Air Track Glider 1 An air-track glider attached to a spring oscillates with a period of $1.5s$. At $t = 0s$ the glider is $5.00cm$ **left** of the equilibrium position and moving to the **right** at $36.3\frac{cm}{s}$.

$\rightarrow +$ $x_{init} = -0.05m$

$V_{init} = 0.363\frac{m}{s}$

a. Write the equation of motion in the form

Let us choose positive position to be to the right of equilibrium, which means the given initial position is negative, while the initial velocity is positive. The angular frequency can be calculated from the given period.

$$x(t) = x_{init}\cos(\omega t) + \frac{v_{init}}{\omega}\sin(\omega t)$$

$$v(t) = -x_{init}\,\omega\sin(\omega t) + V_{init}\cos(\omega t)$$

$$\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi\,\text{rad}}{1.5s} = \frac{4}{3}\pi\,\frac{\text{rad}}{s}$$

$$x(t) = (-0.05m)\cos\left(\left(\frac{4}{3}\pi\frac{\text{rad}}{s}\right)t\right) + \frac{0.363m}{4\pi/3}\sin\left(\left(\frac{4}{3}\pi\frac{\text{rad}}{s}\right)t\right)$$

b. What is the total energy of the glider+spring system? (Answer in terms of m, the glider's mass)

The total energy combines the initial kinetic energy and the initial spring potential energy. We don't know the spring constant, but we know how we defined the characteristic frequency in terms of it.

$$\omega = \sqrt{\frac{k}{m}} \qquad k = m\omega^2$$

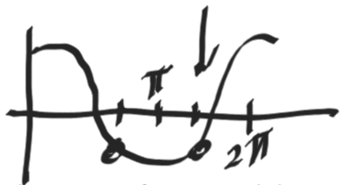
$$E = U_{sp,i} + K_i = \frac{1}{2}kx_i^2 + \frac{1}{2}mv_i^2 = \frac{1}{2}m\omega^2x_i^2 + \frac{1}{2}mv_i^2 \approx \overbrace{\left(0.087\frac{m^2}{s^2}\right)}^{\text{meters}}\underbrace{m}_{\text{mass}}$$

c. Consider the following form for the equation of motion. What is x_{max}?

At the point where the glider is at x_{max}, the glider+spring system still has the same total energy, which we can solve for the amplitude.

$$x(t) = x_{max}\cos(\omega t + \phi_0)$$

$$v(t) = -\omega x_{max}\sin(\omega t + \phi_0)$$



$$\frac{1}{2}kx_{max}^2 = E \Rightarrow \frac{1}{2}\cancel{m}\omega^2x_{max}^2 = \left(0.087\frac{m^2}{s^2}\right)\cancel{m} \Rightarrow x_{max} \approx 0.1m$$

d. What is the phase constant, ϕ_0?

First, we can take information about the possible range of the phase constant from the signs of our initial data. Then, we can solve the position equation in terms of x_{max} at time $t = 0s$, which may have multiple solutions, but only one that is within our constraints.

$$\left.\begin{aligned} x_i = x_{max}\cos\phi_0,\quad & x_i < 0 \Rightarrow \frac{\pi}{2} < \phi_0 < \frac{3\pi}{2} \\ v_i = -\omega x_{max}\sin\phi_0 \quad & v_i > 0 \Rightarrow \pi < \phi_0 < 2\pi \end{aligned}\right\} \Rightarrow \pi < \phi_0 < \frac{3\pi}{2}$$

3rd quadrant

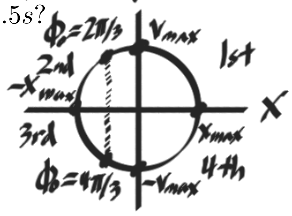
$$x_i = x_{max}\cos\phi_0 \Rightarrow -0.05 = 0.1\cos\phi_0$$

$$-\frac{1}{2} = \cos\phi_0 \Rightarrow \phi_0 = \cancel{\frac{2\pi}{3}}, \frac{4\pi}{3}$$

e. What is the phase $(\omega t + \phi_0)$ at $t = 0s$, $0.5s$, $1.0s$, and $1.5s$?

We only care about phases between zero and 2π, because of the periodicity of the motion. As such, we shall reduce all phases to fall in that range.

t	$0s$	$0.5s$	$1.0s$	$1.5s$
$\phi = (\omega t + \phi_0)$	$\frac{4\pi}{3}$	$\frac{4\pi}{3} + \frac{2\pi}{3} = 2\pi \equiv 0$	$\frac{4\pi}{3} + \frac{4\pi}{3} \equiv \frac{2\pi}{3}$	$\frac{4\pi}{3} + 2\pi \equiv \frac{4\pi}{3}$



Planet X On your first trip to Planet X you happen to take along a 250g mass, a 40cm-long spring, a meter stick, and a stopwatch. You're curious about the free-fall acceleration on Planet X, where ordinary tasks seem easier than on earth, but you can't find this information in your Visitor's Guide. You decided to test it for yourself by suspending the spring from the ceiling in your room and hanging the mass from it. You find that the mass stretches the spring by 19cm. You then pull the mass down and extra 11 cm and release it. With the stopwatch you find that 10 oscillations take 15s. What is the new g? Use your sense-making tools to confirm.

$$10T = 15s \Rightarrow T = 1.5s$$

Hint: Start by drawing a picture, and FBD

This is best considered in two separate parts. First, when the mass is hanging from the spring in equilibrium with gravity, we know that the spring force and the force of gravity are equal.

$$mg = F^g = F^{sp} = k\Delta x, \quad m = 0.250kg, \ \Delta x = 0.19m$$

We need to know the spring constant to proceed any further, and we find it through the oscillation. Now, the amplitude of the oscillation does not affect the characteristic angular frequency, but it is worth discussing. The amplitude of the oscillation is 11 cm, as the equilibrium position of the spring with the mass hanging on it was shifted from the original unstretched equilibrium by gravity, becoming 59 cm rather than 40 cm.

$$k = m\omega^2 = m\left(\frac{2\pi}{T}\right)^2 \Rightarrow \cancel{m}g = \cancel{m}\left(\frac{2\pi}{T}\right)^2\Delta x \Rightarrow \boxed{g \approx 3.33m/s^2}$$

[0]Select problems may be modified from PH 212 course textbook; Knight Physics for Scientists and Engineers

Moon Pendulum A NASA scientist constructs a 2.0 meter long pendulum out of a string and mass. The scientist releases the pendulum from an angle of 0.14 radians.

a. What is the period of the pendulum on earth? $T = 2\pi\sqrt{\frac{L}{g}} \approx 2\pi\sqrt{\frac{2.0m}{9.8m/s^2}} \approx 2.8s$

The period of a pendulum depends on its length and the strength of gravity. The initial angle 0.14 rad (about 8.02 degrees) is small enough for the oscillation to be modeled in the way we intend, but it is otherwise a red herring, having no bearing on the period.

b. Write the equation of motion

$$\theta(t) = \theta_{max}\cos(\omega t) \qquad \omega = \frac{2\pi}{T} = \sqrt{\frac{g}{L}} \approx 2.21\ rad/s$$

$$\theta(t) = (0.14\ rad)\cos\left(\frac{2\pi}{2.8s}t\right) = (0.14\ rad)\cos((2.21\ rad/s)t)$$

c. The scientist then hops onto a space shuttle and takes off for the moon, where $g_{moon} = 1.6m/s^2$. What new length will the pendulum need to have the same period on the moon as it did on the earth in part (a)?

The period is proportional to the square root of the length divided by gravitational acceleration, so to keep the period static, the length must be changed by the same factor as the gravitational acceleration.

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow L = \left(\frac{T}{2\pi}\right)^2 g \Rightarrow \frac{L_{Moon}}{L_{Earth}} = \frac{\cancel{\left(\frac{T}{2\pi}\right)^2} g_{Moon}}{\cancel{\left(\frac{T}{2\pi}\right)^2} g_{Earth}} = \frac{1.6m/s^2}{9.8m/s^2}$$

$$L_{Moon} \approx \frac{1.6}{9.8}(2.0m) \approx 0.32m$$

The Moon pendulum needs to be shorter to have the same period as the Earth pendulum.