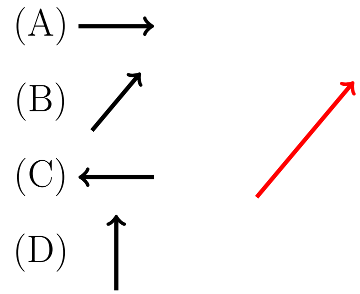
Lecture 15: Energy and Power

Warm-Up Activity

Which of these vectors, when dotted with the red vector below, results in a negative value?



When placed tail to tail, vector (C) forms an obtuse (greater than 90 degrees) angle with the red vector. The dot product of two vectors $\vec{v}$ and $\vec{w}$ is

$$\vec{v} \cdot \vec{w} = vw \cos\theta,$$

where θ is the angle between the two vectors (when they are tail-to-tail). Angles greater than 90 degrees will make the dot product negative.

Types of Energy

The ball is in motion, so it has kinetic energy all to itself. During the bounce, the ball changes shape (deforms), which stores a form of potential energy (elastic potential energy).

There is another form of potential energy—gravitational potential energy—shared between the ball and the Earth. It belongs to neither of them individually.

Finally, collisions jostle molecules and produce heat, so there are separate stores of thermal energy in both the ball and the ground.

Transformations

As the ball falls, the gravitational potential energy it shares with the Earth is transformed into kinetic energy. During the collision, a lot of this energy becomes elastic potential energy as the ball squishes down and comes to a brief stop. There is technically a little change in gravitational potential energy, since the mass distributed in the ball moves as it squishes down. Some of the kinetic energy instead becomes thermal energy that is divided between the ball and the ground. After the bounce, the elastic potential energy has been converted back into kinetic energy (less than it had when it hit the ground) as the ball speeds upward again, gradually getting slower and slower as its kinetic energy returns to gravitational potential.

Calculable

We have the equations necessary to calculate work, and we could figure out the force of gravity and the displacement of the ball, so the work done by gravity is calculable. Through the work-energy theorem, we would then be able to calculate the change in its kinetic energy over the course of the fall. We could use the same techniques to find the kinetic energy after the collision from the ending height of the ball after the bounce. The difference from before and after the bounce would tell us how much was lost to thermal energy, though not how that energy was split between the ball and the ground.

L15-1: The Drop and Bounce

You drop a tennis ball of the top of a tall building. It falls to the ground, bounces, and rises back into the air.

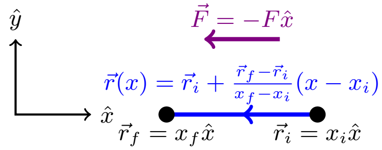
- Identify the different types of energy in this situation and to which object or system these energies belong.
- Describe how energy is transformed within systems and transferred between systems during the drop and bounce.
- Which of the energies, transformations, or transfers do you think you might be able to calculate?

Work vs. Energy

Energy is a property of a system that describes the system's capacity to exert forces that can impact the displacement of other systems. Work describes energy transfer between systems when done through forces.

Coordinate System Care

What if my path from the initial point to the final point goes in the negative direction of my coordinate system?



If force and displacement are in the same direction, the work should be positive, but how does this show up in the math?

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} = \int_{r_i}^{r_f} \vec{F} \cdot \frac{d\vec{r}}{dx} dx = -\int_{x_i}^{x_f} F dx = \int_{x_f}^{x_i} F dx > 0$$

$$\frac{d\vec{r}}{dx} = \frac{\vec{r}_f - \vec{r}_i}{x_f - x_i} = \frac{x_f\hat{x} - x_i\hat{x}}{x_f - x_i} = \frac{x_f - x_i}{x_f - x_i}\hat{x} = \hat{x}$$

Because our path $\vec{r}(x)$ has the same length scale as our parameterization x, the derivative just gives us direction.

An easy mistake is to look at $W = \int_{x_i}^{x_f} \vec{F} \cdot d\vec{x}$, plug in

$$W = \int_{x_i}^{x_f} (-F)(-dx) = \int_{x_i}^{x_f} F dx = -\int_{x_f}^{x_i} F dx,$$

and get a negative work by accidentally introducing an extra negative sign. It is important to make a clear distinction between the path $\vec{r}(x)$ and the parameterization x.

Rather than fixate on the highly mathematical version, the basic take-away from this is that you should either integrate the path from the larger x_i to the smaller x_f against the direction of $d\vec{x}$, or you should integrate from the smaller x_f to the larger x_i and introduce a negative sign ($-d\vec{x}$) to indicate the orientation of the path. Introducing both will cause problems.

Changing a System's Energy

- The total energy of a system can only change through an *interaction* with something external to the system.
- If that interaction is a force, then the energy transferred to the system is known as *work*.

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$$

Let the ball start at height $y_i = h$ and end at $y_f = 0$. The work done by gravity is

$$W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r} = \int_{y_i}^{y_f} \vec{F} \cdot \frac{d\vec{r}}{dy} dy.$$

Just like on the previous page, $\frac{d\vec{r}}{dy} = \hat{y}$, so

$$W = \int_h^0 (-mg\hat{y}) \cdot (dy\hat{y}) = mg \int_0^h \hat{y} \cdot \hat{y} dy = mg \int_0^h dy = mgh.$$

This does not depend on initial speed, only the distance it falls from its initial height, so it does not matter if you throw it! The works are equal.

L15-2: The Drop – Part 1

- Consider the system of the tennis ball.
- Starting at the moment you drop the ball and ending right before the ball hits the ground:
  - How much work does the force of gravity do on the tennis ball if you drop it from rest?
  - If you instead throw the ball with an initial vertical speed of v_0, do you think the work done by the gravitational force is *greater than*, *less than*, or *equal to* the original work?

The Work-Energy Theorem

$$W_{\text{net,ext}} = \Delta E_{\text{total}}$$

- The net external work done on a system is equal to the change in total energy of that system.
- What you decide to put in your system is ***absolutely critical!***

The change in total energy is the change in the kinetic energy of the ball:

$$\begin{aligned} W_{\text{net,ext}} &= \Delta E_{\text{total}} \\ W_{\text{gravity}} &= K_f - K_i \\ mgh &= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 \\ 2gh &= v_f^2 - v_i^2. \end{aligned}$$

Starting from rest ($v_i = 0$), this means $v_f = \sqrt{2gh}$.

Also, note that part of our work looks like the form of our third kinematics equation: $v_f^2 = v_i^2 + 2g\Delta y$ (with h instead of Δy). There is more than one way to do this problem.

L15-2: The Drop – Part 2

- Consider the system of the tennis ball.
- Starting at the moment you drop the ball and ending right before the ball hits the ground:
  - How much work does the force of gravity do on the tennis ball if you drop it from rest?
  - If you instead throw the ball with an initial vertical speed of v_0, do you think the work done by the gravitational force is *greater than*, *less than*, or *equal to* the original work?
  - **What is the speed of the tennis ball right before it hits the ground?**

–

–

–

A Deeper Model for Interactions

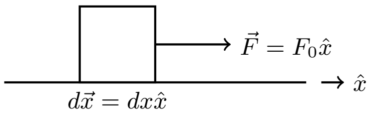
- Quantities
  - Energy E
  - Kinetic Energy $K = \frac{1}{2}mv^2$
- Laws
  - Work-energy theorem $W_{\text{net,ext}} = \Delta E_{\text{total}}$

Power

- When the energy of a system changes, we sometimes want to know how *fast* it changes.
- *Power* is the time rate of change of energy:

$$P = \frac{dE}{dt}.$$

- Power is measured in watts (W).



Total Energy
The block can have kinetic energy, so its change in total energy is the difference of its final and initial kinetic energies. Its initial speed is 0 m/s, so

$$\Delta E = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \frac{1}{2}mv_f^2.$$

Plugging in numbers, we obtain $\Delta E = 9000$ J.
Distance
By the work-energy theorem, $W_{\text{net,ext}} = \Delta E$, so if there is no friction and the winch is the only thing doing work, then we can calculate the distance d from the work it did:

$$W = \int_0^d \vec{F} \cdot d\vec{x} = F_0 d = \Delta E = \frac{1}{2}mv_f^2.$$

Solving for d gives us $d = \frac{m}{2F_0}v_f^2$, and plugging in numbers gives us $d = 0.5$ m.
Power
We know how much energy the winch put into this endeavor, so all we need to find out is the time it took to move the block. For constant acceleration (which we have in this situation of constant force), we know $a = \frac{\Delta v}{\Delta t} = \frac{F_0}{m}$, so $\Delta t = \frac{v_f m}{F_0} = \frac{1}{6}$ s. This means that the power is

$$P = \frac{\Delta E}{\Delta t} = \frac{F_0 d}{v_f m/F_0} = \frac{F_0^2 d}{v_f m}.$$

Given what we know about the distance, this simplifies to

$$P = \frac{F_0^2}{v_f m}\frac{m}{2F_0}v_f^2 = \frac{F_0 v_f}{2} = 54,000 \text{ W}.$$

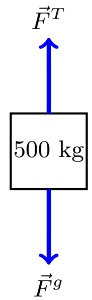
For comparison, the light bulb in my desk lamp in my apartment consumes 9 W while on (this is much more efficient than an equivalently bright incandescent bulb, which would take about 60 W). This much power could light 6,000 desk lamps (or 900 if using an incandescent bulb)!

L15-3: The Winch – Part 1

A winch acts a constant force $F_0 = 18,000$ N on a metal block ($m = 500$ kg) to accelerate it across level ground from rest to a final speed of $v_f = 6$ m/s.

- What is the block's change in total energy?
- How far did the winch move the block?
- How much power does this winch use?

Force
To keep the block at constant speed, the winch must exert a force equal in magnitude to the force of gravity on the block:



$$F^T = F^g = mg \approx (500 \text{ kg})(10 \text{ m/s}^2) = 5000 \text{ N}$$

Distance Lifted
We know the speed of the block and how long it is in motion, so we can calculate its displacement directly from this:

$$\Delta y = v\Delta t = (2 \text{ m/s})(30 \text{ s}) = 60 \text{ m}.$$

Work

$$W = \int_{y_i}^{y_f} \vec{F} \cdot d\vec{y} = \int_{y_i}^{y_f} (F\hat{y}) \cdot (dy\hat{y}) = \int_{y_i}^{y_f} F dy$$
$$= F\Delta y = (5000 \text{ N})(60 \text{ m}) = 300,000 \text{ J}$$

Note that the speed is not changing, so the total energy of the block is not changing. That means $W_{\text{net,ext}} = \Delta E_{\text{total}} = 0$. Something else must be doing work to remove energy from the system! This is being done by gravity.

Power
The winch is transferring 300,000 J of energy over the course of 30 s, so the power is

$$P = \frac{\Delta E}{\Delta t} = \frac{300,000 \text{ J}}{30 \text{ s}} = 10,000 \text{ W}.$$

This much power could light about 1,111 desk lamps (or about 166 if using an incandescent bulb)!

L15-3: The Winch – Part 2

You want to use the winch to lift the block into the air at a constant speed: $v = 2$ m/s.

- What force should you set the winch for?
- How far does the winch move the block from $t = 0$ s to $t = 30$ s?
- How much work does the winch do in $\Delta t = 30$ s?
  - Is anything else doing work on the block?
- How much power does the winch use now?

Energy Analysis

- Understanding: Identify a system and the types of energy within the system.
- Calculating: Is your system's energy conserved or not? Once you know, use the work-energy theorem!
- Sensemaking: All the sensemaking strategies you have will work, but a new strategy is sometimes useful: Solve Multiple Ways.
  - You have kinematics and force techniques at your disposal, so you can solve problems with these and compare their results to the results of your energy approach.

–

–

–

Main Ideas

- Energy is a powerful, ubiquitous concept that can help us solve a wide array of physics problems.
- Energy is a *scalar*—it is not a vector.
- There are different forms of energy, and energy can be transferred between objects and between forms.