Frictionless Pool Lag

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Activity

You are about to play a game of pool, so you and your opponent are lagging for the first shot. To lag, you must bounce the cue ball off of the far end of the table, and get it as close to the near end as possible without touching it. While preparing to strike, you accidentally give the cue ball a light tap, setting it in motion.

Uh-oh! Through some bizarre accident, the pool table has become entirely frictionless! The cue ball slides toward the opposite end without rolling or slowing down.

A spectator across the table from you decides that the midpoint is $x = 0$ cm. Relative to her, the edge near you is at $x = 127$ cm, and the far edge is at $x = -127$ cm.

It takes the cue ball 4 s to slide at constant velocity from $x = 89$ cm to $x = 17$ cm.

a) What is its velocity?

Since the ball is traveling at constant speed, we can find its velocity exactly by dividing the ball's displacement (from 89 cm to 17 cm) by the time it takes to undergo that displacement:

$$v = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{17 \text{ cm} - 89 \text{ cm}}{4 \text{ s}} = \frac{-72 \text{ cm}}{4 \text{ s}} = -18 \text{ cm/s}.$$

Note the sign; velocity is a vector, so it has magnitude (the speed, 18 cm/s) and direction (the negative sign, indicating that it is moving toward the negative side of the pool table, away from you). This is also seen in the displacement, which is negative.

b) How long does it take the cue ball to slide from $x = 17$ cm to $x = -127$ cm?

We can start with the same relationship between velocity, displacement, and elapsed time, and rearrange:

$$v = \frac{\Delta x}{\Delta t} \implies \Delta t = \frac{\Delta x}{v} = \frac{x_f - x_i}{v} = \frac{-127 \text{ cm} - 17 \text{ cm}}{-18 \text{ cm/s}} = \frac{-144 \text{ cm}}{-18 \text{ cm}} = 8 \text{ s}.$$

c) The cue rebounds from the far edge without losing speed. What is its position 10 s after it is at $x = -127$ cm?

Note that $v = +18$ cm/s now. We once again take our expression for velocity and rearrange:

$$\begin{aligned} v &= \frac{x_f - x_i}{\Delta t} \\ x_f - x_i &= v\Delta t \\ x_f &= x_i + v\Delta t = -127 \text{ cm} + (18 \text{ cm/s})(10 \text{ s}) = 53 \text{ cm}. \end{aligned}$$

Position is also a vector, so this positive number tells us that the ball is on the positive end of the table (on the near side of the midpoint, relative to you).