Catapult: You have decided you want to build a projectile launcher using a frictionless ramp and a spring.

The spring (of spring constant k) is attached to the bottom of the ramp (of angle θ) and runs partially up the incline.

The projectile of mass m will start at equilibrium while resting on a both the spring and the ramp.

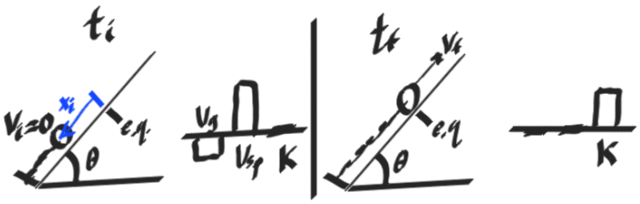
You will then pull the mass back some distance x_i (compressing the spring) and then release the mass to launch it.

In order for it to hit the target, you have calculated that the velocity at this original equilibrium position to be v_f once the mass is released.

How far do you need to pull the mass back (x_i) to reach this velocity?

(Write in terms of m, v_f, k, θ, and other known constants).

There are a few ways to approach this problem. If you treat the Earth and the spring as being external to the system (including only the mass), then the forces they exert are external, and do work on the mass. This position-dependent force can then be integrated across the displacement of the mass to get the work done by the spring and gravity as the spring decompresses, which can then be linked to the mass' change in kinetic energy. Forces are necessary for the next part, so I will leave that method for later, and instead place the Earth and the spring in the system with the mass, making this a conservation of energy problem. Let us write our energies at each time as equations.



	U_g	U_{sp}	K	E_{tot}
t_i	$-mgx_i\sin\theta$	$\frac{1}{2}kx_i^2$	0	$\frac{1}{2}kx_i^2-mgx_i\sin\theta$
t_f	0	0	$\frac{1}{2}mv_f^2$	$\frac{1}{2}mv_f^2$

Note that I chose to place the zero of gravitational potential energy at the equilibrium of the spring, but that is not the only choice. I could have set the zero at the point of compression, which would make gravitational potential energy a positive quantity at the final time. The choice does not affect the outcome. Since energy is conserved, the total energies at each time are equal, giving us $\frac{1}{2}kx_i^2-mgx_i\sin\theta=\frac{1}{2}mv_f^2$.

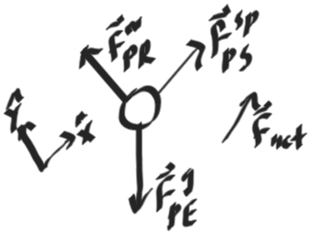
This is a quadratic, which can be placed in the standard form $0 = x_i^2 - 2\frac{mg}{k}\sin\theta\, x_i - \frac{mv_f^2}{k}$, the solutions of which are

$$x_i=\frac{2\frac{mg}{k}\sin\theta \pm \sqrt{4\frac{m^2g^2}{k^2}\sin^2\theta+4\frac{mv_f^2}{k}}}{2}=\frac{mg}{k}\sin\theta + \sqrt{\frac{m^2g^2}{k^2}\sin^2\theta+\frac{mv_f^2}{k}},$$

We must choose the positive solution, to be compatible with how we defined x_i.

Acceleration: Your friend Jamie is trying to build a similar catapult, but she calculated the initial acceleration a_i when the spring is released from being compressed instead of the velocity. How far would you have to pull the mass back for the initial acceleration to be a_i? (Write your answer in terms of known constants and variables).

This is a situation where it is useful to rotate our coordinate axes, which will put the normal and spring forces along the *y* and *x* axes, respectively, and simplify the problem. We already know that the net force is zero in the *y* direction, so we principally concern ourselves with the *x* direction.



$F^g_y=-F^g\cos\theta$

$F^g_x=-F^g\sin\theta$

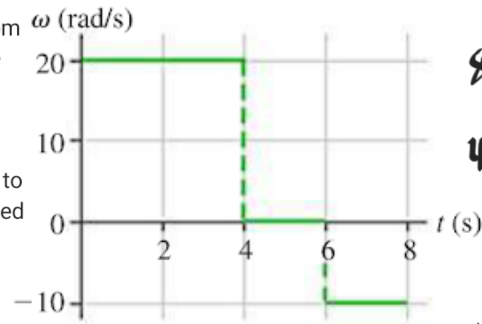
$$ma_i=F_{net}=F^{sp}-F^g\sin\theta=kx_i-mg\sin\theta$$

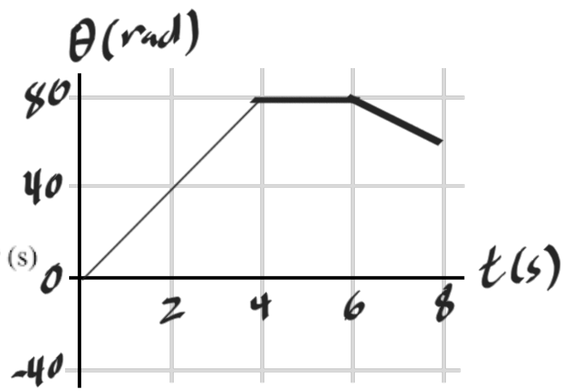
$$a_i=\frac{F_{net}}{m}=\frac{k}{m}x_i-g\sin\theta$$

$$\boxed{x_i=\frac{m}{k}(a_i+g\sin\theta)}$$

ω vs $t \longrightarrow \theta$ vs t:
Below is the angular-velocity-vs-time graph for a particle moving in a circle, starting from $\theta_0 = 0$ rad at $t = 0s$. Draw the angular-position-vs-time graph. Include an appropriate scale on both axes.

Going from angular speed to angular position requires integration, just like going from velocity to position. Here, the graph has three flat regions, which will translate to linear sections on the new graph. These will have slopes equal to the values of the angular speed during these times.





Pebble in a bicycle Your roommate is working on their bicycle and has the bike upside down. They spin the 60 cm diameter wheel, and you notice that a pebble stuck in the tread goes by three times every second. What are the pebble's speed and acceleration? $f = 3$ Hz $r = 0.30$ m

$V = \omega r = 2\pi f r = ?$

$a_c = \omega^2 r = \frac{V^2}{r} = ?$

Before you do any calculations, write down:

What range of values do you predict your answer would fall between? Explain briefly

In one second, the pebble is making three circuits of a wheel that is about 1.8 meters around, so it is going somewhat less than 6 meters per second. Centripetal acceleration is equal to the square of the tangential speed divided by the radius, so we should expect acceleration somewhat less than 120 meters per second squared.

What should the units of your answer be?

The speed should be in meters per second, while the acceleration should be in meters per second squared.

Will your answer be a scalar or a vector?

Speed is a scalar. Acceleration is a vector, but since the wheel appears to be spinning in uniform circular motion (no significant slowing is mentioned), we already know that the acceleration will be centripetal. Thus, we know its direction, and just want its magnitude, a scalar.

Do you know anything else about your answer already?

As was previously said, we have assumed uniform circular motion, so the acceleration is of constant magnitude and centripetal.

While doing your calculations:

Are your units consistent on both sides of the equal sign, at all times? ✓

$\omega = 6\pi$ rad/s ≈ 18.8 rad/s

$V = (0.30\text{m})(6\pi \text{ rad/s}) \approx 5.65$ m/s

$a_c = (6\pi \text{ rad/s})^2 (0.30 \text{ m}) \approx 106.6$ m/s^2

After doing your calculations:

Did your answer fall within the range you predicted? ✓

5.65 m/s < 6 m/s

106.6 m/s < 120 m/s^2

If not, does it make sense anyway and why do you think your prediction was off?

Did you get the units you expected? ✓

Did you get the type of answer you expected (scalar/vector)?

✓