# Magnetic Field of a Partially Curved Wire

Benjamin Bauml and Danielle Skinner

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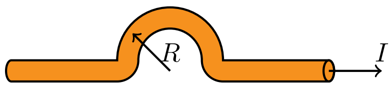
This problem is from the Week 9 Help and Practice Problems for PH 213, and probably was originally sourced from *Physics for Scientists and Engineers*.

## Activity 1

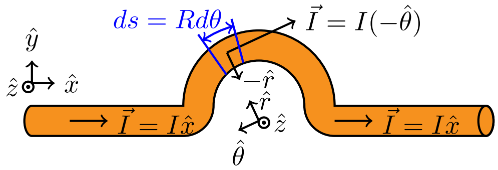
Use the Biot-Savart law to find the magnetic field strength at the center of the semicircle in the figure below.




We will tackle this problem using the Chop-Multiply-Add method. The Biot-Savart law is:

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

Let's first consider the magnetic field due to the two straight sides. The current moves in the $+\hat{x}$ direction, and the $\hat{r}$ is in the $\pm\hat{x}$. Therefore the cross-product in the Biot-Savart law is zero. The straight sides don't contribute a magnetic field to the center of the loop. So we only need to focus on the curved portion!




Starting with the "Chop" part, we can rewrite the cross product:

$$dq\vec{v} \times \hat{r} = ds\vec{I} \times \hat{r} = (Rd\theta)I(-\hat{\theta}) \times (-\hat{r}) = IRd\theta(-\hat{z}).$$

To do this calculation, we switched to a cylindrical coordinate system. The current moves in the $-\hat{\theta}$ direction, and the $\hat{r}$ points from the wire to the center of the semi-circle, making it negative.

$$\boxed{dq \cdot v = dq \cdot \frac{ds}{dt} = ds \cdot \frac{dq}{dt} = ds \cdot I}$$

Now on to "Multiply." The infinitesimal magnetic field can now be written as

$$d\vec{B} = \frac{\mu_o}{4\pi}\frac{dq\vec{v} \times \hat{r}}{r^2} = \frac{\mu_o}{4\pi}\frac{IRd\theta}{R^2}(-\hat{z}).$$

Finally, the "Add":

$$\vec{B} = \frac{\mu_o}{4\pi}\frac{I}{R}(-\hat{z})\int_0^{\pi} d\theta.$$

The final answer is therefore
$$\vec{B} = \frac{\mu_o}{4}\frac{I}{R}(-\hat{z}).$$
We can see that this is half the result of a magnetic field due to a current loop!