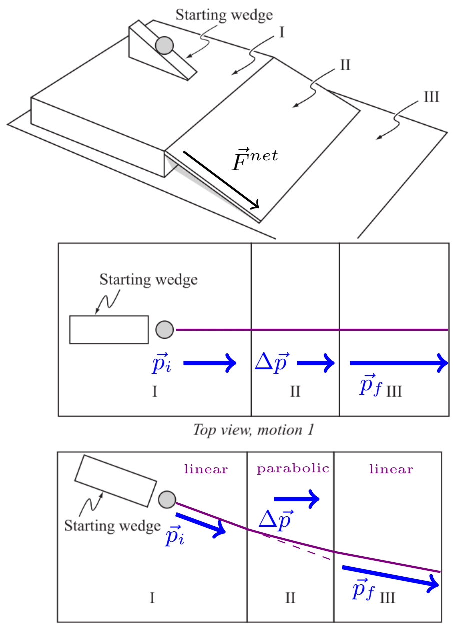
Lecture 23: Combining Physics Concepts II

Announcements

We have learned a lot of topics in this class (kinematics, forces, energy, momentum, and all of their associated equations), and one of the hardest parts of physics can be choosing which method to use. On Friday, we will be going through activities centered around deciding when to choose a particular method.

$\vec{F}^{net} = \vec{0}$ in I and III, where the surface is flat.
$\vec{F}^{net}$ is down the ramp in II, and it doesn't matter which way the ball is moving, so it is the same for both motions.

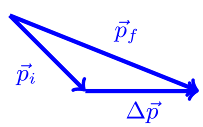


Top view, motion 1

Top view, motion 2

Trajectories are given in violet. Momenta are given in blue. As a side note, we can see just from this that $p_{i,y} = p_{f,y}$, as $\Delta\vec{p}$ is only in the x-direction.

Though the forces are the same, the ball in 2 spends more time on the ramp (it had less of its initial speed pointing straight down the ramp), so it gets more impulse. As such, its change in momentum is larger in magnitude than that of the ball in 1, but both are in the same direction, as the impulse is in the direction of the net force.
Conceptually, the change in momentum in 2 must be larger than the change in momentum in 1, as the momentum change in 2 not only increases the magnitude of the momentum (by the same amount as in 1, since the final momenta are the same; see below), but also must change its direction. This is the reverse triangle inequality:



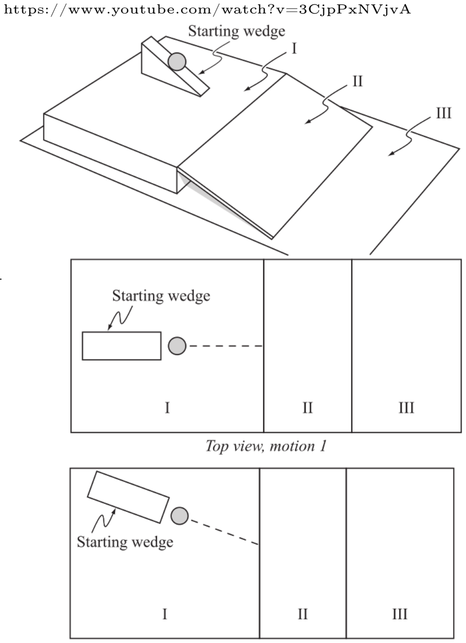
$$|\Delta\vec{p}| = |\vec{p}_f - \vec{p}_i| \geq |\vec{p}_f| - |\vec{p}_i|$$

Regarding energy, if we look at this from a conservation of energy perspective, then the ball descends the same height in both motions, so ΔU_g is the same, and thus $\Delta K_1 = \Delta K_2$. Thus, the balls' final speeds (and magnitudes of final momenta) must be the same.
From a work perspective, the work done on the ball in motion 1 is just $W_1 = F\Delta x$, the force down the ramp times the displacement down the ramp. In motion 2, the displacement along the parabolic path is longer, but not in line with the force, so the dot product in $W_2 = \int \vec{F} \cdot d\vec{x}$ will only keep the parts of the path in line with the force, giving us $W_1 = W_2$.

L23-1: Rolling a Ball

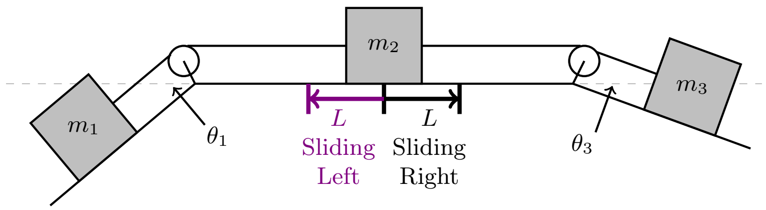
- Sketch the trajectory of the ball in each case.
- Which ball spends more time in region II?
- How does the direction of the net force on the ball in motion 2 compare to the direction of the net force on the ball in motion 1?
- How does the change in kinetic energy of the ball in motion 1 compare to the change in kinetic energy of the ball in motion 2?
  - Is your answer consistent with the net work done on the ball in motions 1 and 2?
  - How does the final speed of the ball in motion 1 compare to the final speed in motion 2?
- Draw vectors that represent the momentum of the ball at the top of the ramp and at the bottom of the ramp in each case.
  - How is the direction of the change in momentum related to the direction of the net force on the ball as it rolls down the ramp?
  - How do the magnitudes of the changes in momentum in the two cases compare to each other?

https://www.youtube.com/watch?v=3CjpPxNVjvA



Top view, motion 1

Top view, motion 2



If we wanted to know about acceleration, forces would make sense, but we just want to know about speed after the objects have moved some distance. We would need to do kinematics to get the final speed. Energy will be more direct, as it works with speed and position directly.

I will do the calculation assuming the boxes slide to the right, but I will mark all changes for sliding left in violet.

Conservation of Energy: $0 = \Delta E_{\text{total}} = \Delta K + \Delta U_{g1} + \Delta U_{g2}$
$\Delta K = K_f - K_i = K_f = \frac{1}{2}(m_1 + m_2 + m_3)v^2$
$\Delta U_{g1} = -m_1 g L \sin\theta_1$ Box 1 rises (lowers) by $h = -L\sin\theta_1$.
$\Delta U_{g3} = \pm m_3 g L \sin\theta_3$ Box 3 lowers (rises) by $h = \pm L \sin\theta_3$.
Our conservation of energy equation becomes

$$\frac{1}{2}(m_1 + m_2 + m_3)v^2 \mp m_1 g L \sin\theta_1 \pm m_3 g L \sin\theta_3 = 0,$$

and therefore

$$v = \sqrt{\frac{-2m_3 g L \sin\theta_3 \pm 2m_1 g L \sin\theta_1}{m_1 + m_2 + m_3}}$$
$$= \sqrt{\frac{2gL}{m_1 + m_2 + m_3}}\sqrt{-m_3\sin\theta_3 \pm m_1 \sin\theta_1}$$

We need $-m_3\sin\theta_3 \pm m_1\sin\theta_1 \geq 0$ (otherwise we are taking the square root of a negative number).

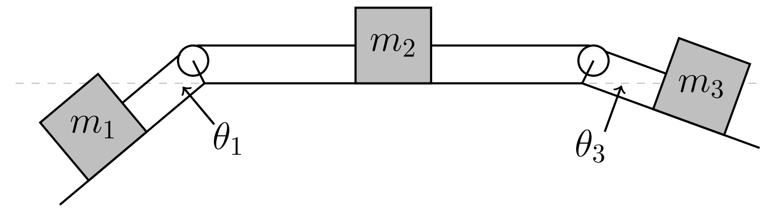
Slides Right: $m_3\sin\theta_3 > m_1\sin\theta_1$ Slides Left: $m_1\sin\theta_1 > m_3\sin\theta_3$

If $m_3\sin\theta_3 = m_1\sin\theta_1$, then the boxes stay still. No kinetic energy is gained by sliding, so potential energy is constant, and thus there is no force pushing the system.

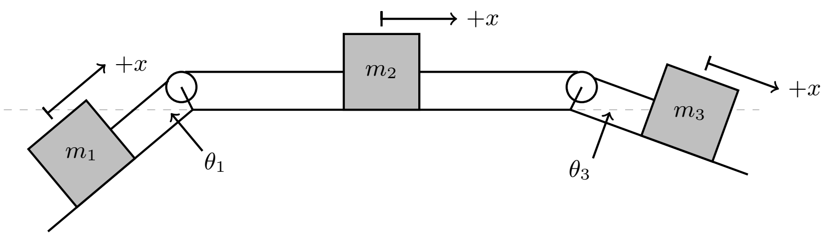
Special Cases

- $\theta_1 = 0°,\ \theta_3 = 90° \implies v = \sqrt{\frac{2gLm_3}{m_1+m_2+m_3}}$ sliding right
- $\theta_1 = 90°,\ \theta_3 = 0° \implies v = \sqrt{\frac{2gLm_1}{m_1+m_2+m_3}}$ sliding left
  - In both of these cases, we have a single block hanging while two are on a flat surface. There is no condition to keep them stationary, as there is no way to balance the force on the one hanging block. We have also done single hanging block problems that we can compare this to.
- $m_1 = m_3 \implies v = \sqrt{\frac{2gLm_1}{2m_1+m_2}}\sqrt{-\sin\theta_3 \pm \sin\theta_1}$
  - In this case, the condition for the boxes to remain stationary is $\theta_3 = \theta_1$, which makes sense, as for equal masses, the forces on m_2 will be balanced if the other two masses hang symmetrically.

L23-2: Angled Ramps



- The three boxes shown at right are connected by ideal strings and pulleys. Assume the surface is frictionless. The boxes are initially at rest.
  - Do you think this situation will be easier to analyze using **forces** or **energy**? Does your answer depend on what quantity you are looking for?
- What is the speed of box 2 after it has moved a distance L?
  - Evaluate your answer in at least 3 special cases.
  - Is there any relationship between the three masses that will cause the blocks to remain at rest?



Forces The forces in the y-direction of each coordinate system are balanced, so I will leave those off. Let the tension in the rope attached to m_1 be F_1^T, and the tension in the rope attached to m_3 be F_3^T

$$m_1 a\hat{x} = \vec{F}_1^{net} = \left(F_1^T - m_1 g\sin\theta_1\right)\hat{x}$$

$$m_2 a\hat{x} = \vec{F}_2^{net} = \left(F_3^T - F_1^T\right)\hat{x}$$

$$m_3 a\hat{x} = \vec{F}_3^{net} = \left(m_3 g\sin\theta_3 - F_3^T\right)\hat{x}$$

$$(m_1 + m_2 + m_3)a\hat{x} = (-m_1 g\sin\theta_1 + m_3 g\sin\theta_3)\hat{x}$$

$$a = \frac{g}{m_1 + m_2 + m_3}(-m_1\sin\theta_1 + m_3\sin\theta_3)$$

Kinematics

$$v_f^2 = v_i^2 + 2a\Delta x \qquad \Delta x = -L$$

$$= -2aL$$

$$= -\frac{2gL}{m_1 + m_2 + m_3}(-m_1\sin\theta_1 + m_3\sin\theta_3)$$

$$v = \sqrt{\frac{2gL}{m_1 + m_2 + m_3}}\sqrt{-m_3\sin\theta_3 \pm m_1\sin\theta_1}$$

Same answer!

Main Ideas

- The work-energy and impulse-momentum theorems can be used to solve a broad array of problems.