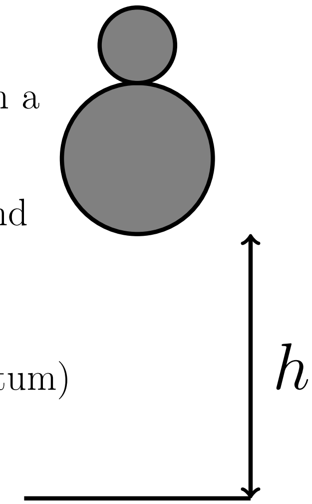
Energy is (C)
The work-energy theorem ($W_{\text{net,ext}} = \Delta E_{\text{total}}$) determines whether energy is conserved. Everything that can do significant amounts of work is in our system.

Momentum is (B)
The impulse-momentum theorem ($\int_{t_i}^{t_f} \vec{F}^{net} dt = \vec{J}_{net} = \Delta\vec{p}$) determines whether momentum is conserved. Everything that exerts a significant force is in our system.

Studio 8: Combining Physics Concepts

<u>Warm-Up Activity</u>



- A tennis ball and a basketball are dropped from a height h as shown.
- Our system is the tennis ball, the basketball, and the Earth.

Energy/Momentum is …
(answer twice; once for energy and once for momentum)

(A) … conserved, because energy/momentum is always conserved.

(B) … conserved, because the net force on the system is zero.

(C) … conserved, because the work done on the system is zero.

(D) … not conserved, because energy is never conserved.

(E) … not conserved, because the net force on the system is not zero.

(F) … not conserved, because the work done on the system is not zero.

A Deeper Model for Interactions

- Quantities
  - Energy E
  - Work $W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$
  - Kinetic Energy $K = \frac{1}{2}mv^2$
  - Potential Energy $U =$ depends on interaction
    You have to tell everyone where zero PE is!
    * Gravity $U_g = mgy$
    * Spring $U_{sp} = \frac{1}{2}kx^2$
  - Momentum $\vec{p} = m\vec{v}$
  - Impulse $\vec{J}_{net} = \int_{t_i}^{t_f} \vec{F}^{net} dt$
- Laws
  - Work-energy theorem $W_{\text{net,ext}} = \Delta E_{\text{total}}$
  - Impulse-momentum theorem $\vec{J}_{net} = \Delta\vec{p}$

	U_{gTB}	K_{TB}	U_{gBB}	K_{BB}	E_{total}
t_i	$mg(h+2R)$	0	Mgh	0	$mg(h+2R)+Mgh$
t_f	mgh_{max}	0	0	0	mgh_{max}

$$\begin{aligned} mgh_{max} &= mg(h+2R) + Mgh \\ h_{max} &= h + 2R + \frac{M}{m}h \\ &= \left(\frac{M}{m}+1\right)h + 2R \end{aligned}$$

You don't necessarily need to include the radius of the basketball, as the tennis ball never gets lower than $2R$. If you leave off $2R$, you are effectively setting the zero of potential for the tennis ball at $2R$ above the ground, and so the h_{max} you would find would actually be measured from this higher point.

Plugging in the given numbers, note that $\frac{M}{m} = 15$, so $h_{max} = 8.23$ m. Now, this would exceed the ceiling height in our classroom, but when we actually do the demonstration, the non-negligible bounce of the basketball leads to this being a large overestimate; the tennis ball won't hit the ceiling after being dropped from this height.

However, if you wanted to find out the maximum drop height in the ideal case, it would only take a small rearrangement of the equation:

$$h = \frac{h_{max} - 2R}{\frac{M}{m} + 1}.$$

Assuming a ceiling height of 3 m, we should drop the ball from less than $h \approx 0.17$ m. In practice, this will be nowhere near the ceiling.

S8-1: Ball Drop – Height

- A tennis ball and a basketball are dropped from a height h as shown.
- Our system is the tennis ball, the basketball, and the Earth.
- If the basketball does not bounce at all, how high does the tennis ball go?
  - Basketball Mass: $M = 0.6$ kg
  - Basketball Radius: $R \approx 11.5$ cm
  - Tennis Ball Mass: $m = 0.04$ kg
  - Drop Height: $h = 0.5$ m

Let us add two times to our table: t_{bb} (right before the bounce) and t_{ab} (right after the bounce).

	U_{gTB}	K_{TB}	U_{gBB}	K_{BB}	E_{total}
t_i	$mg(h+2R)$	0	Mgh	0	$mg(h+2R)+Mgh$
t_{bb}	$mg(2R)$	$\frac{1}{2}mv_{bb}^2$	0	$\frac{1}{2}Mv_{bb}^2$	$mg(2R)+\frac{1}{2}(m+M)v_{bb}^2$
t_{ab}	$mg(2R)$	$\frac{1}{2}mv_{ab}^2$	0	0	$mg(2R)+\frac{1}{2}mv_{ab}^2$
t_f	mgh_{max}	0	0	0	mgh_{max}

Note that the extra starting height of the tennis ball really doesn't matter in some of these calculations. It drops the same height as the basketball, and that change in height is what provides the kinetic energy; the extra $2R$ that remains does nothing. This can be seen in how it will cancel out in equations relating E_i to E_{bb} or E_{ab}.

To find the pre-collision speed, I will choose to set $E_i = E_{bb}$ (which isn't the only choice I could make), which gives

$$\begin{aligned} mg(h+2R)+Mgh &= mg(2R)+\frac{1}{2}(m+M)v_{bb}^2 \\ (m+M)gh &= \frac{1}{2}(m+M)v_{bb}^2 \\ v_{bb} &= \sqrt{2gh}. \end{aligned}$$

To find the post-collision speed, I will choose to set $E_f = E_{ac}$ (which isn't the only choice I could make), which gives

$$\begin{aligned} mgh_{max} &= mg(2R)+\frac{1}{2}mv_{ab}^2 \\ v_{ab} &= \sqrt{2g(h_{max}-2R)} = \sqrt{2gh\left(\frac{M}{m}+1\right)} \end{aligned}$$

Now, from the impulse-momentum theorem (simplifying the impulse as $\vec{J}_{net} = \vec{F}^{net}_{avg}\Delta t$, and choosing a coordinate system with $\hat{y}$ pointing upward), we have

$$\begin{aligned} \vec{F}^{net}_{avg}\Delta t &= \vec{p}_{ab} - \vec{p}_{bb} \\ F^{net}_{avg}\Delta t &= mv_{ab} - (-(m+M)v_{bb}) \\ &= m\sqrt{\frac{M}{m}+1}\sqrt{2gh} + (m+M)\sqrt{2gh} \\ F^{net}_{avg} &= \frac{\sqrt{\frac{M}{m}+1}+(m+M)}{\Delta t}m\sqrt{2gh}. \end{aligned}$$

If we want to get just the normal force from the ground, we need to know the force of gravity (which is constant, and therefore equal to its average). Since our system is both balls, the force should be on both, meaning

$$\begin{aligned} F^{net}_{avg} &= F^{N}_{avg} - F^{g} \\ &= F^{N}_{avg} - (m+M)g \\ F^{N}_{avg} &= F^{net}_{avg} + (m+M)g \\ &= \frac{\sqrt{\frac{M}{m}+1}+(m+M)}{\Delta t}m\sqrt{2gh} + (m+M)g. \end{aligned}$$

S8-2: Ball Drop – Force

- A tennis ball and a basketball are dropped from a height h as shown.
- During the collision with the ground, which takes 0.2 s, you may want to choose a different system.
  - What is the speed of each ball right before the basketball hits the ground?
  - What is the speed of the tennis ball right after it leaves the basketball?
  - What force did the ground exert on the basketball?

Blue Track
The blue track changes potential energy to kinetic energy soonest, so the ball accelerates more at the outset and has more speed over more of the distance of the track.

This can also be thought about in terms of forces, as the region with the most slope accelerates the ball the most and happens earliest on this track.

Either way, the ball on the blue track reaches the end first.

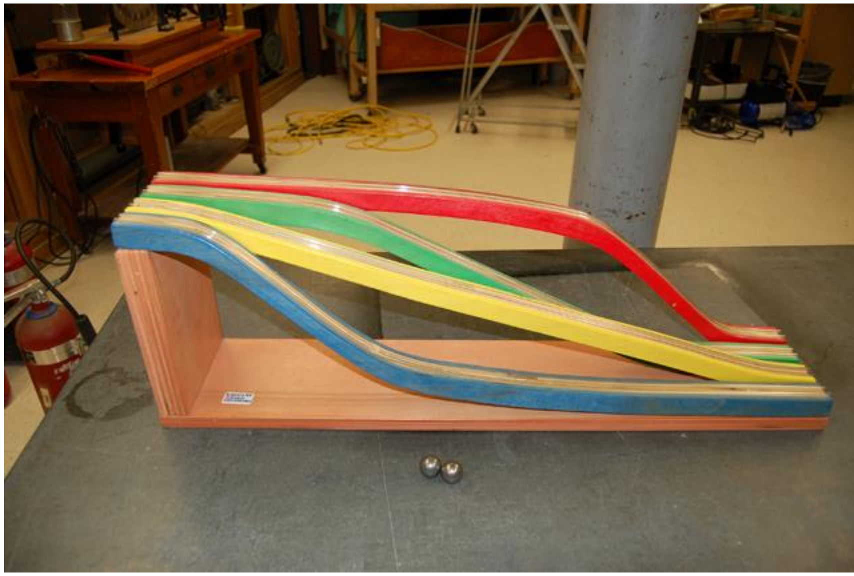
Yellow Track
The yellow track gives the ball constant acceleration.

Green Track
The green track as less slope (and therefore less acceleration) near both ends, and more slope (and therefore more acceleration) in the middle.

Red Track
The red track is the slowest. Most of the conversion from U_g to K occurs near the end (where the slope, and therefore the force, is greatest), and the ball spends most of its time at a lower speed.

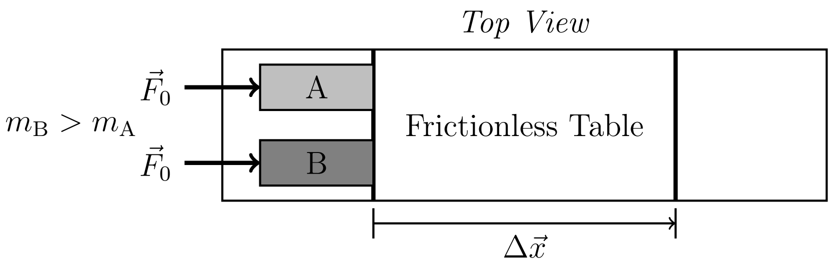
All Four
All four tracks have the same change in height, so the balls lose the same amount of potential energy. As such, they all end with the same speed.

S8-3: Frictionless Track



Use the understanding you have built about the energy for a ball moving along a frictionless track to predict **which ball will reach the end of the track first.**

- Explain your reasoning.
- Each ball starts at rest at the top of the track, and the tracks are nearly the same length.



Three Students

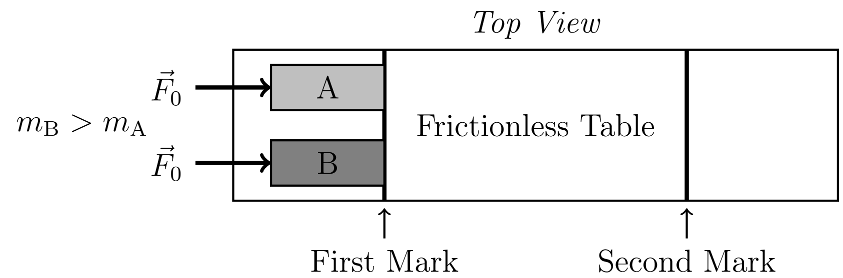
(1) Same force, $m_{\mathrm{B}} > m_{\mathrm{A}}$, therefore m_{A} goes faster than m_{B}.
True by application of Newton's second law.
$\vec{p} = m\vec{v}$ True (definition of momentum).

(2) Speed compensates for mass, so $p_{\mathrm{A},f} = p_{\mathrm{B},f}$.
False. m_{A} goes faster, so $\Delta t_{\mathrm{A}} < \Delta t_{\mathrm{B}}$ to reach the end. $\vec{J} = \int_{t_i}^{t_f} \vec{F}^{net} dt$, so for the same force, $J_{\mathrm{A}} < J_{\mathrm{B}}$, and since $p_i = 0$ for both, this means $p_{\mathrm{A},f} < p_{\mathrm{B},f}$.

(3) K is proportional to m and to v^2, so a bigger speed will have more effect than a smaller mass, thus $K_{\mathrm{A}} > K_{\mathrm{B}}$.
False. $W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F} \cdot d\vec{r}$ means $W_{\mathrm{A}} = W_{\mathrm{B}}$, as the blocks have the same force and displacement. The work-energy theorem then tells us that $K_{\mathrm{A}} = K_{\mathrm{B}}$.
Also, note that $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$, so if $K_{\mathrm{A}} = K_{\mathrm{B}}$, then $\frac{p_{\mathrm{A}}^2}{2m_{\mathrm{A}}} = \frac{p_{\mathrm{B}}^2}{2m_{\mathrm{B}}}$, which can be rearranged into

S8-4: Carts on a Track

- Two carts, A and B, are initially at rest on a level, frictionless table. A constant force of magnitude F_0 is exerted on each cart as it travels between two marks. Cart B has a greater masss than cart A.



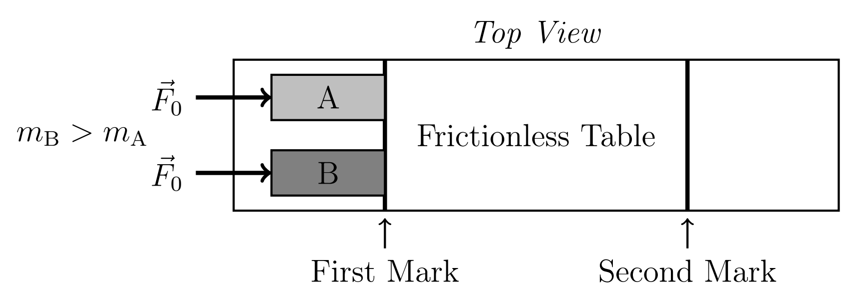
- Three students discuss the final momentum and kinetic energy of each cart.

  (1) *"Since the same force is exerted on both carts, the cart with the smaller mass will move quickly, while the cart with the larger mass will move slowly. The momentum of each cart is equal to its mass times its velocity."*

  (2) *"This must mean that the speed compensates for the mass and the two carts have equal final momenta."*

  (3) *"I was thinking about the kinetic energies. Since the velocity is squared to get the kinetic energy, but mass isn't, the cart with the bigger speed must have more kinetic energy."*

- Do you agree or disagree with the statements made by each student?
- Which cart takes longer to travel between the two marks? Explain your reasoning.
- Determine if the magnitude of the final momentum of cart A is *greater than*, *less than*, or *equal to* that of cart B.
- Determine if the final kinetic energy of cart A is *greater than*, *less than*, or *equal to* that of cart B.
- Reflect on your initial thoughts about the three students above.



Main Ideas

- The work-energy and impulse-momentum theorems can be used to solve a broad array of problems.