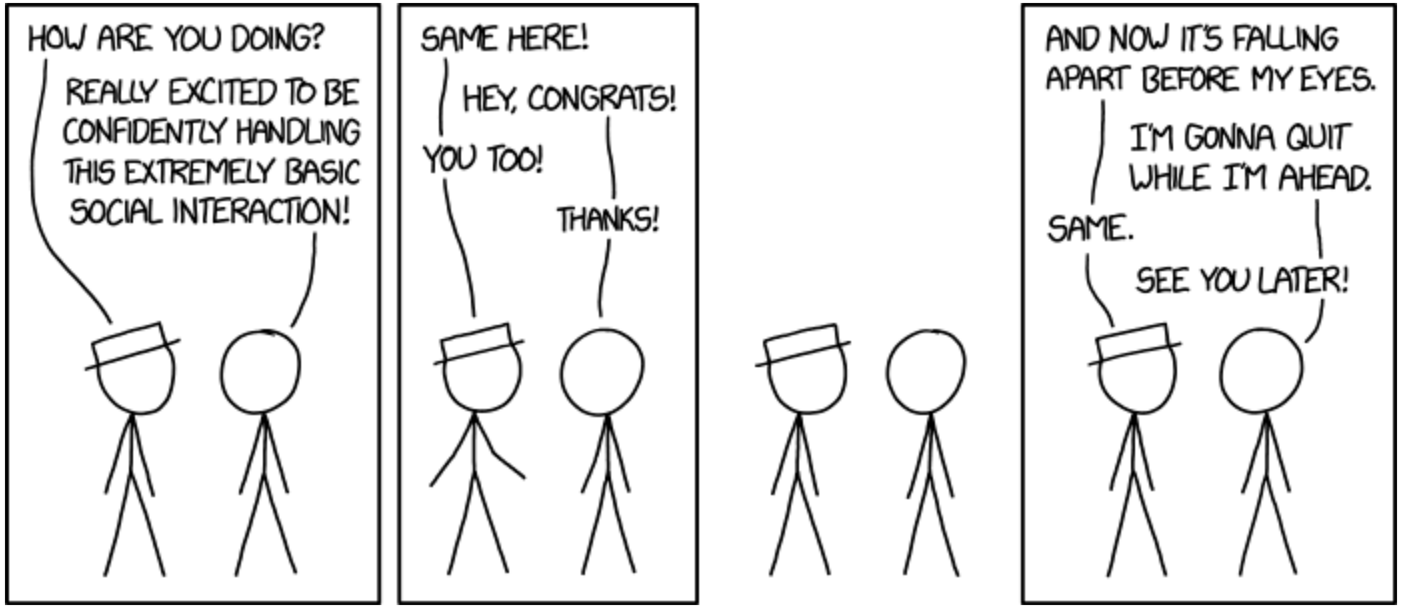
Studio Week 6
Interacting Systems



Picture credit: xkcd.com

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Make sense of everything.**

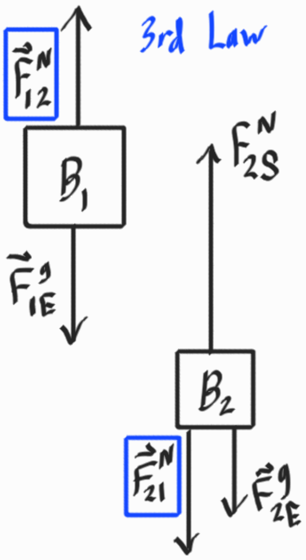
Activity 6-1: Book Stack

- The stack of books below is sitting in an elevator. Consider the following situations:
  A. The elevator is at rest
  B. The elevator is moving downwards at a constant velocity
  C. The elevator is accelerating upwards
- For each situation:
  - Draw a free-body diagram for each book
  - Identify all third-law pairs
  - Determine if any forces are equal in magnitude

6-1 Book Stack

Let Book 1 (B_1) be the larger book on top, and let Book 2 (B_2) be the smaller book on the bottom. S will symbolize the Surface of the elevator floor.

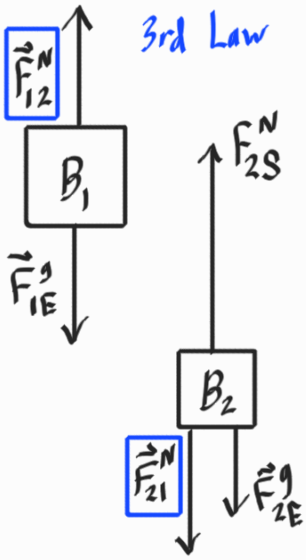
Elevator at Rest



$F^N_{21} = F^N_{12}$ by the 3rd law.

$F^g_{1E} = F^N_{12}$ by Newton's 2nd law as $\vec{a} = \vec{0}$.

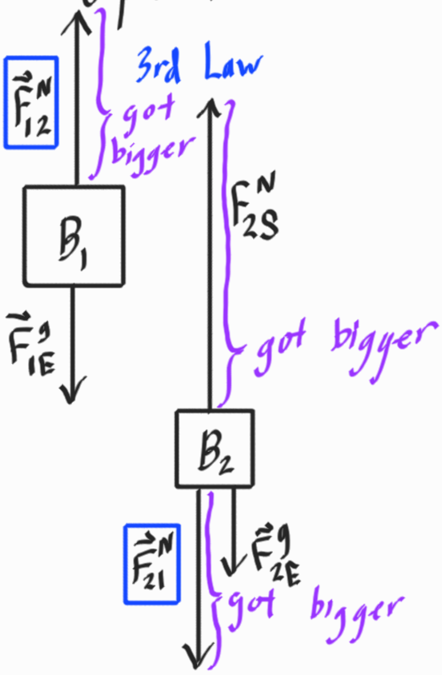
Moving Downward at Constant Velocity



$F^N_{21} = F^N_{12}$ by the 3rd law.

$F^g_{1E} = F^N_{12}$ by Newton's 2nd law as $\vec{a} = \vec{0}$.

Accelerating Upward

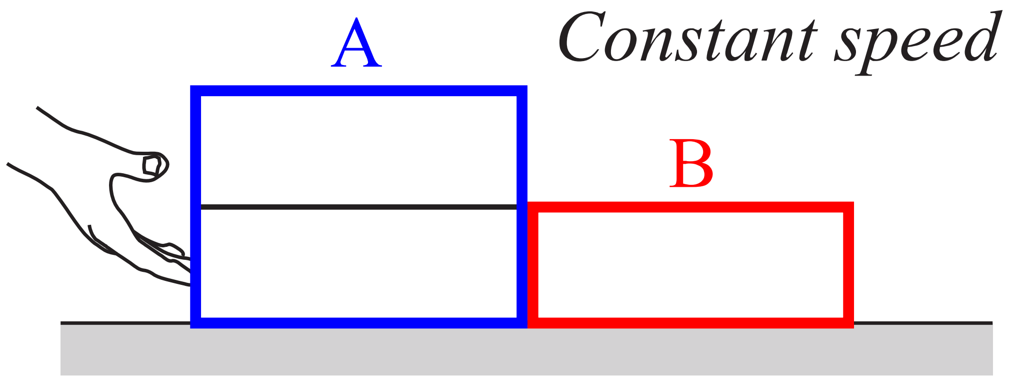


$F^N_{21} = F^N_{12}$ by the 3rd law.

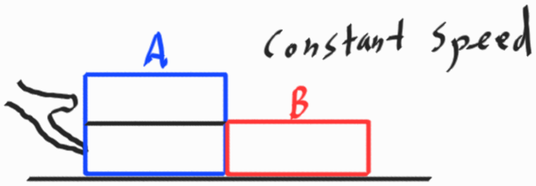
$F^N_{12} > F^g_{1E}$ and $F^N_{2S} > F^N_{21} + F^g_{2E}$ to get $\vec{a} \neq \vec{0}$.

Activity 6-2: Moving Bricks

- Three identical bricks are pushed across a table at *constant speed* as shown. The hand pushes horizontally. There is friction.
- System A is the left (stacked) bricks and system B is the right brick.
  - Compare the *net force* on system A to that on system B.
  - Draw separate free-body diagrams for system A and system B.
  - Identify all the Newton's third law (action-reaction) force pairs.
  - Rank the *horizontal* forces by magnitude, from largest to smallest.
  - Explain how you used Newton's Laws.

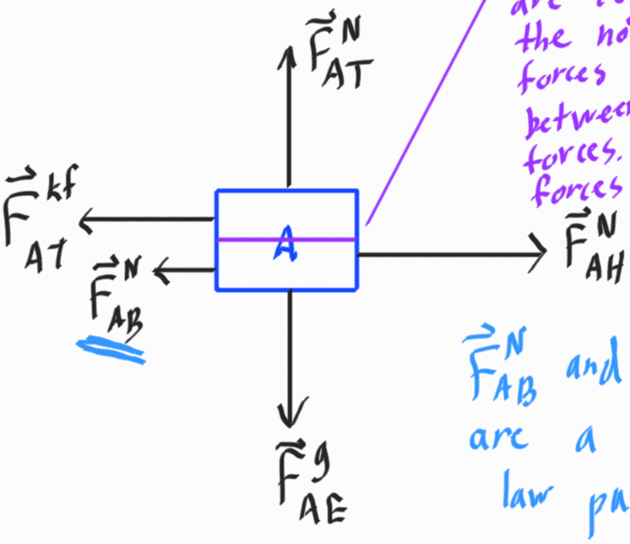


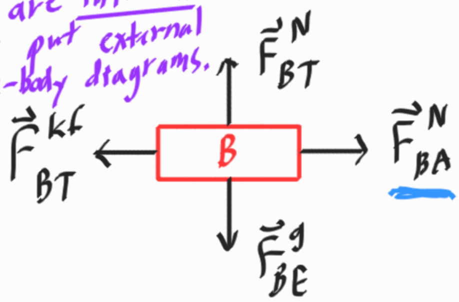
6-2 Three Bricks, Two Systems



At constant speed, $a_A = a_B = 0$, so $F_A^{net} = F_B^{net} = 0$.

Since these two bricks are together in the system, the normal forces and forces of static friction between them are internal forces. We only put external forces on free-body diagrams.





$\vec{F}_{AB}^{N}$ and $\vec{F}_{BA}^{N}$ are a 3rd law pair.

$$F_{AH}^{N} > F_{AT}^{kf} > F_{BT}^{kf} = F_{BA}^{N} = F_{AB}^{N}$$

2nd
$F_{Ax}^{net} = F_{AH}^{N} - F_{AT}^{kf} - F_{AB}^{N} = m_A a_{Ax} = 0$

2nd & kinetic friction

2nd
$F_{Bx}^{net} = F_{BA}^{N} - F_{BT}^{kf} = m_B a_{Bx} = 0$

3rd

$$F_{AT}^{kf} = \mu_k F_{AT}^{N} = \mu_k m_A g = 2\mu_k m_B g = 2\mu_k F_{BT}^{N} = 2F_{BT}^{kf}$$

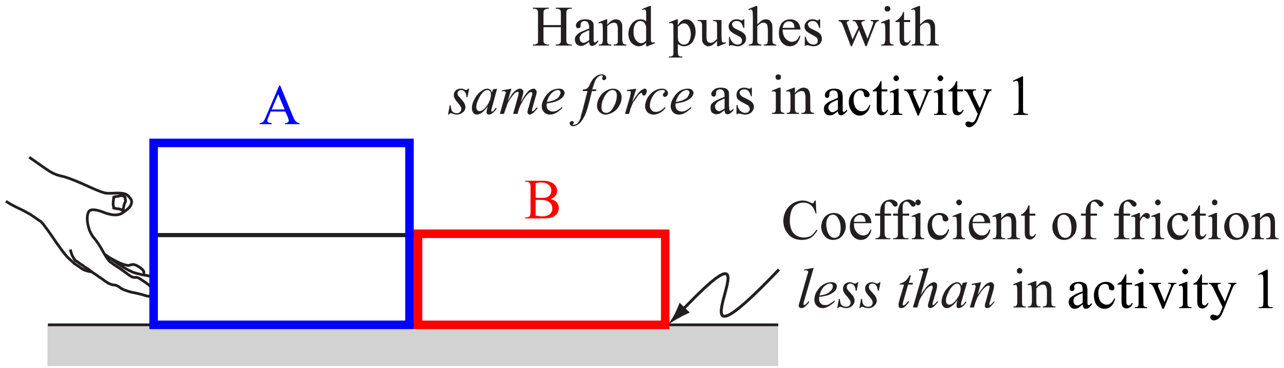
2nd law y direction System A

A has 2 bricks.

2nd law y direction System B

Activity 6-3: Moving Bricks II

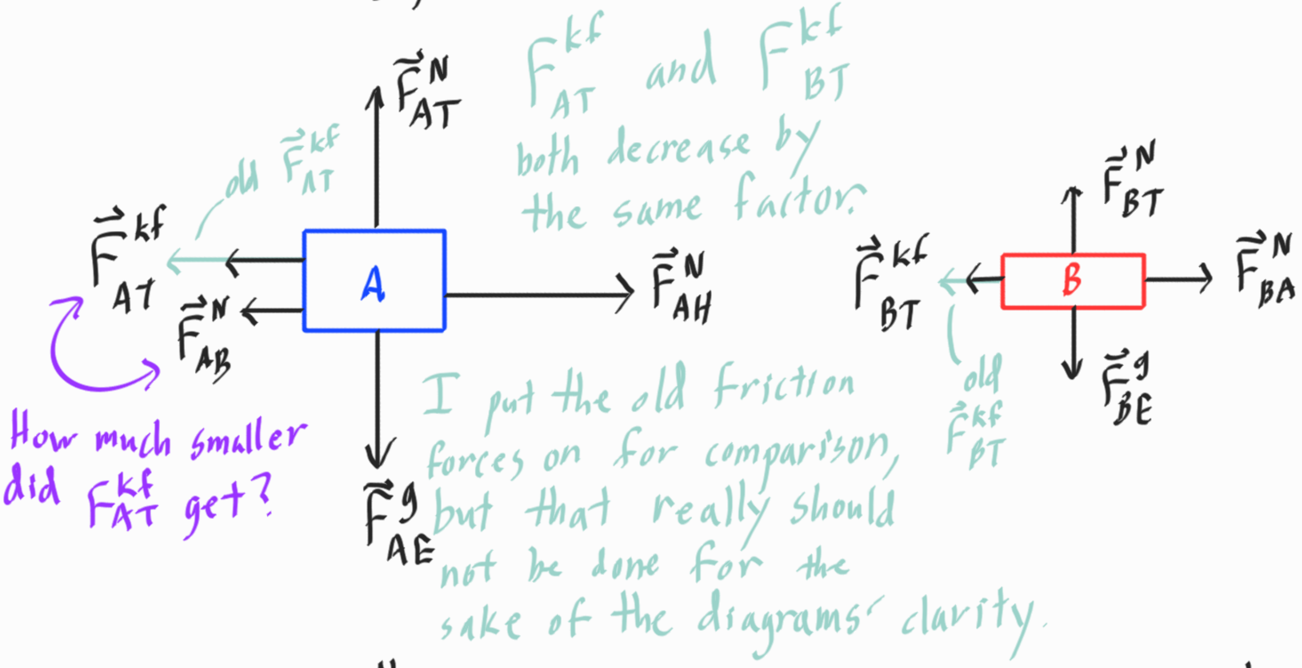
- The hand still pushes horizontally, but the coefficient of friction is less than it was in Activity 6-2.
  - How does the motion of the blocks change, if at all?
  - Compare the *net force* on system A to that on system B.
  - Draw separate free-body diagrams for system A and system B.
  - Rank the *horizontal* forces by magnitude, from largest to smallest. (Is it possible to rank the horizontal forces *completely*?)
  - Explain how you used Newton's Laws.



6-3 Three Bricks, Two Systems, One Change

With μ_k lower, the forces of friction decrease, therefore the boxes will now be accelerating together.

$$\vec{a}_A = \vec{a}_B, \; m_A > m_B \Rightarrow F_A^{net} > F_B^{net}$$



F^N_{AB} and F^N_{BA} do not change when the friction changes. The argument is a bit involved, and requires both the 2nd and 3rd laws:

$$m_B a = F^{net}_{Bx} = F^N_{BA} - F^{kf}_{BT} = F^N_{AB} - \mu_k m_B g$$

3rd

$$m_A a = F^{net}_{Ax} = F^N_{AH} - F^{kf}_{AT} - F^N_{AB} = F^N_{AH} - \mu_k m_A g - F^N_{AB} = F^N_{AH} - 2\mu_k m_B g - F^N_{AB}$$

$$= 2m_B a$$

$$\Rightarrow F^N_{AH} - 2\cancel{\mu_k m_B g} - F^N_{AB} = 2(F^N_{AB} - \cancel{\mu_k m_B g})$$

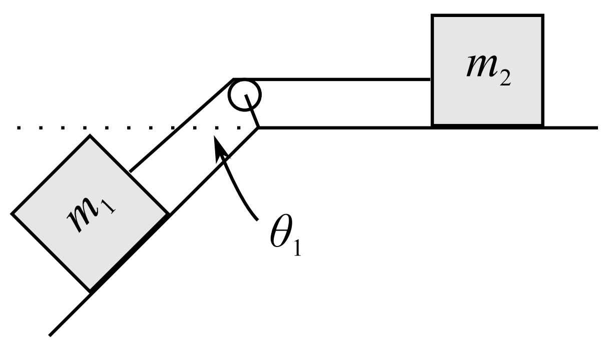
$$\Rightarrow F^N_{AH} = 3F^N_{AB} \quad \text{(no dependence on } \mu_k\text{)}$$

However, we need to know exactly how much μ_k changes in order to know if F^{kf}_{AT} got smaller than F^{kf}_{BT}, so we cannot completely rank the forces anymore. Other than that, the reasoning is the same:

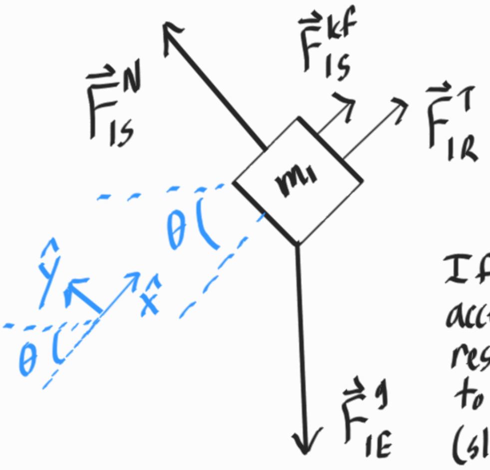
$$F^N_{AH} \overset{2nd}{>} F^N_{AB} \overset{3rd}{=} F^N_{BA} \overset{2nd}{>} F^{kf}_{BT} \qquad F^N_{AH} \overset{2nd}{>} F^{kf}_{AT} \underset{\&\,kf}{\overset{2nd}{>}} F^{kf}_{BT} \qquad F^{kf}_{AT} \text{ vs. } F^N_{AB} \; ?$$

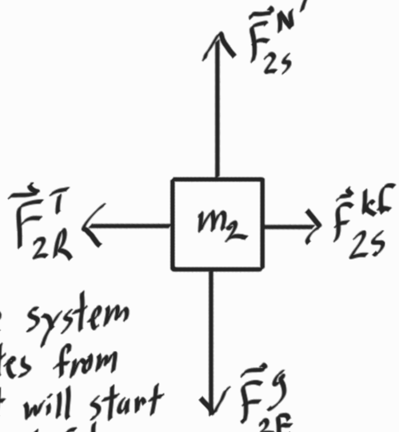
Activity 6-~~3~~4: Angled Ramp

- Draw separate free-body diagrams for both objects in the situation below.
  - Assume there is friction between all blocks and surfaces.
  - Assume each system accelerates from rest.
- Identify all the Newton's third law (action-reaction) force pairs.



6-4 Two Blocks and a Pulley on a Half-Ramp





If the system accelerates from rest, it will start to slide left (sliding right makes no physical sense in this situation), so kinetic friction will point to the right, along the surface.

You do not need to assume that $m_1 > m_2$ for this to work. You can imagine that, in the frictionless case, this system would certainly start to slide, and low friction cases should approach the outcome of the frictionless version.

There are no 3rd law pairs!

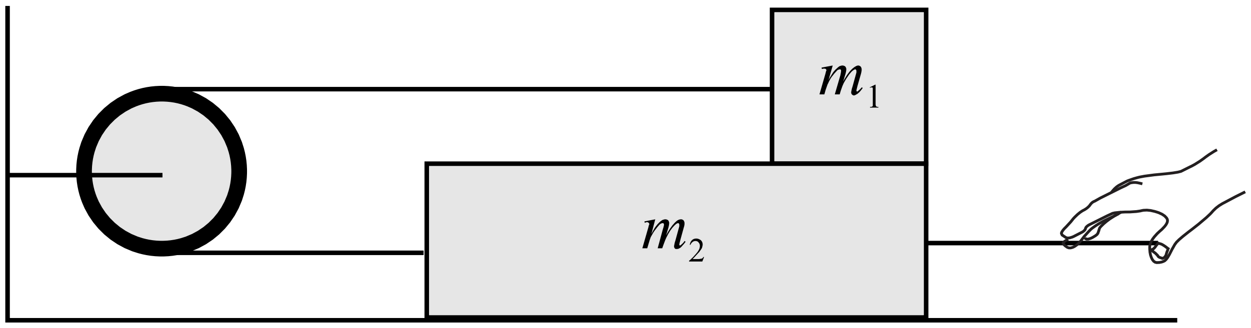
The two tensions are equal ($F^T_{1R} = F^T_{2R}$), but not directly because of the third law. Third law pairs must be between two directly interacting objects (in the notation, the subscripts should reverse: $F_{AB} = F_{BA}$). Third law pairs must also be equal and opposite, but $\vec{F}^T_{1R}$ and $\vec{F}^T_{2R}$ do not point in opposite directions: $\vec{F}^T_{1R} \nearrow \quad \leftarrow \vec{F}^T_{2R}$

- F^T_{1R} is in a 3rd law pair with F^T_{R1} (the force of tension from m_1 on rope 1).
- F^T_{2R} is in a 3rd law pair with F^T_{R2} (the force of tension from m_2 on rope 2).
- $F^T_{R2} = F^T_{R1}$ due to two assumptions: (i) an ideal (massless & inextensible) rope, and (ii) a massless and frictionless pulley.
  - (i) keeps the tensions constant throughout lengths of free rope
  - (ii) keeps the tensions equal across the pulley (see PH 212 for more)

In summary: $F^T_{1R} \overset{3rd}{=} F^T_{R1} \underset{(ii)}{\overset{(i)}{=}} F^T_{R2} \overset{3rd}{=} F^T_{2R}$

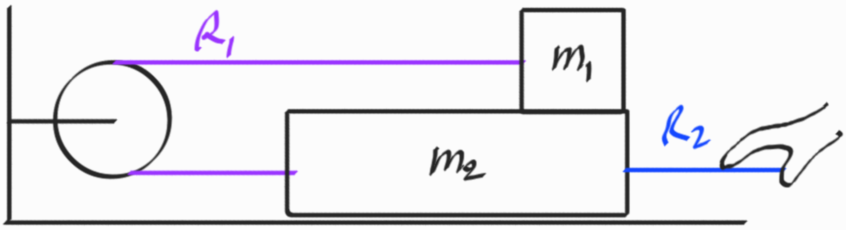
Activity 6-5: Horizontal Pulley

- Draw separate free-body diagrams for both objects in the situation below.
  - Assume there is friction between all blocks and surfaces.
  - Assume the strings and the pulley are ideal.
  - Assume each system accelerates from rest.
- Identify all the Newton's third law (action-reaction) force pairs.



6-5 Two Stacked Blocks and a Pulley

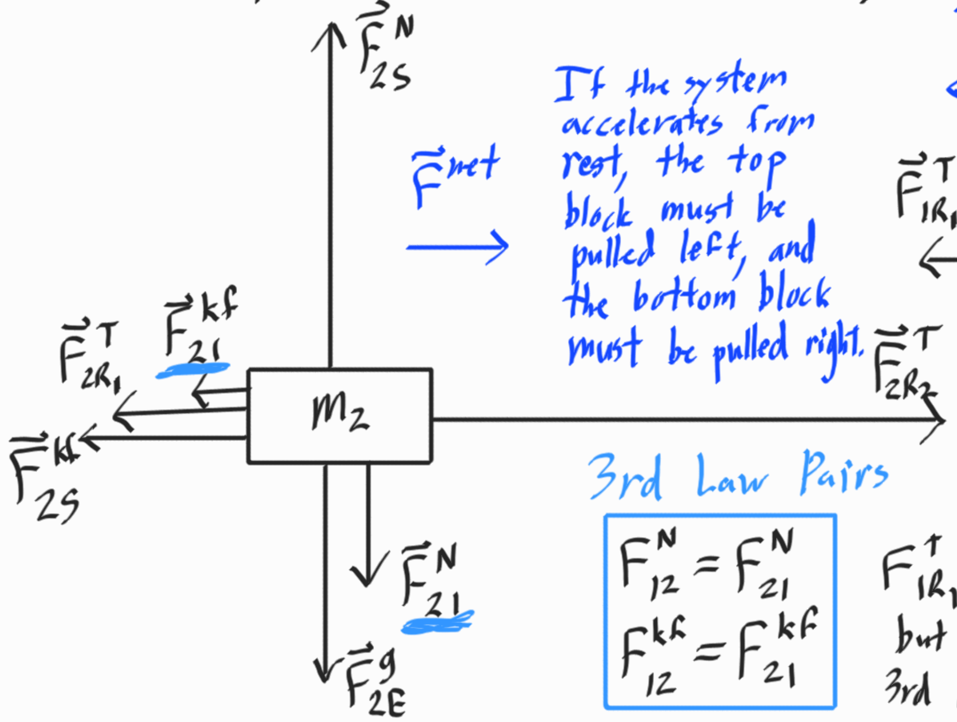
Since the ideal string and massless, frictionless pulley assumptions keep tension constant in this rope, I am fine referring to the entire thing as R_1. I might give each side across the pulley a different name in more general circumstances.



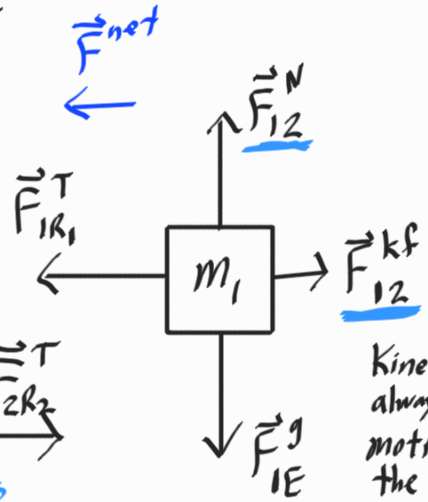
The rope held by the hand will be R_2.

Counting Forces

- Each block has one long-range force (gravity).
- m_1 has 3 contact forces. The rope is pulling it (that's one), and it is touching m_2 (that's two, normal and friction).
- m_2 has 6 contact forces. There are two ropes pulling it, and it touches both the surface below (two more forces) and m_1 above (2 more forces).



If the system accelerates from rest, the top block must be pulled left, and the bottom block must be pulled right.



Kinetic friction always points against motion relative to the surface of contact. The bottom of m_1 is sliding to the left across the top of m_2, so F^{kf}_{12} points right.

3rd Law Pairs

$$\boxed{\begin{aligned} F^{N}_{12} &= F^{N}_{21} \\ F^{kf}_{12} &= F^{kf}_{21} \end{aligned}}$$

$F^{T}_{1R_1} = F^{T}_{2R_1}$ but not a 3rd law pair