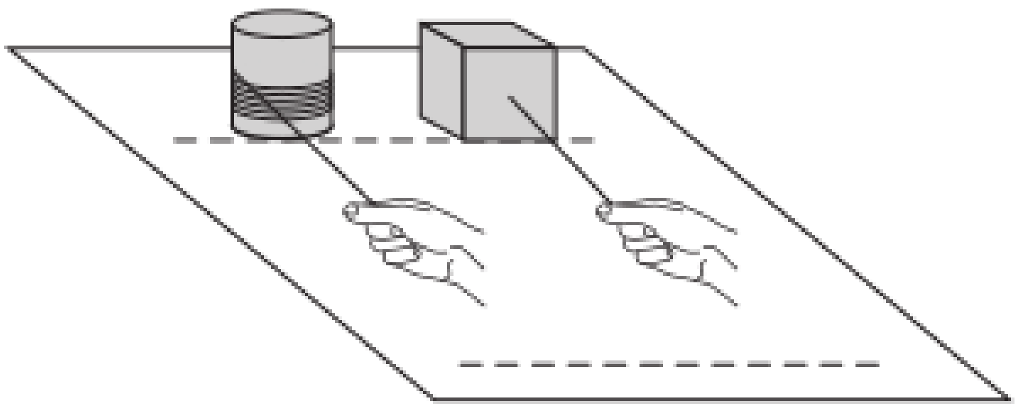
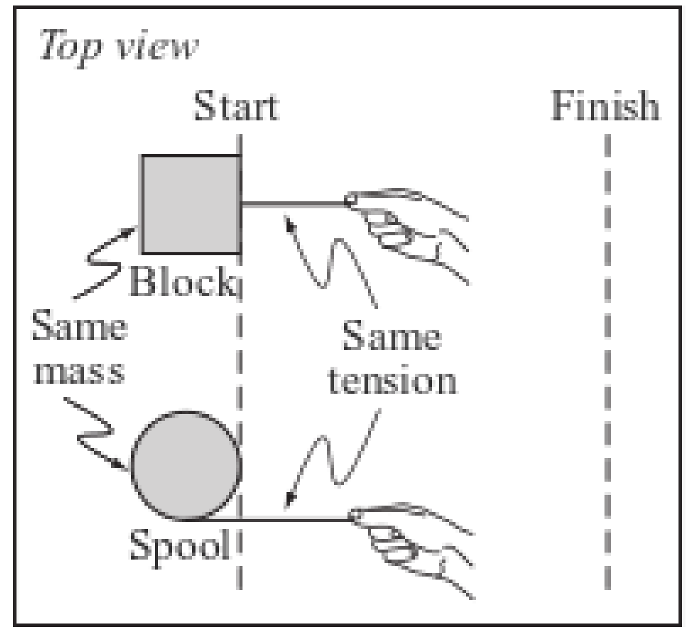
Spools and Wheels





Picture credit: Tutorials in Introductory Physics.

Principles for Success

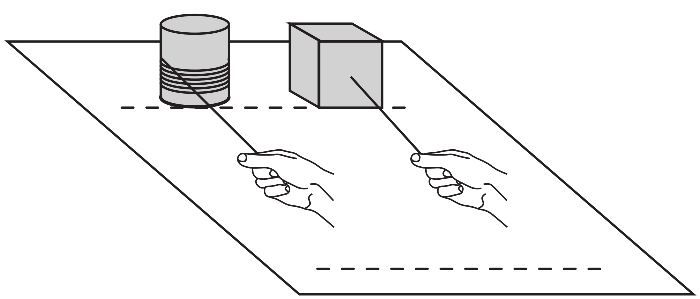
- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

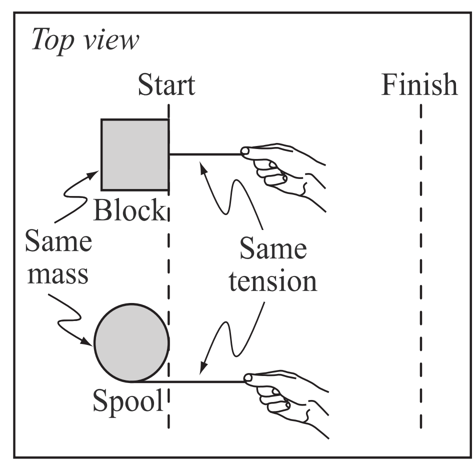
Activity 4-1 – The Block-and-Spool Problem

- A block and a spool are each pulled across a level, frictionless surface by a string.
- The block and the spool have the same mass. The strings start pulling with the same constant force at the same instant.
- *Predict* whether the spool will cross the finish line *before, after,* or *at the same instant as* the block.

Three students debate

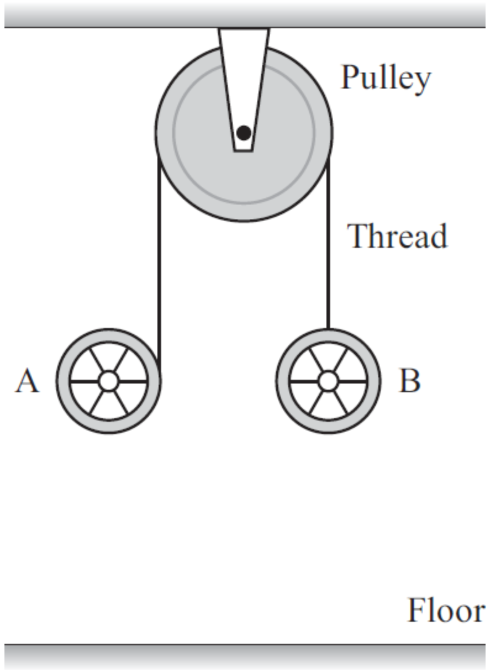
1. *"The spool will rotate and cross the finish line at the same time as the block. They have the same mass and the same net force, so their centers of mass will have the same acceleration. It doesn't matter that the spool rotates and the block doesn't—the tension will have the same effect on translational motion."*
2. *"I disagree. The spool will cross after the block since some of the tension is used to rotate the spool. When a force causes a rotation, it has less of an effect on translational motion."*
3. *"I agree that the spool will rotate and cross after the block, but I was thinking about energy. They will have the same total kinetic energy when they get to the finish line. Since the spool will have some rotational kinetic energy, it must have less translational kinetic energy than the block. Therefore, it will be slower and arrive at the finish line later."*





Activity 4-2 – Spool Drop

- The two spools shown at right are identical, but spool A is free to rotate and spool B is not.
- The spools are dropped at the same time from the same height.
  - Predict whether spool A will hit the floor *before, after,* or *at the same time* as spool B?
  - Draw an extended free body diagram for each spool.
  - Recall the student debate from Activity 3-1. With your group, determine what prediction each of these students would make regarding this new experiment.



4-1 The Block-and-Spool Problem

Short 4-1 talk at 2:20 10:20
Start 4-2 2:25 10:25
Demo 2:40 10:40
Wrap 2:45 10:45
Collision slide during break.

Student 1: Same mass & same net force ⇒ same accel ⇒ same time

Student 2: Some tension "used up" by rotation ⇒ spool takes longer

Student 3: same force & same distance ⇒ same work
⇒ same kinetic energy at end
Spool has K_{rot} ⇒ $K^{spool}_{trans} < K^{block}_{trans}$ ⇒ $V_{spool} < V_{block}$ ⇒ spool takes longer

4-2 Spool Drop

<u>A 1st</u> <u>B 1st</u> <u>Same Time</u>

Energy Method: For conservation, system must have A, B, Earth, and string, but that gives too many energies.
Putting the string outside of the system allows it to do work, which moves energy from B into A.

4-1 The Block-and-Spool Problem

Start 4-1 talk at 2:20 / 10:20 Start 4-2 2:25 / 10:25 Demo 2:40 / 10:40 Wrap 2:45 / 10:45 Collision slide during break.

✓ Student 1: Same mass & same net force ⇒ same accel ⇒ same time

force is not reduced just by causing rotation (but it may be more difficult to pull at constant force when the object can rotate)

Student 2: Some tension "used up" by rotation ⇒ spool takes longer

spool slides & unwinds, so hand moves farther, doing more work

point of contact rotates, so

$W^{net} = \vec{F}\cdot\Delta\vec{x} + \vec{T}\cdot\Delta\vec{\theta}$

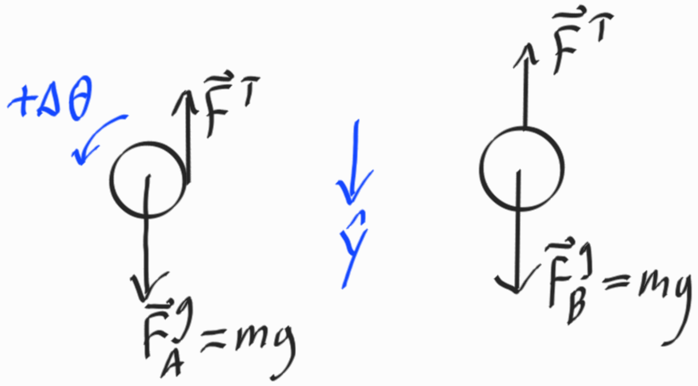
Student 3: same force & same distance ⇒ same work

⇒ same kinetic energy at end

Spool has K_{rot} ⇒ $K^{spool}_{trans} < K^{block}_{trans}$ ⇒ $v_{spool} < v_{block}$ ⇒ spool takes longer

4-2 Spool Drop

A 1st	B 1st	Same Time
student 2	Student 3	Student 1



$$ma_A = F^{netA}_y = mg - F^T$$

$$I\alpha = \tau^{netA} = rF^T$$

$$ma_B = F^{netB}_y = mg - F^T$$

$$\Delta\theta r - \Delta y_A = \Delta y_B$$

$$\alpha r - a_A = a_B$$

$$\Rightarrow \begin{cases} a_A = a_B \\ F^T = \dfrac{I\alpha}{r} = \dfrac{I(r\alpha)}{r^2} \end{cases}$$

Both spools drop at same time

$$\alpha r = 2a_A$$

$$ma_A = mg - \frac{2Ia_A}{r^2} \Rightarrow a_A = g\Big/\left(1 + \frac{2I}{mr^2}\right)$$

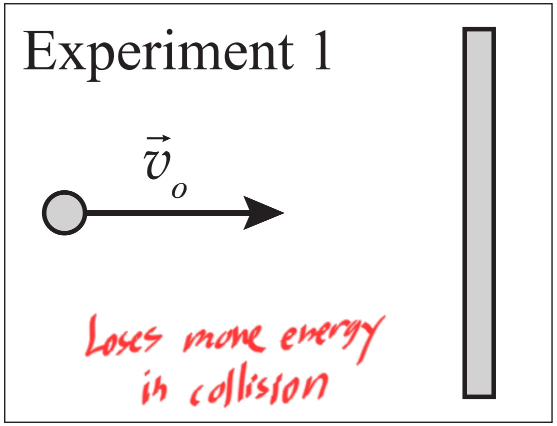
Energy Method: For conservation, system must have A, B, Earth, and string, but that gives too many energies.
Putting the string outside of the system allows it to do work, which moves energy from B into A.

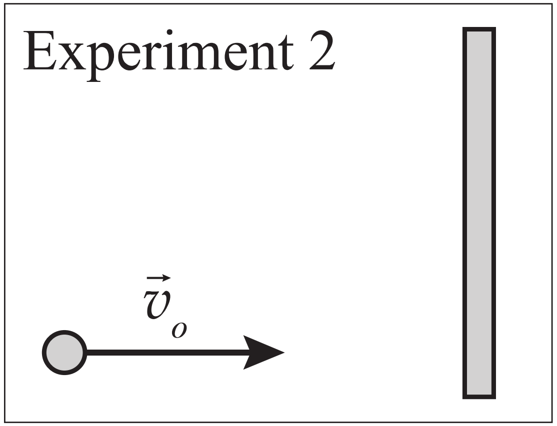
Collisions

- A puck collides with and sticks to a rod that is free to move.
  inelastic
- Which system will have a larger translational velocity after the collision?
  A. Experiment 1
  B. Experiment 2
  C. Same velocity

Conservation of linear momentum

Top-view diagrams

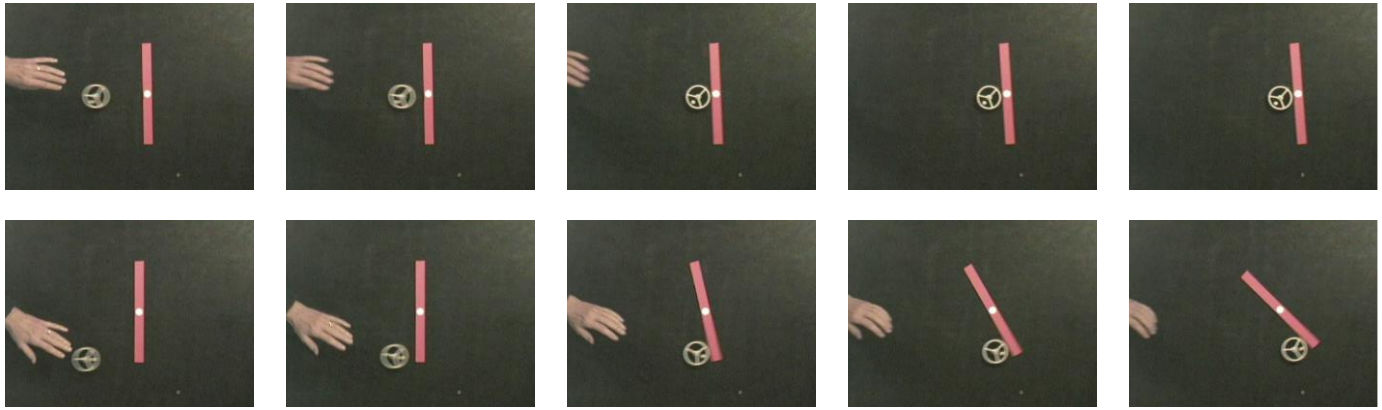




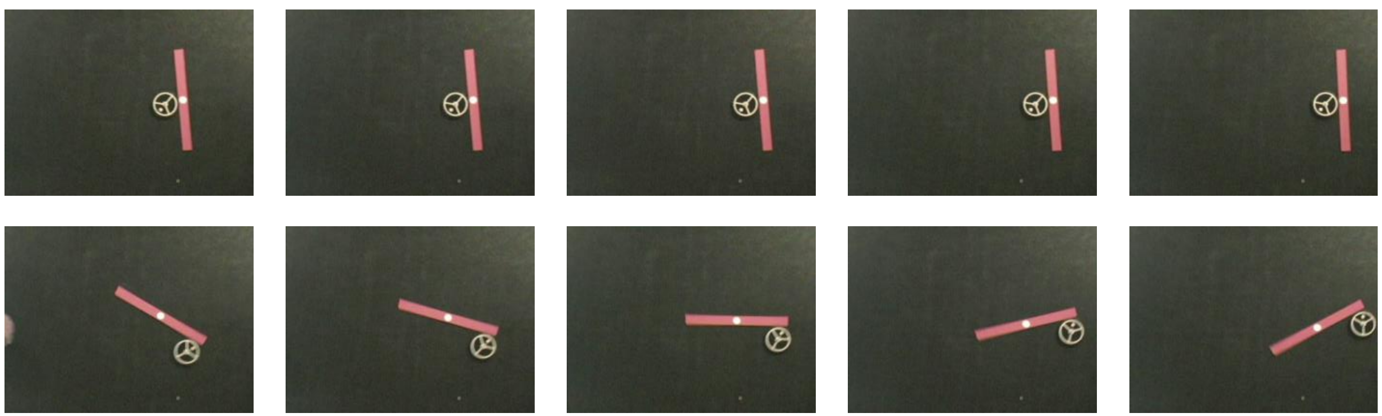
Collisions

Experiment 1

Experiment 2



1	2	3	4	5



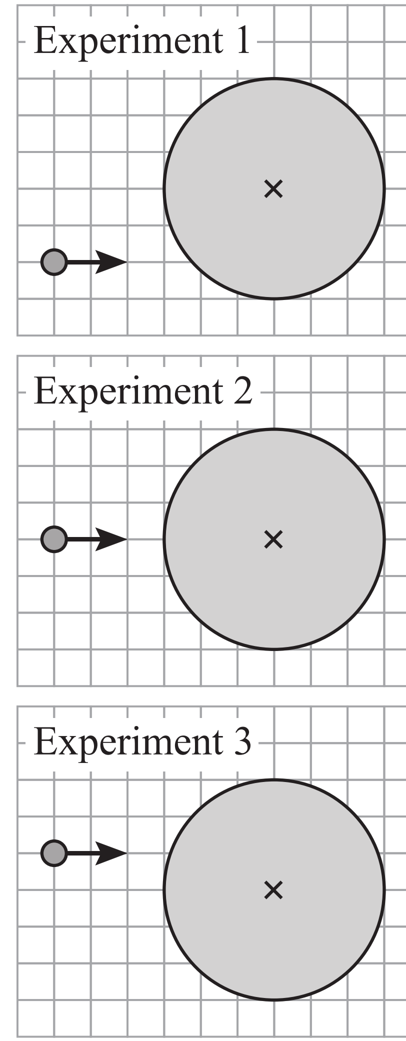
6	7	8	9	10

Activity 4-3 – The Puck and the Disc

In each case, the small puck sticks to the large disc, which is fixed in place but free to rotate.

- Describe the motion of each system after the collision.
- Specify the directions of the **final** angular momentum of each system.
- Rank the magnitudes of the **final** angular momenta of the systems.
- Rank the magnitudes of the **initial** angular momenta of the systems.

Top-view diagrams

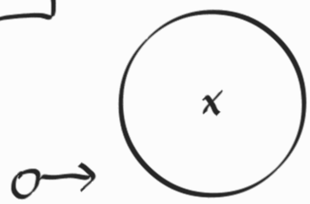


4-3 The Puck and the Disc

Finish collision slides talk & start 4-3 3:10 11:10

Finish 3:20 11:20

1



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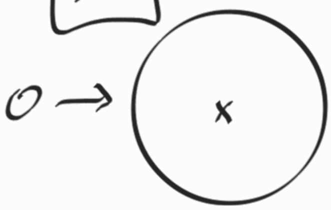

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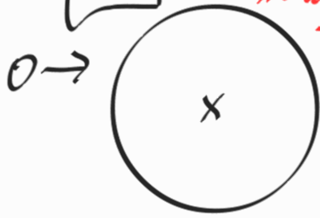


Answer: C by conservation of linear momentum
Inelastic collision
⇒ Exp1 loses more energy than Exp2

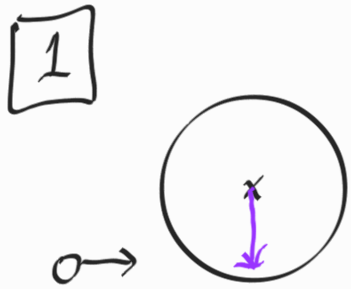
4-3 The Puck and the Disc

Finish collision slides talk & start 4-3 3:10 11:10

Finish 3:20 11:20

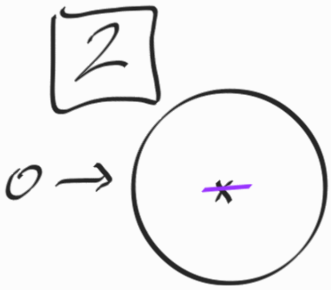
Answer: C by conservation of linear momentum

Inelastic collision $\Rightarrow$ Exp 1 loses more energy than Exp 2



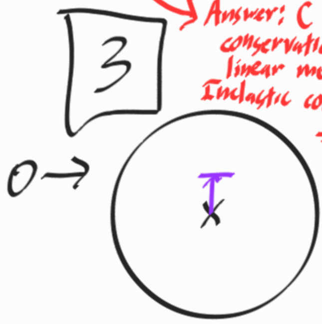
Rotates ccw after impact

$\circlearrowleft \odot \vec{L}_{f1}$



Does not rotate

$\vec{L}_{f2} = \vec{0}$



Rotates cw after impact

$\circlearrowright \otimes \vec{L}_{f3}$

$$L_{f1} > L_{f3} > L_{f2} = 0$$

Puck strikes farther from axis in 1, causing greater rotation.

OR

$$\vec{L}_i = \vec{r} \times \vec{p}$$

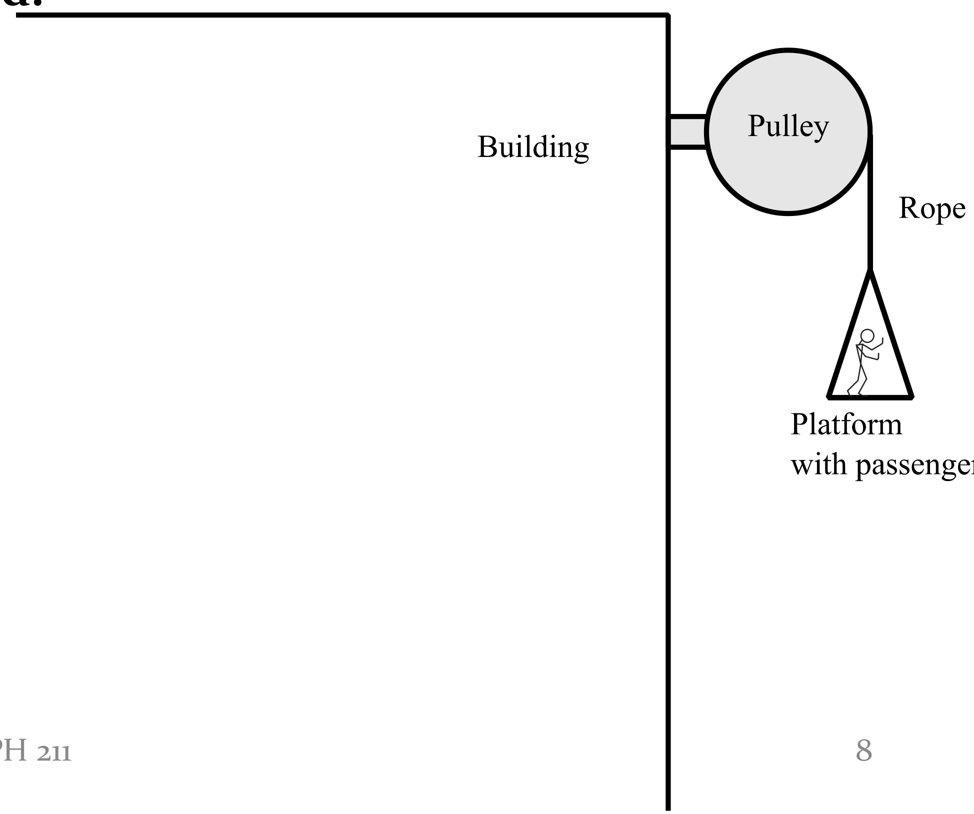
$$\vec{L}_i = \vec{L}_f \quad \text{b/c no external torque}$$

$$L_{i1} > L_{i3} > L_{i2} = 0$$

$$r_1 > r_2 > r_3 = 0$$

Activity 4-4 – The Fire Escape

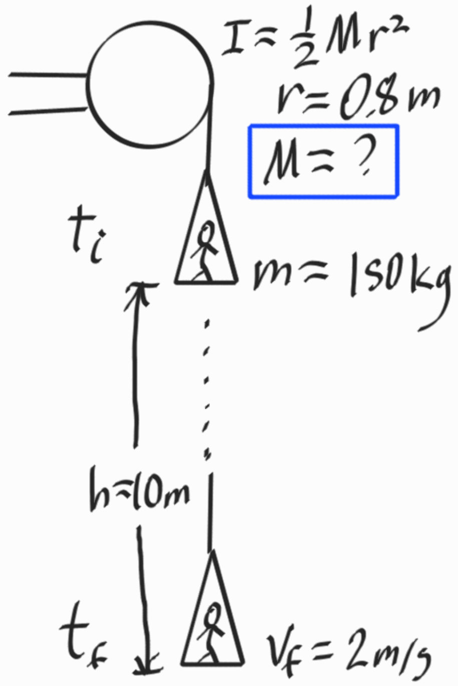
- You are helping to design a new fire escape for your third-floor apartment. You step out the window onto a platform that is attached to a rope, which is wound around a massive solid cylindrical pulley that can rotate on its axis. As you fall at increasing speed, the rope causes the pulley to turn at increasing angular speed. Your task is to choose a pulley that will enable you to be moving a safe 2 m/s when you reach the ground 10 m below your starting position. The combined mass of you and the platform is 150 kg. Your task is to determine the mass of the pulley needed. The pulley's radius is 0.8 m. The moment of inertia for a solid disk (pulley) is $\frac{1}{2}mr^2$ where m is its mass and r its radius.
- **There is more than one way to solve this problem. Use conservation of energy to solve the problem—if you want practice using forces and torques, retry the problem again later using that method.**



4-4 The Fire Escape

Start 3:30
11:30

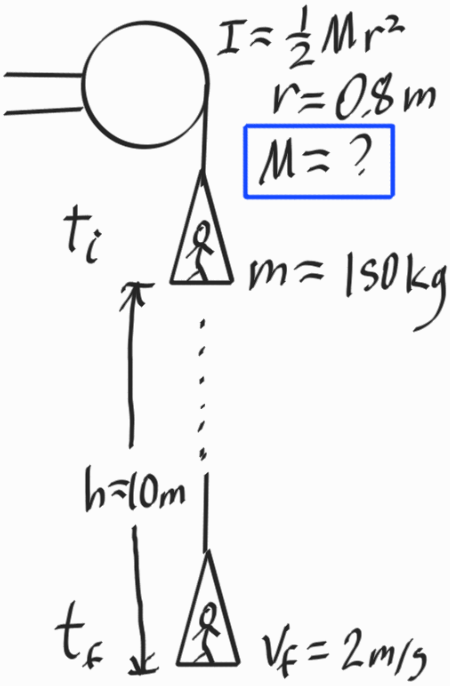
Finish 3:45
11:45



	U_g	K_{plat}	K_{pull}	E_{tot}
t_i				
t_f				

4-4 The Fire Escape

Start 3:30 11:30
Finish 3:45 11:45



	U_g	K_{plat}	K_{pull}	E_{tot}
t_i	mgh	0	0	mgh
t_f	0	$\frac{1}{2} m v_f^2$	$\frac{1}{2} I \omega_f^2$	$\frac{1}{2} m v_f^2 + \frac{1}{4} M v_f^2$

$$\omega_f = \frac{v_f}{r}$$

$$\left(\frac{1}{2} m + \frac{1}{4} M\right) v_f^2 = mgh$$

$$2m + M = \frac{4mgh}{v_f^2}$$

$$M = 2m\left(\frac{2gh}{v_f^2} - 1\right)$$

$$\approx (300\,kg)\left(\underbrace{\frac{200\,m^2/s^2}{4\,m^2/s^2}}_{50} - 1\right)$$

$$= 14{,}700\ kg \quad \left(\text{more than } 16 \text{ tons!}\right)$$

16 tons, and what do you get?
Another day older and a bad idea for a fire escape.