RC Concepts in Current

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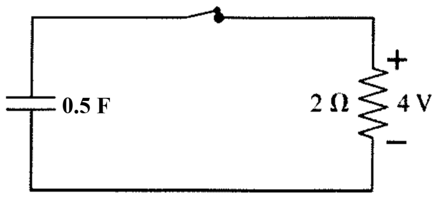
Winter 2024

This problem is borrowed from Chapter 29 of the *Student Workbook* for *Physics for Scientists and Engineers*.

Activity

The capacitor in this circuit was initially charged, then the switch was closed. At this instant of time, the potential difference across the resistor is $\Delta V_R = 4$ V.



(a) At this instant of time, what is the current through the resistor?

The current through the resistor at the instant the switch closes is

$$I_0 = \frac{\Delta V_R}{R} = \frac{4 \text{ V}}{2 \ \Omega} = 2 \text{ A}.$$

(b) What is the current through the resistor over time?

A discharging RC circuit has current of the form

$$I(t) = I_0 e^{-t/\tau}.$$

We just found I_0, and for the given circuit, $\tau = RC = (2 \ \Omega)(\frac{1}{2} \text{ F}) = 1$ s. As such,

$$I(t) = (2 \text{ A})e^{-t/(1 \text{ s})}.$$

(c) What is the charge in the capacitor over time?

A discharging RC circuit has charge of the form

$$Q(t) = Q_0 e^{-t/\tau}.$$

The initial charge $Q_0 = \Delta V_C C$, and the initial voltage difference across the capacitor must be $\Delta V_C = 4$ V by Kirchhoff's loop law. As such, $Q_0 = (4 \text{ V})(\frac{1}{2} \text{ F}) = 2$ C, and the charge varies over time as

$$Q(t) = (2 \text{ C})e^{-t/(1 \text{ s})}.$$

(d) If the capacitor consists of two parallel plates of area A, symbolically calculate the electric flux between them, and the time derivative of this flux.

Assuming uniform distribution of charge over the plate, the surface charge density across the positive side is

$$\sigma(t) = \frac{Q(t)}{A} = \frac{Q_0}{A} e^{-t/\tau}.$$

Thus, the electric field magnitude between the plates is

$$E = \frac{\sigma(t)}{\epsilon_0} = \frac{Q_0}{A\epsilon_0} e^{-t/\tau}.$$

As such, the flux through the cross-sectional area A of the region between the plates is

$$\Phi_e = EA = \frac{Q_0}{\epsilon_0} e^{-t/\tau}.$$

This flux changes in time as

$$\frac{d\Phi_e}{dt} = -\frac{Q_0}{\tau\epsilon_0} e^{-t/\tau} = -\frac{Q_0}{RC\epsilon_0} e^{-t/\tau} = -\frac{\Delta V_R}{R\epsilon_0} e^{-t/\tau} = -\frac{I_0}{\epsilon_0} e^{-t/\tau}.$$