Lecture 9: Static Special Cases

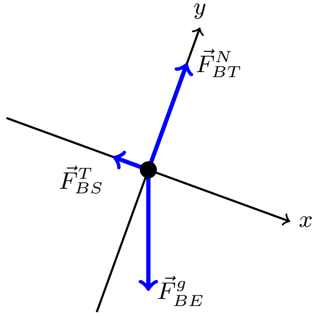
In-Class Quizzes on Monday

- Two Quizzes: Motion and Forces
  - Formatted just like the practice quizzes in the Week 3 module.
  - Not just calculations; could be assumptions, sensemaking, explanations, etc.
- One sheet (front and back) of notes allowed.
- Scientific calculators only (no graphing calculators).
  - There will be some some numerical calculation, but it is not a major emphasis.
- For simplicity, $g \approx 10\ \text{m/s}^2$.

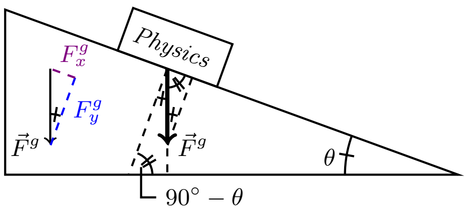
Other Announcements

- Working on another back-up lecture location.
  - Main classroom will still be WNGR 212.
    * Hopefully here for quizzes, but maybe BEXL 103 if it is still too warm–(won't know until Monday).
  - May have to change "official" classroom to reserve a back-up room for the term.
- Don't forget the Address Assessment portion of your lab reports.
  - This will make your lab feedback more effective by helping your TA see how you are trying to improve.
- Let us know how if you have issues with the feedback you receive on assignments.
  - Is it clear? Is it enough? How is the tone? Other thoughts?

(A)



(B) In this problem, it makes sense to tilt the axes. By aligning the normal force with the y-axis and the tension force with the (negative) x-axis, we now don't have to break them into components: $F^N_{BT,y} = F^N_{BT}$ and $F^T_{BS,x} = -F^T_{BS}$. That leaves just one vector to break up: the force of gravity.



There are a number of right triangles in the figure above that can help us determine that

$$F^g_{BE,x} = F^g_{BE}\sin\theta, \qquad F^g_{BE,y} = -F^g_{BE}\cos\theta.$$

We can also determine this with special-case analysis. We should get $\vec{F}^g = -F^g\hat{y}$ when $\theta = 0°$ (our standard coordinate system) and $\vec{F}^g = F^g\hat{x}$ when $\theta = 90°$ (tilted such that $\hat{x}$ points down). We can guess where the signs and trigonometric functions go and check these special cases, which will eliminate equations for $\vec{F}^g$ that don't behave correctly.

(C) The net force on the book should be zero, as the book is not accelerating. The string is keeping the book from moving along the surface, and the book doesn't fall through or leap off of the table.

(D) We know that $\vec{F}^{net} = \vec{0}$. In the x-direction, this tells us:

$$\begin{aligned} 0 = F^{net}_x &= F^g_{BE,x} + F^T_{BS,x} \\ &= F^g_{BE}\sin\theta - F^T_{BS} \\ &\implies F^T_{BS} = F^g_{BE}\sin\theta \end{aligned}$$

In the y-direction, this tells us:

$$\begin{aligned} 0 = F^{net}_y &= F^g_{BE,y} + F^N_{BT,y} \\ &= -F^g_{BE}\cos\theta + F^N_{BT} \\ &\implies F^N_{BT} = F^g_{BE}\cos\theta \end{aligned}$$

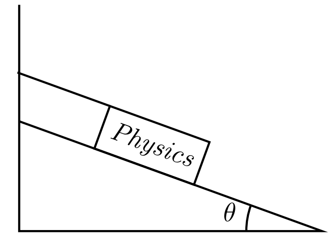
L9-1: Textbook on a Tilted Table

A physics textbook is on a tilted, frictionless table, supported by a string.

(A) Sketch a free-body diagram for the system.

(B) What coordinate system do you think will make analyzing this situation easiest?

(C) Should the net force on the book be *zero* or *not zero*?



(D) Write an expression for the magnitude of each force acting on the system in terms of the gravitational force F^g.

Special-Case Analysis

After you solve for a quantity:

- Choose a case that is special, not arbitrary.
- Figure out what your quantity **should** be in the case you chose.
- Identify the value of one or more other quantities that corresponds to your **case**.
- Evaluate your answer in the special case.
- Check whether or not your symbolic answer for the case matches what you expected the answer should be.

Increased Verticality (Covariational Sensemaking)
As the table tilts more, the string has to support the book more and more to keep it from sliding, and the surface of the table correspondingly needs to support the book less and less. Thus, the magnitude of the normal force will decrease, and the magnitude of the tension force will increase.

Note that $\cos\theta$ decreases and $\sin\theta$ increases as θ increases, so our symbolic answers agree with our physical prediction.

Horizontal Case (Special-Case Analysis)
When the table is flat ($\theta = 0°$), the string does not need to pull on the book to keep it from sliding. The tension will be 0 N, and the normal force will be equal in magnitude to the force of gravity. Our symbolic answers agree with this expectation:

$$F^T_{BS} = F^g_{BE}\sin(0°) = 0 \text{ N}$$
$$F^N_{BT} = F^g_{BE}\cos(0°) = F^g_{BE}$$

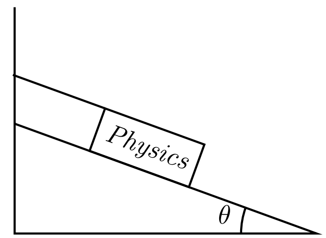
Vertical Case (Special-Case Analysis)
When the table is vertical ($\theta = 90°$), the string is supporting the entire weight of the book, and the surface of the table does not support the boook at all (the book just hangs beside it from the string). The tension is equal in magnitude to the force of gravity, and the normal force will be 0 N. Our symbolic answers agree with this expectation:

$$F^T_{BS} = F^g_{BE}\sin(90°) = F^g_{BE}$$
$$F^N_{BT} = F^g_{BE}\cos(90°) = 0 \text{ N}$$

L9-2: Tilted Table Sensemaking

A physics textbook is on a tilted, frictionless table, supported by a string.

- Suppose the table is slanted so that it becomes *steeper*. What happens to the magnitudes of the normal force and the tension force?



- Consider the following special cases:
  - What if the table were horizontal?
  - What if the table were vertical?

  For each of these cases, answer the following questions:
  - How big **should** each force be?
  - What angle corresponds to this **case**?
  - Does our symbolic answer for the case match what the answer should be?

Main Ideas

- Tension and normal force don't have models with which to be directly calculated, so we need to find them via Newton's 2nd law.
- Picking a special case for which the answer is clear by physical reasoning can be a powerful tool for checking symbolic solutions.