Lecture 13: Newton' 3rd Law of Motion
(and a Bit of Springs)

Warm-Up Activity

Which of the following statements, if any, are true about Newton's 3rd law pairs?

(A) They appear on different free-body diagrams.

(B) They are the same type of force.

(C) They appear on the same free-body diagram.

(D) $\vec{F}^t_{AB} = -\vec{F}^t_{AB}$

Feedback Question

Do you prefer feedback through documents uploaded to Canvas, or through Gradescope (as was done on the quizzes)?

(A) PDFs on Canvas

(B) Comments in Gradescope

Newton's 3rd Law of Motion

- If A exerts a force on B, then B exerts a force of the same magnitude on A in the opposite direction:

$$\vec{F}^{t}_{AB} = -\vec{F}^{t}_{BA}$$

- These two forces make a *Newton's 3rd law pair*, or an *action-reaction pair*.
- 3rd law pair forces...
  - are the same type of force;
  - appear on different free body diagrams.

Spring Forces

- Many objects resist changes in physical configuration (*i.e.* deformations).
- For small deformations, we can model the object as a spring.
- The forces caused by springs obey Hooke's law: $\vec{F}^S = -k(\vec{x} - \vec{x}_{eq})$.
  - $\Delta\vec{x} = (\vec{x} - \vec{x}_{eq})$ is displacement from equilibrium.
  - k is the spring constant.
  - What does the negative sign mean?

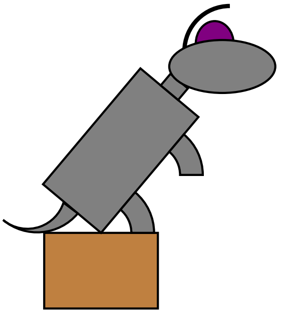
Types of Forces

- Gravity $\vec{F}^g_{AB} = m_A \vec{g}_B$
  - Newtonian $\vec{g}_B = G\frac{M_B}{r^2}(-\hat{r})$, $G = 6.67408 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
  - Near-Earth $\vec{g}_E = g(-\hat{y}),\ g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2} \approx 10\frac{\mathrm{m}}{\mathrm{s}^2}$
- Normal $\vec{F}^N$ always $\perp$; varies in magnitude
- Tension $\vec{F}^T$ uniform (massless, inextensible rope)
- Spring $\vec{F}^S = -k(\vec{x} - \vec{x}_{eq})$
- Friction
  - Static Friction $F^{sf} \leq \mu_s |\vec{F}^N|$
  - Kinetic Friction $F^{kf} = \mu_k |\vec{F}^N|$

Not Forces

- Momentum
- Inertia
- Velocity
- Acceleration

A*R*C*S: Uh-Oh Dr. Paws



In the video in Section 3.16 of our textbook, Paul pushes a footstool (mass m_1) across the floor with a constant force so that the footstool speeds up. Dr. Paws (a dog with mass m_2) is sitting on the footstool. The coefficient of static friction between the dog and the footstool is μ (assume no friction between the footstool and the ground). How much force can Paul exert on the footstool before the dog begins sliding?

L13-1: Uh-Oh Dr. Paws – Analyze and Represent

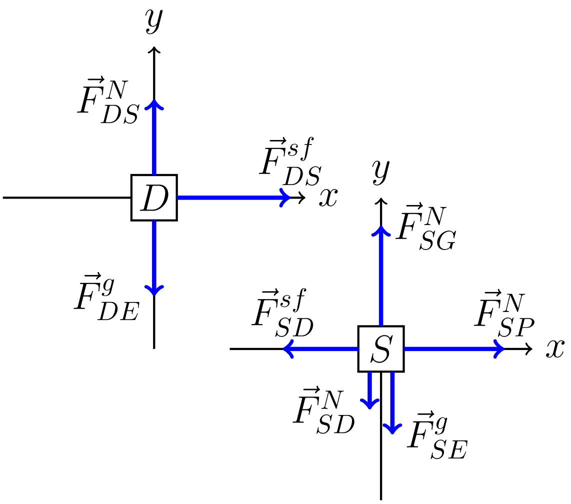
(1a) Understand the Problem

- Mass of the footstool: $m_1 = 10$ kg
- Mass of the dog: $m_2 = 30$ kg
- Gravity: $g = 10$ m/s^2
- Coefficient of static friction: $\mu = 0.4$

(1b) Identify Assumptions

- Near-Earth
- Particle model
- Neglect air-resistance
- Dr. Paws doesn't move.

(1c) Represent Physically



- Why are the assumptions reasonable?
- Identify any problems with these free body diagrams.
- Identify the third-law pairs.

L13-2: Uh-Oh Dr. Paws – Sensemake

You have three friends who each come up with a different equation for the maximum allowable force that Paul can apply:

(A) $F^{N}_{SP} = \mu \frac{m_1}{m_1+m_2} g$

(B) $F^{N}_{SP} = \mu(m_2 - m_1)g$

(C) $F^{N}_{SP} = \mu \frac{m_1 m_2}{m_1+m_2} g$

Which of the above equations, if any, are correct? How can you tell?

Uh-Oh Dr. Paws – Calculate

(2a) Represent Principles

$$\vec{F}^{net} = m\vec{a} \qquad \vec{F}_{AB} = -\vec{F}_{BA} \qquad F^{sf} \leq \mu F^{N} \qquad \vec{F}^{g} = m\vec{g}$$

(2b) Solve Symbolically

Dog

$$\begin{aligned} m_2 a = F_x^{net} &= F_{DS}^{sf} \\ 0 = F_y^{net} &= F_{DS}^{N} - F_{DE}^{g} \\ F_{DS}^{N} &= m_2 g \end{aligned}$$

Stool

$$\begin{aligned} F_x^{net} &= m_1 a \\ F_{SP}^{N} - F_{SD}^{sf} &= m_1 a \\ F_{SP}^{N} &= m_1 a + F_{SD}^{sf} = m_1 a + F_{DS}^{sf} \\ F_{SP}^{N} &= m_1 a + m_2 a = (m_1 + m_2) a \end{aligned}$$

$$\begin{aligned} F_{DS}^{sf} &\leq \mu F_{DS}^{N} = \mu m_2 g \\ m_2 a &\leq \mu m_2 g \\ a &\leq \mu g \end{aligned}$$

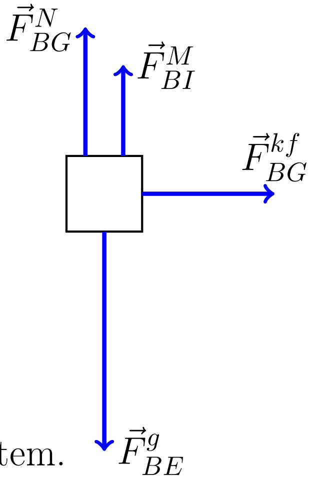
$$F_{SP}^{N} \leq (m_1 + m_2)\mu g$$

(2c) Plug in Numbers

$$\begin{aligned} F_{SP}^{N} &\leq (m_1 + m_2)\mu g \\ &= (10\ \text{kg} + 30\ \text{kg})(0.4)\left(10\frac{\text{m}}{\text{s}^2}\right) \\ &= 160\ \text{N} \end{aligned}$$

A Model for Interactions

- Quantities
  - Mass m
  - Force $\vec{F}$
- Laws
  - Net force is proportional to acceleration: $\vec{F}^{net} = m\vec{a}$
  - Forces come in pairs: $\vec{F}_{AB} = -\vec{F}_{BA}$
- Asssumptions
  - We can treat multiple objects as a system.
  - All forces act as if on the center of the system.
- Diagram



Solving Problems Using Forces

- Identify a system.
- Identify the (external) forces acting on the system.
  - Draw a free-body diagram.
- Identify the acceleration (**not a force**).
  - Static/dynamic equilibrium (acceleration = 0)
  - Dynamics (acceleration not 0)
- Use the laws of motion.
- Reflect on your answer (check units and evaluate special cases).

Main Ideas

- Newton's 3rd law of motion can be used to relate the forces acting on *different* objects or systems.