Spring Reasoning

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XX-1: Spring Reasoning

A spring has an unstretched length of 10 cm. It exerts a restoring force F when stretched to a length of 11 cm.

(a) For what length of the spring is its restoring force $3F$?

Let us keep this very general for now. An ideal spring obeys Hooke's law:

$$\vec{F}_{sp} = -k\Delta\vec{s},$$

where k is the spring constant, and $\Delta\vec{s}$ is the displacement of the end of the spring from its equilibrium position. Taking components along the s-direction, we maintain the sign relationship between the force and the displacement, but remove the vector notation:

$$F_{sp,s} = -k\Delta s.$$

Let us have two situations: Case A, where we have some displacement Δs_A and a restoring force $F_{sp,s,A} = -k\Delta s_A$, and Case B, where we have another displacement Δs_B and a restoring force $F_{sp,s,B} = -k\Delta s_B$. Consider what happens when we divide one equation by the other:

$$\begin{aligned} F_{sp,s,A} &= -k\Delta s_A \\ \div(F_{sp,s,B} &= -k\Delta s_B) \\ \implies \frac{F_{sp,s,A}}{F_{sp,s,B}} &= \frac{-k\Delta s_A}{-k\Delta s_B} = \frac{\Delta s_A}{\Delta s_B}. \end{aligned}$$

From this, we can see that if we triple the force from Case A to Case B, we must also triple the displacement. For our particular situation, we have a positive displacement $\Delta s = 11\text{ cm} - 10\text{ cm} = 1\text{ cm}$ and therefore a negative restoring force $-F$. If we desire triple the restoring force ($-3F$), we get

$$\frac{-F}{-3F} = \frac{1\text{ cm}}{\Delta s_B} \implies \Delta s_B = 3\text{ cm}.$$

Thus, the spring's total length must be 13 cm.

(b) At what compressed length is the restoring force $2F$?

For a compressed spring, the displacement is negative and the restoring force is positive. We have doubled the magnitude of the force, so the displacement must also be doubled in magnitude:

$$\frac{-F}{2F} = \frac{1\text{ cm}}{\Delta s_B} \implies \Delta s_B = -2\text{ cm}.$$

Thus, the spring must be 2 cm shorter than its equilibrium length, or 8 cm long.