Studio Week 9
Collisions

Picture credit: http://30318-physics-carcollisions.blogspot.com/.

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Make sense of everything.**

Impulse and Momentum

- Definitions

  - Impulse $$\vec{J}_{net} = \int_{t_i}^{t_f} \vec{F}_{net} dt$$

  - Momentum $$\vec{p} = m\vec{v}$$

  - Impulse-Momentum Theorem $$\vec{J}_{net} = \Delta\vec{p}$$

Activity 9-1: Momentum Conservation

- In each scenario below, if possible, identify a system for which momentum is conserved. If not possible, explain why not.
    1. Two cars crash into each other in an intersection.
    2. A baseball player catches a baseball while standing at second base.
    3. A block attached to a spring on a horizontal table oscillates back and forth.
    4. A firework bursts into pieces in the sky.
    5. A bird dives through the air, speeding up.

9-1 Systems & Momentum Conservation

1) System: Both Cars

For the short duration and large forces involved in a crash, the external impulse (for example, from friction with the road) would be insignificant.

2) System: Baseball, Catcher, 2nd Base, Earth

The forces between the catcher and 2nd base (which is firmly rooted in the Earth) probably cannot reasonably be ignored.

3) System: Block, Spring, Table, Earth

Unless the table is on frictionless wheels (which would allow it to oscillate too), it is interacting with the Earth in a significant way.

4) System: Firework (not in homework!)

The strong forces in a fast explosion should vastly outmatch any impulse from gravity.

In the homework, the explosion takes a non-negligible amount of time, and impulse cannot be ignored!

5) System: Bird, Air, Earth

Gravity is changing the bird's momentum, and air resistance may have an appreciable effect.

Putting Earth in your system is not conducive to solving a momentum problem. It is better to leave it out to exert an impulse.

Activity 9-2: Bumper Cars

- Two bumper cars collide with each other and get tangled together.
- Car 1 (m_1) moves north at v_1. Car 2 (m_2) moves south at v_2.
- Case 1
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south at 3 m/s.
  - Find the final velocity of the cars.
  - Determine the initial and final kinetic energies of the cars
  - Compare the total kinetic energy before and after the collision.
- Case 2
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south.
  - Find the initial velocity of Car 2 assuming they both end at rest.
  - Determine the initial and final kinetic energies of the cars
  - Compare the total kinetic energy before and after the collision.

9-2 Bumper Cars

Coordinate System:

N

$\uparrow \hat{y}$

S

Case 1

$m_1 = 100\,kg \quad \vec{v}_1 = (4\,m/s)\hat{y}$

$m_2 = 200\,kg \quad \vec{v}_2 = -(3\,m/s)\hat{y}$

$m_f = m_1 + m_2 \quad \vec{v}_f = ?$

$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 v_1 - m_2 v_2)\hat{y}$$

$$\vec{p}_f = (m_1 + m_2)\vec{v}_f = (m_1 + m_2)v_f\,\hat{y}$$

$$\vec{p}_i = \vec{p}_f$$

$$m_1 v_1 - m_2 v_2 = (m_1 + m_2)v_f$$

$$v_f = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$$

$$= \frac{(100\cancel{kg})(4\,m/s) - (200\cancel{kg})(3\,m/s)}{100\cancel{kg} + 200\cancel{kg}}$$

$$= \frac{400 - 600}{300}\,m/s$$

$$= -\frac{2}{3}\,m/s$$

The cars are going $\frac{2}{3}$ m/s to the south.

$$K_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$= \frac{1}{2}(100\,kg)(4\,m/s)^2 + \frac{1}{2}(200\,kg)(3\,m/s)^2$$

$$= 800\,J + 900\,J$$

$$= 1700\,J$$

$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2$$

$$= \frac{1}{2}(300\,kg)\left(\frac{2}{3}\,m/s\right)^2$$

$$= \frac{200}{3}\,J \approx 66.7\,J$$

A lot of energy is lost to heat and deformation of metal.

Case 2

$m_1 = 100\,kg \quad \vec{v}_1 = (4\,m/s)\hat{y}$

$m_2 = 200\,kg \quad \vec{v}_2 = ?$

$m_f = m_1 + m_2 \quad \vec{v}_f = 0$

$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 v_1 - m_2 v_2)\hat{y}$$

$$\vec{p}_f = (m_1 + m_2)\vec{v}_f = 0$$

$$\vec{p}_i = \vec{p}_f$$

$$m_1 v_1 - m_2 v_2 = 0$$

$$v_2 = \frac{m_1}{m_2}v_1$$

$$= \frac{1}{2}v_1$$

$$= 2\,m/s$$

Car 2 was going 2 m/s (to the south).

$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2 = 0\,J$$

$$K_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

$$= \frac{1}{2}(100\,kg)(4\,m/s)^2 + \frac{1}{2}(200\,kg)(2\,m/s)^2$$

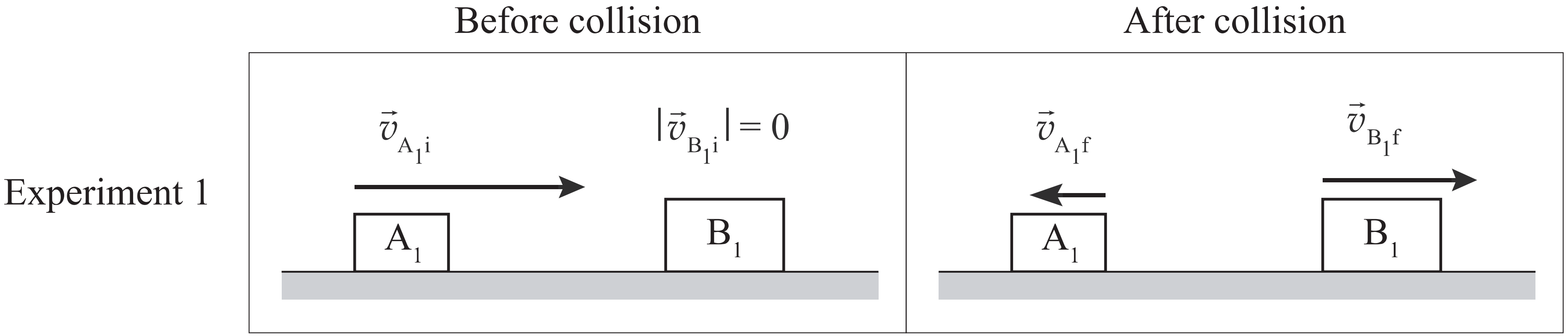
$$= 800\,J + 400\,J$$

$$= 1200\,J$$

All of the energy was lost to heat and deformation of metal.

Prediction

- Experiment 1 is conducted with two carts, A_1 and B_1, on a level, frictionless track. The mass of cart B_1 is greater than that of cart A_1 (*i.e.*, $m_{B1} > m_{A1}$).
- In experiment 1, cart A_1 moves toward cart B_1, which is initially at rest. Magnets are attached to the carts so that the carts repel each other without touching. After the collision, cart A_1 has reversed direction and cart B_1 moves to the right.
- *Predict* whether the magnitude of the final momentum of cart B_1 is *greater than, less than,* or *equal to* that of the system S_1 of both carts. Briefly explain.



A. Greater than
B. Less than
C. Equal to
D. Not enough information

Prediction: Two Carts

Without external impulse, the momentum of the system will not change. The inital momentum of the system is $\vec{p}_{S,i} = \vec{p}_{A,i}$, and the final momentum is $\vec{p}_{S,f} = \vec{p}_{A,f} + \vec{p}_{B,f}$, where $\vec{p}_{A,f}$ points opposite $\vec{p}_{A,i}$.

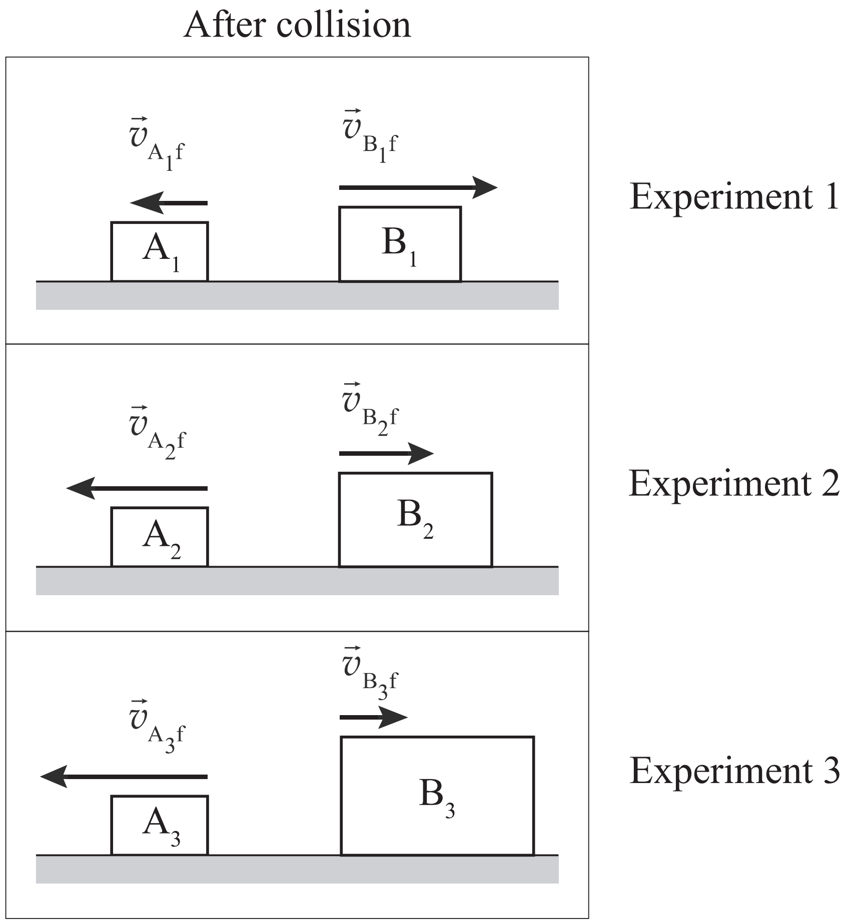
$$\vec{p}_{S_1} = \vec{p}_{A,i} = \vec{p}_{A,f} + \vec{p}_{B,f}$$

$$p_{S_1} = p_{A,i} = -p_{A,f} + p_{B,f} < p_{B,f}$$

The final momentum of B, is greater than the momentum of the system. The vector $\vec{p}_{B,f}$ had to be longer to account for $\vec{p}_{A_1}$ changing directions when conserving momentum.

Activity 9-3: Collision Experiments

- Two additional experiments are performed that are identical to experiment 1 with one exception: the mass of the target cart is larger in each subsequent experiment (*i.e.*, $m_{B3} > m_{B2} > m_{B1}$).
- In each experiment, cart A moves toward cart B, which is initially at rest.
- The incoming carts A_1–A_3 are identical and have the same initial velocity to the right.
  - Create momentum vector diagrams for experiments 1, 2, and 3. (your vectors only need to be qualitatively accurate)
  - Use your vector diagrams to rank the final momenta of the B carts according to magnitude
  - Consider what would happen in the special case that the mass of cart B is made *very large*



9-3 Collision Experiments

Experiment 1

	A_1	B_1	A_1 & B_1
$\vec{p}_i$	$\longrightarrow$	$\cdot$	$\longrightarrow$
$\Delta\vec{p}$	$\longleftarrow$	$\longrightarrow$	$\cdot$
$\vec{p}_f$	$\leftarrow$	$\longrightarrow$	$\longrightarrow$

Experiment 2

	A_2	B_2	A_2 & B_2
$\vec{p}_i$	$\longrightarrow$	$\cdot$	$\longrightarrow$
$\Delta\vec{p}$	$\longleftarrow$	$\longrightarrow$	$\cdot$
$\vec{p}_f$	$\leftarrow$	$\longrightarrow$	$\longrightarrow$

Experiment 3

	A_3	B_3	A_3 & B_3
$\vec{p}_i$	$\longrightarrow$	$\cdot$	$\longrightarrow$
$\Delta\vec{p}$	$\longleftarrow$	$\longrightarrow$	$\cdot$
$\vec{p}_f$	$\longleftarrow$	$\longrightarrow$	$\longrightarrow$

$\vec{p}_{fB_1}$ $\longrightarrow$

$\vec{p}_{fB_2}$ $\longrightarrow$

$\vec{p}_{fB_3}$ $\longrightarrow$

$$p_{fB_1} < p_{fB_2} < p_{fB_3}$$

Even though V_{fB} is decreasing with each experiment, p_{fB} is increasing.

The increase in m_B must be faster than the decrease in V_{fB} to keep $p_{fB} = m_B V_{fB}$ increasing. Still, V_{fB} could be decreasing fast enough that $K_{fB} = \frac{1}{2} m_B V_{fB}^2$ would decrease, thanks to the square.

If $m_B >> m_A$, object A will reflect at its original speed and object B will barely move. ($V_{fB} \to 0$ at minimum, and $V_{fA} \to V_{iA}$ at most, otherwise energy would not be conserved.)

Activity 9-4: Carnival Game

- A carnival game requires you to knock over a wood post by throwing a ball at it. You're offered a very bouncy rubber ball and a very sticky clay ball of equal mass. Assume that you can throw them with equal speed and equal accuracy. You only get one throw.
  - Which ball do you want to choose? Why?
  - Create a momentum vector diagram for this situation to defend your answer.
  - Ask an instructor to help you check your answers.

9-4 Carnival Game

We should choose the bouncy ball, as it will have a greater momentum change when rebounding, which means it will impart more momentum to the post.

	Ball (Clay / Rubber)	Post	System
$\vec{p}_i$	$\rightarrow$	$\cdot$	$\longrightarrow$
$\Delta\vec{p}$	Clay: $\leftarrow$ / Rubber: $\longleftarrow$	Clay: $\rightarrow$ / Rubber: $\longrightarrow$	$\cdot$
$\vec{p}_f$	Clay: $\cdot$ / Rubber: $\leftarrow$	Clay: $\rightarrow$ / Rubber: $\longrightarrow$	$\longrightarrow$

No significant external impulse during the brief collision.

In this diagram, like in the in-class demonstration, the non-bouncy ball just stops when it hits the post.

The rubber ball gives more momentum to the post, making it more likely to tip over.

If the ball stuck to the post (moving with it after the collision), the diagram would really look like this.

	Ball	Post	System
$\vec{p}_i$	$\longrightarrow$	$\cdot$	$\longrightarrow$
$\Delta\vec{p}$	$\leftarrow$	$\rightarrow$	$\cdot$
$\vec{p}_f$	$\rightarrow$	$\rightarrow$	$\longrightarrow$

The post gets even less speed from the collision, as the momentum is shared between both masses.