Studio Week 10
Combining Physics Concepts

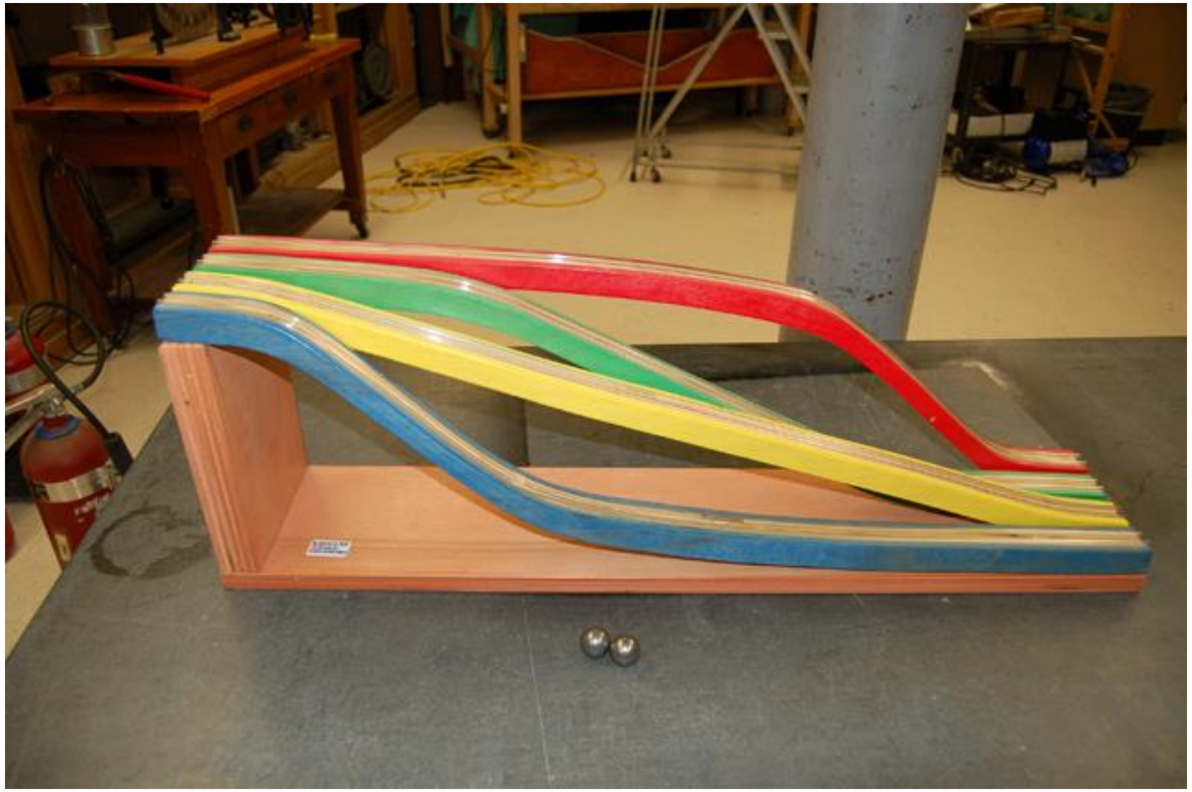
Picture credit: xkcd.com.

Principles for Success

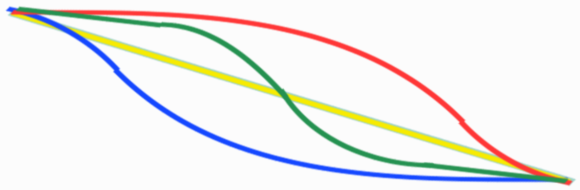
- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Make sense of everything.**

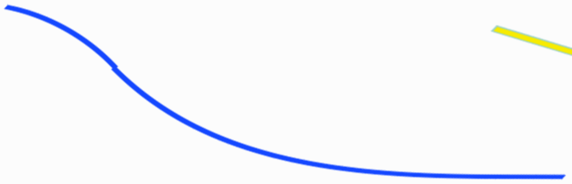
Activity 10-1: Frictionless Track

- Use the understanding you have built about the energy for a ball moving along a frictionless track to predict **which ball will reach the end of the track first**.
  - Explain your reasoning.
  - Each ball starts at rest at the top of the respective track, and the tracks are nearly the same length.



10-1 Four Tracks

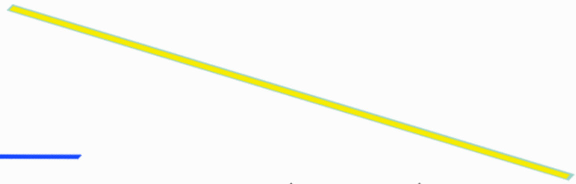




The blue track changes potential energy to kinetic energy soonest, so the ball accelerates more at the outset and has more speed over more of the distance of the track.

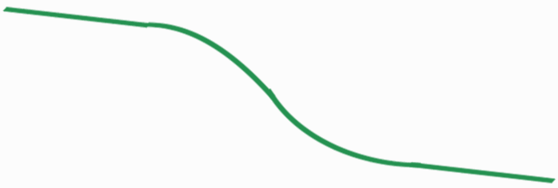
This can also be thought about in terms of forces, as the region with the most slope accelerates the ball the most and happens earliest on this track.

Either way, the ball on the blue track reaches the end first.



The yellow track gives the ball constant acceleration.

The red track is the slowest. Most of the conversion from U_g to K occurs near the end (where the slope, and therefore the force, is greatest), and the ball spends most of its time at a lower speed.

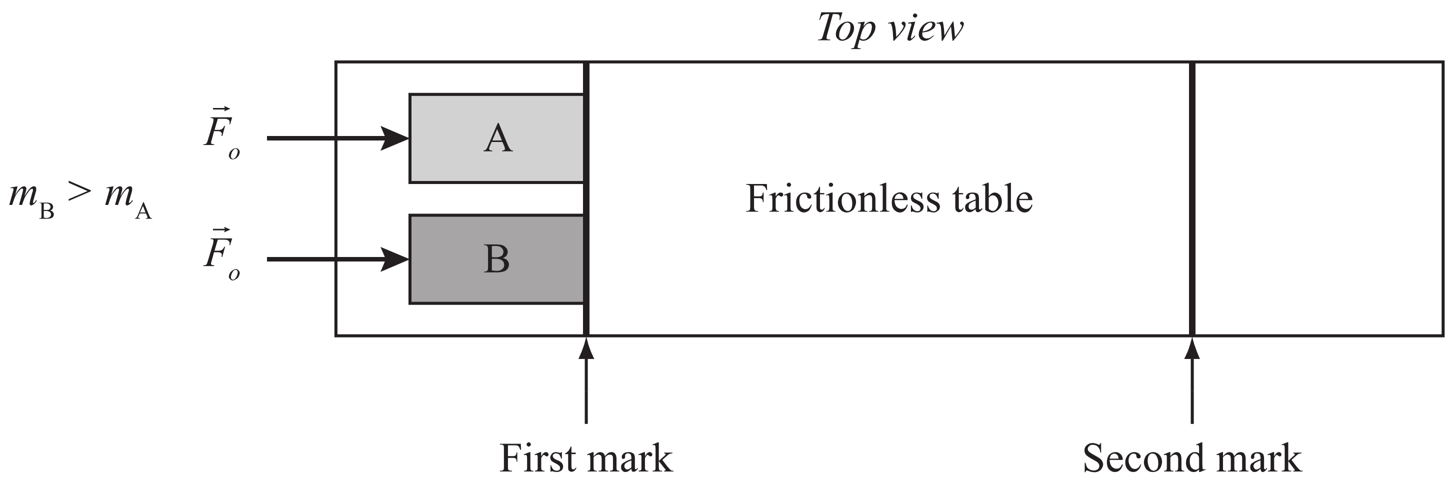


The green track has less slope (and therefore less acceleration) near both ends, and more slope (and therefore more acceleration) in the middle.

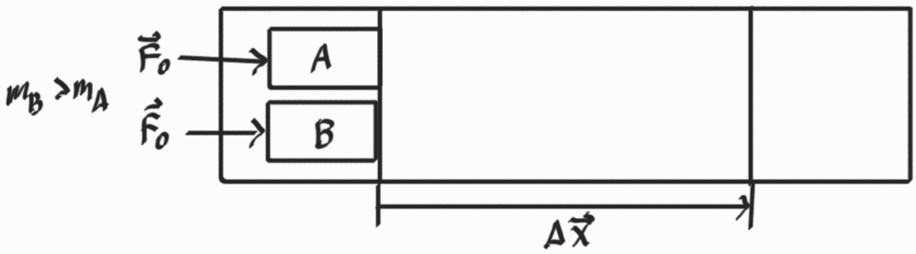
All four tracks have the same change in height, so the balls lose the same amount of potential energy. As such, they all end with the same speed.

Activity 10-2: Carts on a Track

- Two carts, A and B, are initially at rest on a level frictionless table. A constant force of magnitude F_o is exerted on each cart as it travels between two marks. Cart B has a greater mass than cart A.
- Three students discuss the final momentum and kinetic energy of each cart.
  1. *"Since the same force is exerted on both carts, the cart with the smaller mass will move quickly, while the cart with the larger mass will move slowly. The momentum of each cart is equal to its mass times its velocity."*
  2. *"This must mean that the speed compensates for the mass and the two carts have equal final momenta."*
  3. *"I was thinking about the kinetic energies. Since the velocity is squared to get the kinetic energy but mass isn't, the cart with the bigger speed must have more kinetic energy."*
- Do you agree or disagree with the statements made by each student?
- Which cart takes longer to travel between the two marks? Explain your reasoning.
- Determine if the magnitude of the final momentum of cart A is *greater than, less than,* or *equal to* that of cart B.
- Determine if the final kinetic energy of cart A is *greater than, less than,* or *equal to* that of cart B.
- Reflect on your initial thoughts about the three students above.



10-2 Two Carts Reasoning



3 Students

1) Same force, $m_B > m_A \Rightarrow m_A$ goes faster than m_B

True by application of Newton's 2nd law

$\vec{p} = m\vec{v}$ True (definition of momentum)

Since m_A goes faster than m_B, B will take longer to travel between the two marks.

2) Speed compensates for mass, so $p_{A,f} = p_{B,f}$

False. m_A goes faster, so $\Delta t_A < \Delta t_B$ to reach end.

$J = \int_{t_i}^{t_f} F_{net}\,dt$, so for the same force, $J_A < J_B$, and since $p_i = 0$ for both, this means $p_{A,f} < p_{B,f}$.

3) K proportional to m and to v^2, so bigger speed will have more effect than smaller mass, thus $K_A > K_B$.

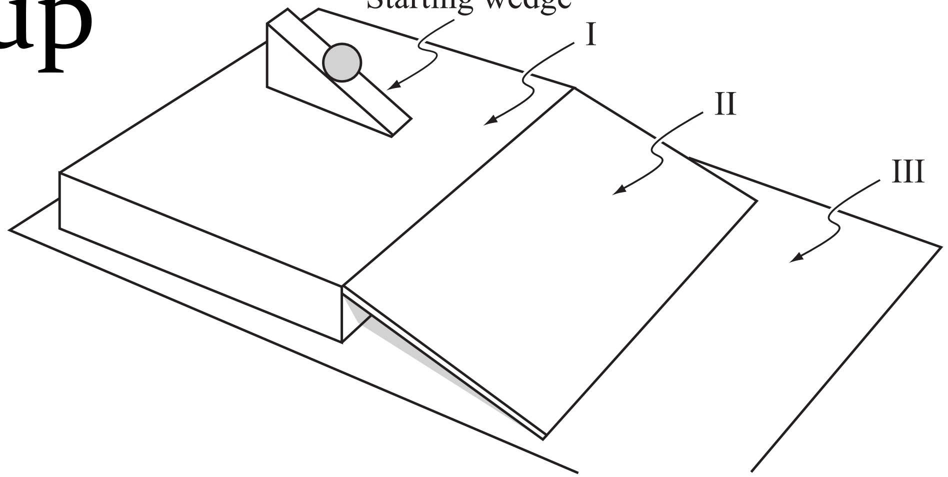
False. $W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$ means $W_A = W_B$ for same force & displacement, and work-energy theorem then tells us that $K_A = K_B$.

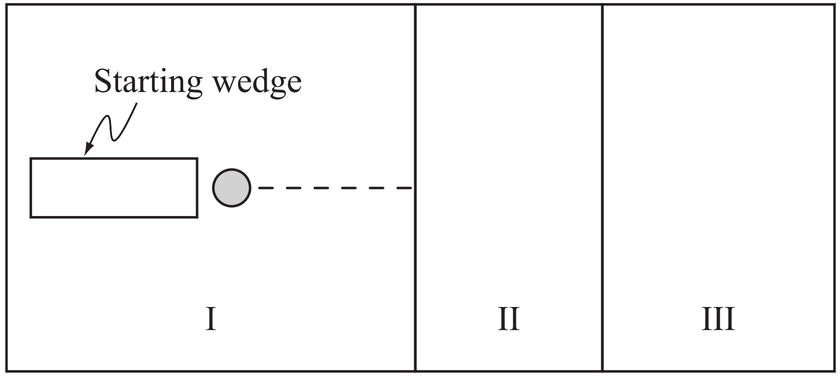
Note also: $K = \frac{1}{2}mv^2 = \frac{p^2}{2m}$, so if $K_A = K_B$, then

$$\frac{p_A^2}{2m_A} = \frac{p_B^2}{2m_B} \Rightarrow 1 > \frac{m_A}{m_B} = \frac{p_A^2}{p_B^2} \Rightarrow p_A < p_B$$

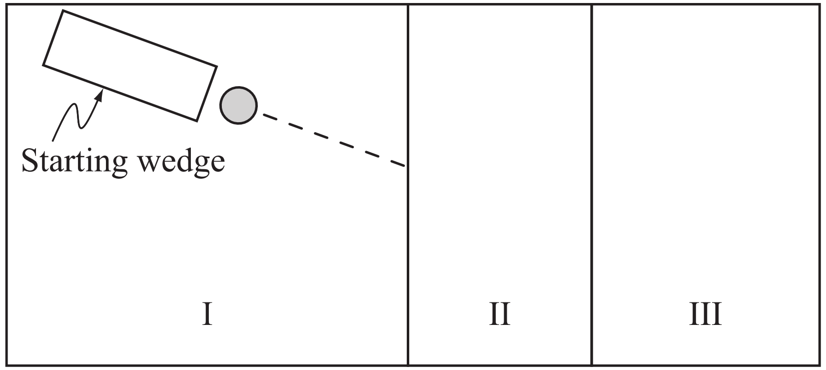
Extra confirmation that 2 is wrong.

Activity 10-3 Setup





Top view, motion 1

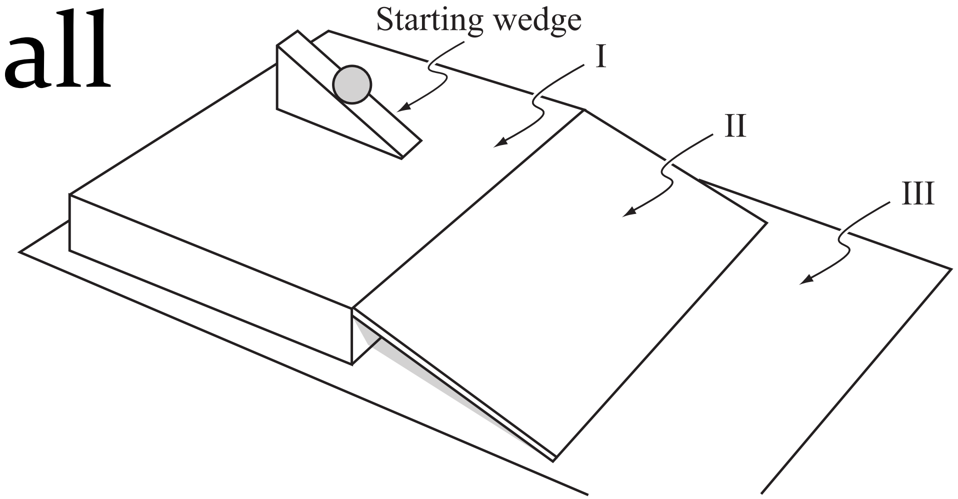


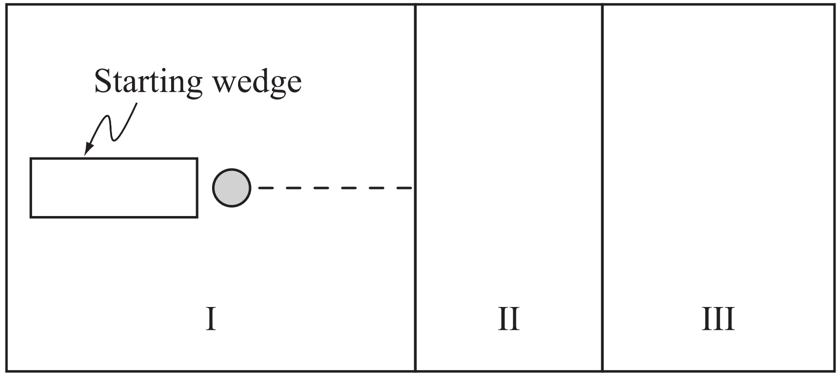
Top view, motion 2

https://www.youtube.com/watch?v=3CjpPxNVjvA

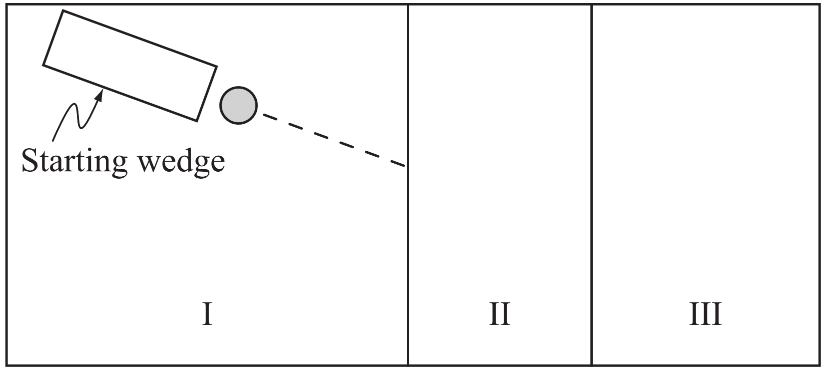
Activity 10-3: Rolling a Ball

- Sketch the trajectory of the ball in each case.
- Which ball spends more time in region II?
- How does the direction of the net force on the ball in motion 2 compare to the direction of the net force on the ball in motion 1?
- How does the change in kinetic energy of the ball in motion 1 compare to the change in kinetic energy of the ball in motion 2?
  - Is your answer consistent with the net work done on the ball in motions 1 and 2?
  - How does the final speed of the ball in motion 1 compare to the final speed in motion 2?
- Draw vectors that represent the momentum of the ball at the top of the ramp and at the bottom of the ramp in each case.
  - How is the direction of the change in momentum related to the direction of the net force on the ball as it rolls down the ramp?
  - How do the magnitudes of the changes in momentum in the two cases compare to each other?



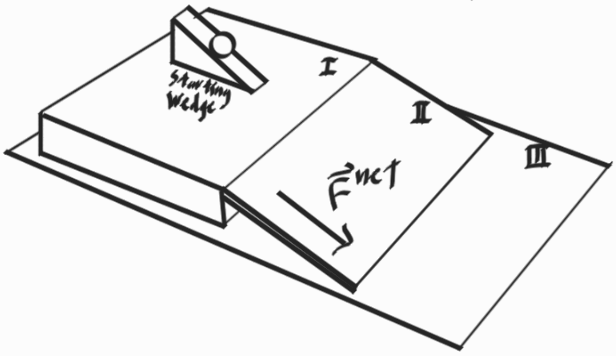


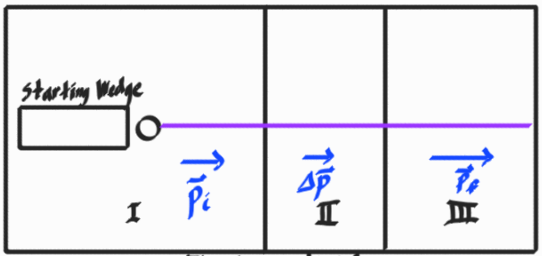
Top view, motion 1



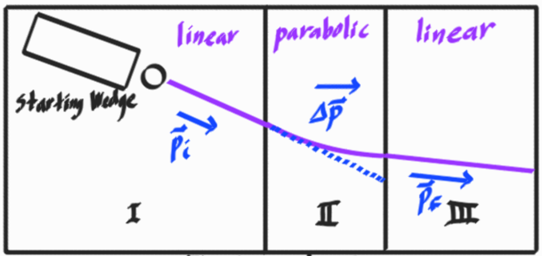
Top view, motion 2

10-3 Two Ramps, Two Trajectories





Top view motion 1



Top view motion 2

Side note: $p_{i,y} = p_{f,y}$, as $\Delta\vec{p}$ is only in the x-direction.

$\vec{F}^{net} = \vec{0}$ in I and III, where the surface is flat.

$\vec{F}^{net}$ is down the ramp in II, and it doesn't matter which way the ball is moving, so it is the same for both motions.

Ball 2 spends more time in II (see next page).

The ball descends the same height in both motions, so ΔU_g is the same, which means $\Delta K_1 = \Delta K_2$ by conservation of energy. The balls' final speeds must be the same.

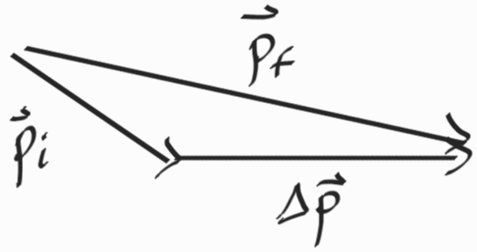
Regarding work, the work done on the ball in motion 1 is just $W_1 = F\Delta x$, the force down the ramp times the displacement down the ramp. In motion 2, the displacement along the parabolic path is longer, but not in line with the force, so the dot product in $W_2 = \int \vec{F} \cdot d\vec{x}$ will only keep the parts of the path in line with the force, giving us $W_1 = W_2$.

The momentum change is parallel to the force, so it always points to the right.

$$\Delta\vec{p} = \vec{J} = \int \vec{F}\,dt \Rightarrow \Delta\vec{p} \parallel \vec{F}$$

$\Delta p_1 < \Delta p_2$, as the momentum change in motion 2 not only increases the magnitude of the momentum, but also changes its direction.

This is the reverse triangle inequality:



$$|\Delta \vec{p}| = |\vec{p}_f - \vec{p}_i| \geq |\vec{p}_f| - |\vec{p}_i|$$

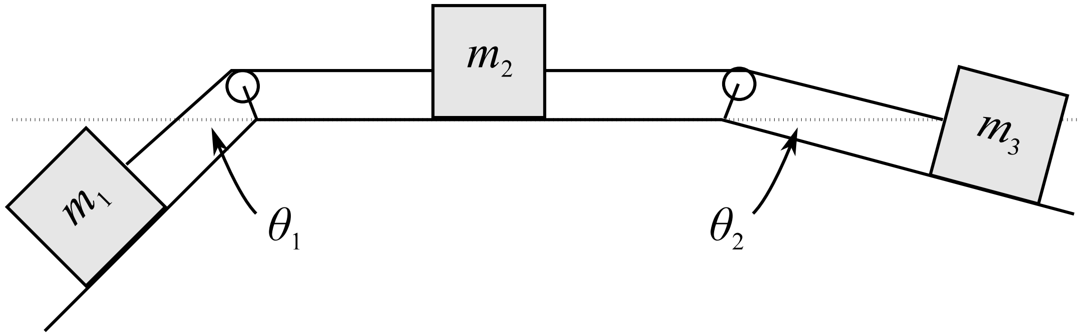
This can also be seen from an impulse perspective:

$$\Delta p_1 = J_1 < J_2 = \Delta p_2$$

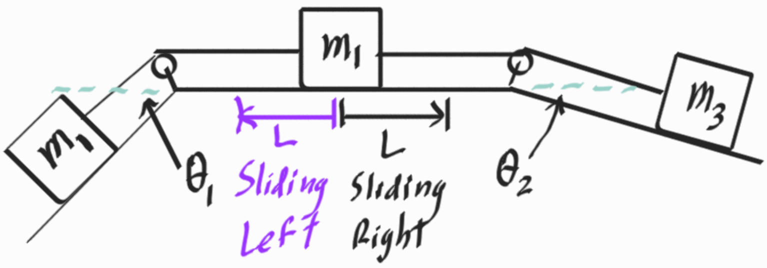
Though the forces are the same, the ball in 2 spends more time on the ramp (it had less of its initial speed pointing straight down the ramp), so it gets more impulse.

Activity 10-4: Angled Ramps

- The three boxes shown at right are connected by ideal strings and pulleys. Assume the surface is frictionless. The boxes are initially at rest.
  - Do you think this situation will be easier to analyze using **forces** or using **energy**? Does your answer depend on what quantity you are looking for?
- What is the speed of box 2 after it has moved a distance L?
  - Evaluate your answer in at least 3 special cases.
  - Is there any relationship between the three masses that will cause the blocks to remain at rest?



10-4 Three Linked Boxes



If we wanted to know about acceleration, forces would make sense, but we just want to know about speed after the objects have moved some distance. We would need to do kinematics to get the final speed. Energy will be more direct, as it works with speed and position directly.

I will do the calculation assuming the boxes slide to the right, but I will mark all changes for sliding left in purple.

Conservation of Energy: $\Delta E_{tot} = 0$

$$\Delta K + \Delta U_{g_1} + \Delta U_{g_3} = 0$$

$$\Delta K = K_f - \cancel{K_i}^{\,0} = \frac{1}{2}(m_1 + m_2 + m_3)\, V^2$$

$\Delta U_{g_1} = -m_1 g L \sin\theta_1$ Box 1 rises (lowers) by $h = -L\sin\theta_1$.

$\Delta U_{g_3} = \pm m_3 g L \sin\theta_2$ Box 2 lowers (rises) by $h = \pm L\sin\theta_2$

$$\frac{1}{2}(m_1 + m_2 + m_3) \mp m_1 g L \sin\theta_1 \pm m_3 g L \sin\theta_2 = 0$$

$$\Rightarrow V = \sqrt{\frac{-2m_3 g L \sin\theta_2 \pm 2m_1 g L \sin\theta_1}{m_1 + m_2 + m_3}} = \sqrt{\frac{2gL}{m_1+m_2+m_3}}\sqrt{-m_3 \sin\theta_2 \pm m_1 \sin\theta_1}$$

We need $-m_3 \sin\theta_2 \pm m_1 \sin\theta_1 \geq 0$ (otherwise we are taking the square root of a negative number).

Slides Right: $m_3 \sin\theta_2 > m_1 \sin\theta_1$ Slides Left: $m_1 \sin\theta_1 > m_3 \sin\theta_2$

If $m_3 \sin\theta_2 = m_1 \sin\theta_1$, then the boxes stay still. No kinetic energy is gained by sliding, so potential energy is constant, and thus there is no force pushing the system to slide.

Special Cases

$$\theta_1 = 0^\circ,\ \theta_2 = 90^\circ \Rightarrow V = \sqrt{\frac{2gLm_3}{m_1+m_2+m_3}}$$ sliding right

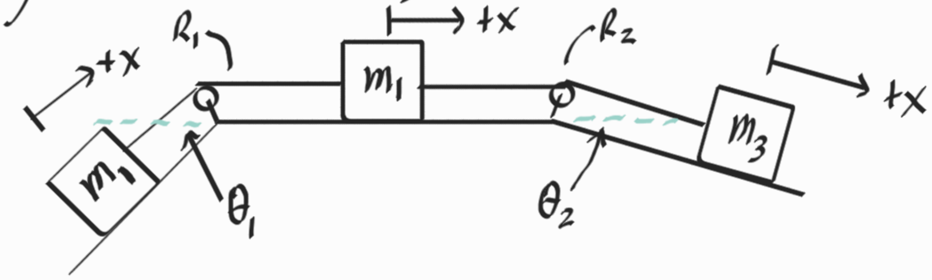
$$\theta_1 = 90^\circ,\ \theta_2 = 0^\circ \Rightarrow V = \sqrt{\frac{2gLm_1}{m_1+m_2+m_3}}$$ sliding left

In both of these cases, we have a single block hanging while two are on a flat surface. There is no condition to keep them stationary, as there is no way to balance the force on the one hanging block.

$$m_1 = m_3 \Rightarrow V = \sqrt{\frac{2gLm_1}{2m_1+m_2}}\sqrt{-\sin\theta_2 \pm \sin\theta_1}$$

In this case, the condition for the boxes to remain stationary is $\theta_2 = \theta_1$, which makes sense, as for equal masses, the forces on m_2 will be balanced if the two other masses hang symmetrically.

Solving with Forces



$$m_1 \vec{a} = \vec{F}_1^{net} = \left(F^T_{R_1} - m_1 g \sin\theta_1\right)\hat{x}$$

$$m_2 \vec{a} = \vec{F}_2^{net} = \left(F^T_{R_2} - F^T_{R_1}\right)\hat{x}$$

$$m_3 \vec{a} = \vec{F}_3^{net} = \left(m_3 g \sin\theta_2 - F^T_{R_2}\right)\hat{x}$$

$$(m_1 + m_2 + m_3)\vec{a} = \left(-m_1 g \sin\theta_1 + m_3 g \sin\theta_2\right)\hat{x}$$

$$\vec{a} = a\hat{x}$$

$$a = \frac{g}{m_1 + m_2 + m_3}\left(-m_1 \sin\theta_1 + m_3 \sin\theta_2\right)$$

Kinematics: $V_f^2 = V_i^2 + 2a\Delta x$ $\qquad \Delta x = -L$

$$V^2 = -2aL$$

$$= -\frac{2gL}{m_1+m_2+m_3}\left(-m_1 \sin\theta_1 + m_3 \sin\theta_2\right)$$

$$V = \sqrt{\frac{2gL}{m_1+m_2+m_3}}\sqrt{-m_3 \sin\theta_2 \pm m_1 \sin\theta_1}$$

Same answer!