Lecture 13: Newton' 3rd Law of Motion

(and a Bit of Springs)

Warm-Up Activity

Which of the following statements, if any, are true about Newton's 3rd law pairs?

(A) They appear on different free-body diagrams.

(B) They are the same type of force.

(C) They appear on the same free-body diagram.

(D) $\vec{F}^{t}_{AB} = -\vec{F}^{t}_{AB}$

Feedback Question

Do you prefer feedback through documents uploaded to Canvas, or through Gradescope (as was done on the quizzes)?

(A) PDFs on Canvas

(B) Comments in Gradescope

Newton's 3rd Law of Motion

- If A exerts a force on B, then B exerts a force of the same magnitude on A in the opposite direction:

$$\vec{F}^{t}_{AB} = -\vec{F}^{t}_{BA}$$

- These two forces make a *Newton's 3rd law pair*, or an *action-reaction pair*.
- 3rd law pair forces...
  - are the same type of force;
  - appear on different free body diagrams.

The Negative Sign
The negative sign tells us that the force of the spring points opposite the displacement from equilibrium. The spring wants to return to its unstretched/uncompressed length. Forces that try to return a system to equilibrium are called *restoring forces*.

Spring Forces

- Many objects resist changes in physical configuration (*i.e.* deformations).
- For small deformations, we can model the object as a spring.
- The forces caused by springs obey Hooke's law: $\vec{F}^S = -k(\vec{x} - \vec{x}_{eq})$.
  - $\Delta\vec{x} = (\vec{x} - \vec{x}_{eq})$ is displacement from equilibrium.
  - k is the spring constant.
  - What does the negative sign mean?

Types of Forces

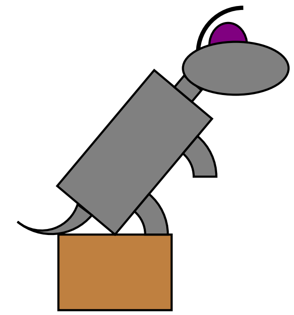
- Gravity $\vec{F}^g_{AB} = m_A \vec{g}_B$
  - Newtonian $\vec{g}_B = G\frac{M_B}{r^2}(-\hat{r})$, $G = 6.67408 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
  - Near-Earth $\vec{g}_E = g(-\hat{y}),\ g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2} \approx 10\frac{\mathrm{m}}{\mathrm{s}^2}$
- Normal $\vec{F}^N$ always $\perp$; varies in magnitude
- Tension $\vec{F}^T$ uniform (massless, inextensible rope)
- Spring $\vec{F}^S = -k(\vec{x} - \vec{x}_{eq})$
- Friction
  - Static Friction $F^{sf} \leq \mu_s |\vec{F}^N|$
  - Kinetic Friction $F^{kf} = \mu_k |\vec{F}^N|$

Not Forces

- Momentum
- Inertia
- Velocity
- Acceleration

A*R*C*S: Uh-Oh Dr. Paws

In the video in Section 3.16 of our textbook, Paul pushes a footstool (mass m_1) across the floor with a constant force so that the footstool speeds up. Dr. Paws (a dog with mass m_2) is sitting on the footstool. The coefficient of static friction between the dog and the footstool is μ (assume no friction between the footstool and the ground). How much force can Paul exert on the footstool before the dog begins sliding?



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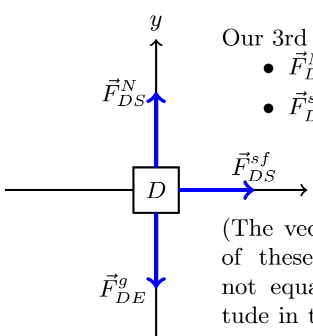
(1a) Understand the Problem

- Mass of the dog: $m_2 = 30$ kg
  - A typical golden retriever is 25-34 kg, so a nice round 30 kg is reasonable and helpful to us.
- Mass of the footstool: $m_1 = 10$ kg
  - A footstool can reasonably have less mass than a sizable dog.
- Gravity: $g = 10\ \mathrm{m/s^2}$
  - This comes from our near-Earth approximation.
- Coefficient of static friction: $\mu = 0.4$
  - This is a reasonable coefficient of static friction in general (wood-on-wood can range from 0.25-0.5, by some sources), and I don't have a better guess for the specific situation of dog-on-footstool.

(1b) Identify Assumptions

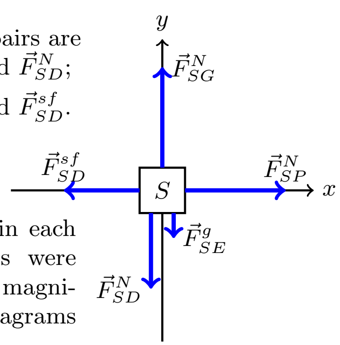
- Near-Earth
  - This can reasonably be expected to occur in a home on Earth's surface, where elevation does not appreciably affect gravitational acceleration.
- Neglect air-resistance
  - Air resistance is a much weaker effect at low speeds, and we probably are not going to get the dog truly racing in our living room.
- Dr. Paws doesn't move.
  - This is probably the weakest assumption (we saw what Dr. Paws did in the video), but it is necessary, as the problem becomes far more difficult if the dog and the stool do not move perfectly together. With this, we can say that $\vec{a}_D = \vec{a}_S = a\hat{x}$.
- Particle model
  - If the dog doesn't move, then its shape doesn't change, and it doesn't rotate, so we can ignore those complicated aspects of motion and just concentrate on the motion of the dog's center of mass.

(1c) Represent Physically



Our 3rd law pairs are
- $\vec{F}^N_{DS}$ and $\vec{F}^N_{SD}$;
- $\vec{F}^{sf}_{DS}$ and $\vec{F}^{sf}_{SD}$.

(The vectors in each of these pairs were not equal in magnitude in the diagrams on the right.)



L13-1: Uh-Oh Dr. Paws – Analyze and Represent

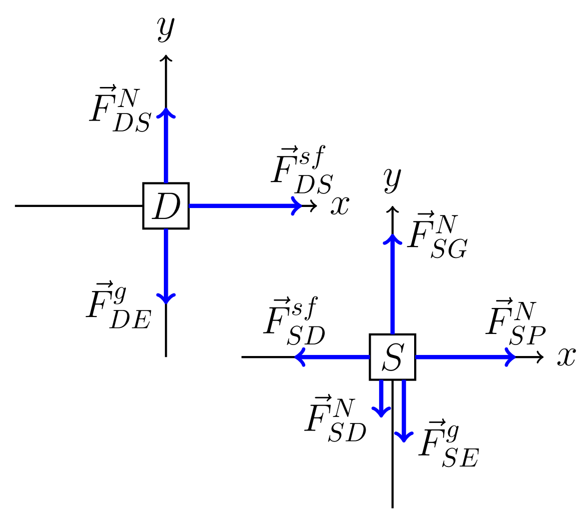
(1a) Understand the Problem

- Mass of the footstool: $m_1 = 10$ kg
- Mass of the dog: $m_2 = 30$ kg
- Gravity: $g = 10\ \mathrm{m/s^2}$
- Coefficient of static friction: $\mu = 0.4$

(1b) Identify Assumptions

- Near-Earth
- Particle model
- Neglect air-resistance
- Dr. Paws doesn't move.

(1c) Represent Physically



- Why are the assumptions reasonable?
- Identify any problems with these free body diagrams.
- Identify the third-law pairs.

None of these equations is correct, and we can figure this out with different sensemaking techniques.

(A) $F_{SP}^{N} = \mu \frac{m_1}{m_1+m_2} g$
Unit Check: The fraction $\frac{m_1}{m_1+m_2}$ is unitless (kilograms in the numerator and denominator cancel), so the right hand side has units of $\mathrm{m/s^2}$, which is not a force.

(B) $F_{SP}^{N} = \mu(m_2 - m_1)g$
Special-Case Analysis: If the dog and the footstool are equal in mass ($m_1 = m_2$), then $F_{SP}^{N} = 0$, which means pushing with any force will cause the dog to slide. It does not seem reasonable that even the smallest amount of force would break the bonds of static friction.

(C) $F_{SP}^{N} = \mu \frac{m_1 m_2}{m_1+m_2} g$
Special-Case Analysis: Suppose that the dog is much more massive than the footstool ($m_2 \gg m_1$). That would mean that

$$\frac{m_1 m_2}{m_1 + m_2} \approx \frac{m_1 m_2}{m_2} = m_1.$$

As such, in this special case, $F_{SP}^{N} = \mu m_1 g$. It does not seem reasonable that increasing the mass of the dog (which proportionally increases the maximum static friction between the dog and the stool) would eventually cease to have an effect on the feasibility of pushing the stool out from under the dog.

Sensemaking for the Correct Answer
The actual answer is $F_{SP}^{N} \leq \mu(m_1+m_2)g$ (see the next page for the Calculate steps that obtain this).
(3a) Units $\left[F_{SP}^{N}\right] = [\mu]\,[(m_1+m_2)]\,[g] = 1 \cdot (\mathrm{kg}+\mathrm{kg}) \cdot \frac{\mathrm{m}}{\mathrm{s}^2} = \frac{\mathrm{kg\,m}}{\mathrm{s}^2} = \mathrm{N}$
(3b) Numbers Is a 160 N force reasonable?

- To hold the dog and the stool stationary in the air, Paul would have to exert a normal force equal to the force of gravity on the stool-dog system: $F_{SD,P}^{N} = F_{SD,E}^{g} = (m_1+m_2)g = 40\ \mathrm{kg} \cdot 10 \frac{\mathrm{m}}{\mathrm{s}^2} = 400\ \mathrm{N}$.
  A force of 160 N is much less than the force required to hold the stool-dog system stationary in the air. This is reasonable—it would be easier to push on the stool than to hold everything up.

(3c) Symbols

- If either the mass of the dog or the stool increase, the maximum allowable force increases.
  - Increasing the mass of the dog increases the maximum static friction between the dog and the stool, which should make it harder for the dog to slide.
  - Increasing the mass of the stool makes the dog-stool system more resistant to changes to its motion, so a push that would have caused the dog to slide off of a less massive stool would not accelerate the system as much when applied to a more massive stool, and less friction would be required to give the dog the new, lesser acceleration.
- If the coefficient of friction increases, the maximum allowable force increases.
  - Increasing the coefficient of friction means that the dog and stool are grippier to each other, making it harder for the dog to slide off.

L13-2: Uh-Oh Dr. Paws – Sensemake

You have three friends who each come up with a different equation for the maximum allowable force that Paul can apply:

(A) $F_{SP}^{N} = \mu \frac{m_1}{m_1+m_2} g$

(B) $F_{SP}^{N} = \mu(m_2 - m_1)g$

(C) $F_{SP}^{N} = \mu \frac{m_1 m_2}{m_1+m_2} g$

Which of the above equations, if any, are correct? How can you tell?

(2a) Represent Principles

- Newton's 2nd law: $\vec{F}^{net} = m\vec{a}$
- Newton's 3rd law: $\vec{F}_{AB} = -\vec{F}_{BA}$
- Static friction: $F^{sf} \leq \mu F^N$
- Gravity: $\vec{F}^g = m\vec{g}$

(2b) Solve Symbolically

For the dog,

$$\begin{aligned} m_2 a = F_x^{net} &= F_{DS}^{sf}, \\ 0 = F_y^{net} &= F_{DS}^N - F_{DE}^g \\ F_{DS}^N = F_{DE}^g &= m_2 g. \end{aligned}$$

By the nature of static friction, this means that

$$\begin{aligned} m_2 a = F_{DS}^{sf} \leq \mu F_{DS}^N = \mu m_2 g \\ a \leq \mu g. \end{aligned}$$

For the stool,

$$\begin{aligned} F_x^{net} &= m_1 a \\ F_{SP}^N - F_{SD}^{sf} &= m_1 a. \end{aligned}$$

If we solve for F_{SP}^N, use the fact that $\vec{F}_{SD}^{sf}$ and $\vec{F}_{DS}^{sf}$ are an action-reaction pair, and recall what we know about the static friction on the dog, we find

$$\begin{aligned} F_{SP}^N &= m_1 a + F_{SD}^{sf} \\ &= m_1 a + F_{DS}^{sf} \\ &= m_1 a + m_2 a \\ &= (m_1 + m_2) a. \end{aligned}$$

Now, recall that we found $a \leq \mu g$, so

$$F_{SP}^N \leq (m_1 + m_2)\mu g.$$

(2c) Plug in Numbers

- Mass of the footstool: $m_1 = 10$ kg
- Mass of the dog: $m_2 = 30$ kg
- Gravity: $g = 10 \text{ m/s}^2$
- Coefficient of static friction: $\mu = 0.4$

$$\begin{aligned} F_{SP}^N &\leq (m_1 + m_2)\mu g \\ &= (10 \text{ kg} + 30 \text{ kg})(0.4)\left(10 \frac{\text{m}}{\text{s}^2}\right) \\ &= 160 \text{ N} \end{aligned}$$

Uh-Oh Dr. Paws – Calculate

(2a) Represent Principles

$$\vec{F}^{net} = m\vec{a} \qquad \vec{F}_{AB} = -\vec{F}_{BA} \qquad F^{sf} \leq \mu F^N \qquad \vec{F}^g = m\vec{g}$$

(2b) Solve Symbolically

Dog

$$\begin{aligned} m_2 a = F_x^{net} &= F_{DS}^{sf} \\ 0 = F_y^{net} &= F_{DS}^N - F_{DE}^g \\ F_{DS}^N &= m_2 g \end{aligned}$$

Stool

$$\begin{aligned} F_x^{net} &= m_1 a \\ F_{SP}^N - F_{SD}^{sf} &= m_1 a \\ F_{SP}^N &= m_1 a + F_{SD}^{sf} = m_1 a + F_{DS}^{sf} \\ F_{SP}^N &= m_1 a + m_2 a = (m_1 + m_2) a \end{aligned}$$

$$\begin{aligned} F_{DS}^{sf} &\leq \mu F_{DS}^N = \mu m_2 g \\ m_2 a &\leq \mu m_2 g \\ a &\leq \mu g \end{aligned}$$

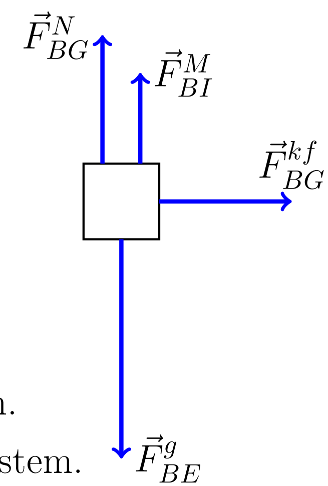
$$F_{SP}^N \leq (m_1 + m_2)\mu g$$

(2c) Plug in Numbers

$$\begin{aligned} F_{SP}^N &\leq (m_1 + m_2)\mu g \\ &= (10 \text{ kg} + 30 \text{ kg})(0.4)\left(10 \frac{\text{m}}{\text{s}^2}\right) \\ &= 160 \text{ N} \end{aligned}$$

A Model for Interactions

- Quantities
  - Mass m
  - Force $\vec{F}$
- Laws
  - Net force is proportional to acceleration: $\vec{F}^{net} = m\vec{a}$
  - Forces come in pairs: $\vec{F}_{AB} = -\vec{F}_{BA}$
- Asssumptions
  - We can treat multiple objects as a system.
  - All forces act as if on the center of the system.
- Diagram



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Solving Problems Using Forces

- Identify a system.
- Identify the (external) forces acting on the system.
  - Draw a free-body diagram.
- Identify the acceleration (**not a force**).
  - Static/dynamic equilibrium (acceleration = 0)
  - Dynamics (acceleration not 0)
- Use the laws of motion.
- Reflect on your answer (check units and evaluate special cases).

Main Ideas

- Newton's 3rd law of motion can be used to relate the forces acting on *different* objects or systems.