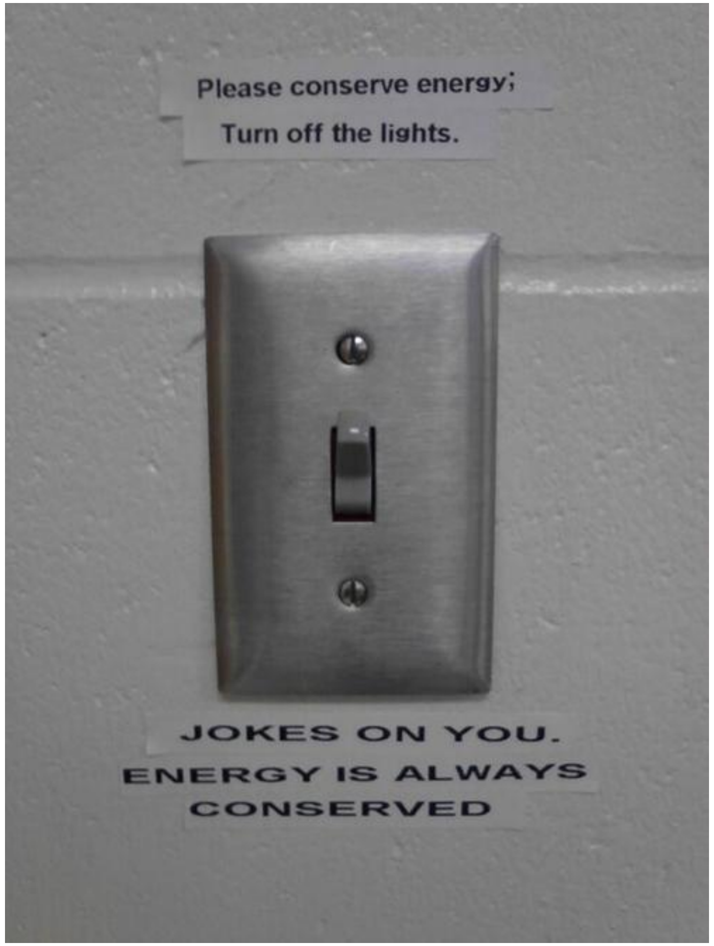
Studio Week 8
Energy Conservation

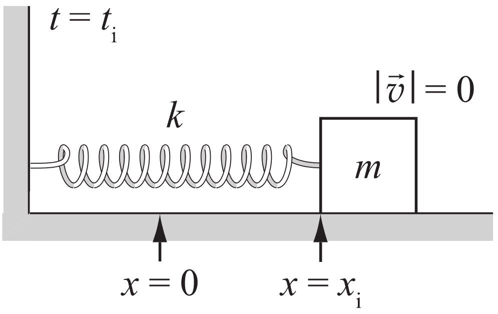


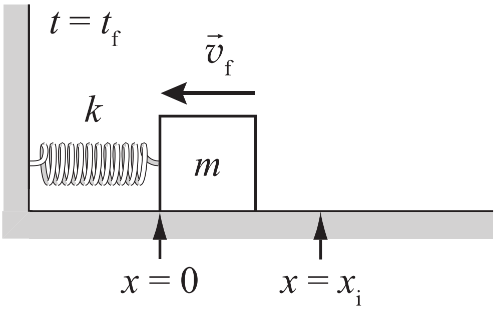
Picture credit: the Internet.

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Make sense of everything.**

Activity 8-1: Block on a Spring - Forces





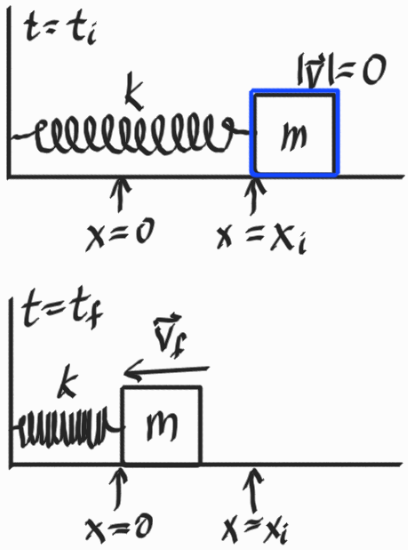
- A block of mass m on a level, frictionless surface is attached to an ideal massless spring of constant k that is initially stretched.
- At time t_i, the block is **released from rest** at $x = x_i$.
- At time t_f, the block reaches $x = 0$ moving to the left with speed v_f.
- System B consists of the block alone.
- System SB consists of the spring and the block.
- For the interval from t_i to t_f (for each system):
  1. list all external forces acting on the system
  2. identify the point of contact associated with each force
  3. draw a vector to indicate the **displacement** associated with each force
  4. determine if the work done by **each force** is positive, negative, or zero
- What do you want to remember about this situation for future problems?

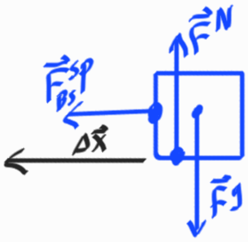
System	B	SB
Forces		
Displacements		
Works		

8-1 Block on a Spring – Forces

For this activity, I will identify the points of contact for forces on my free-body diagrams, which will violate our normal FBD conventions.

System B





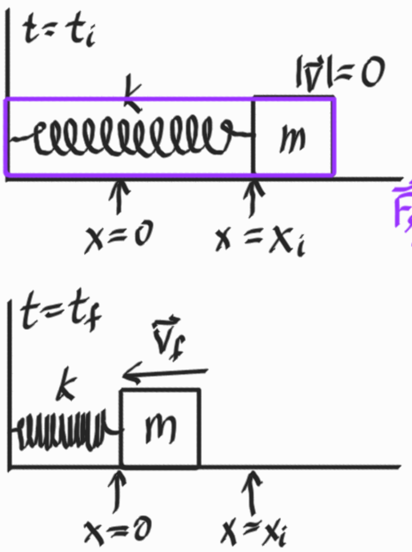
$W^g = 0$ & $W^N = 0$

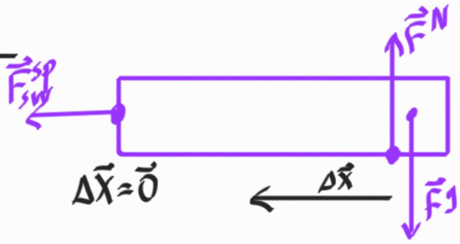
The points of contact for F^g and F^N are moving, but the forces are perpendicular to the displacement, so the work is zero.

$W^{sp} > 0$

The spring pulls to the left, in the direction of the displacement of its point of contact.

System SB





$W^N = 0$ & $W^g = 0$

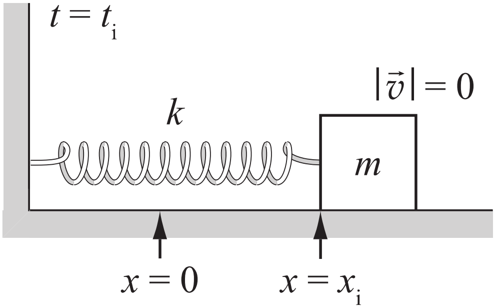
Same as before

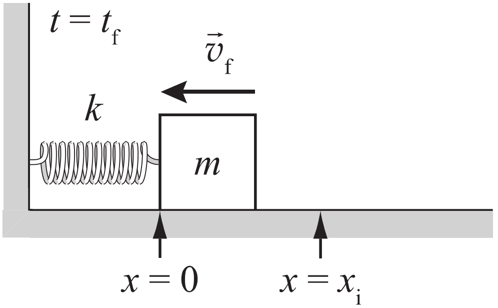
$W^{sp} = 0$

The work done on the spring by the wall is zero, as the point of contact at the stationary wall undergoes no displacement.

System	B			SB		
Forces	F^g	F^N	F^{sp}	F^g	F^N	F^{sp}
Displacements	Δx	Δx	Δx	Δx	Δx	0
Works	0	0	W^{sp}	0	0	0

Activity 8-2: Block on a Spring - Energy





- A block of mass *m* on a level, frictionless surface is attached to an ideal massless spring of constant *k* that is initially stretched.
- At time t_i, the block is **released from rest** at $x = x_i$.
- At time t_f, the block reaches $x = 0$ moving to the left with speed v_f.
- System B consists of the block alone.
- System SB consists of the spring and the block.
- For the interval from t_i to t_f (for each system):
  - determine if the **net external work** is positive, negative, or zero (based on Activity 8-1)
  - determine if the **change in total energy** is positive, negative, or zero
  - determine if the **change in kinetic energy** is positive, negative, or zero
  - determine if the **change in potential energy** is positive, negative, or zero
- What is different about the two systems?

System	B	SB
Net external work		
Change in total energy		
Change in kinetic energy		
Change in potential energy		

8-2 Block on a Spring - Energy

System	B	SB
$W_{net,ext}$	+	0
ΔE_{tot}	+	0
ΔK	+	+
ΔU	0	−

For B, there is net external work, but for SB there is not. This follows from the previous activity.

By the work-energy theorem, $\Delta E_{tot} = W_{net,ext}$.

The block starts at rest and ends with nonzero speed, so it must have gained kinetic energy, regardless of our choice of system.

A block by itself that cannot change elevation can only have kinetic energy. It has no way of storing energy (no potential).

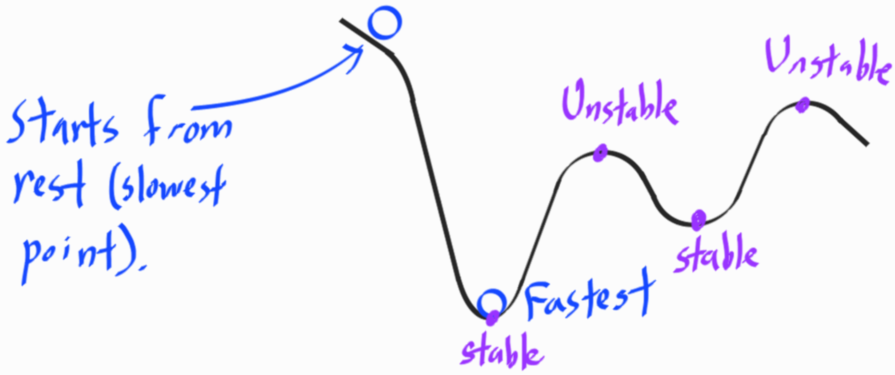
Without external work, the ΔK in SB must come from somewhere, and $\Delta E_{tot} = 0$, so there must be energy stored in the system that is transformed into kinetic energy. By putting the spring in the system, the system now can store potential energy.

Activity 8-3: Ball on a Ramp

- Consider the ball that can roll on the track in the center of the room. Ignore friction and air resistance. The ball starts from rest at the top of the track. Use the apparatus to see how the ball moves.

1. At what point will the ball be moving the fastest? How do you know?
2. At what point will it be moving the slowest? How do you know?
3. Are there any stable equilibrium points? If so, then where are they located?
4. Are there any unstable equilibrium points? If so, then where are they located?
5. Make a graph of the gravitational potential energy of the ball vs. horizontal position. Remember to define the origin.
6. On the same set of axes, make a graph of the total mechanical energy of the ball *vs.* horizontal position. Assume that the ball starts from rest.
7. Where are the points determined previously for questions 1 through 4 above located on the graph?
8. How can stable and unstable equilibrium points be defined in terms of the graph of potential energy *vs.* horizontal position?

8-3 Equilibrium on a Track



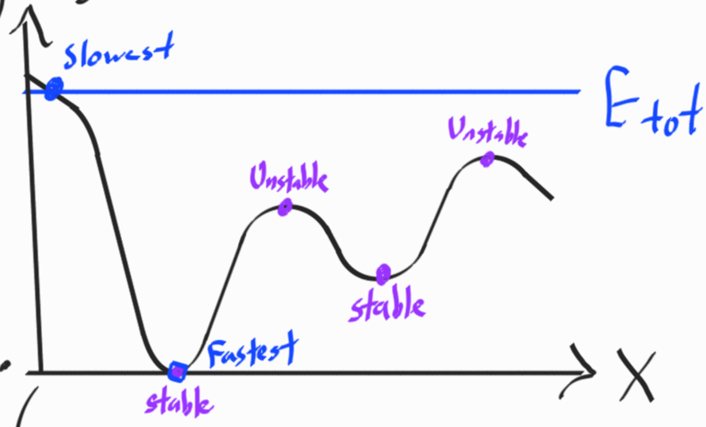
1) The ball will be moving fastest at the lowest point on the track, where the most potential energy has been transformed into kinetic energy.
2) The ball will be moving slowest at the top of the track, where it starts with zero speed.

   It won't be going slowest at the two small peaks, but those will be local minima for its speed, as its kinetic energy will be at a local minimum in these places.

3) There are stable equilibrium points at the bottoms of the two dips in the track. If the ball is placed slightly off of one of these points, it will want to fall downhill toward it.
4) There are unstable equilibrium points at the tops of the two peaks on the track. If the ball is placed perfectly at one of these points, it will stay, but if it is placed slightly off of one of these points, it will want to fall downhill away from it.

5) $U_g = mgh$

Since $U_g = mgh$, this graph will have the same shape as the physical track.



We can choose the origin here, setting the initial position at $x=0$ and putting $U_g = 0$ at the bottom of the track.

8) Equilibrium occurs where the slope is zero:

$$\frac{\partial U_g}{\partial x} = 0.$$

Stable equilibrium occurs where the graph is concave up: $\frac{\partial^2 U_g}{\partial x^2} > 0$.

Unstable equilibrium occurs where the graph is concave down:

$$\frac{\partial^2 U_g}{\partial x^2} < 0.$$

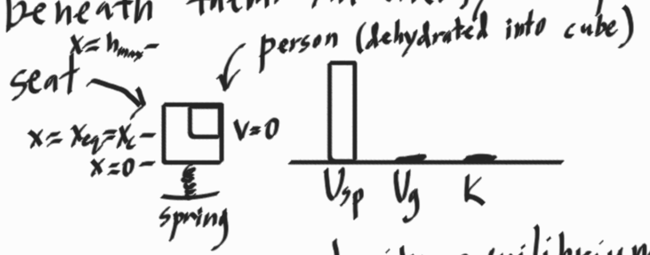
Activity 8-4: Stunt Car

- A stationary stunt car has an ejector seat that rests on a compressed vertical spring.
- When the spring is released, the seat with its passenger is launched out of the car and into the air.
  - explain why you might want to include the Earth and the spring in your system
- Consider three instants in time: (1) just before the seat is released, (2) when the spring is at equilibrium, and (3) the person reaches maximum height.
  - Write a qualitative description of how the energy of the system transforms through these three instants.
- For each instant:
  - draw a physical diagram
  - construct an energy bar chart
  - write each energy symbolically
- How high does the person go?
- What is the person’s speed when the ejector seat leaves the spring?
  - Don’t forget to make sense of your answers!

8-4 Ejector Seat

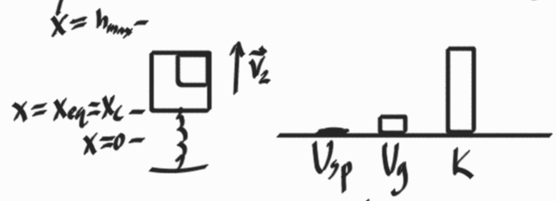
We want to include the Earth and the spring in our system with the seat and passenger in order to use conservation of energy. If we don't then we have to go to the trouble of calculating work done by gravity and the spring.

t_1: The person and seat are stationary, and the spring is compressed beneath them. All energy is spring potential energy.



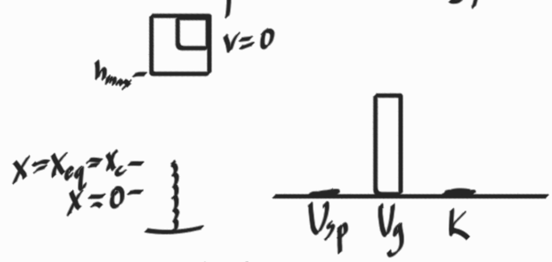
Here, I define the amount by which the spring was compressed as x_c, and I set $x=0$ at the top of the spring.

t_2: The spring is at its equilibrium length, and the person and seat have been lifted up slightly and now have some speed. The spring potential energy has all been transformed into kinetic energy and a little bit of gravitational potential energy.



I use the label v_2 for the speed at this moment. It is not v_{max}, which would actually happen before the spring reaches equilibrium. $v_2 < v_{max}$

t_3: The person and seat stop momentarily at their maximum height. The kinetic energy has all been transformed into gravitational potential energy.



Use the following helpful table!

Write the instants down the side.

List every possible type of energy as a column, and put a symbolic representation of each energy at each time into the table entries.

Make a final column for total energy, and write the sum of the individual energies for that row in each entry.

	U_{sp}	U_g	K	E_{tot}
t_1	$\frac{1}{2}kx_c^2$	0	0	$\frac{1}{2}kx_c^2$
t_2	0	mgx_c	$\frac{1}{2}mv_2^2$	$mgx_c+\frac{1}{2}mv_2^2$
t_3	0	mgh_{max}	0	mgh_{max}

If there is no external work between any two times, then the E_{tot} entries for those times are equal!

$$E_{tot,1} = E_{tot,3}$$

$$\frac{1}{2}kx_c^2 = mgh_{max}$$

$$\boxed{h_{max} = \frac{kx_c^2}{2mg}}$$

$$E_{tot,1} = E_{tot2} \Rightarrow \frac{1}{2}kx_c^2 = mgx_c + \frac{1}{2}mv_2^2$$

$$\frac{1}{2}kx_c^2 - mgx_c = \frac{1}{2}mv_2^2$$

$$\left(\frac{k}{m}x_c - 2g\right)x_c = v_2^2$$

$$\boxed{v_2 = \sqrt{\left(\frac{k}{m}x_c - 2g\right)x_c}}$$

Sense making

$$h_{max} = \frac{kx_c^2}{2mg} \quad \text{Units: } [h_{max}] = \frac{[k][x_c]^2}{[mg]} = \frac{\frac{N}{\cancel{m}} \cdot m^{\cancel{2}}}{\cancel{N}} = m \checkmark$$

Covariation

- k increases or x_c increases $\Rightarrow h_{max}$ increases

  A stiffer spring (higher k) stores more energy for the same compression, and a more compressed spring (higher x_c) stores more energy. Imparting more kinetic energy to the seat and person will allow them to go higher.

- m increases or g increases $\Rightarrow h_{max}$ decreases

  A more massive object will store more gravitational potential energy in less of an elevation change, as would an object in stronger gravity. As such, the object would transform its kinetic energy entirely at a smaller maximum height.

$$v_2 = \sqrt{\left(\frac{k}{m}x_c - 2g\right)x_c} \quad \text{Units: } [v_2] = \sqrt{\left(\frac{[k]}{[m]}[x_c] - [g]\right)[x_c]} = \sqrt{\left(\frac{N/m}{kg} \cdot m - \frac{m}{s^2}\right) \cdot m}$$

$$= \sqrt{\left(\frac{N}{kg} - \frac{m}{s^2}\right) \cdot m}$$

$$= \sqrt{\left(\frac{m}{s^2} - \frac{m}{s^2}\right) \cdot m}$$

$$= \sqrt{\frac{m^2}{s^2}} = \frac{m}{s} \checkmark$$

Special Case

- What if $\frac{k}{m}x_c = 2g$?

  The equation would say $v_2 = 0$.

  $$\frac{k}{m}x_c = 2g \Rightarrow kx_c = 2mg \Rightarrow F^{sp}_{initial} = 2F^g$$

  $$\Rightarrow x_c = \frac{2mg}{k} \Rightarrow h_{max} = \frac{k\left(\frac{2mg}{k}\right)^2}{2mg} = \frac{2mg}{k} = x_c$$

  If $v_2 = 0$, the seat does not leave the spring. Our equations are consistent with each other.

- What if $\frac{k}{m}x_c < 2g$?

  Then v_2 is not real (we are taking the square root of an imaginary number).

  Physically, if $x_c < \frac{2mg}{k}$, then the spring never fully decompresses, and v_2 is defined based on the assumption that it does, so it makes sense that we would get an invalid answer.