesson here. Even if we provide you with an equation that is mathematically valid for all values of t, that does not mean it should be assumed to be an accurate model for all t.

For instance, $G(t)$ increases for $t < 0$, which does not make sense while driving. The model probably should only be applied to $t > 0$.

Furthermore, note that the slope gets steeper as t increases. This tells us that gas is being consumed faster as time goes on. Should we expect fuel efficiency to vary like this?



Here is another very important question: when do we reach Portland? It is possible that we should have stopped graphing before $G(t) = 0$ (out of gas). Graphing beyond the point at which the model applies may show us predictions that don't make sense.