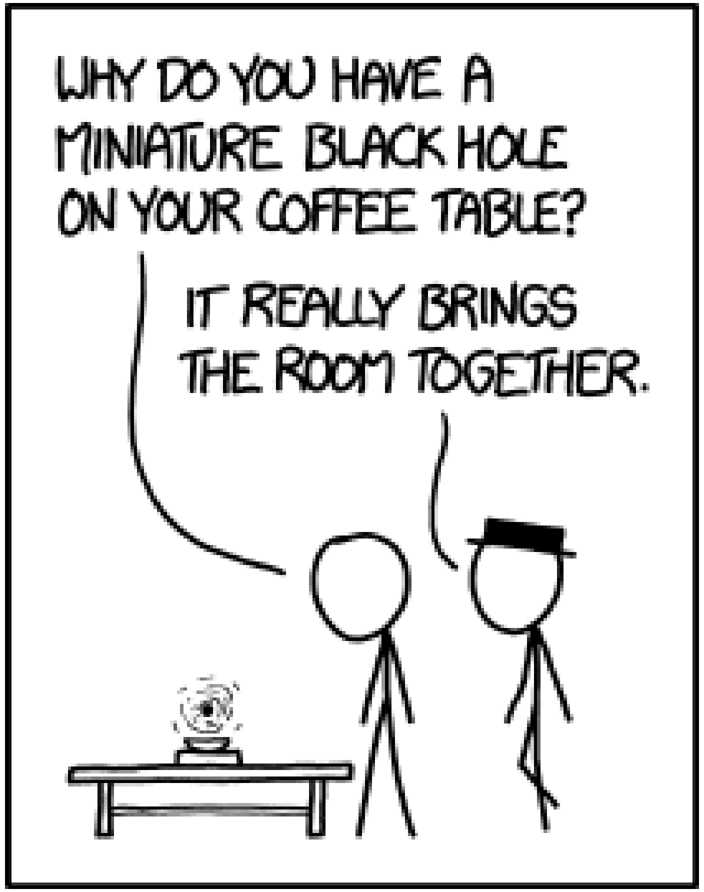
Studio Week 1
Representations and Sensemaking



Picture credit: xkcd.com

Principles for Success

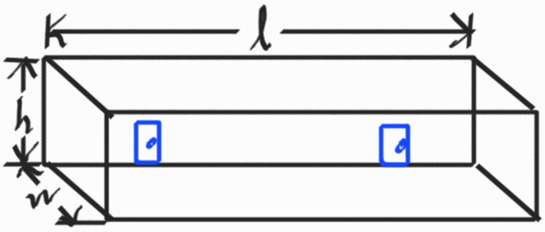
- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Make sense of everything.**

Activity 1-1: Representations of Volume

- You want to determine the volume of the room you are in.
    1. Write a description of how to find the volume of the room in words.
    2. Sketch a diagram that would help you find the volume of the room.
    3. Write a symbolic expression that would allow you to find the volume of the room.
        a. Check the units of your expression.
    4. Without standing up, estimate the volume of the room as a number.
        a. Make sense of the number by comparing it to something.

1-1 Representations of Volume

- If we measure the dimensions of the room (ceiling height, length, width), and multiply them together, we can get a decent estimate of the volume of the room.
    - Of course, it won't be perfect. The floor isn't even, the walls have a few pillars embedded in them, the doors lie in slight recesses, and there are air ducts sticking into the room. We could find their volumes and subtract, perhaps, if we need that level of accuracy.
    - Instead, we will assume for simplicity that the room is rectangular and empty.
- Diagram:



I put the doors in to orient the drawing. They are otherwise not critical, and their sizes and positions are not quantitatively accurate.

- Symbols: $V = lwh$

  Units: $m^3 = m \times m \times m$ ✓
- Estimate: There are several ways to estimate the dimensions of the room without measuring it directly.
    - Guess your local TA's height (or just ask) and compare them to the height of the ceiling, like a living measuring stick.
    - Look up the height of a standard interior door (Home Depot says 80 inches, which is about 2.032 meters) and estimate how many doors high the ceiling is.
    - The carpet tiles are a regular size. Measure one nearby and count how many you see from one wall to another.
- Compare: An Olympic pool is about 2500 m^3 (technically, there is no standard depth, so this can vary). Comparing this to the size of the room can give you some idea about how accurate your estimate is (for example, the room is certainly not larger than an Olympic pool).

Activity 1-2: Driving to Portland

1. Discuss "sensemaking" with your group.
   a. Identify several ways of making sense of answers or contexts you have used in math or science courses.
2. You are driving from Corvallis to Portland, and you measure how full your gas tank is (in gallons) as a function of time (in hours): $G(t) = G_o - \beta t^2$.
   a. Make sense of this expression with your group in as many different ways as you can, making use of as many different representations as you can.

1-2 Driving to Portland

- Sensemaking:
  - Compare a numerical result against a reference number.
  - Check the units of your expression.

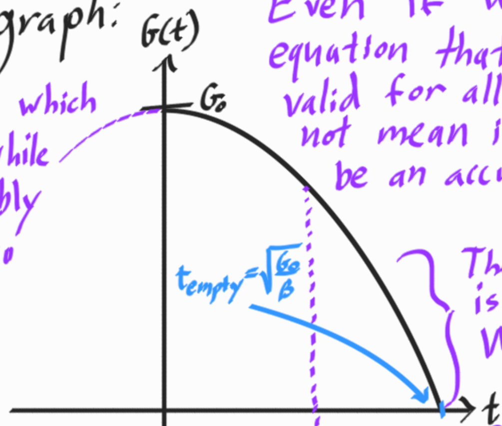
    Example: The units of β must be $[\beta] = \frac{gal}{h^2}$ for βt^2 to have units of gallons.
  - Use "covariation." See how changing the variables changes the output of your expression. See what the signs in your expression tell you.

    Example: The minus sign in $G_0 - \beta t^2$ tells us that the tank is getting emptier as time progresses (assuming $\beta > 0$), which we expect of a vehicle consuming gas as an energy source.
  - Choose values for variables which correspond to "special cases," where the physical expectation is obvious and the math is simpler.

    Example: We should have the most gas in the tank when we start, and if we plug in an initial time of $t=0$, then we get the largest possible value $G(t=0) = G_0$.

Important! Do not just comment on the behavior of an expression without comparing to physical expectations. For example, do NOT just say "G(t) is decreasing, which makes sense." Say why: "G(t) is decreasing, which makes sense, as gas is consumed during travel."

Also, consider this graph:



G(t) increases for $t<0$, which does not make sense while driving. The model probably should only be applied to $t>0$.

Even if we provide you with an equation that is mathematically valid for all values of t, that does not mean it should be assumed to be an accurate model for all t.

The slope is steeper, so gas is being consumed faster. Would we expect fuel efficiency to vary like this?

When do we reach Portland? Maybe we should have stopped graphing before $G(t) = 0$ (out of gas).

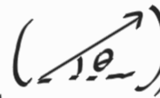
Activity 1-3: Representations of Vectors

1. Discuss **vectors** with your group.
2. Write down a list of things about vectors on your shared whiteboard. You can write:
   a. words or sentences
   b. numbers or symbols
   c. pictures or diagrams

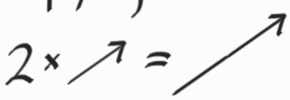
- Write large!

1-3 Representations of Vectors

The following list only contains my thoughts about vectors. It should not be assumed to be definitive. There are many other things about vectors that could have been included.

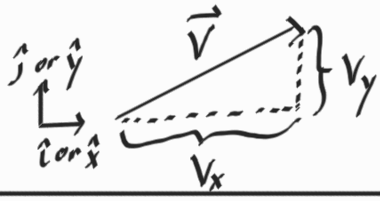
- A vector has direction and magnitude.
- A vector is often represented with an arrow (); the length is its magnitude, and the orientation of the arrow (its angle) indicates direction.
- Vectors add "tip-to-tail."

- Multiplying a vector by a scalar changes its length.



- Multiplying a vector by -1 reverses its direction.

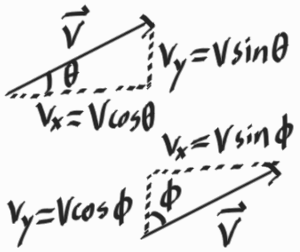
- Vectors can be broken into components.



$$\vec{V} = (v_x, v_y) = \langle V_x, V_y \rangle = V_x \hat{i} + V_y \hat{j} = V_x \hat{x} + V_y \hat{y}$$

This sort of notation was probably used in your math classes. It is okay, but parentheses and angle brackets can mean a lot of things, so it may not always be clear.

We recommend unit vector notation, as it is much clearer. Using $\hat{x}, \hat{y}, \hat{z}$ instead of $\hat{i}, \hat{j}, \hat{k}$ helps to more clearly associate the Cartesian unit vectors with the coordinate axes.

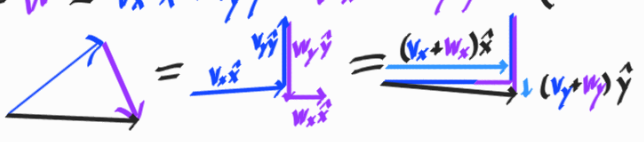


Warning!

Cosine is not always x! It depends on where the angle is.

- Vectors add "componentwise."

$$\vec{V} + \vec{W} = V_x \hat{x} + V_y \hat{y} + W_x \hat{x} + W_y \hat{y} = (V_x + W_x)\hat{x} + (V_y + W_y)\hat{y}$$

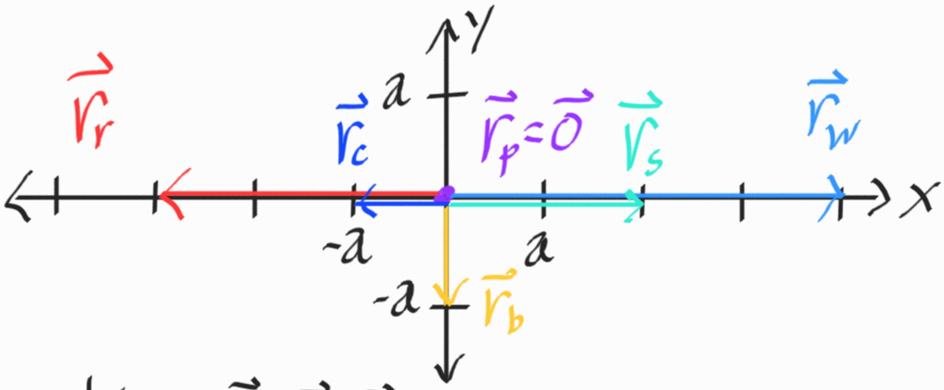


Activity 1-4: Vectors in a Garden

- You visit a garden with a trail that includes the following landmarks.
  a. Red roses at $\vec{r}_{\mathrm{r}} = -3a\hat{x}$
  b. White roses at $\vec{r}_{\mathrm{w}} = +4a\hat{x}$
  c. A pond at $\vec{r}_{\mathrm{p}} = 0\hat{x}$
  d. A bench at $\vec{r}_{\mathrm{b}} = -a\hat{y}$
  e. A bridge over a creek at $\vec{r}_{\mathrm{c}} = -a\hat{x}$
  f. A statue at $\vec{r}_{\mathrm{s}} = +2a\hat{x}$

1. Sketch and label the garden and its landmarks.
2. Find the following displacement vectors using both symbols and diagrams.
   a. From the red roses to the white roses
   b. From the pond to the red roses
   c. From the bench to the statue

1-4 Vectors in a Garden



Displacements $\Delta\vec{r} = \vec{r}_f - \vec{r}_i$

A displacement vector, $\Delta\vec{r}$, can be thought of as the difference of the final position vector, $\vec{r}_f$, and the initial position vector, $\vec{r}_i$:

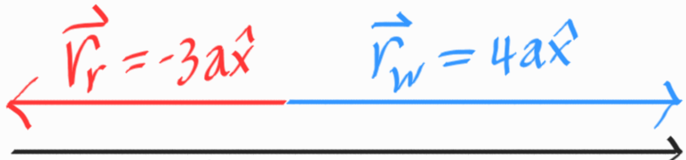
$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i.$$

It can also be thought of as the vector that, when added to $\vec{r}_i$, gives you $\vec{r}_f$:

$$\vec{r}_f = \vec{r}_i + \Delta\vec{r}.$$

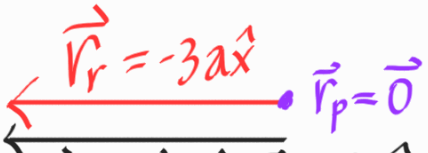
As such, $\Delta\vec{r}$ points from the tip of $\vec{r}_i$ to the tip of $\vec{r}_f$:

— Red Roses to White Roses



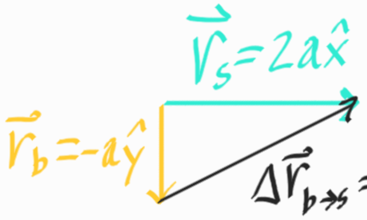
$$\Delta\vec{r}_{r\to w} = \vec{r}_w - \vec{r}_r = 4a\hat{x} - (-3a\hat{x}) = 7a\hat{x}$$

— Pond to Red Roses



$$\Delta\vec{r}_{p\to r} = \vec{r}_r - \vec{r}_p = \vec{r}_r = -3a\hat{x}$$

— Bench to Statue



$$\Delta\vec{r}_{b\to s} = \vec{r}_s - \vec{r}_b = 2a\hat{x} - (-a\hat{y}) = 2a\hat{x} + a\hat{y}$$

$$|\Delta\vec{r}_{b\to s}| = \sqrt{(2a)^2 + a^2} = \sqrt{5}\,a$$