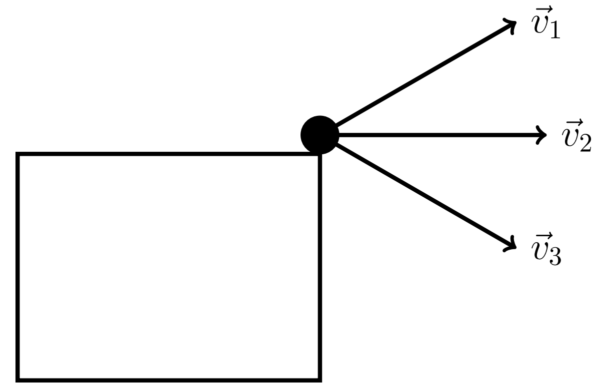
Studio 3: Motion and Forces

Warm-Up Activity

Which marble is in the air for the most time?



(A) Marble 1

(B) Marble 2

(C) Marble 3

(D) They are all in the air for the same amount of time.

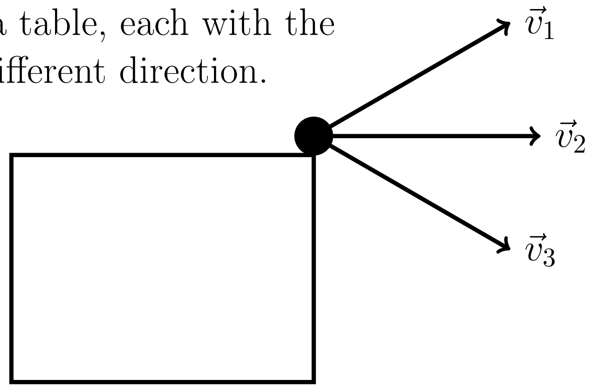
Projectile Motion

- Acceleration in the x-direction is equal to zero.
- Acceleration in the y-direction is only due to gravity.

$$\begin{aligned} a_x(t) &= 0 & a_y(t) &= -g \\ v_x(t) &= v_{ix} & v_y(t) &= v_{iy} - gt \\ x(t) &= x_i + v_{ix}t & y(t) &= y_i + v_{iy}t - \frac{1}{2}gt^2 \end{aligned}$$

Three Marbles

- You throw three marbles off a table, each with the same initial speed, but in a different direction.
  - Which is in the air for the most time?
  - Which travels the most horizontal distance?
  - Which is moving fastest when it hits?



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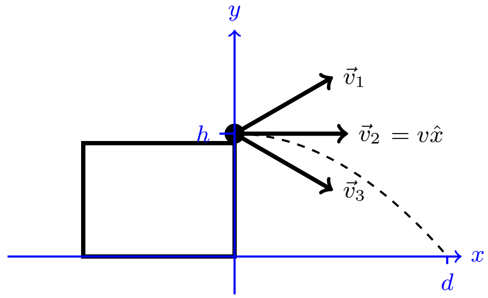


To find the final time with the given information, it makes sense to use the position kinematics equations:

$$x(t) = vt \qquad y(t) = h - \frac{1}{2}gt^2$$

The time of flight can be determined based on how far the marble has to fall.

$$0 = y(t_f) = h - \frac{1}{2}gt_f^2$$
$$\frac{1}{2}gt_f^2 = h$$
$$t_f = \sqrt{2\frac{h}{g}}$$

The horizontal distance traveled can then be determined using this time.

$$d = x(t_f) = vt_f$$
$$d = v\sqrt{2\frac{h}{g}}$$

We know $v_x(t) = v$, so $v_{fx} = v$. We also know

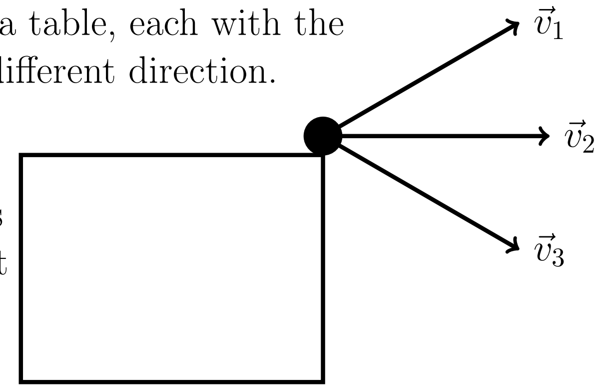
$$v_{fy} = -gt_f = -g\sqrt{2\frac{h}{g}} = -\sqrt{2gh}.$$

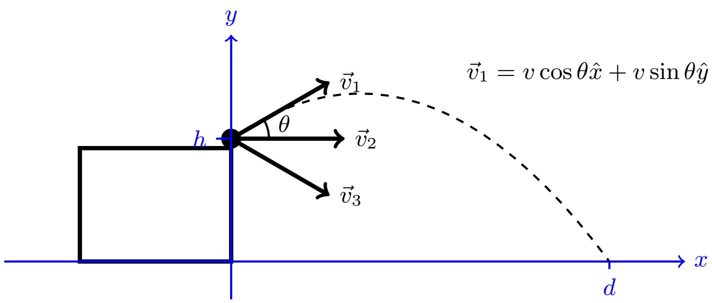
Therefore, the final speed is

$$v_f = \sqrt{v_{fx}^2 + v_{fy}^2}$$
$$= \sqrt{v^2 + 2gh}.$$

S3-1: Marble 2

- You throw three marbles off a table, each with the same initial speed, but in a different direction.
  - If the height of the table is h, how much time does it take for marble 2 to hit the ground?
  - How far has the marble traveled horizontally in that amount of time?
  - What is the marble's speed when it hits the ground?





We use the same starting place as before, albeit with more complicated terms:

$$x(t) = v\cos\theta t, \qquad y(t) = h + v\sin\theta t - \frac{1}{2}gt^2.$$

The time of flight can be determined based on how far the marble has to fall. This time, we need the quadratic formula:

$$\begin{aligned}
0 = y(t_f) &= h + v\sin\theta t_f - \frac{1}{2}gt_f^2 \\
t_f &= \frac{-v\sin\theta \pm \sqrt{v^2\sin^2\theta + 4\cdot\frac{1}{2}g\cdot h}}{-g} \\
t_f &= \frac{v}{g}\sin\theta \mp \sqrt{\frac{v^2}{g^2}\sin^2\theta + 2\frac{h}{g}}
\end{aligned}$$

Which sign do we use? Note that $\sqrt{\frac{v^2}{g^2}\sin^2\theta + 2\frac{h}{g}} \geq |\frac{v}{g}\sin\theta|$, and t_f occurs in the future, so we need to pick the sign that keeps the expression positive (a negative time does have a physical interpretation; this is the time the marble would have started at to undergo this motion, had it been launched from the ground). As such, we want to use $+$, as it is the only way to make t_f positive:

$$t_f = \frac{v}{g}\sin\theta + \sqrt{\frac{v^2}{g^2}\sin^2\theta + 2\frac{h}{g}}.$$

The distance traveled is just $d = v\cos\theta t_f$, so the above expression can be plugged in. The final speed is much more interesting:

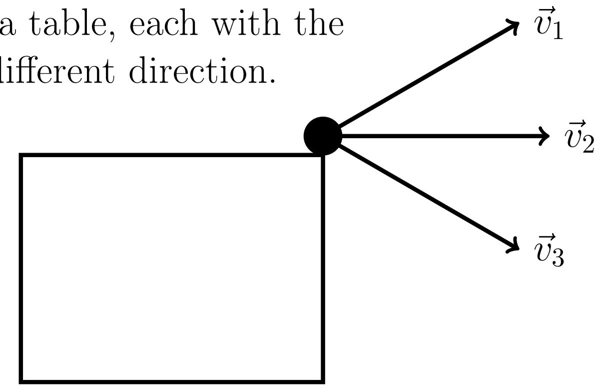
$$v_{fx} = v_{ix} = v\cos\theta \qquad \begin{aligned}
v_{fy} &= v\sin\theta - gt_f \\
&= v\sin\theta - g\left[\frac{v}{g}\sin\theta + \sqrt{\frac{v^2}{g^2}\sin^2\theta + 2\frac{h}{g}}\right] \\
&= -\sqrt{v^2\sin^2\theta + 2gh}
\end{aligned}$$

$$\begin{aligned}
v_f = \sqrt{v_{fx}^2 + v_{fy}^2} &= \sqrt{v^2\cos^2\theta + v^2\sin^2\theta + 2gh} \\
&= \sqrt{v^2\left(\cos^2\theta + \sin^2\theta\right) + 2gh} = \sqrt{v^2 + 2gh}
\end{aligned}$$

Final speed is independent of the angle!
We also got marble 3 for free! Just plug in a negative quantity for θ.

S3-2: Marble 1

- You throw three marbles off a table, each with the same initial speed, but in a different direction.



  - If the angle is θ, how long does it take for marble 1 to hit the ground?
  - How far has the marble traveled horizontally in that amount of time?
  - What is the marble's speed when it hits the ground?

$$
\begin{aligned}
d &= v\cos\theta\left(\frac{v}{g}\sin\theta+\sqrt{\frac{v^2}{g^2}\sin^2\theta+\frac{2h}{g}}\right)\\
&= \frac{v^2}{g}\cos\theta\sin\theta+\sqrt{\frac{v^4}{g^2}\cos^2\theta\sin^2\theta+\frac{2hv^2}{g}\cos^2\theta}\\
&= \frac{v^2}{2g}\sin(2\theta)+\sqrt{\frac{v^4}{4g^2}\sin^2(2\theta)+\frac{2hv^2}{g}\cos^2\theta}
\end{aligned}
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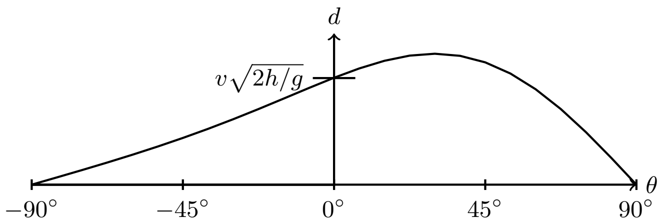
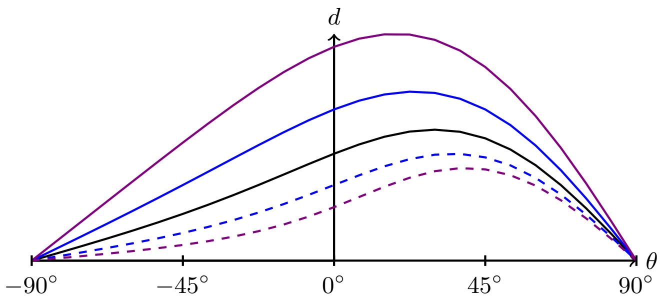


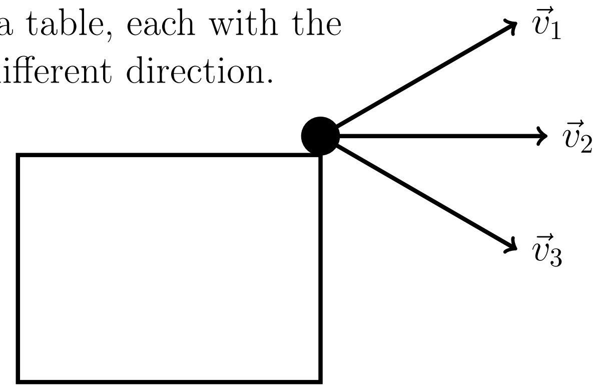


Each curve shows the distance d as a function of the launch angle θ. These plots assumed that $v = 1$ m/s and $g = 1$ m/s^2. The black curve is for $h = 1$ m, while the solid blue and purple curves are for $h = 2$ m and $h = 4$ m, respectively, and the dashed blue and purple curves are for $h = 0.5$ m and $h = 0.25$ m, respectively. The distance always increases with increasing h, but the angle $\theta_{\max}$ at which the maximum distance occurs decreases with increasing h.

It appears that, as h decreases, $\theta_{\max}$ approaches $45°$, which is the angle for maximum range when the projectile starts and ends at the same height.

Three Marbles

- You throw three marbles off a table, each with the same initial speed, but in a different direction.
  - If h is the height of the table, v is the initial speed, and θ is the angle up from the horizontal:



$$t_f = \frac{v}{g}\sin\theta+\sqrt{\frac{v^2}{g^2}\sin^2\theta+\frac{2h}{g}} \qquad d = vt_f\cos\theta \qquad v_f = \sqrt{v^2+2gh}$$

- What does it mean to say that two objects are interacting?
  - We will be characterizing interactions in terms of forces between objects, which is to say we are looking at how objects push and pull each other.
- What happens to objects when they interact?
  - Interacting objects affect each other's motion. Objects that do not interact say at a constant velocity, while interacting objects may accelerate.
- What if multiple objects interact with the same object?
  - We associate each force with a pair of interacting objects, so we account for (at least) one force for each object interacting with the object of interest. All of these forces are added together (as vectors) to find the net force on the object of interest.

What are Interactions/Forces?

- What does it mean to say that two objects are interacting?
- What happens to objects when they interact?
- What if multiple objects interact with the same object?
- **What different kinds of interactions are there?**

This Week: Gravity and Normal Force
Next Week: Friction (kinetic and static), Tension, and Springs
PH 213: Electric and Magnetic Force

Types of Forces

- Gravity: $\vec{F}^g$
- Normal: $\vec{F}^N$
- Tension: $\vec{F}^T$
- Spring: $\vec{F}^{sp}$
- Friction: $\vec{F}^f$ ($\vec{F}^{kf}$, $\vec{F}^{sf}$)
- Electric: $\vec{F}^E$
- Magnetic: $\vec{F}^M$

Normal Forces

- Normal forces are contact forces that act perpendicular to the surface of contact.
- There is no formula for determining normal forces—the magnitude can change depending on the circumstances.
- Too much normal force can cause objects to break!

Tension Forces

- Tension forces are kind of like normal forces, except they pull in the direction of the rope.
- There is no formula for determining tension forces—the magnitude can change depending on the circumstances.
- Too much tension force can cause a rope to break!
- Tension is uniform throughout a single rope.
  - . . . if the rope is massless, inextensible, and the middle of the rope isn't in contact with anything.

Free-Body Diagrams and Systems

- Choose a system.
  - Make sure you know what is internal to your system and what is external to your system.
- Identify and describe each external force:

$$\vec{F}^{\,\text{type}}_{\text{on,by}}$$

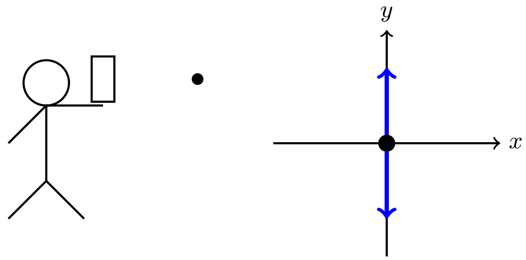
  - Say what kind of force it is.
  - Determine the object the force is being acted on.
  - Determine the object that is exerting the force.
  - Write a symbolic version of the force that includes the information above.
  - Represent all the forces acting on a single object or system using a **free-body diagram**.

In all three cases, there are two forces acting on the bag of groceries:

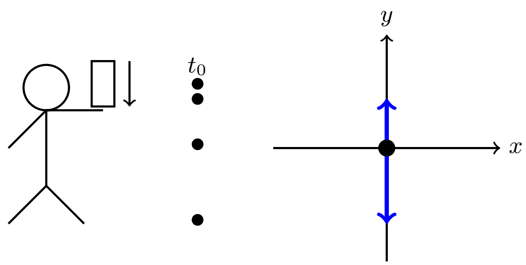
- $\vec{F}^N_{BH} = F^N \hat{y}$: normal force of the hand pushing up on the bag.
- $\vec{F}^g_{BE} = F^g(-\hat{y})$: gravitational force of Earth pulling down on the bag.

In cases (A) and (C), the forces are equal in magnitude, as the motion of the bag is not changing. In (B), the bag is accelerating downward, so the normal force must be smaller than the gravitational force.

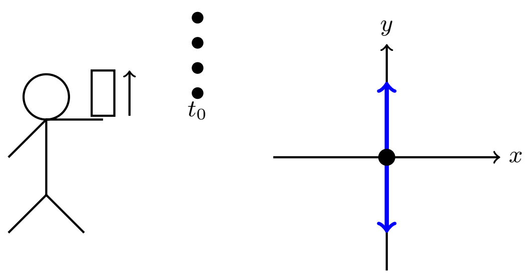
(A)
There isn't really a motion diagram in this case, as the bag is not moving.



(B)



(C)



S3-3: The Bag of Groceries

For each situation to the right:

(1) Sketch a picture of the object of interest.

(2) Make a strobe or motion diagram.

(3) Identify and describe the forces acting on the object.

(4) Draw a free-body diagram for the object.

(A) You hold a bag of groceries in your hand.

(B) You lower the bag of groceries; the bag moves downward faster and faster.

(C) You lift the bag of groceries; the bag moves upward at constant speed.

Main Ideas

- Motion in 2 dimensions can be broken down into independent motion in each dimension.
- Solving a problem symbolically allows you to solve many problems with one set of algebra.
- Forces arise from interactions between objects.
- There are many different kinds of forces that we can analyze differently.
- Objects can only change their motion when acted upon by an external force.