Torque

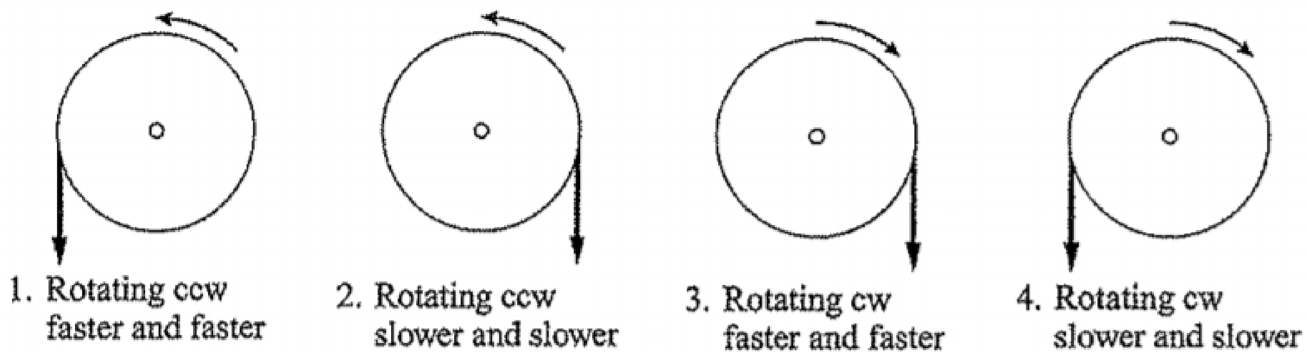
Picture credit: *The Swiss Family Robinson* (1960).

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

Activity 2-1 – Torque

Observe and Explain. For each situation shown in the figure below, determine the signs of the net torque, $\Sigma\boldsymbol{\tau}$, the initial angular velocity, $\boldsymbol{\omega}$, and the angular acceleration, $\boldsymbol{\alpha}$. Sign convention is + for counterclockwise, − for clockwise, and 0 for zero net torque or 0 for the angular velocity of a stationary disk. The curved arrows represent the initial direction of rotation. The straight arrows represent the direction of a rope pulling on each pulley. The axis for each pulley is fixed in place and is frictionless.

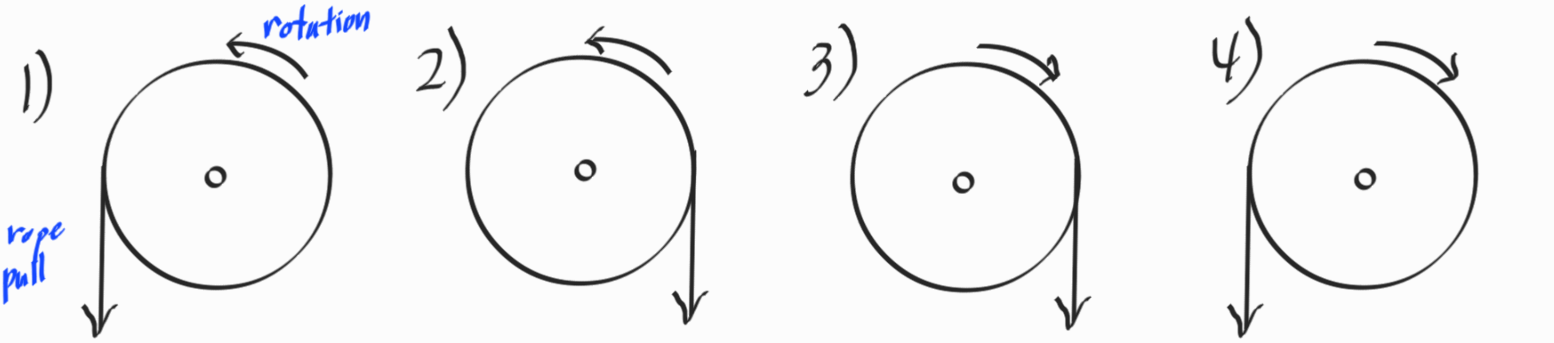


Remember that the sign of the angular acceleration, $\boldsymbol{\alpha}$, is the same as the sign of the initial angular velocity, $\boldsymbol{\omega}$, if the magnitude of the angular velocity is increasing, opposite the sign of the initial angular velocity if its magnitude is decreasing, and zero if the magnitude of the angular velocity is not changing.

a. Fill in the appropriate signs in the following table:

situation	sign of $\Sigma\boldsymbol{\tau}$	sign of $\boldsymbol{\omega}$	sign of $\boldsymbol{\alpha}$
1			
2			
3			
4			

2-1 Torque

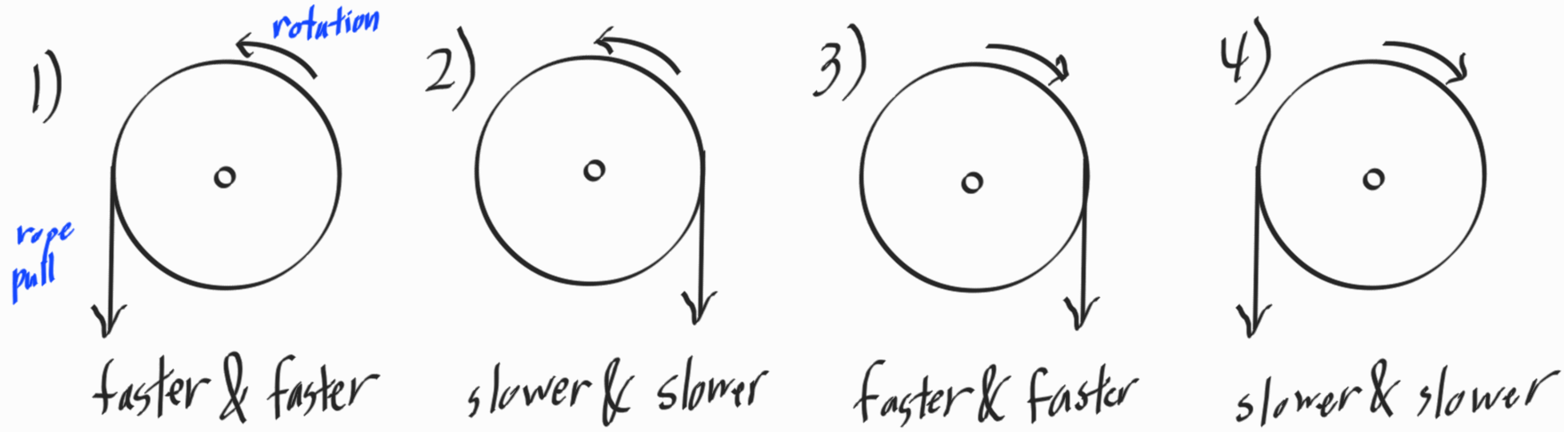


faster & faster | slower & slower | faster & faster | slower & slower

Situation	sign of $\Sigma \tau$	sign of ω	sign of α
1			
2			
3			
4			

2-1 Torque

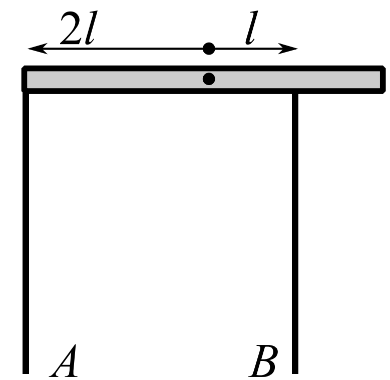
Finish by T 3:10
W 10:10



Situation	sign of $\Sigma\tau$	sign of ω	sign of α
1	+	+	+
2	−	+	−
3	−	−	−
4	+	−	+

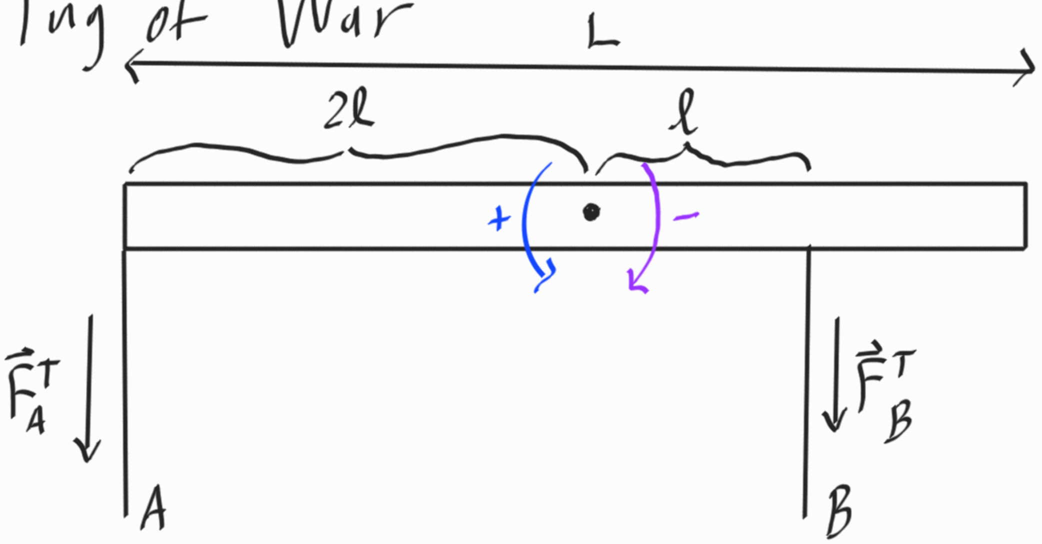
Activity 2-2 – Tug-of-War

Top-down view



- Two athletes are engaged in a modified game of Tug-of-War. Each athlete has a rope that is connected to a long wooden board (mass m) that is free to rotate about its center, as shown in the figure.
  - If the wooden board does not rotate, how does the tension from athlete A compare to the tension from athlete B?
  - Which one will win if athlete B can exert a tension force that is 1.5 times bigger than athlete A?
    - Write the moment of inertia for the board in terms of given quantities by looking at a moment of inertia table. You will need to decide what kind of object to use as a model for the board and choose an appropriate axis of rotation.
    - Write a symbolic expression for the board's angular acceleration.

2-2 Tug of War



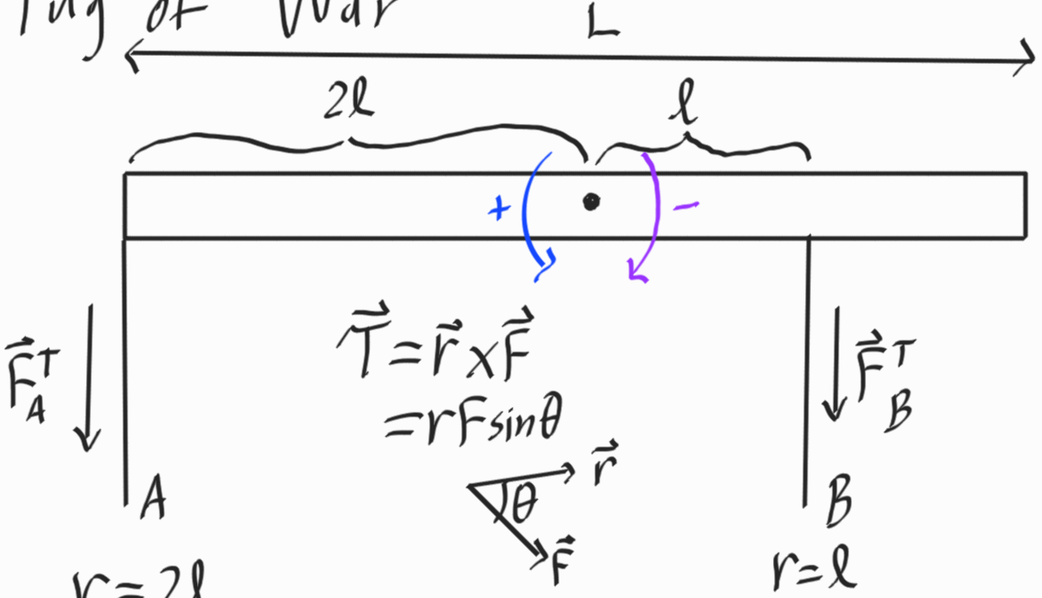
$I = \frac{1}{12} m L^2$

Board does not move?

B pulls more strongly than A?

Finish by T 3:35
W 10:35

2-2 Tug of War



$$\vec{\tau} = \vec{r} \times \vec{F} = rF\sin\theta$$

$$I = \frac{1}{12}mL^2 = \frac{1}{12}m(4l)^2 = \frac{4}{3}ml^2$$

$r = 2l$

$\tau_A = 2lF_A^T$

$r = l$

$\tau_B = -lF_B^T$

Board does not move?

$$\sum \tau = I\alpha$$

$$2lF_A^T - lF_B^T = I(0\ \mathrm{rad/s^2})$$

$$2lF_A^T = lF_B^T$$

$$\boxed{F_A^T = \frac{1}{2}F_B^T}$$

B pulls more strongly than A?

$$F_B = 1.5F_A$$

$$2lF_A^T - lF_B^T = I\alpha$$

$$2lF_A^T - l(1.5F_A^T) = \frac{3}{4}ml^2\alpha$$

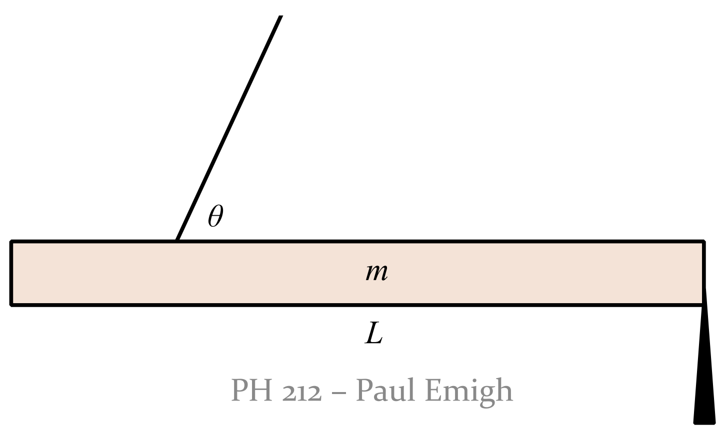
$$\frac{1}{2}F_A^T = \frac{4}{3}ml\alpha$$

$$\boxed{\alpha = \frac{3}{8}\frac{F_A^T}{ml}}$$

$$\frac{kg\frac{m}{s^2}}{kg\,m} \downarrow 1/s^2$$

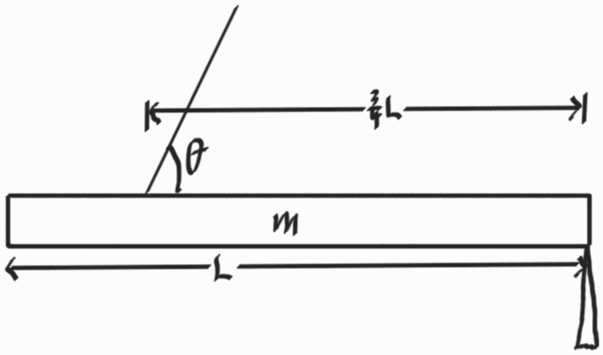
Activity 2-3 – Balancing Board

- Shown below is a wooden board of mass m and length L balanced on a pivot point by the rope attached a distance $3L/4$ from the pivot point.
  - Draw an extended free-body diagram for the board.
  - Choose an origin of coordinates.
  - Write equations representing Newton's 2nd Law for both translational and rotational motion for the board.
  - Solve for the tension in the rope.
  - How would you make sense of your answer?



2-3 Balancing Board

Start by T 3:55
W 10:55



$F_x^{net} =$

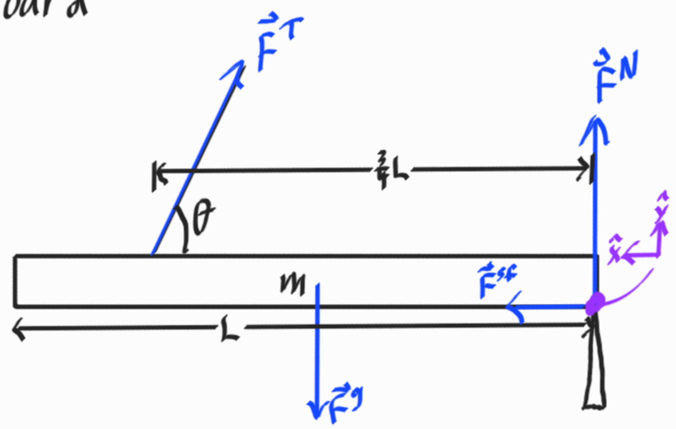
$F_y^{net} =$

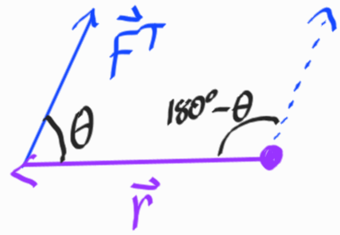
$\tau^{net} =$

Special Cases

2-3 Balancing Board

Finish by T 4:10 W 11:10

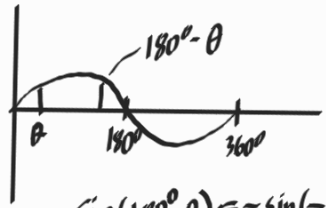




$$F_x^{net} = F^{sf} - F^T \cos\theta \qquad F_y^{net} = F^T \sin\theta + F^N - F^g \qquad \tau^{net} = \frac{L}{2}F^g - \left(\frac{3}{4}L\right)F^T \sin(180° - \theta)$$

$= I\alpha$

$0 \ \frac{rad}{s^2}$



$\sin(180° - \theta) = -\sin(-\theta)$
$= \sin\theta$

$$0 = \frac{L}{2}\left(F^g - \frac{3}{2}F^T \sin\theta\right)$$

$$mg = F^g = \frac{3}{2}F^T \sin\theta$$

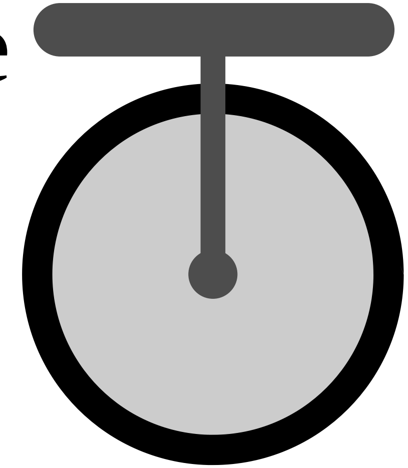
$$\boxed{F^T = \frac{2mg}{3\sin\theta}}$$

Special Cases

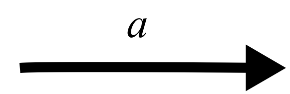
$\theta \to 0° \Rightarrow F^T \to \infty$ b/c pulling to the side means only a small portion of the force is acting against gravity, so more overall is required.

$\theta = 90° \Rightarrow F^T = \frac{2}{3}mg$ b/c pulling farther out from the pivot is more effective, requiring less force.

Activity 2-4 – Unicycle



- Shown at right is a unicycle (a vehicle with one wheel), which has mass m, radius R, and axle radius r that is accelerating to the right.
  - How is the tire's angular acceleration related to the tire's translational acceleration?
  - Draw an extended free-body diagram for the unicycle.
  - What direction is the net force on the unicycle?
  - What direction is the net torque on the unicycle?
  - Think carefully about your answers above
    - are they consistent with each other?
    - what do you think is the nature of the *internal* interaction between the top of the unicycle and the wheel.



2-4 Unicycle

Angular & Tangential

$$a_t = \alpha R$$

$s = \theta R \Rightarrow a_t = \frac{d^2 s}{dt^2} = \frac{d^2\theta}{dt^2} R = \alpha R$

Tangential vs. Translational

a_t — acceleration of rim of wheel around axis of rotation

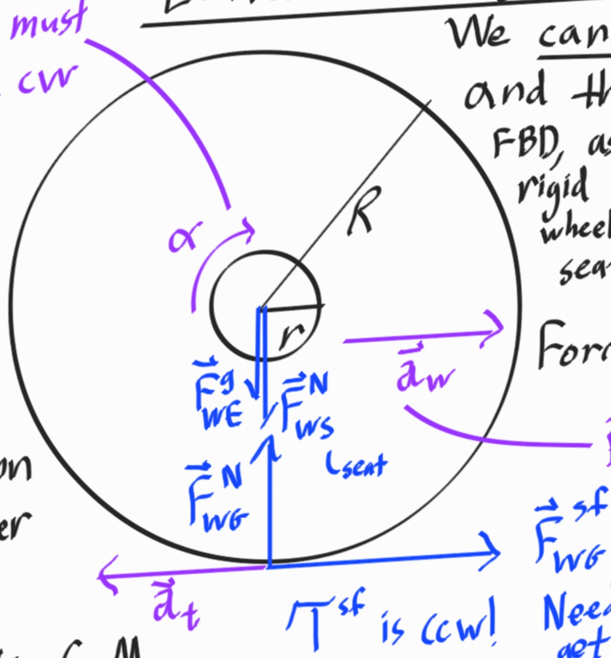
a_w — translational acceleration of the wheel's center of mass (CoM)

$a_t = a_w$ (as magnitudes) b/c CoM accelerates over the ground while the rim of the wheel does not slip against the ground

Extended FBD for Wheel

We cannot put the wheel and the seat in the same FBD, as that would violate the rigid body model (since the wheel spins independent of the seat).

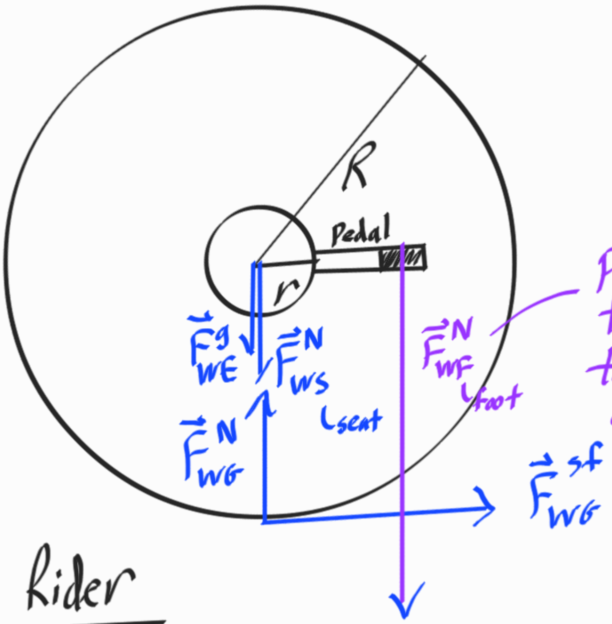
Forces are not to scale.



Need another force to get the correct torque.

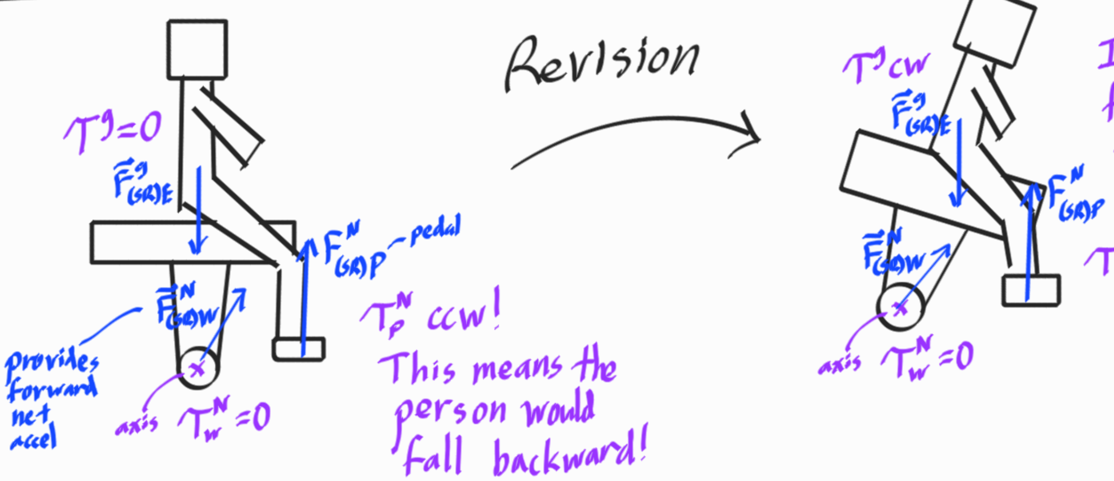
We forgot the pedals!

Revised FBD for Wheel



Provides the torque needed to make τ_{net} clockwise.

Bonus: FBD of Seat and Rider



If the rider leans forward, the torques from gravity and the pedal can cancel out, keeping the rider steady.