Rolling Solid Sphere

A uniform solid sphere rolls without slipping along a flat, level, frictionless horizontal surface. It then rolls up a frictionless inclined plane. The angle of incline is θ. Its speed is momentarily zero after it has rolled a distance *d* along the ramp. Determine its speed on the way up when it is at the base of the inclined plane in terms of θ, *d*, and/or *g*.

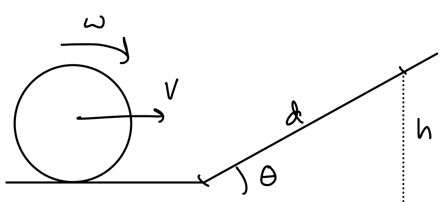
Before you do any calculations, write down what approach to solving this problem you are going to use (hint: what did we learn about in class on Thursday?):

Conservation of Energy! $K_T = \frac{1}{2}mv^2$ $K_R = \frac{1}{2}I\omega^2$

$U = mgh$

What units should your answer have? m/s

Rolling without slipping: $\omega = \frac{v}{r}$ $I = \frac{2}{5}mr^2$



$h = d\sin\theta$

Final energy: $U_f = mgh = mgd\sin\theta$

$K_{T,f} = 0$ $K_{R,f} = 0$

Initial energy: $U_i = 0$ $K_{T,i} = \frac{1}{2}mv^2$ $K_{R,i} = \frac{1}{2}I\omega^2$

$$\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgd\sin\theta$$

$$\frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}\cancel{r^2}\right)\left(\frac{v^2}{\cancel{r^2}}\right) = \cancel{m}gd\sin\theta$$

$$\frac{7}{5}v^2 = 2gd\sin\theta \rightarrow v^2 = \frac{10}{7}gd\sin\theta$$

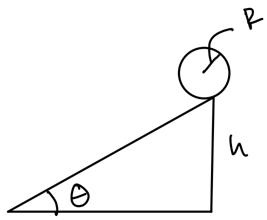
$$\Rightarrow \boxed{v = \sqrt{\frac{10}{7}gd\sin\theta}} \qquad \left[\frac{m}{s}\right] = \sqrt{\left[\frac{m}{s^2}\right][m]} = \left[\frac{m}{s}\right] ✓$$

Solid Sphere vs. Hoop

A solid sphere of radius *R* is placed at a height of 0.3 m on a 15° slope. It is released and rolls, without slipping, to the bottom of the ramp. From what height should a circular hoop of radius *R* be released on the slope to have the same speed as the sphere when they reach the bottom?

$$I_{sphere} = \frac{2}{5}mR^2$$
$$I_{hoop} = mR^2$$

$I = AmR^2$



$K_i = 0$ $\quad K_{T,f} = \frac{1}{2}mv^2$ $\quad U_f = 0$

$U_i = mgh$ $\quad K_{R,f} = \frac{1}{2}I\omega^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 \qquad \omega = \frac{v}{r}$$

Find v for the sphere:

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\frac{2}{5}\cancel{m}\cancel{R^2}\right)\frac{v^2}{\cancel{R^2}} \rightarrow 2gh = \frac{7}{5}v^2 \rightarrow v^2 = \frac{10}{7}gh$$

Find h for the hoop (h_h)

$$\cancel{m}gh_h = \frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\cancel{m}\cancel{R^2}\left(\frac{v^2}{\cancel{R^2}}\right) \rightarrow gh_h = v^2$$

$$\Rightarrow h_h = \frac{v^2}{g} = \frac{10}{7}\frac{g}{g}h = \frac{10}{7}h$$

$$\Rightarrow h_h = \frac{10}{7}(0.3m) = \frac{10}{7}\left(\frac{3}{10}\right)m = \boxed{\frac{3}{7}m = h}$$

Moment of Inertia

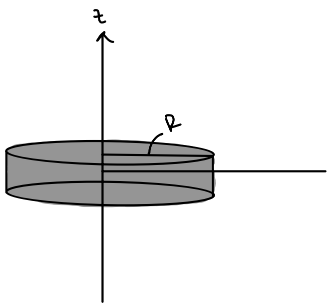
You have a circular plate of radius *R*, thickness *d*, mass *M*, and uniform density. Placing its axis of rotation through the center of the face, perpendicular to the plane of its surface, what is its moment of inertia?

a) In general, $dm = \rho dxdydz$. How can this be simplified to get the required mass-element *dm* for the moment of inertia expression?

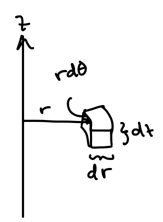
$r^2 = x^2 + y^2$. So we can integrate of the z dependence.

$$dm = \rho dx dy \int_{-\frac{d}{2}}^{d/2} dz = d\rho dx dy$$

b) Rewrite *dm* in cylindrical coordinates. Hint: Draw a tiny chunk of the circular plate to determine what the volume is.



Little chunk:



Volume in cylindrical: $dV = r dr d\theta dz$

$$\Rightarrow dm = d \cdot \rho \cdot r dr d\theta$$

$$\rho = \frac{M}{\pi R^2 d}$$

c) Determine the moment of inertia.

$$I = \int r^2 dm = \int_0^{2\pi}\int_0^R r^2 (d \cdot \rho \cdot r dr d\theta) = 2\pi \cdot d\rho \int_0^R r^3 dr = 2\pi d\rho \frac{R^4}{4}$$

$$I = 2\cancel{\pi}\cancel{d} \frac{M}{\cancel{\pi}\cancel{R^2}\cancel{d}} \frac{\cancel{R^4}^{\,2}}{4} = \frac{1}{2} MR^2$$

$$\Rightarrow \boxed{I = \frac{1}{2} MR^2}$$