Sliding Cube in a Bowl

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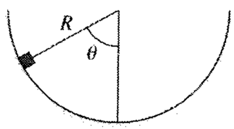
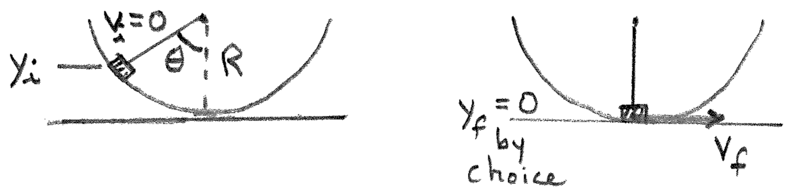
This material is borrowed/adapted from Chapter 10 of the *Student Workbook* for *Physics for Scientists and Engineers.*

XX-1: Sliding Cube in a Bowl

A small cube of mass m slides back and forth in a frictionless, hemispherical bowl of radius R. Suppose the cube is released at angle θ. What is the cube's speed at the bottom of the bowl?



(a) Begin by drawing a before-and-after pictorial representation. Let the cube's initial position and speed be y_i and v_i. Use a similar notation for the final position and speed.



(b) At the initial position, are either K_i or U_{Gi} zero? If so, which?

The cube is released from rest at the initial position, so $K_i = 0$ J.

(c) At the final position, are either K_f or U_{Gf} zero? If so, which?

We can choose the zero of gravitational potential energy to be at the bottom of the bowl, so $U_{Gf} = 0$ J.

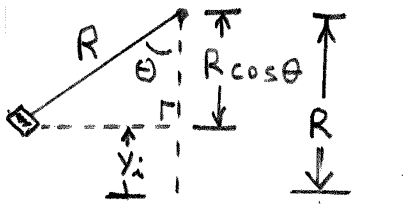
(d) Does thermal energy need to be considered in this situation? Why or why not?

No, because the bowl is frictionless. There are no dissipative forces stealing energy from the system.

(e) Write the conservation of energy equation in terms of position and speed variables, omitting any terms that are zero.

$$\cancel{K_i} + U_{Gi} = K_f + \cancel{U_{Gf}}$$
$$mgy_i = \frac{1}{2}mv_f^2$$

(f) You're given not the initial position but the initial angle. Do the geometry and trigonometry to find y_i in terms of R and θ.



$$y_i = R(1-\cos\theta)$$

(g) Use your result of part (f) in the energy conservation equation, and then finish solving the problem.

$$mgy_i = \frac{1}{2}mv_f^2$$
$$\cancel{m}gR(1-\cos\theta) = \frac{1}{2}\cancel{m}v_f^2$$
$$\implies v_f = \sqrt{2gR(1-\cos\theta)}$$