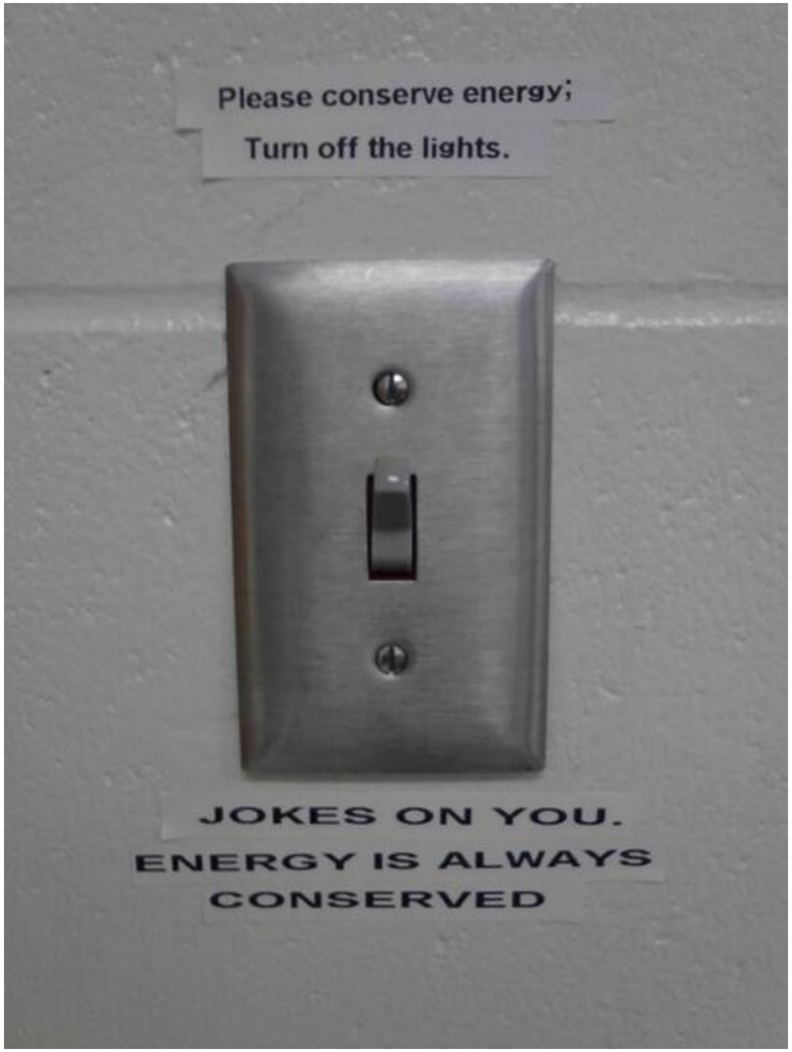
Studio Week 4
Energy and Potential



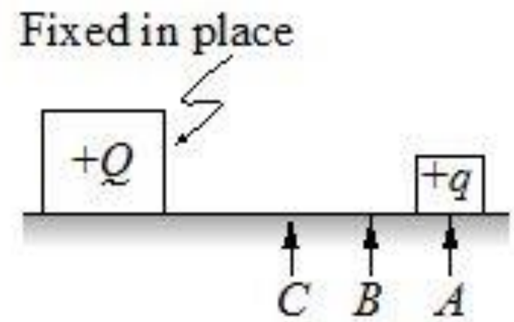
Picture credit: The Internet.

Principles for Success

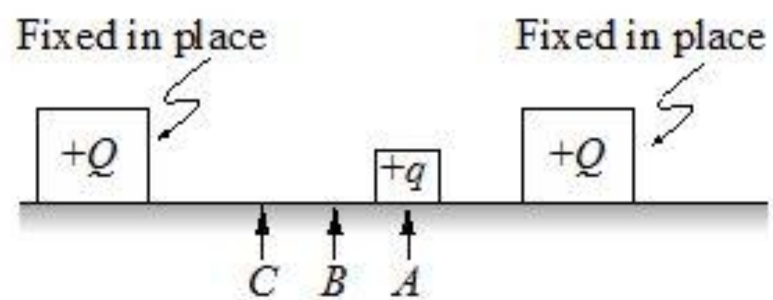
- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

Activity 4-1 – Charged Blocks

1. A hand moves the small $+q$ block from point *A* to point *B*. The block begins and ends at rest. Consider the system of the two blocks.
   a. For the system of the two blocks, is ΔU_E *positive, negative,* or *zero?*
   b. Would ΔU_E due to the hand moving $+q$ from point *A* to point *B* be *greater than, less than,* or *equal to* ΔU_E due to the hand moving $+q$ from point *B* to point *C?*
   c. How would your answers differ if the small block were negatively charged?



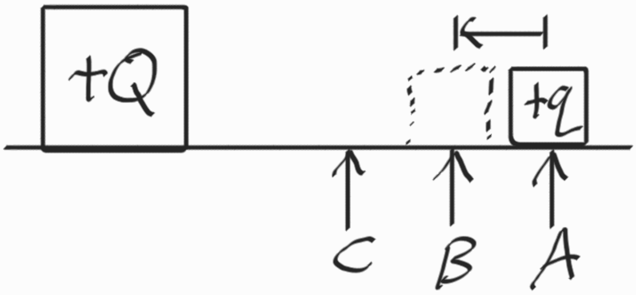
2. Another large, fixed block is added.
   a. Suppose the hand again moves the $+q$ charge from point *A* to point *B*. Would ΔU_E be *positive, negative,* or *zero?*



Take a picture!

4-1 Charged Blocks

1)



$V_B > V_A$

$V_Q = \frac{kQ}{r}$ & $r_B < r_A$

a) $\Delta U_E^{A\to B} > 0$ Positive $\quad \Delta U_E = q\Delta V_Q$

$\vec{F}^E_{qQ}$ repels +q from +Q, so work must be done on +q (adding energy to the system) to move it closer.

b) $\Delta U_E^{A\to B} < \Delta U_E^{B\to C}$

This explanation uses math consistent with point charges. These blocks are not point charges, but from a distance, the effect will be qualitatively similar.

The force is greater from B to C $\left(F = \frac{kqQ}{r^2},\ r_C < r_B < r_A\right)$, so more work must be done to cover that same distance.

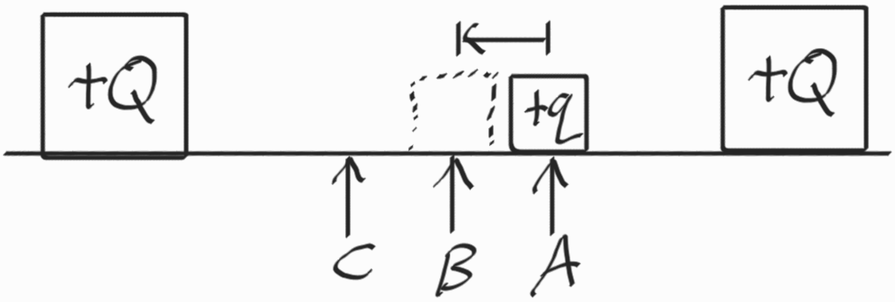
$$\Delta U_E^{A\to B} = q\Delta V_Q^{A\to B} = qkQ\left(\frac{1}{r_B} - \frac{1}{r_A}\right) < qkQ\left(\frac{1}{r_C} - \frac{1}{r_B}\right) = q\Delta V_Q^{B\to C} = \Delta U_E^{B\to C}$$

$$\frac{1}{r_B} - \frac{1}{r_A} = \frac{r_A - r_B}{r_B r_A} = \underbrace{\frac{r_B - r_C}{r_B r_A}}_{r_B - r_A = r_C - r_B} < \underbrace{\frac{r_B - r_C}{r_B r_C}}_{\frac{1}{r_A} < \frac{1}{r_C}} = \frac{1}{r_C} - \frac{1}{r_B}$$

c) $+q \to -q$ The magnitudes of the changes would be the same, but the signs would be different ($\Delta U_E = +q\Delta V \to \Delta U_E = -q\Delta V$).

(a) $\Delta U_E^{A\to B} < 0$ $\quad$ (b) $\Delta U_E^{A\to B} > \Delta U_E^{B\to C}$ more negative number

2)



$$\Delta U_E^{A\to B} = q\Delta V^{A\to B} = q\left(\underbrace{\Delta V_{Q\,left}^{A\to B}}_{>0} + \underbrace{\Delta V_{Q\,right}^{A\to B}}_{<0}\right) < 0$$

Negative

>0 two positives getting closer

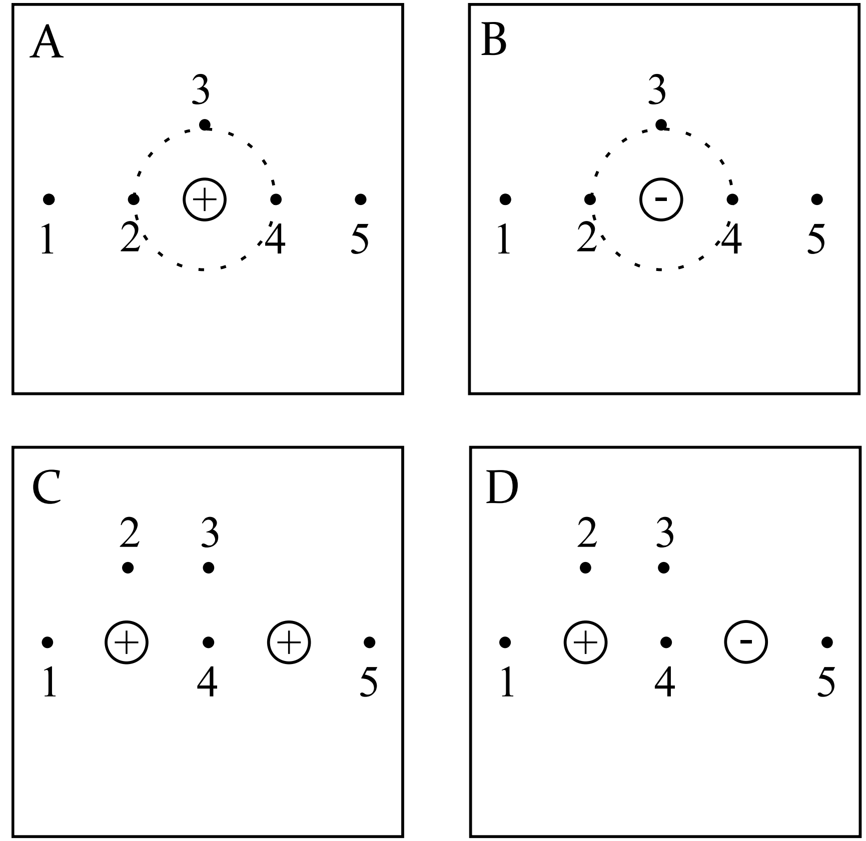
<0 two positives getting farther

Assuming B is in the center, $|\Delta V_{Q\,left}^{A\to B}| < |\Delta V_{Q\,right}^{A\to B}|$, as $+q$ is farther from the lefthand $+Q$. Therefore $\Delta V_{Q\,left}^{A\to B} + \Delta V_{Q\,right}^{A\to B} = |\Delta V_{Q\,left}^{A\to B}| - |\Delta V_{Q\,right}^{A\to B}| < 0$.

In the middle, B will be the point of minimum potential along the line between the two $+Q$ charges. As such, any motion from another point inward to B will be through decreasing potential, and thus decreasing potential energy for a positive charge.

Activity 4-2 – Point Charge Potentials

- For each of the cases at right, rank the electric potentials at each point from greatest to least. For each, explain how you arrived at your ranking.



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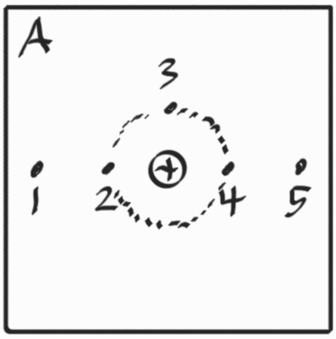
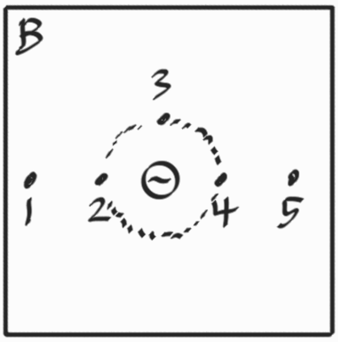
4-2 Point Charge Potentials



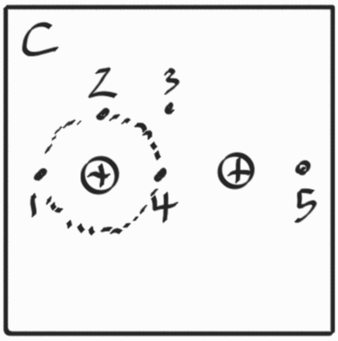
$$V_2 = V_3 = V_4 > V_1 = V_5$$

$V = \frac{kq}{r}$, so points at the same distance from a point charge have the same potential. If $q > 0$, then V decreases as r increases.



$$V_1 = V_5 > V_2 = V_3 = V_4$$

For $q < 0$, V increases (gets less negative) as r increases.

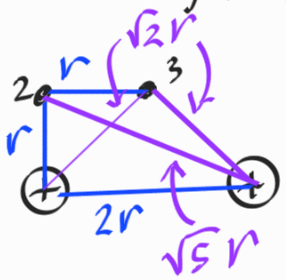


$$V_4 > V_2 > V_3 > V_1 = V_5$$

By superposition, $V = V^{left} + V^{right}$

- $V_1^{left} = V_2^{left} = V_4^{left} > V_3^{left} > V_5^{left}$

  $V_1^{right} < V_2^{right} < V_3^{right} < V_4^{right} = V_5^{right}$

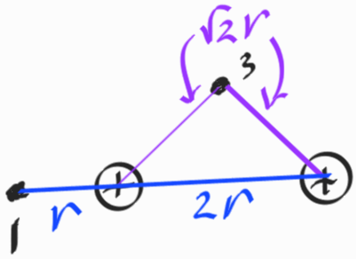
  $\Rightarrow V_1 < V_2 < V_4$ and $V_3 < V_4$
- $V_1^{left} = V_5^{right}$ and $V_1^{right} = V_5^{left}$ as the distances and charges are the same, so $V_1 = V_5$
- 

$$V_2 = \frac{kQ}{r} + \frac{kQ}{\sqrt{5}\,r} = \left(1 + \frac{1}{\sqrt{5}}\right)\frac{kQ}{r}$$

$$V_3 = \frac{kQ}{\sqrt{2}\,r} + \frac{kQ}{\sqrt{2}\,r} = \sqrt{2}\,\frac{kQ}{r}$$

$$2 = 1 + \frac{1 + 2\cdot 2}{5} < 1 + \frac{1 + 2\sqrt{5}}{5} = \left(1 + \frac{1}{\sqrt{5}}\right)^2$$

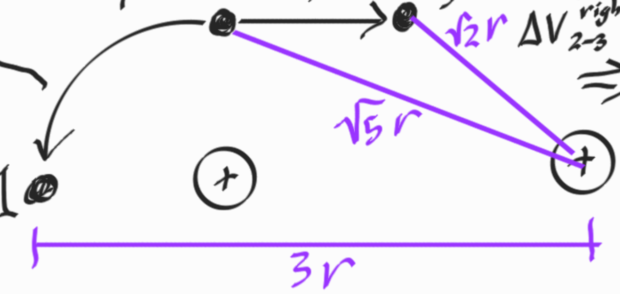
$$\Rightarrow \sqrt{2} < 1 + \frac{1}{\sqrt{5}} \Rightarrow V_3 < V_2$$



$$V_1 = \frac{kQ}{r} + \frac{kQ}{3r} = \frac{4}{3}\frac{kQ}{r}$$

$$\frac{16}{9} < 2 \Rightarrow \frac{4}{3} < \sqrt{2} \Rightarrow V_1 < V_3$$

- Alternatively



$$|\Delta V^{left}_{2\to3}| > |\Delta V^{right}_{2\to3}|$$

$$\Delta V^{left}_{2\to3} < 0$$

$$\Delta V^{right}_{2\to3} > 0$$

$$\Rightarrow V_2 > V_3$$

$$\Delta V^{left} = 0$$

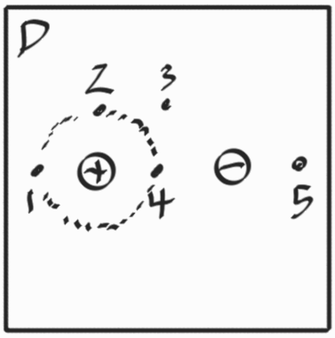
$$\Delta V_{2\to1} = \Delta V^{right}_{2\to1} = kq\left(\frac{1}{3r} - \frac{1}{\sqrt{5}r}\right)$$

$$\Delta V_{2\to3} = \Delta V^{left}_{2\to3} + \Delta V^{right}_{2\to3} = kq\left(\frac{1}{\sqrt{2}r} - \frac{1}{r}\right) + kq\left(\frac{1}{\sqrt{2}r} - \frac{1}{\sqrt{5}r}\right) = kq\left(\underbrace{\frac{2}{\sqrt{2}r} - \frac{1}{r}}_{\frac{\sqrt{2}-1}{r}} - \frac{1}{\sqrt{5}r}\right)$$

$$= \frac{3\sqrt{2}-3}{3r} > \frac{1}{3r}$$

$$\Rightarrow \Delta V_{2\to3} > \Delta V_{2\to1}$$ more negative

$$\Rightarrow V_3 > V_1$$



$$V_1 > V_2 > V_3 = V_4 > V_5$$

- 3 & 4 are the same distance from each charge, and one charge is negative, so $V^{right}_{3/4} = -V^{left}_{3/4}$, and thus $V_3 = V_4 = 0$
- $V_1^{left} = V_2^{left} = V_4^{left}$, $V_1^{right} > V_2^{right} > V_4^{right}$

  getting farther from negative charge

  $\Rightarrow V_1 > V_2 > V_4$
- $|V_5^{right}| > |V_5^{left}| \Rightarrow V_5 < 0$

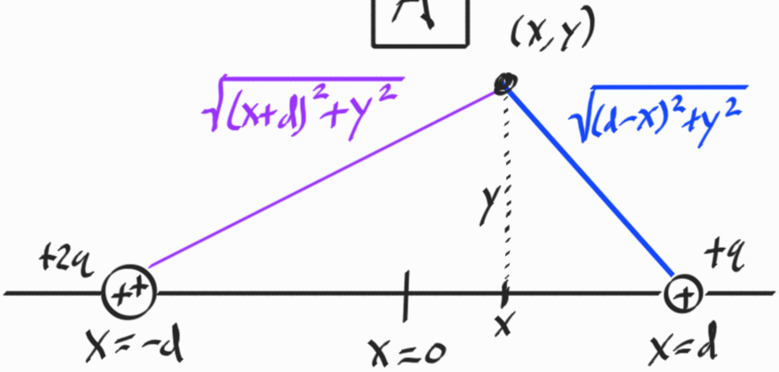
Activity 4-3

- Consider the following situations:
  A. Charge $+q$ is located at $x = +d$ and charge $+2q$ is located at $x = -d$.
  B. Charge $-q$ is located at $x = +d$ and charge $+2q$ is located at $x = -d$.
- For each situation:
  - draw and label a diagram showing the charges
  - identify any locations where the electric potential is equal to zero
  - sketch a graph of the electric potential along the x-axis
  - sketch a graph of the electric potential along the y-axis

Take a picture!

4-3 Potential Graphs

A

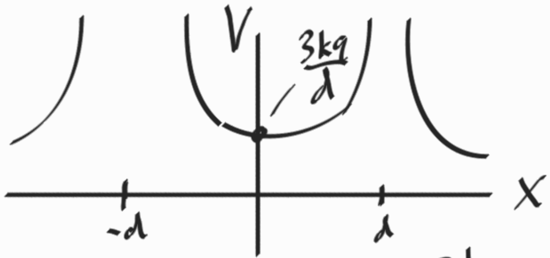


$$V(x,y) = \frac{2kq}{\sqrt{(x+d)^2+y^2}} + \frac{kq}{\sqrt{(d-x)^2+y^2}}$$

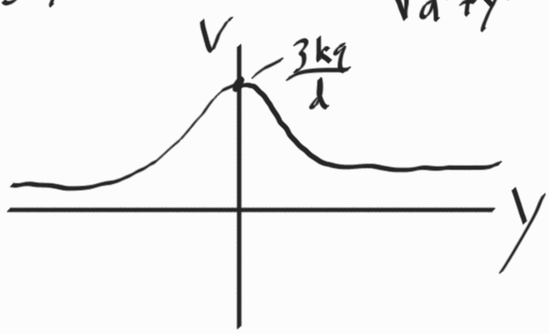
V is nonzero (and positive) everywhere, though $V \to 0$ as x and/or $y \to \infty$.

Graphs

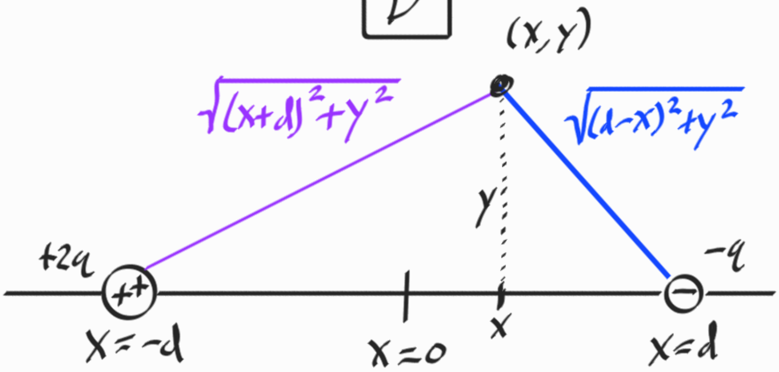
Along x-axis: $y=0 \Rightarrow V = \frac{2kq}{|x+d|} + \frac{kq}{|d-x|}$



Along y-axis: $x=0 \Rightarrow V = \frac{3kq}{\sqrt{d^2+y^2}}$



B



$$V(x,y) = \frac{2kq}{\sqrt{(x+d)^2+y^2}} - \frac{kq}{\sqrt{(d-x)^2+y^2}}$$

$$0 = kq\left(\frac{2}{\sqrt{(x+d)^2+y^2}} - \frac{1}{\sqrt{(d-x)^2+y^2}}\right)$$

$$\Rightarrow \frac{2}{\sqrt{(x+d)^2+y^2}} = \frac{1}{\sqrt{(d-x)^2+y^2}}$$

$$\Rightarrow 4\left[(d-x)^2+y^2\right] = (x+d)^2+y^2$$

$$4\left[d^2-2dx+x^2+y^2\right] = d^2+2dx+x^2+y^2$$

$$\Rightarrow 0 = 3d^2-10dx+3x^2+3y^2$$

$$\Rightarrow 0 = x^2-2x\tfrac{5}{3}d+\tfrac{25}{9}d^2-\tfrac{16}{9}d^2+y^2$$

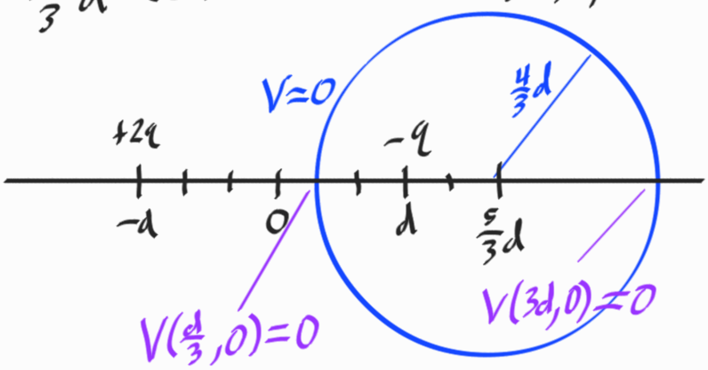
complete the square

$$= \left(x-\tfrac{5}{3}d\right)^2+y^2-\tfrac{16}{9}d^2$$

$$\Rightarrow \left(x-\tfrac{5}{3}d\right)^2+y^2 = \left(\tfrac{4}{3}d\right)^2$$

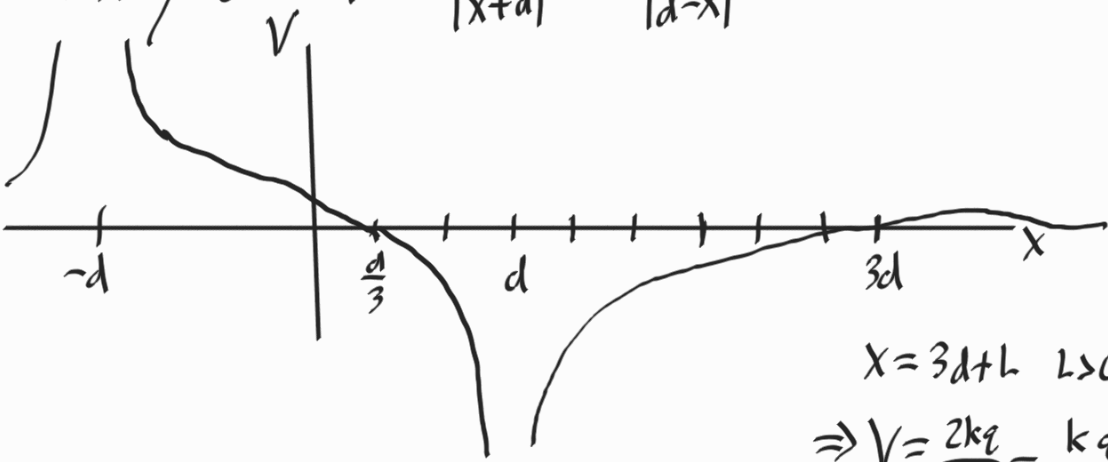
equation for a circle

$V=0$ along a circle of radius $\frac{4}{3}d$ centered at $x=\frac{5}{3}d$, $y=0$.



$\boxed{B}$ Graphs

Along x-axis: $y=0 \Rightarrow V=\frac{2kq}{|x+d|}-\frac{kq}{|d-x|}$



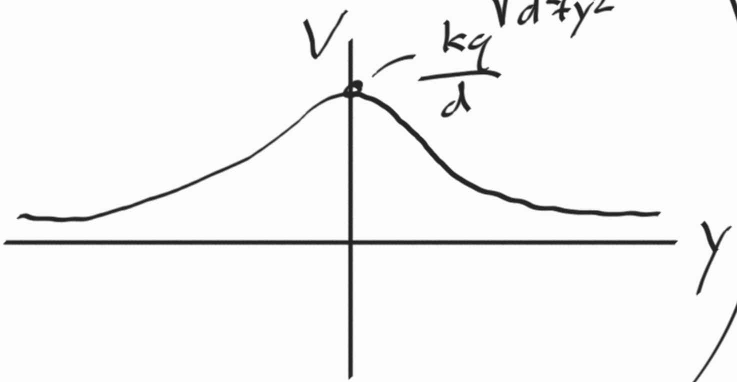
$x=3d+L \quad L>0$

$\Rightarrow V=\frac{2kq}{4d+L}-\frac{kq}{2d+L}$

$=kq\underbrace{\left(\frac{1}{2d+\frac{L}{2}}-\frac{1}{2d+L}\right)}_{>0}$

V positive for $x>3d$

Along y-axis: $x=0 \Rightarrow V=\frac{kq}{\sqrt{d^2+y^2}}$



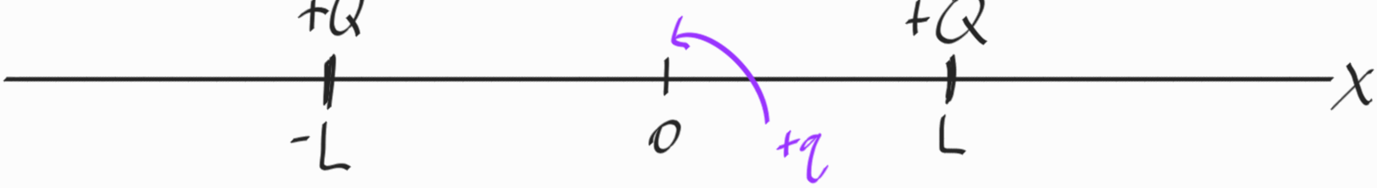
From a long distance away ($r>>d$), the distance between the charges is negligible, so the potential should be approximately the same as that of a single $+q$ charge at $x=0$. That is consistent with V being positive for $x>3d$.

Activity 4-4

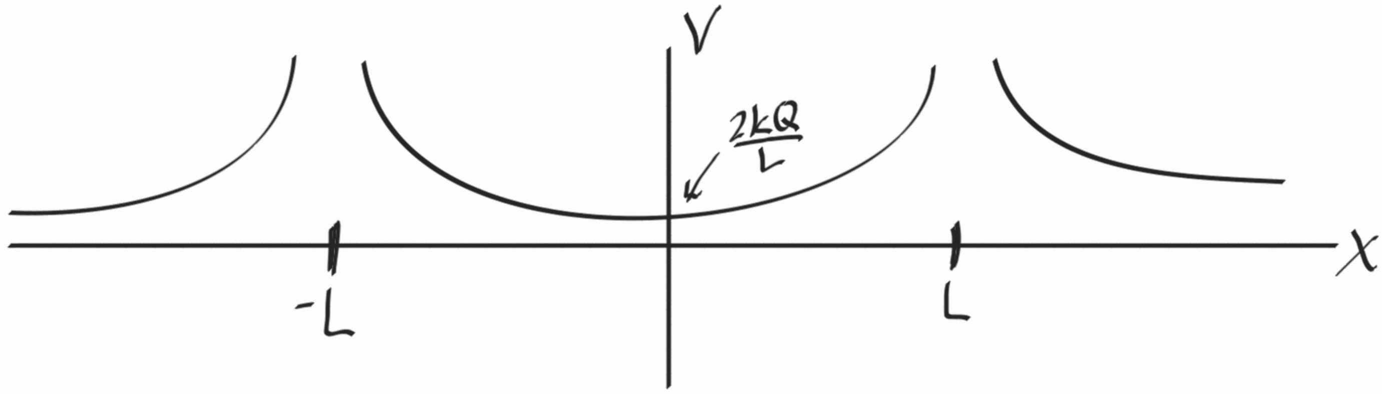
1. Two identical point charges ($+Q$) are fixed in place at $x = +L$ and $x = -L$.
   a. Sketch a graph of the potential (V) *vs.* x along the line between the two charges.
2. A small positive point charge ($+q$) is added at the origin.
   a. What is U_i, the initial electric potential energy of this system? Sketch a graph of U *vs.* x. How would your graph change if the small point charge were negative?
   b. What is the initial net force on the point charge?
3. You give the point charge an initial kinetic energy K_i to the left.
   a. What will happen to the point charge? How do you think your answer would differ if the small point charge had been $-q$ instead of $+q$?
   b. Set up an equation that you could solve to find the maximum distance that the point charge will move to the left. *Hint:* write your work in terms of U_i and K_i whenever you can.
   c. How do you think your maximum distance should change if you increase K_i?

Take a picture!

4-4 Between a Charge and a Hard Place



1)

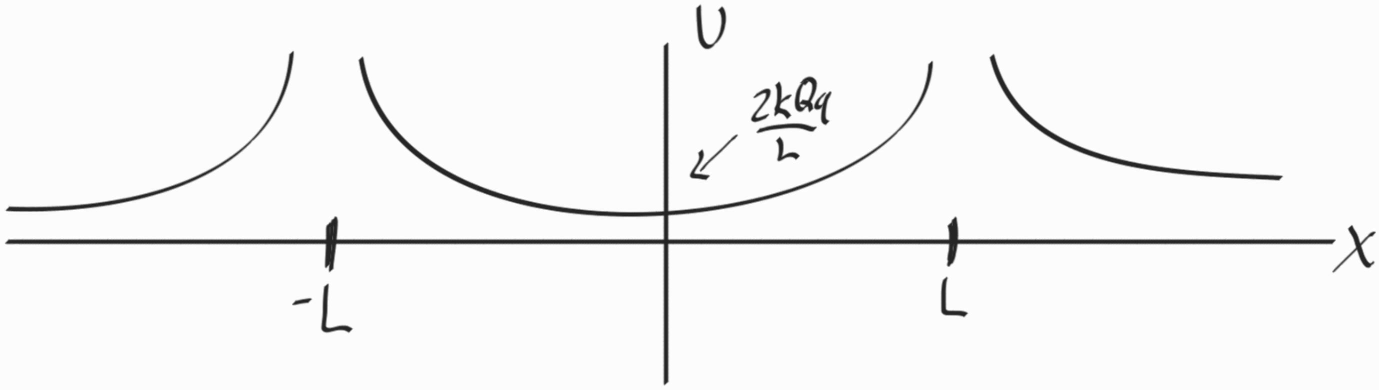


2a) Initial electric potential energy of +q:

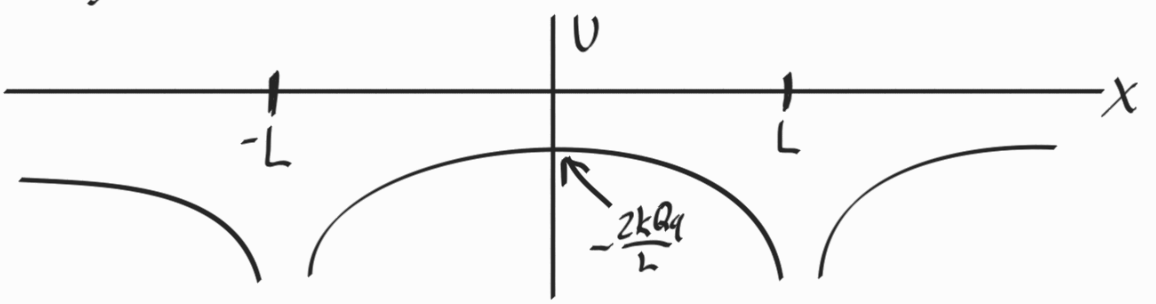
$$U_i = qV_i = q\left(V_{+Q}^{left} + V_{+Q}^{right}\right) = q\left(\frac{kQ}{L} + \frac{kQ}{L}\right) = \frac{2kQq}{L}$$

We do not add in the potential energy between the +Q charges. They are fixed in place, so that energy will not change, and is thus not relevant.

$U = qV$, so a potential energy graph will look very similar to a potential graph:



For a <u>negative</u> charge (-q), everything changes sign:



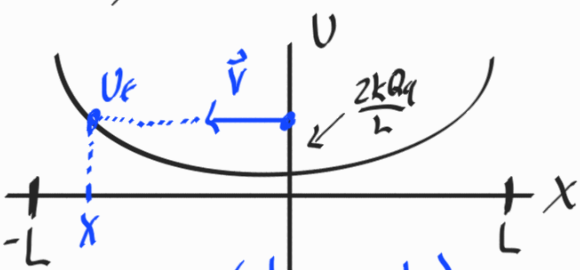
2b) The particle is at a local extremum (minimum for $q>0$, maximum for $q<0$), where the slope is zero, so $F_i^{net} = -\frac{d}{dx}U\Big|_{x=0} = 0$.

3) $+q$ has initial K_i, with $\vec{v}$ to the left

3a) A positive point charge will move left, slowing down until it stops, then turn around and start going the other way, entering oscillatory motion.

A negative charge would accelerate to the left, as it is attracted to the nearer lefthand positive charge.

3b) Conservation of energy: $\underset{\substack{\text{unknown} \\ \text{(depends on} \\ \text{final } x)}}{U_f} + \underset{\substack{\text{zero} \\ \text{(charge stops} \\ \text{moving)}}}{K_f} = \underset{\text{constant}}{U_i} + \underset{\text{given}}{K_i}$



$$U_f = kQq\Big(\underbrace{\frac{1}{L+x}}_{\substack{\text{from charge} \\ \text{at } -L}} + \underbrace{\frac{1}{L-x}}_{\substack{\text{from charge} \\ \text{at } +L}}\Big) = kQq\left(\frac{L-x+L+x}{(L+x)(L-x)}\right) = kQq\frac{2L}{L^2-x^2} = \underbrace{\frac{2kQq}{L}}_{U_i}\frac{1}{1-(\frac{x}{L})^2}$$

c) Prediction: Final x should get farther from 0 (more negative) as K_i increases. More energy will allow the charge to get farther from the local minimum.

Solve:

$$U_f = K_i + U_i$$

$$\frac{U_i}{1-(\frac{x}{L})^2} = K_i + U_i$$

$$1-\left(\frac{x}{L}\right)^2 = \frac{U_i}{K_i+U_i}$$

$$x^2 = L^2\left(1-\frac{U_i}{K_i+U_i}\right)$$

K_i increases $\Rightarrow \frac{U_i}{K_i+U_i}$ decreases

$\Rightarrow 1-\frac{U_i}{K_i+U_i}$ increases

$\Rightarrow |x|$ increases

Matches prediction!

Activity 4-4

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   b. Set up an equation that you could solve to find the maximum distance that the point charge will move to the left. *Hint:* write your work in terms of U_i and K_i whenever you can.
   c. How do you think your maximum distance should change if you increase K_i?

$$x^2 = L^2\left(1 - \frac{U_i}{K_i + U_i}\right) = L^2\left(1 - \frac{\frac{2kQq}{L}}{\frac{1}{2}mv^2 + \frac{2kQq}{L}}\right)$$

Take a picture!