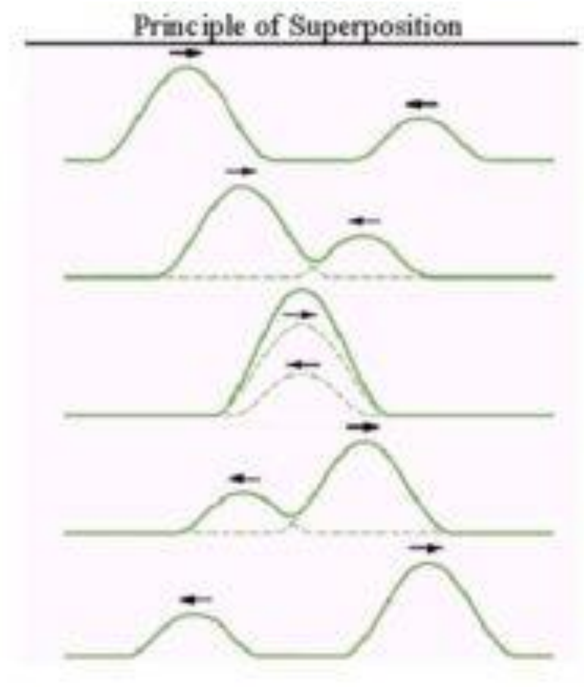
Studio Week 7
Standing Waves



Picture credit: toppr.com.

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

Activity 7-1

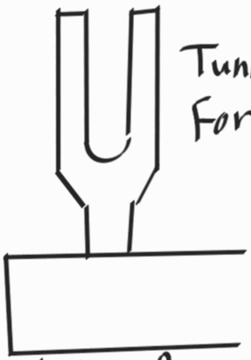
- After observing the demonstration, use a graphing calculator (such as desmos.com/calculator) to explore superposition of waves that have differing frequencies.
  - Graph a wave with frequency 256 Hz.
  - Graph a second wave with a different frequency.
  - Graph the sum of the two waves above.
  - Adjust the frequencies of the two waves to see if you can replicate what you observed during the demonstration.

7-1 Beats

Wrap 2:20
10:20

Beats Demonstration

Do this before 7-1, then repeat afterward.

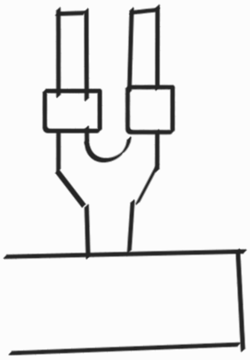


Tuning Fork

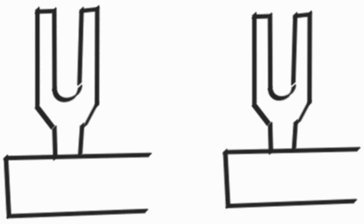
Hollow Resonator Box

When the tuning fork is struck, an audible tone is produced.

Weights can be attached to the fork to change its frequency.

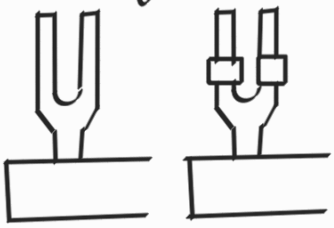


① Strike both forks.



Produce the same tone.

② Add weights most of the way down one fork and strike both again.

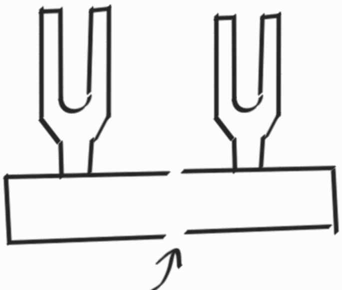


This one will produce a slightly different tone.

One tone is heard (its frequency is the average of the two), plus a periodic "beating" as its amplitude changes.

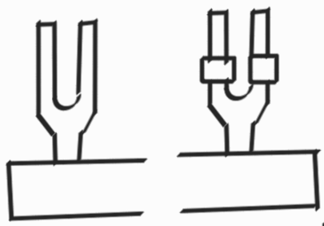
Resonance Demonstration

Do this after 7-1.

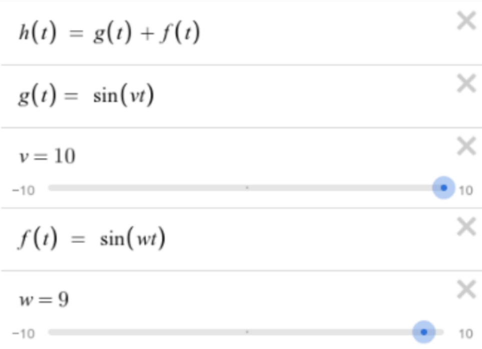


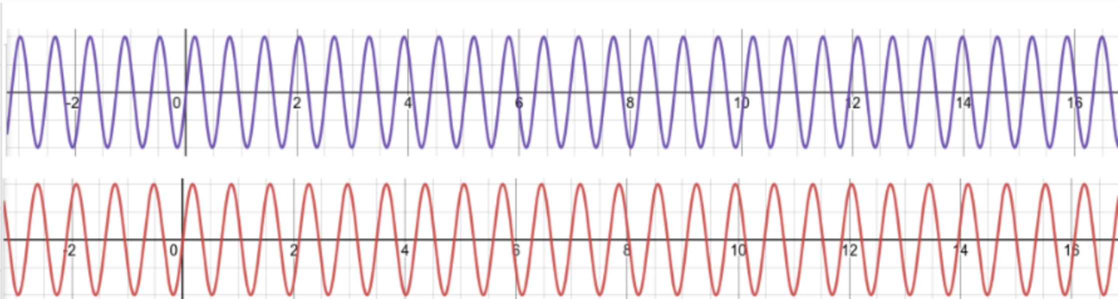
They do not have to touch, but the effect is better if they do.

① Strike just one fork, then immediately grab it to dampen the sound.
The sound will still continue, as the unstruck fork, tuned to the same frequency, was set vibrating by the oscillations moving through the air and the wood of the boxes.



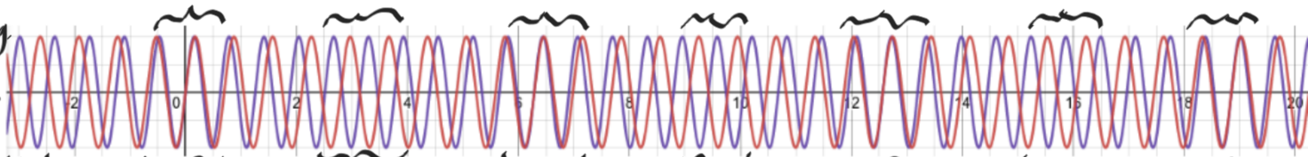
② If the forks have different frequencies, the second fork will not vibrate as much (some for close frequencies, and practically not at all for very different frequencies).





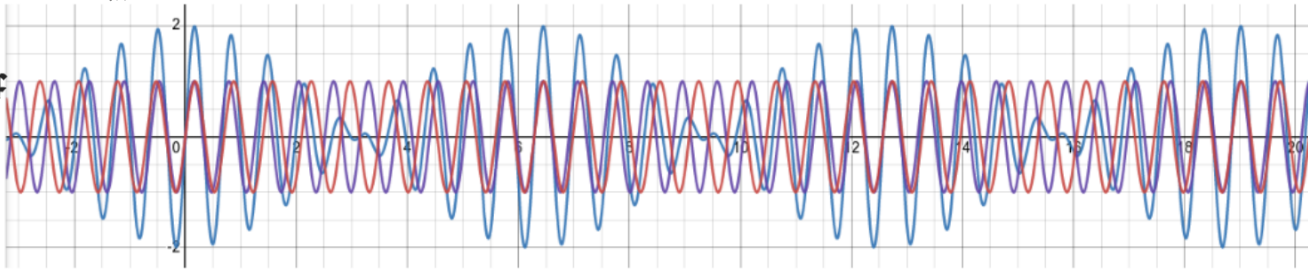
Above: The waves by themselves. Similar, but with slightly different frequencies.

The waves overlapping. Due to the different frequencies, they sometimes line up constructively, and sometimes destructively.

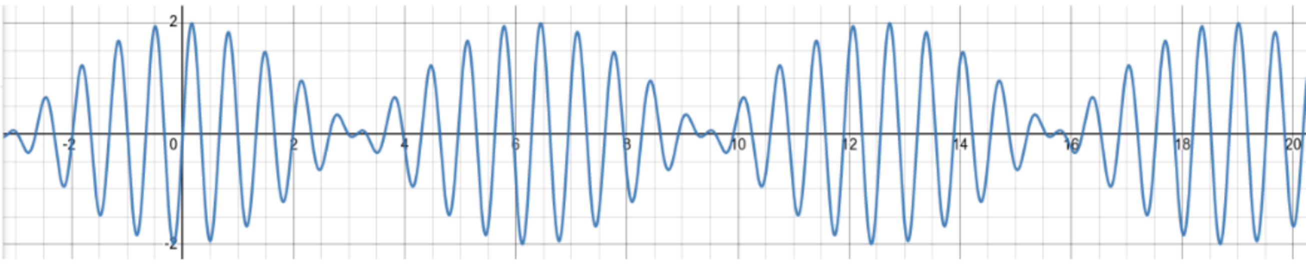


Lined up nearly perfectly for constructive interference. Lined up nearly perfectly for destructive interference. Constructive Destructive Constructive Destructive Constructive

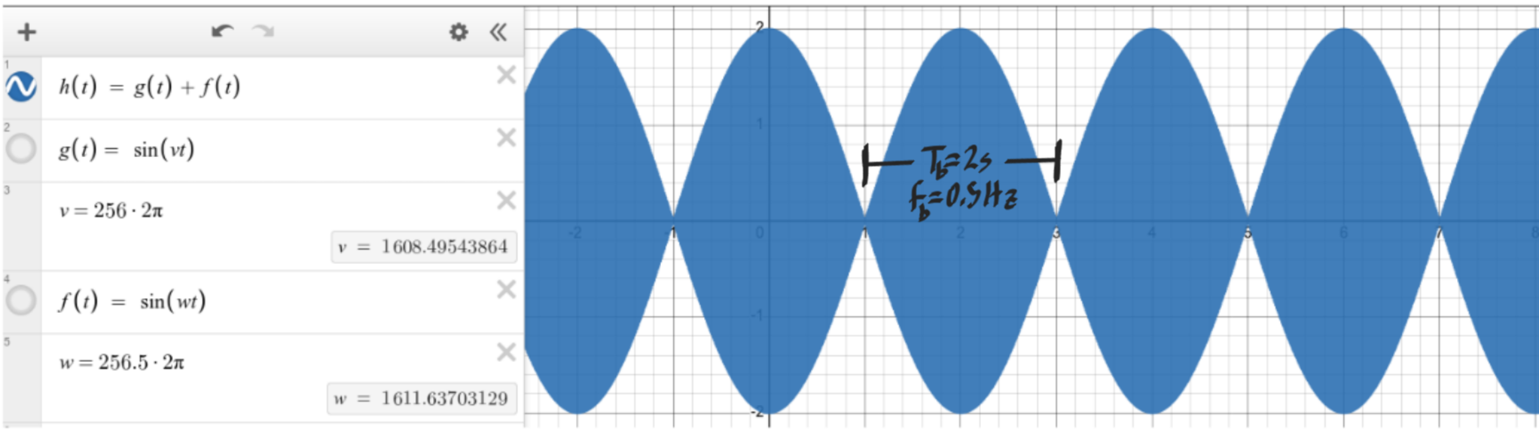
The sum of the two waves is also graphed here, with its regions of greatest amplitude located where the constituent waves almost perfectly constructively align, and its regions of smallest amplitude located where the constituent waves almost perfectly destructively align.



The sum of the waves graphed alone. A periodic increase and decrease of its amplitude can be seen—this is the "beating" phenomenon one hears when musical notes are out of tune.



https://www.desmos.com/calculator/hldfrbocct



Here, the oscillations are too dense to see, but the beats are very clear. Beats occur at a frequency equal to the difference of the frequencies of the two waves (subtract the smaller frequency from the larger frequency to get the beat frequency). Here, for waves of 256.5 Hz and 256 Hz, the beat frequency is 0.5 Hz. Thus each beat has a period of 2 seconds.

Wave Superposition

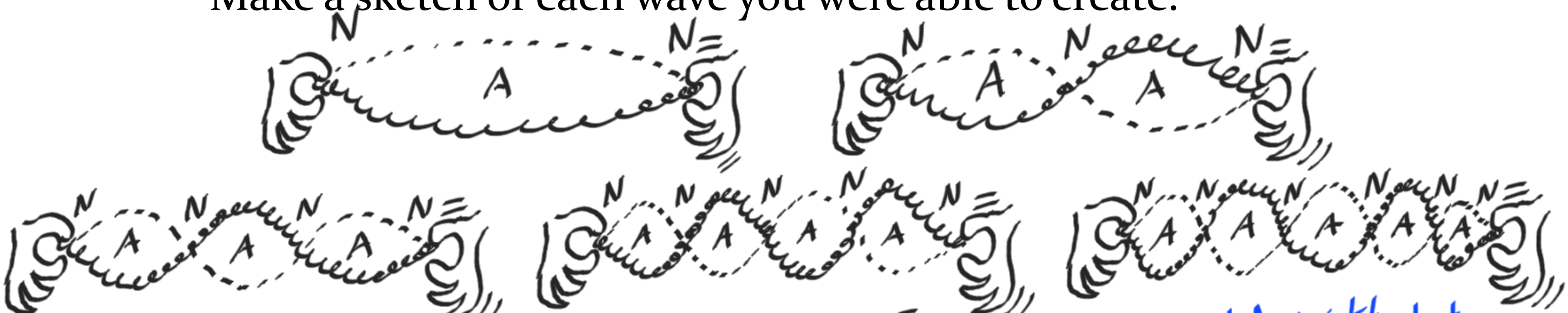
- When two waves (or pulses) meet, both waves (or pulses) will influence the shape of the spring.
- The total displacement of the spring can be determined by adding the displacements caused by each wave individually.
- The displacements have to be added **separately at every point**.
- Sometimes, there are special points that have zero displacement at all times, which are known as **nodes**. There are also special points that have the maximum possible displacement, which are known as **antinodes**.

Activity 7-2

- Perform the following experiment with the given equipment: 7-2 Standing Wave Experiment

Start 2:25 10:25
Wrap 2:35 10:35

  - Please be careful with the springs, as they can break.
  - Have a different member of your group hold each end of the spring.
  - Have one person wiggle the end of the spring to try to make a wave that has at least one **node** and at least one **antinode**.
  - How many different such waves can you make? (Remember not to break the springs!)
  - Make a sketch of each wave you were able to create.



This hand, held still, is certainly a node.

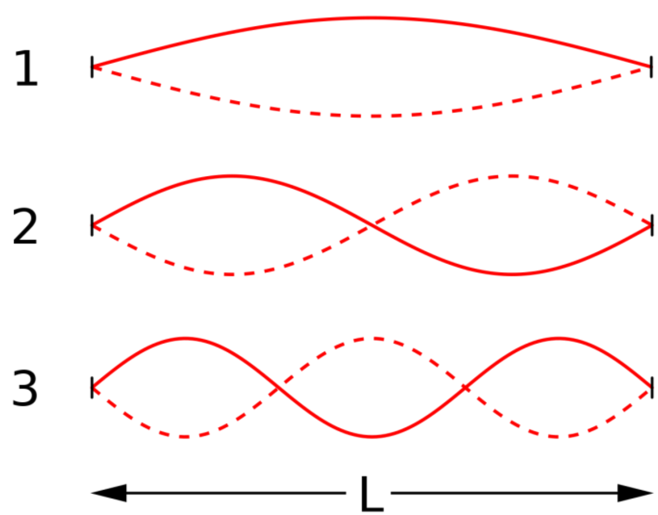
This hand, wiggled quickly but without much displacement, is practically a node, compared to the displacements of the antinodes.

Activity 7-3

You have a string with length *L*. Both ends of the string are pinned so that they cannot move (many musical instruments, such as guitars, have strings like this).

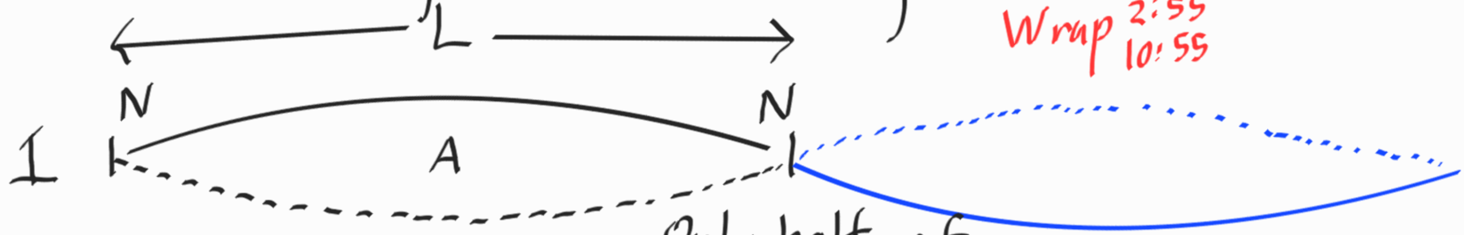
Shown below are three different standing waves that can be formed on such a string. Try to make these waves using the equipment.

- Identify all **nodes** and **antinodes** in the sketches below.
- For each sketch, determine the number of wavelengths that fit on the string. Use your answer to determine the wavelength (λ) in terms of the length of the string (L) and indicate it on your sketch.
- Do you notice any patterns in the wavelengths that you found? Can you come up with a general rule for all possible wavelengths?
- Assuming that you know the wave speed v for your string, determine the frequencies (f) in terms of v and the length of the string (L).
- Sketch another wave on this string that has a *higher* frequency than the three waves shown.



7-3 Wavelengths on a String

Start 2:40 / 10:40
Wrap 2:55 / 10:55



$\frac{1}{2}\lambda_1 = L$

Only half of one wavelength on the string!

$\lambda_2 = L$

$\frac{3}{2}\lambda_3 = L$

Wavelengths

$\lambda_1 = 2L$

$\lambda_2 = L$

$\lambda_3 = \frac{2}{3}L$

General Rule

$$\boxed{\lambda_n = \frac{2L}{n}} \qquad n = 1, 2, 3, \ldots$$

n is the "harmonic" number, which here corresponds to the number of antinodes, NOT the number of nodes.

Frequencies

wave speed $V = f\lambda$

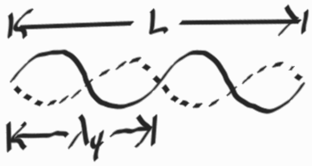
$\Rightarrow V = f_n \lambda_n \qquad (f_1 = \frac{V}{2L})$

$\Rightarrow \boxed{f_n} = \frac{V}{\lambda_n} = \boxed{\frac{nV}{2L}} = nf_1$

$f_1 = \frac{V}{2L} \qquad f_2 = \frac{V}{L} \qquad f_3 = \frac{3V}{2L}$

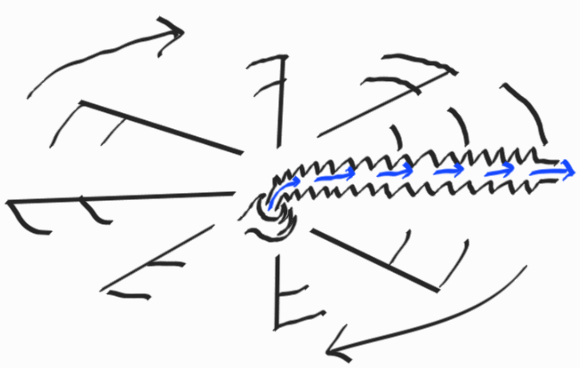
Higher Frequency

$\lambda_4 = \frac{L}{2} \quad f_4 = \frac{2V}{L}$



Corrugaphone (Whirly Tube) Demonstration

This instrument is a flexible, corrugated tube: 



When swung in a circle, air is pulled through the tube. As the air passes the corrugated sides, its flow becomes turbulent, which can lead to sound if the frequency of the turbulent vortex is resonant with the tube's harmonics. } These details go beyond our course.

Faster spinning generates higher frequencies, allowing one to explore the harmonics of the tube. } This is the big take-away.

Activity 7-4

- A banjo string is 0.7 m long and has a minimum (fundamental) frequency of 392 Hz (G). Determine:
  - The wave speed along this string.
  - Three other frequencies at which this string can vibrate.
  - How long should you make the string to increase the minimum (fundamental) frequency to 494 Hz (B)?

7-4 Banjo String

Start 3:15 11:15
Wrap 3:30 11:30

Fundamental (1st harmonic)

$L = 0.7m$

$\Rightarrow \lambda_1 = 1.4m$

$f_1 = 392\ Hz$

$$V = f_1 \lambda_1 = \left(392 \tfrac{1}{s}\right)(1.4\,m) = 548.8\ m/s$$

Other frequencies?

$$f_n = \frac{nv}{2L} = n f_1$$

$f_2 = 784\ Hz$

$f_3 = 1176\ Hz$

$f_4 = 1568\ Hz$

New Fundamental: $f_1 = 494\ Hz$

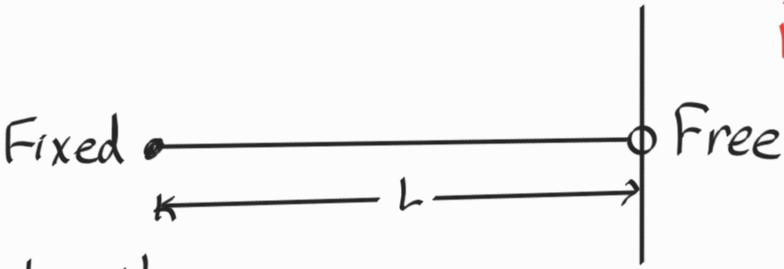
$$f_1 = \frac{V}{2L} \Rightarrow L = \frac{V}{2f_1} = \frac{548.8\ m/s}{2(494\ 1/s)} = \frac{548.8}{988}\,m \approx 0.555m$$

Activity 7-5

- Consider a string with one end fixed but the other end is not fixed—instead, it is free to move back and forth.
  - Sketch the standing wave consistent with the ends of the string that has the *largest wavelength*.
  - Find the frequency of this standing wave.
  - Sketch at least two additional waves and find their wavelengths and frequencies.

7-5 String with One Free End

Stretch Goal
Start 3:35 11:35
Wrap 3:45 11:45

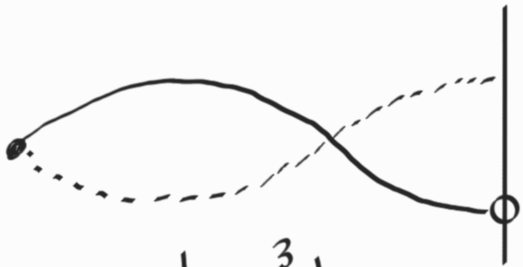


Largest Wavelength

$L = \frac{\lambda_1}{4}$

$\lambda_1 = 4L$

$V = f_1 \lambda_1 \Rightarrow f_1 = \frac{V}{\lambda_1} = \frac{V}{4L}$



$\lambda_3 = \frac{4L}{3}$

$f_3 = \frac{3V}{4L}$

$L = \frac{3}{4}\lambda_3$

$\lambda_5 = \frac{4L}{5}$

$f_5 = \frac{5V}{4L}$

$L = \frac{5}{4}\lambda_5$

$\lambda_n = \frac{4L}{n}$ $\quad n = 1, 3, 5, \ldots$

$f_n = \frac{nV}{4L}$

Only the odd harmonics can manifest when one end is free.