Vectors in a Garden

Benjamin Bauml
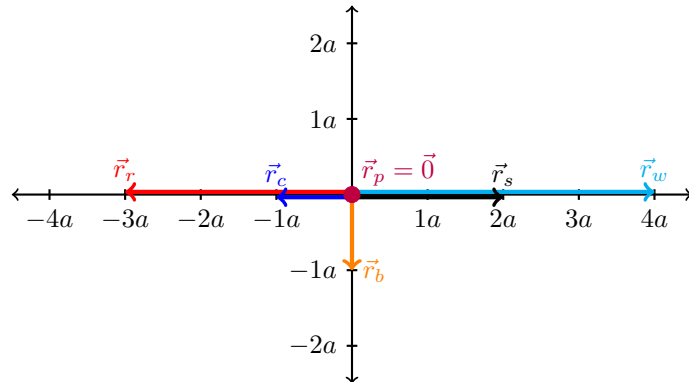
Summer 2024

XX-1: Vectors in a Garden

You visit a garden with a trail that includes the following landmarks.

- Red roses at $\vec{r}_r = -3a\hat{x}$

- White roses at $\vec{r}_w = +4a\hat{x}$

- A pond at $\vec{r}_p = 0\hat{x}$

- A bench at $\vec{r}_b = -a\hat{y}$

- A bridge over a creek at $\vec{r}_c = -a\hat{x}$

- A statue at $\vec{r}_s = +2a\hat{x}$

(1) Sketch and label the garden and its landmarks.

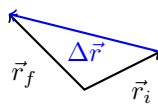(2) Find the following displacement vectors using both symbols and diagrams.

A displacement vector, $\Delta\vec{r}$, can be thought of as the difference of the final position vector, $\vec{r}_f$, and the initial position vector, $\vec{r}_i$:

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i.$$

It can also be thought of as the vector that, when added to $\vec{r}_i$, gives you $\vec{r}_f$:

$$\vec{r}_f = \Delta\vec{r} + \vec{r}_i.$$

As such, $\Delta\vec{r}$ points from the tip of $\vec{r}_i$ to the tip of $\vec{r}_f$:

(a) From the red roses to the white roses

$$\vec{r}_r = -3a\hat{x} \qquad\qquad \vec{r}_w = 4a\hat{x}$$

$$\Delta\vec{r}_{r\to w} = \vec{r}_w - \vec{r}_r = 4a\hat{x} - (-3a\hat{x}) = 7a\hat{x}$$

(b) From the pond to the red roses

$$\vec{r}_r = -3a\hat{x} \qquad\qquad \vec{r}_p = \vec{0}$$

$$\Delta\vec{r}_{p\to r} = \vec{r}_r - \vec{r}_p = \vec{r}_r = -3a\hat{x}$$

(c) From the bench to the statue

$$\vec{r}_s = 2a\hat{x}$$

$$\vec{r}_b = -a\hat{y}$$

$$\Delta\vec{r}_{b\to s} = \vec{r}_s - \vec{r}_b = 2a\hat{x} - (-a\hat{y}) = 2a\hat{x} + a\hat{y}$$

$$|\Delta\vec{r}_{b\to s}| = \sqrt{(2a)^2 + a^2} = \sqrt{5}a$$