In groups of 3, do the following problems together. Be prepared to share your solutions (not just answers!) with the class.

1. A uniform solid sphere rolls without slipping along a flat, level, frictionless horizontal surface. It then rolls up a frictionless inclined plane. The angle of incline is θ. Its speed is momentarily zero after it has rolled a distance d along the ramp. Determine its speed on the way up when it is at the base of the inclined plane in terms of θ, d and/or g.

Before you do any calculations, write down:

What approach to solving this problem are you going to use?

This is a situation in which conservation of energy is best to use. We have gravitational potential energy and two kinetic energies (translational and rotational) to account for.

	U_g	K_t	K_r
Bottom	0	$\frac{1}{2}mv^2$	$\frac{1}{2}I\omega^2$
Top	mgh	0	0

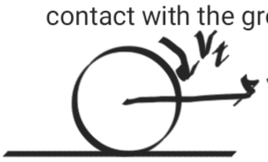
$I = \frac{2}{5}mr^2$

What units should your answer have?

We are looking for an initial speed, so we should have units of meters per second.

Solution:

Since the ball rolls without slipping, we know that its tangential speed must equal its linear speed. That way, its point of contact with the ground will have no speed relative to the ground.



$v_\ell = v_t = \omega r$

$h = d\sin\theta$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

$$\frac{1}{2}\cancel{m}v^2 + \cancel{\frac{1}{2}}\left(\frac{\cancel{2}}{5}\cancel{m}\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2 = \cancel{m}gd\sin\theta$$

$$\frac{7}{10}v^2 = gd\sin\theta$$

$$\boxed{v = \sqrt{\frac{10}{7}gd\sin\theta}}$$

$\sqrt{\frac{m}{s^2}m} = m/s$ ✓

2. A solid sphere of radius R is placed at a height of 0.3 m on a 15° slope. It is released and rolls, without slipping, to the bottom of the ramp. From what height should a circular hoop of radius R be released on the slope to have the same speed as the sphere when they reach the bottom?

In the previous part, we *retrodicted* the speed based on the later distance we found. However, due to the lack of dissipative forces, this will also be the speed the sphere attains upon rolling back to the base of the ramp. As such, we can use our previous answer here, with the adjustment that this problem asks for height, not distance along the ramp's surface, which will simplify the expression.

$h_{hoop} = ?$

$$I_{sphere} = \frac{2}{5}mR^2$$

$$I_{hoop} = mR^2$$

$$v = \sqrt{\frac{10}{7}gh_{sphere}} \Rightarrow \frac{v^2}{g} = \frac{10}{7}h_{sphere}$$

Solving for the final velocity of the hoop involves the same conservation of energy process as above, except with the new moment of inertia:

$$\frac{1}{2}\cancel{m}v^2 + \frac{1}{2}\left(\cancel{m}\cancel{r^2}\right)\left(\frac{v}{\cancel{r}}\right)^2 = \cancel{m}gh_{hoop}$$

$$v^2 = gh_{hoop} \Rightarrow \boxed{h_{hoop} = \frac{v^2}{g} = \frac{10}{7}h_{sphere}}$$

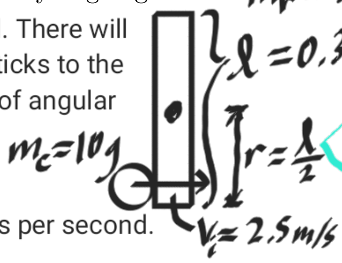
We have to release the hoop higher up, as its inertia causes its rotation to take more energy. For the numbers given, the release height is 3/7 meters, or just shy of 43 centimeters.

3. A 75 gram, 0.30 m long rod hangs vertically on a frictionless, horizontal axle passing through its center. A 10 g ball of clay traveling horizontally at 2.5 m/s hits and sticks to the very bottom tip of the rod.

   What angular speed does the rod have immediately after the clay sticks to it?

   Before you do any calculations, write down:
   What approach to solving this problem are you going to use?

In an inelastic collision problem, energy is not conserved. There will be energy lost to heat and deforming the clay ball as it sticks to the rod. As such, we should consider this as a conservation of angular momentum problem.



$$L_i \begin{cases} L_{ic} = |\vec{r} \times \vec{p}| = \frac{l}{2} m_c v_i \\ L_{ir} = \frac{1}{12} m_r l^2 \omega_i = 0 \end{cases}$$

$$L_f = \left(\frac{1}{12} m_r l^2 + m_c \left(\frac{l}{2}\right)^2\right)\omega_f$$

$$\omega_f = \frac{\frac{l}{2} m_c v_i}{\frac{1}{12} m_r l^2 + m_c \left(\frac{l}{2}\right)^2} = \boxed{\frac{v_i}{l} \Big/ \left(\frac{1}{6}\frac{m_r}{m_c} + \frac{1}{2}\right) \approx 4.76 \frac{rad}{s}}$$

   What units should your answer have?

We are seeking an angular speed, which will be in radians per second.

   Solution:

The initial angular momentum of the rod is the product of the moment of inertia of a rod rotating about its center, and the initial angular speed, which is zero. That is the easy one. For the clay ball, we can calculate its angular momentum relative to the axle as the cross product of its position (right below the axle as it strikes the rod) and its linear momentum, which are conveniently at right angles. The final angular momentum requires us to add the moment of inertia of a point mass to that of the rod to make a new moment of inertia, which we then multiply by the unknown final speed we wish to find.

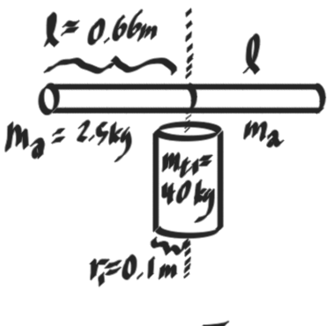
4. A 45 kg figure skater is spinning on the toes of her skates at 1.0 rev/s. Her arms are outstretched as far as they will go. In this orientation, the skater can be modeled as a cylindrical torso (40 kg, 0.1 m radius) plus two rods (2.5 kg each, 0.66 m long) for the arms. Then she puts her arms at her side, which we will model as a single cylinder of mass 45 kg, and radius 0.2m. What is her new angular velocity (in rad/s)?

This is a very rough model of a human body, but it will do for our purposes. This is another conservation of momentum problem, where we will have to find the moments of inertia with arms out and arms in, then the angular momenta to calculate the angular speeds. For these rods, we will use the moment of inertia of a rod with the axis of rotation at its end, which is different from what we used before. However, by stacking them end to end, we are creating one rod of twice the mass and length rotating through its center (see the purple handwriting).

$$I_r = \frac{1}{3} m_a l^2$$

$$I_c = \frac{1}{2} m_t r^2$$

Arms Out



Arms In

$f = 1.0$ rev/s

$\omega_1 = 2\pi$ rad/s

$$\underbrace{I_1 = 2I_{rod} + I_c}_{= \frac{1}{12}(2m_a)(2l)^2}$$

$$= \frac{2}{3} m_a l^2 + \frac{1}{2} m_{t1} r_1^2$$

$$\approx 0.726 \text{ kgm}^2 + 0.2 \text{ kgm}^2$$

$$= 0.926 \text{ kgm}^2$$

$$I_2 = I_c = \frac{1}{2} m_{t2} r_2^2$$

$$= 0.9 \text{ kgm}^2$$

$$L = \begin{cases} I_1 \omega_1 \\ I_2 \omega_2 \end{cases} \Rightarrow \boxed{\omega_2 = \frac{I_1}{I_2}\omega_1}$$

$$\approx 1.029\,\omega_1$$

$$\boxed{\approx 2.058\pi \text{ rad/s}}$$

[0]Select problems may be modified from PH 212 course textbook; Knight Physics for Scientists and Engineers