Lecture 3: Acceleration

Warm-Up Activity

For an object moving in one dimension (in a straight line), what does it mean if the acceleration and the velocity are in opposite directions? The object is…

(A) Slowing Down

(B) Speeding Up

(C) Staying Still

(D) Not Enough Information

The members of the instructional team who are performing assessment have limits on their time. We cannot guarantee that we will be able to take time to assess late work, or that we will be able to give feedback on all parts of an assignment. We may elect to assess a subset of the problems in a given assignment.

Homework

- Physics homework is not (just) about getting correct answers.
- The homework format is designed to help you **communicate the depth of your physics knowledge and understanding** and **learn how to break down a problem**.
- Make sure you submit an organized and clear submission!

Late Work

- If you need an extension, ask first.
- If you miss a deadline, you should still turn it in when you are ready.
- Assignments turned in after the deadline without an approved extension may not receive feedback.
- If you need more feedback on a problem (even a problem that missed the deadline), reach out to us in the Wormhole and in office hours.

A Model for Motion

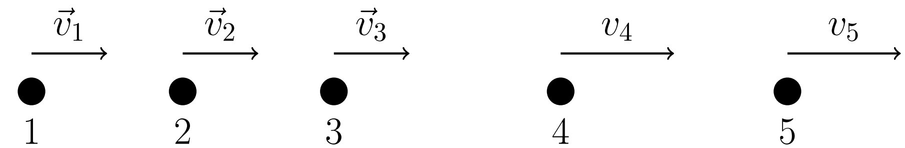
Quantities

- Position: $\vec{r}$
- Velocity: $\vec{v} = \frac{d\vec{r}}{dt}$
- Acceleration: $\vec{a} = \frac{d\vec{v}}{dt}$

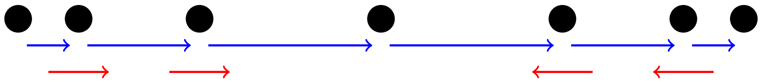
Assumptions

- Use the Particle Model

Motion Diagram



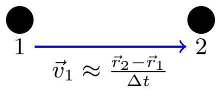
L3-1: Direction of Velocity and Acceleration



We can approximate the velocity vectors using the definition of average velocity:

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$

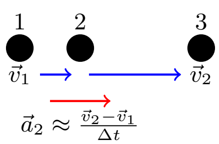
Using a forward approximation, we will take $\Delta \vec{r}$ to be the displacement from one dot to the next, and estimate the velocity of the first dot to be the calculated average velocity.



We can approximate the acceleration vectors using the definition of average acceleration:

$$\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}.$$

This will be a rougher approximation, as we are using approximate velocities instead of instantanous velocities. It appears rather natural to determine $\Delta \vec{v}$ at a dot by taking the vector pointing to it from the previous dot and subtracting it from the vector pointing from it to the next dot—which is technically a backward approximation.



(a) When acceleration is zero, the velocity does not change. This happens in the middle of the motion diagram.

(b) When acceleration and velocity are in opposite directions, the object slows down (the velocity decreases in magnitude). This happens on the right side of the motion diagram.

(c) When acceleration and velocity are in the same direction, the object speeds up (the velocity increases in magnitude). This happens on the left side of the motion diagram.

L3-1: Direction of Velocity and Acceleration

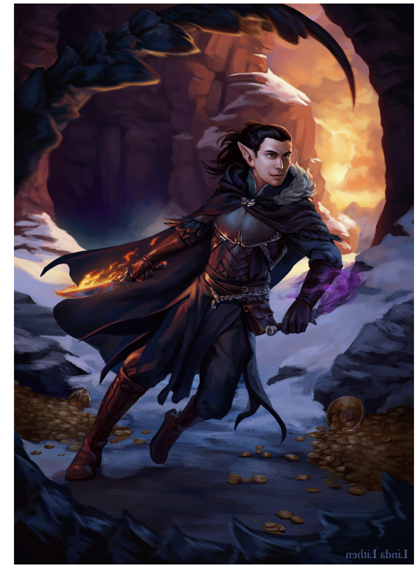
- The motion diagram above shows a ball moving from left to right.
  - Draw velocity and acceleration vectors at each instant.
- For an object moving in one dimension (in a straight line), what does it mean if:

  (a) The acceleration is zero?

  (b) The acceleration and the velocity are in opposite directions?

  (c) The acceleration and the velocity are in the same direction?

L3-2: Vax'ildan's Acceleration



- Vax'ildan Vessar is initially located at position x_i, running to the right with initial speed v_i.
- At $t = 0$, Vax clicks his *boots of haste*, which provide an acceleration:
$$\vec{a}(t) = a_0 \left(1 - \frac{t}{T}\right) \hat{x}$$
- Our goals are:
  - Find how much time it takes for Vax to return to his initial velocity.
  - Find Vax's position at this time.

Solving an ARCS Problem

1. Analyze and Represent

1a. **Understand the problem** – identify quantities by symbol and number.
1b. **Identify Assumptions** – identify important simplifications and assumptions.
1c. **Represent physically** – draw and label one or more appropriate diagrams and/or graphs that might help you solve the problem.

2. Calculate

2a. **Represent principles** – identify relevant concepts, laws, or definitions.
2b. **Find unknown(s) symbolically** – without numbers, find any unknown(s) in terms of symbols representing known quantities.
2c. **Plug in numbers** – plug numbers (with units) into your symbolic answer!

3. Sensemake

3a. **Units** – check that the units of your answer agree with the units you expect
3b. **Numbers** – compare your answer to other numbers in the problem or in the everyday world; if relevant, check the sign or direction.
3c. **Symbols** – use a strategy like covariation or special cases to check that your answer makes physical sense.

We need $1 - \frac{t}{T}$ to be unitless, because 1 is unitless, so t/T must also be. Since t is in seconds, it follows that T must also be.

Note that, when we plug in $t = T$, we find that $\vec{a}(T) = 0\hat{x}$, so the acceleration burst stops after T passes, at which point the acceleration changes direction to bring Vax back to his initial velocity. As such, T can be thought of as the duration of the acceleration burst.

Since $\left(1 - \frac{t}{T}\right)\hat{x}$ is unitless, the overall units of the right hand side come from a_0. We need the right hand side to be an acceleration to match the left, so a_0 has units of $\mathrm{m/s^2}$.

Note that, when we plug in $t = 0$ s, we find that $\vec{a}(0\text{ s}) = a_0\hat{x}$, so a_0 is the magnitude of the initial acceleration.

The unit vector $\hat{x}$ carries all of the direction information. It tells us that the acceleration is in the x-direction (though left or right depends on the sign).

Understand and Plan

Knowns

- Initial Position: $x_i = 0$ m (simplifying assumption)
- Initial Velocity: $v_i = 2$ m/s (a reasonable speed to estimate for a half-elf)
- $T = 6$ s (rounds in *Dungeons & Dragons* last six seconds)
- $a_0 = 0.5\ \mathrm{m/s^2}$ (significantly less than free-fall acceleration)

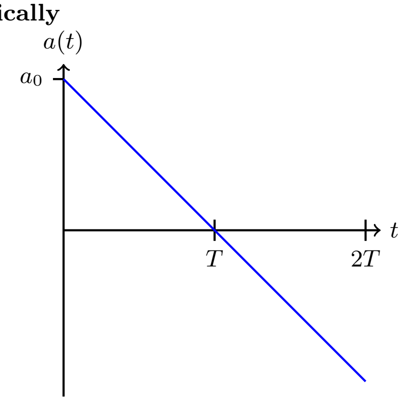
Unknowns

- When Vax returns to his initial velocity: t_f
- Vax's final position: x_f
- Equations of motion for velocity and position: $\vec{v}(t)$ and $\vec{x}(t)$

Identify Assumptions

- Particle Model
  - We do not wish to handle the complexities of how Vax's arms, legs, and wings move as he runs, so we will treat him as a point mass.
- 1-D Motion
  - We will assume Vax is travelling over relatively level ground so we do not have to consider Vax's vertical motion.
- Vax is not obstructed in his movement.
  - If Vax were to bump into something, that brief interaction would alter his motion in additional ways that the acceleration of the boots does not account for.

Represent Physically



L3-2: Vax'ildan's Acceleration – Analyze & Represent

- Vax'ildan Vessar is initially located at position x_i, running to the right with initial speed v_i.
- At $t = 0$, Vax clicks his *boots of haste*, which provide an acceleration:

$$\vec{a}(t) = a_0\left(1 - \frac{t}{T}\right)\hat{x}$$

- With your group:
  - What are the units of T and a_0? What do they represent physically?
  - What does $\hat{x}$ mean?
  - **Understand and Plan:** Identify quantities by symbol and number.
  - **Identify Assumptions**
  - **Represent Physically**
    - What does $\vec{a}(t)$ look like?

Main Ideas

- If we know the acceleration of an object as a function of time, we can determine the velocity as a function of time.
- If we know the velocity as a function of time, we can determine the position as a function of time.