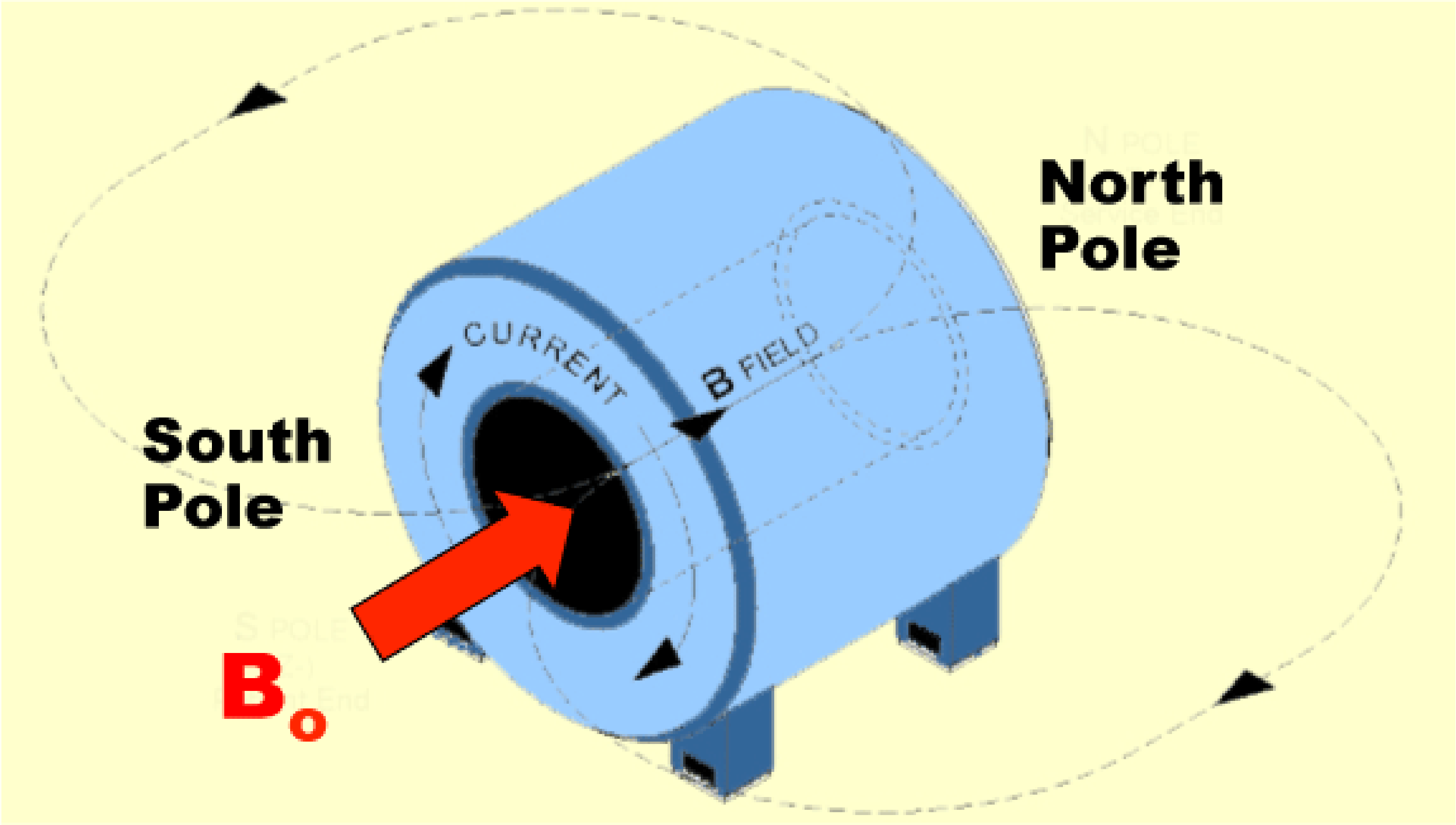
Studio Week 9
Magnetic Flux



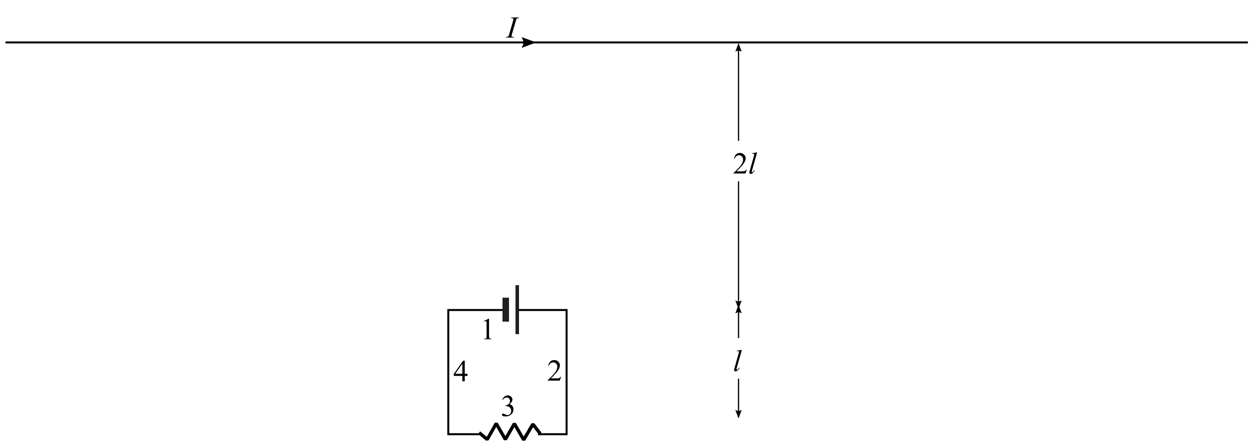
Picture credit: mri-q.com.

Principles for Success

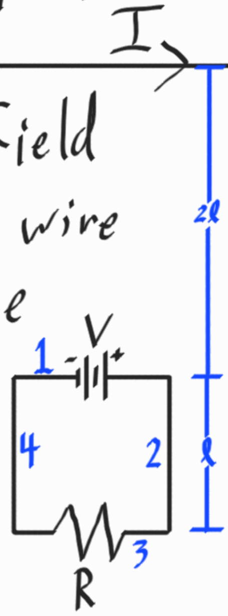
- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

Activity 9-1 – Circuit Force I

- The square circuit below (voltage V and resistance R) is located below a long straight wire carrying current I. Each side of the circuit has length l.
  A. What is the direction of the magnetic field in the region below the long straight wire?
  B. What is the direction of the magnetic force on each side of the square circuit?
  C. Rank the magnitudes of the magnetic forces on the four sides of the loop.
  D. Does the magnetic field exert a net nonzero magnetic force on the loop? If so, what direction is the net force?
  E. Does the magnetic field exert a net nonzero magnetic torque on the loop? If so, what direction would you expect the loop to rotate (starting from rest)?



9-1 Circuit Force I



A) The magnetic field below the long wire points into the page. ⊗ $\vec{B}$

B) The magnetic force on a wire of length l carrying a current I is $\vec{F}^B = l\,\vec{I}\times\vec{B}$.
We only need the direction of current in each side of the loop and the direction of the magnetic field to find the magnetic force.

Side 1:
$\vec{F}_1^B \propto \vec{I}\times\vec{B} = \rightarrow \times \otimes = \uparrow$

Side 2:
$\vec{F}_2^B \propto \vec{I}\times\vec{B} = \downarrow \times \otimes = \rightarrow$

Side 3:
$\vec{F}_3^B \propto \vec{I}\times\vec{B} = \leftarrow \times \otimes = \downarrow$

Side 4:
$\vec{F}_4^B \propto \vec{I}\times\vec{B} = \uparrow \times \otimes = \leftarrow$

C) $F_1^B > F_2^B = F_4^B > F_3^B$

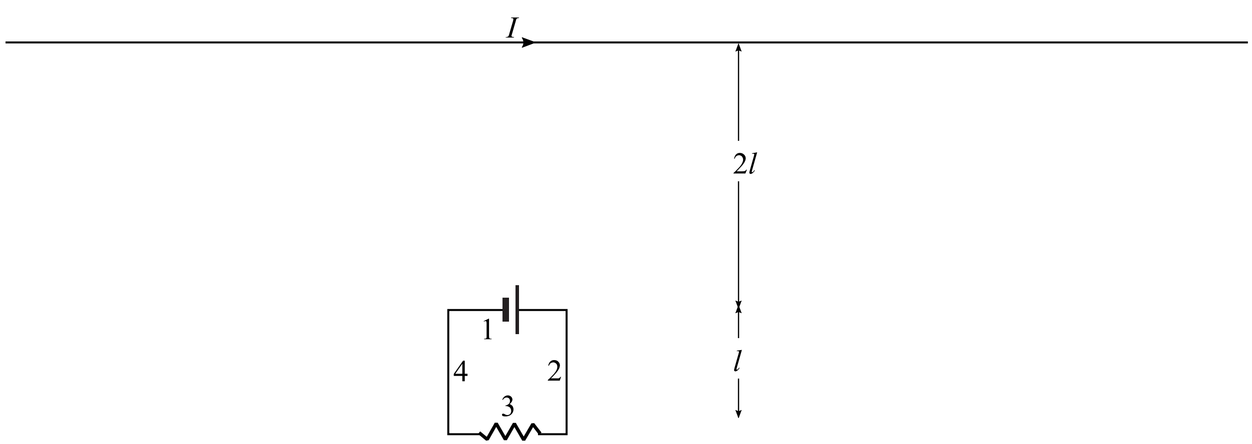
Side 1 is closest to the wire, so the external magnetic field is strongest there.
Side 3 is farthest from the wire, so the external magnetic field is weakest there.
Sides 2 and 4 each have one end a distance $2l$ from the wire, and the other end $3l$ from the wire. The magnetic field along each varies between the extremes of the other two sides. With the same current and the same variation of field strength over the length of each side, both 2 and 4 should have the same magnitude of force on them.

D) $\vec{F}_2^B$ and $\vec{F}_4^B$ will cancel, but $\vec{F}_1^B$ is stronger than $\vec{F}_3^B$, so there will be a net force upward.

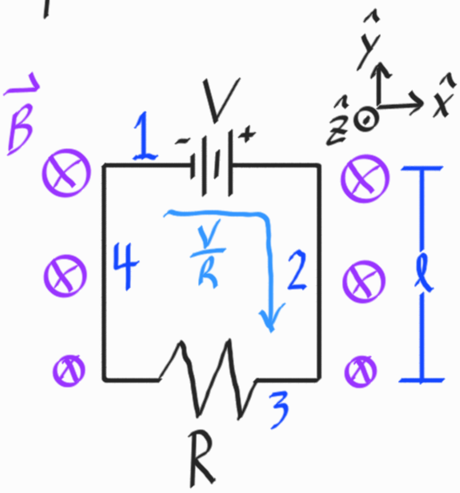
E) The loop can be modeled as an electric dipole $\vec{\mu} = I\vec{A}$, which points into the page. The torque on a dipole is $\vec{\tau} = \vec{\mu}\times\vec{B}$, and since $\vec{\mu}$ and $\vec{B}$ point the same way, this cross product is zero. This is complicated by the fact that $\vec{B}$ is not uniform over the loop, but the principle is still valid. From an energy perspective, the potential energy of a dipole in a magnetic field is $U = -\vec{\mu}\cdot\vec{B}$, so the lowest potential energy occurs when $\vec{\mu}$ and $\vec{B}$ point the same way. No torque is required to twist the loop into a lower energy orientation. However, the potential can still get lower by making $\vec{B}$ larger, hence there is a force pulling the loop toward the wire, where the field is stronger.

Activity 9-2 – Circuit Force II

- The square circuit below (voltage V and resistance R) is located below a long straight wire carrying current I. Each side of the circuit has length l.
    A. Calculate the magnetic force on <u>each side</u> of the circuit in terms of known variables.
    B. Use your answers to find the net magnetic force and net magnetic torque on the circuit.
    C. Check to see if your answers agree with the prediction you made in question C of Activity 9-1.



9-2 Circuit Force II



$$\vec{B}_{wire} = \frac{\mu_0}{2\pi} \frac{\vec{I} \times \hat{s}}{s} \qquad \left.\begin{array}{l} \vec{I} = I\hat{x} \\ \hat{s} = -\hat{y} \end{array}\right\} \vec{B} = \frac{\mu_0 I}{2\pi s}(-\hat{z})$$

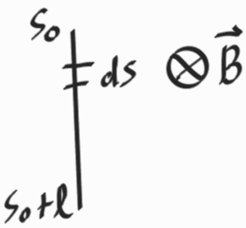
A) Side 1

$$\vec{F}_1^B = \ell\left(\frac{V}{R}\hat{x}\right) \times \left(-\frac{\mu_0 I}{2\pi(2\ell)}\hat{z}\right) = \frac{\mu_0 I V}{4\pi R}\hat{y}$$

Side 3

$$\vec{F}_3^B = \ell\left(-\frac{V}{R}\hat{x}\right) \times \left(-\frac{\mu_0 I}{2\pi(3\ell)}\hat{z}\right) = -\frac{\mu_0 I V}{6\pi R}\hat{y}$$

Sides 2 and 4 have a varying $\vec{B}$, so we need to do chop-multiply-add!



$$d\vec{F}^B = ds\,\vec{I} \times \vec{B}$$

Side 2

$$\vec{F}_2^B = \int_{2\ell}^{3\ell} ds\left(-\frac{V}{R}\hat{y}\right) \times \left(-\frac{\mu_0 I}{2\pi s}\hat{z}\right) = \frac{\mu_0 I V}{2\pi R}\hat{x} \overbrace{\int_{2\ell}^{3\ell} \frac{ds}{s}}^{\ln(s)\Big|_{s=2\ell}^{3\ell} = \ln\left(\frac{3}{2}\right)} = \frac{\mu_0 I V}{2\pi R}\ln\left(\frac{3}{2}\right)\hat{x}$$

Side 4

$$\vec{F}_4^B = \int_{2\ell}^{3\ell} ds\left(\frac{V}{R}\hat{y}\right) \times \left(-\frac{\mu_0 I}{2\pi s}\hat{z}\right) = -\frac{\mu_0 I V}{2\pi R}\ln\left(\frac{3}{2}\right)\hat{x}$$

B) $\vec{F}^{net} = \vec{F}_1^B + \vec{F}_2^B + \vec{F}_3^B + \vec{F}_4^B$

$$= \frac{\mu_0 I V}{\pi R}\left(\frac{1}{4}\hat{y} + \cancel{\frac{\ln(3/2)}{2}\hat{x}} - \cancel{\frac{\ln(3/2)}{2}\hat{x}} - \frac{1}{6}\hat{y}\right)$$

$$= \frac{\mu_0 I V}{12\pi R}\hat{y}$$

C) $\ln\left(\frac{3}{2}\right) \approx \frac{2}{5}$

$$\Rightarrow F_{2/4}^B \approx \frac{\mu_0 I V}{5\pi R}$$

$$\frac{1}{4}\frac{\mu_0 I V}{\pi R} > \frac{\ln(3/2)}{2}\frac{\mu_0 I V}{\pi R} > \frac{1}{6}\frac{\mu_0 I V}{\pi R}$$

$$F_1^B > F_2^B = F_4^B > F_3^B$$

$$\vec{\tau}^{net} \approx \vec{\mu} \times \vec{B} = (-IA\hat{z}) \times (-B\hat{z}) = 0$$

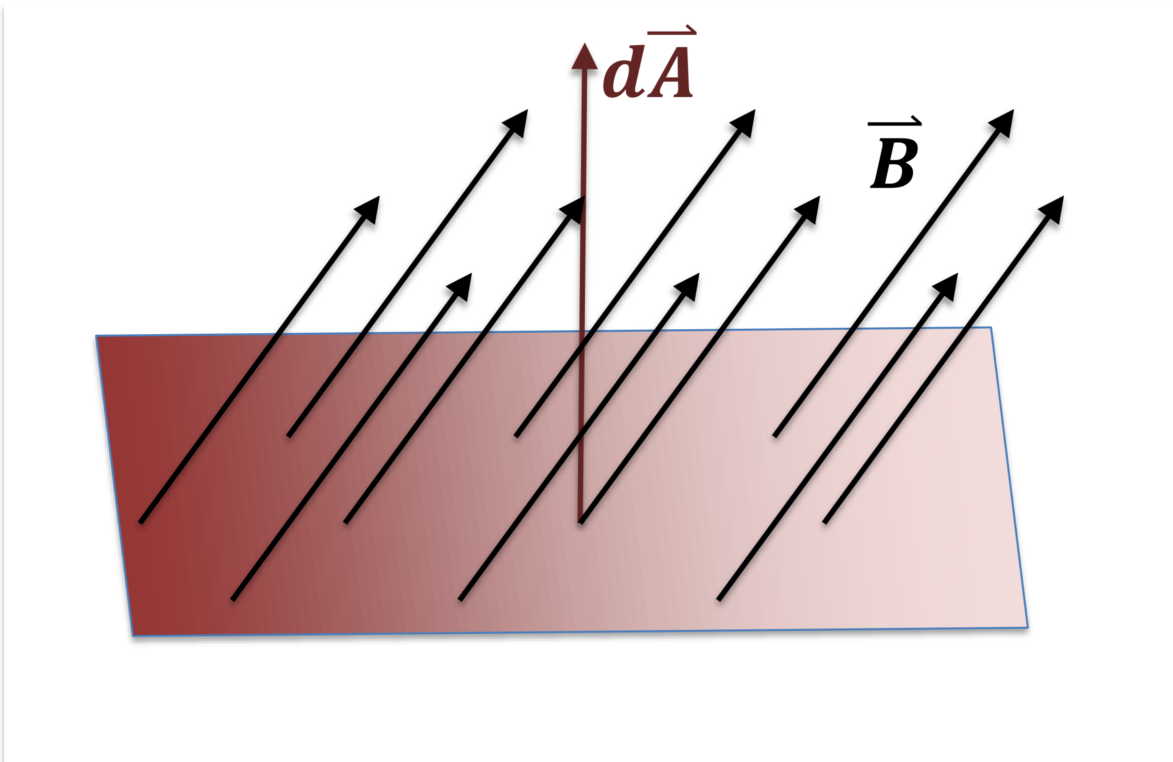
Here I approximate $\vec{B}$ as a uniform field, but the critical point that the magnetic moment is already aligned with $\vec{B}$ is still valid.

Electric Flux

- How do you find the total electric flux?
  - First you choose a small piece of the whole surface: $d\vec{A}$
  - Then you find the electric flux through the small piece: $d\Phi = \vec{E} \cdot d\vec{A}$
  - Then you add up all the fluxes:

$$\Phi = \oiint \vec{E} \cdot d\vec{A}$$

Magnetic Flux



$$\Phi_B = \iint \vec{B} \cdot d\vec{A}$$

Activity 9-3 – Flux Loops

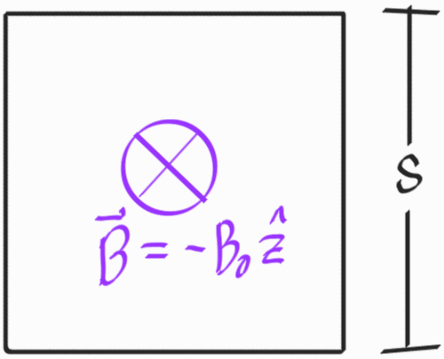
A square loop (side length s) in the plane of the screen is in a region of magnetic field. Find the magnetic flux through the loop if:

A. The magnetic field is a uniform B_o and points *into the screen*
B. The magnetic field is a uniform B_o and points *to the right*
C. The magnetic field points *into the screen* and increases linearly from 0 along the left edge to B_o along the right edge

9-3 Flux Loops

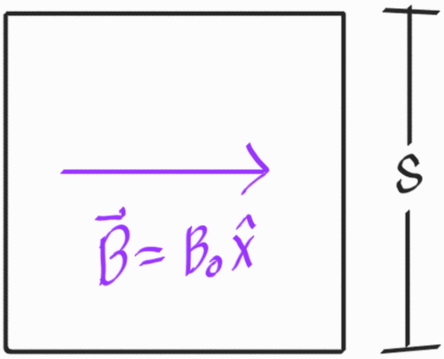
Case A

To make the fluxes non-negative, let us assume that $d\vec{A}$ points into the screen as well $(d\vec{A} = -dx\,dy\,\hat{z})$.



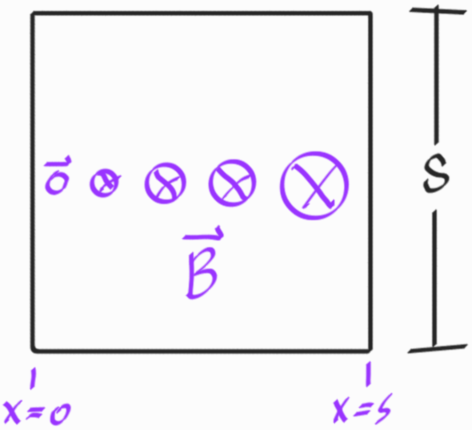
$$\Phi_{BA} = \int_0^s\int_0^s (-B_0\hat{z})\cdot(-dx\,dy\,\hat{z})$$
$$= B_0\int_0^s dx\int_0^s dy = B_0 s^2$$

Case B



$$\Phi_{BB} = \int_0^s\int_0^s (B_0\hat{x})\cdot(-dx\,dy\,\hat{z}) = 0$$
$$\hat{x}\cdot\hat{z} = 0$$

Case C



$$\vec{B} = -B_0\left(\frac{x}{s}\right)\hat{z}$$

$$\Phi_{BC} = \int_0^s\int_0^s \left(-B_0\frac{x}{s}\hat{z}\right)\cdot(-dx\,dy\,\hat{z})$$
$$= \frac{B_0}{s}\underbrace{\int_0^s x\,dx}_{\frac{1}{2}s^2}\underbrace{\int_0^s dy}_{s} = \frac{1}{2}B_0 s^2$$

Activity 9-4 – The Pickup Coil

Run the simulation: https://phet.colorado.edu/en/simulation/faraday. Go to the Pickup Coil tab.

A. Play with the simulation for a few minutes. Record a list of observations on your large whiteboard.
B. How many different kinds of changes can you make that will cause the light bulb to light up?
C. Consider the statement below:

   "While the magnetic flux through the loop changes, there is a voltage difference induced in the loop."
D. What evidence is there to support this statement? Can you find any evidence to refute the statement? Can you find any evidence that suggests the statement is incomplete?
E. Identify what properties the induced voltage in the loop depends on. Justify your answers with evidence from the simulation.

9-4 The Pickup Coil

B/E) There are five things that you can change that affect the bulb (and affect the induced voltage).

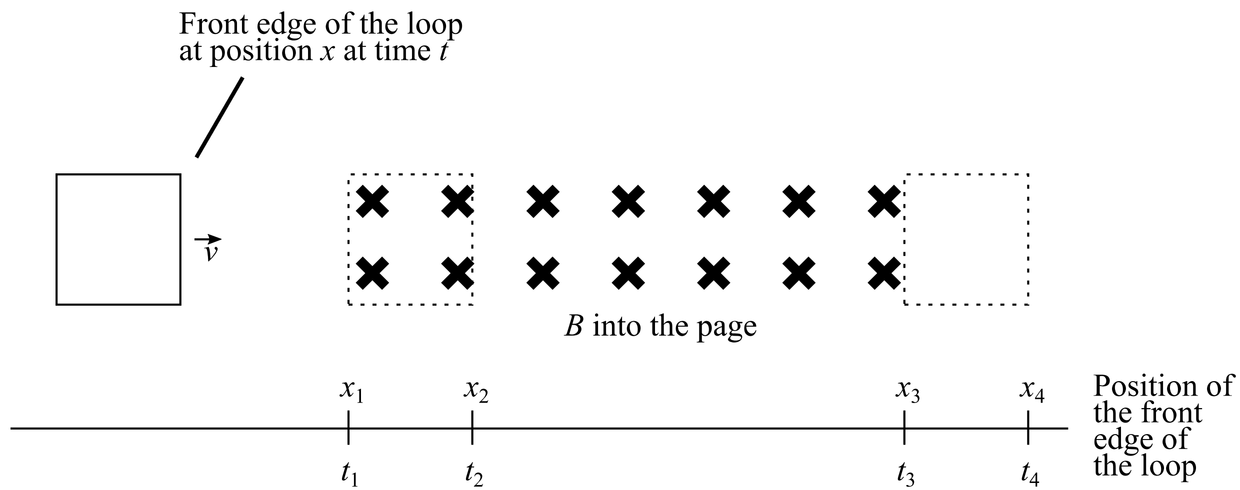
Magnet Position	Moving the magnet in any way that changes the field through the loop (especially toward or away from the loop) lights up the bulb. Moving the magnet faster makes the bulb brighter. It is also brighter if the bar is closer to the loop when it moves (and the other changes have a stronger effect when the magnet is closer too).
Magnet Polarity	Flipping the polarity (swapping south and north) also lights up the bulb. The effect does not depend on which pole is closer to start, and which pole is closer also does not affect the strength of the effects of the other changes.
Magnet Strength	Changing the strength of the magnet also lights up the bulb. Making the magnet stronger also makes the other changes light up the bulb more brightly.
Number of Loops	Changing the number of loops lights up the bulb. More loops cause the other changes to light up the bulb more brightly.
Loop Area	Changing the loop area lights up the bulb. Larger loops cause the other changes to light up the bulb more brightly.

C/D) This statement is absolutely true. All of the above changes affect the magnetic flux through the loop and light the bulb, which means there must be current and thus a voltage difference.

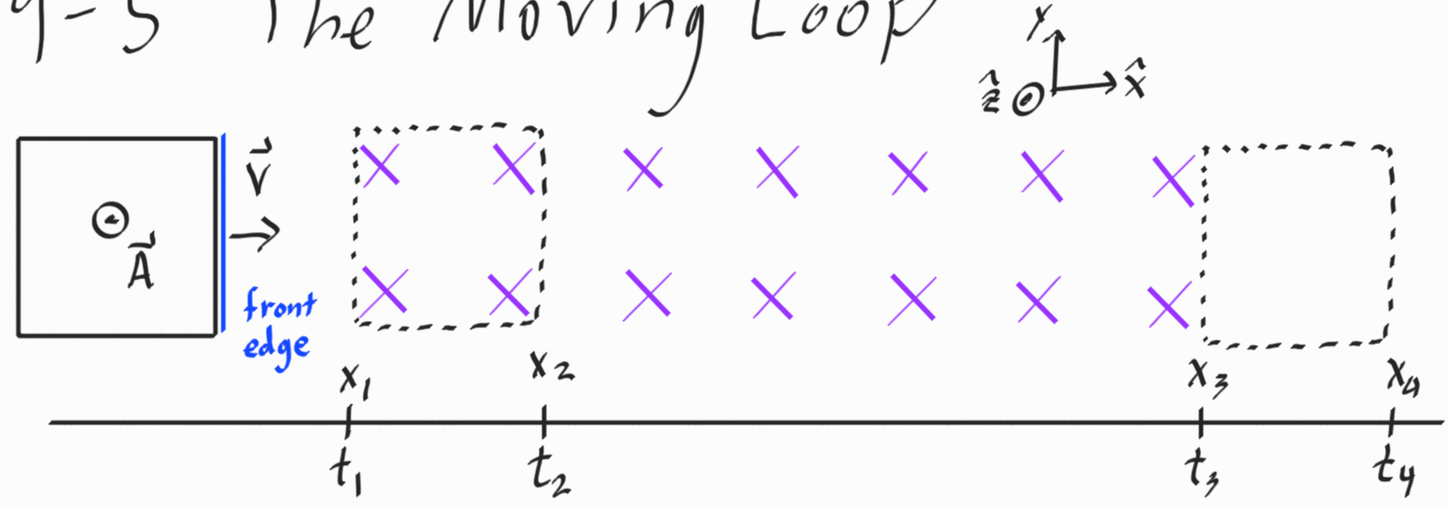
Activity 9-5 – The Moving Loop

Below is a rectangular wire loop (area A) that moves at a constant velocity into the region of external uniform magnetic field (into the page) shown below.

1. Choose an area vector for the wire loop.
2. Sketch and label a graph of the magnetic flux (Φ_B) through the loop as a function of time. Your graph should include all four labeled values of t in the figure below.
3. Consider a positive charge in the wire. Will this positive charge experience a magnetic force? If so, what direction will the magnetic force be? Does your answer depend on where the loop is located? Does your answer depend on which section of the wire the charge is located in?
4. Based on your answer about the force, predict whether or not there will be a current through the loop (and its direction) during each of the following intervals:
   A. Between t_1 and t_2
   B. Between t_2 and t_3
   C. Between t_3 and t_4



9-5 The Moving Loop



The front edge reaches the edge of the field at t_1.

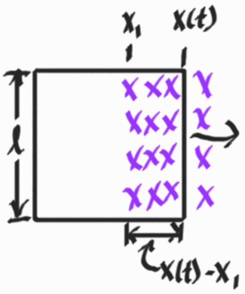
The front edge is here, with the entire loop in the field at t_2.

The front edge reaches the other side of the field at t_3.

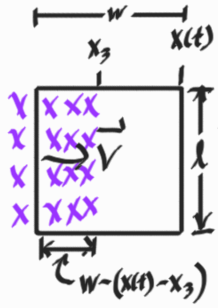
The front edge is here with the entire loop out of the field at t_4.

1) Let us choose the area vector to be out of the page (along $+\hat{z}$).

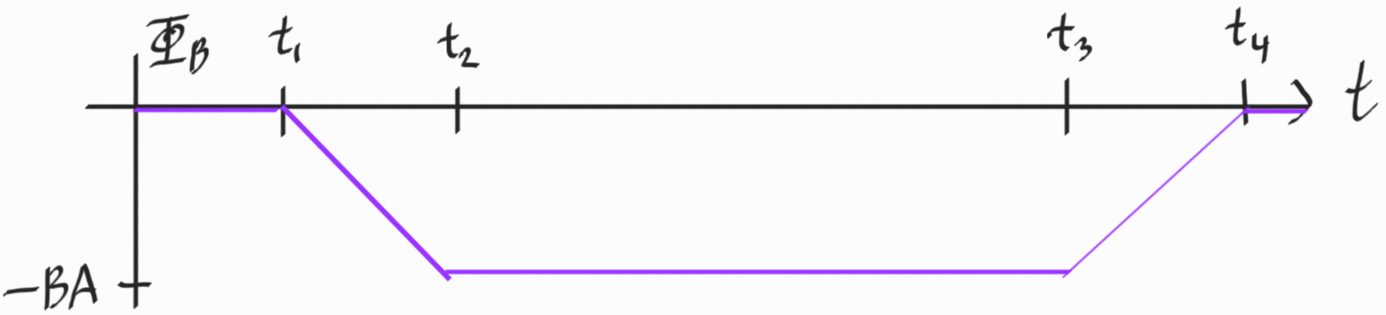
2) The flux through the loop is zero when it is outside of the field (before t_1 and after t_4). The flux is $\vec{B}\cdot\vec{A} = (-B\hat{z})\cdot(A\hat{z}) = -BA$ when the loop is entirely within the field (between t_2 and t_3). Let the loop have vertical length ℓ and horizontal length w (thus $A = \ell w$).



Entering, only an area of the loop equal to $\ell(x(t)-x_1)$ is in the field, so $\Phi_B = -B\ell(x(t)-x_1)$. Speed is constant, so $x(t)$ (and therefore Φ_B) is linear.



Leaving, only an area of the loop equal to $\ell[w-(x(t)-x_3)]$ is in the field, so $\Phi_B = -B\ell[w-(x(t)-x_3)]$. Speed is constant, so $x(t)$ (and therefore Φ_B) is linear.



3) A positive charge moving with the loop is moving to the right, so if the charge is in a part of the loop that is in the field, the force on it will be $\vec{F}^B = q\vec{v}\times\vec{B} \parallel \rightarrow \times \otimes = \uparrow$. The force on the charge points upward. This is almost irrelevant for charges in the top and bottom of the loop, since there is no wire to flow into that way (but look up the Hall effect). For charges in the left and right sides, however, this force could cause the charges to flow through the loop, creating a current.

Side Note: It is fair to wonder if the current would change the answer. In the top and bottom, charges in a current would still be moving horizontally, but no longer at speed v. The forces on them from the magnetic field would still be completely vertical. In the left and right sides, the horizontal component of a charge's velocity is still $\vec{v}$, so the vertical force is the same. The vertical component of velocity added by the current would cause additional horizontal magnetic force, which is nearly irrelevant, as it is perpendicular to the wire.

4) A) Between t_1 and t_2, the front edge of the loop is in the field, so the positive charges in it feel an upward force and a counterclockwise current will flow through the loop.

$$V_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(-B\ell\,(x(t)-x_1)\right) = B\ell v$$

Positive V_{ind} means current will flow counterclockwise relative to our chosen area vector:

B) Between t_2 and t_3, both the front (right) and back (left) edges are in the field, so positive charges in both edges feel an upward force. The upward force on the charges in the left side works against the other force, preventing current flow. $V_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}(-BA) = 0$

C) Between t_3 and t_4, only the back edge is in the field, so the upward force on the positive charges will create a clockwise current.

$$V_{ind} = -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(-B\ell\left[w-(x(t)-x_3)\right]\right) = -B\ell v$$

Negative V_{ind} means current will flow clockwise relative to our chosen area vector:

<u>Magnetic Induction</u>

A changing magnetic flux through a loop will induce an electric potential around that loop!

Faraday's Law

$$V_{\text{induced}} = -\frac{d\Phi_B}{dt}$$