

# Representations of Vectors

Benjamin Bauml

Summer 2024

## XX-1: Representations of Vectors

Discuss **vectors** with your group.

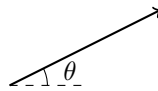
Write down a list of things about vectors on your shared whiteboard. You can write:

- words or sentences
- numbers or symbols
- pictures or diagrams

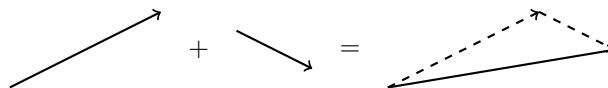
Write large!

The following list only contains my thoughts about vectors. It should not be assumed to be definitive. There are many other things about vectors that could have been included.

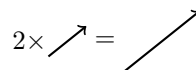
- A vector has direction and magnitude.
- A vector is often represented with an arrow; the length is its magnitude, and the orientation of the arrow (its angle) indicates direction.



- Vectors add “tip-to-tail.”



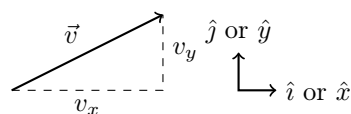
- Multiplying a vector by a scalar changes its length.



- Multiplying a vector by  $-1$  reverses its direction.



- Vectors can be broken into components.



Parenthetical or angle-bracket notation was probably used in your math classes:

$$\vec{v} = (v_x, v_y) = \langle v_x, v_y \rangle.$$

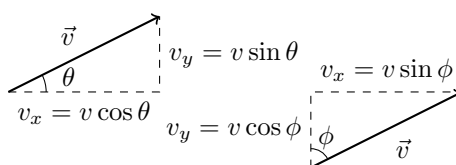
It is okay, but parentheses and angle brackets can mean a lot of things, so it may not always be clear.

We recommend unit vector notation, as it is much clearer:

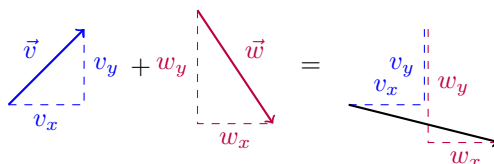
$$\vec{v} = v_x \hat{i} + v_y \hat{j} = v_x \hat{x} + v_y \hat{y}.$$

Using  $\hat{x}$ ,  $\hat{y}$ ,  $\hat{z}$  instead of  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$  helps to more clearly associate the Cartesian unit vectors with the coordinate axes.

**Warning!** Cosine is not always  $x$ ! It depends on where the angle is.



- Vectors add “componentwise.”



$$\vec{v} + \vec{w} = v_x \hat{x} + v_y \hat{y} + w_x \hat{x} + w_y \hat{y} = (v_x + w_x) \hat{x} + (v_y + w_y) \hat{y}$$