

# Studio 1: Representations and Sensemaking

## THREE IMPORTANT STEPS:

1. Grab a small whiteboard for yourself! (These are by the doors.)
2. Sit at tables 2, 3, 6, and 7 (the four closest to the center).
3. Grab three large whiteboards for your table!

## S1-1: Representations of Volume

You want to determine the volume of the room you are in.

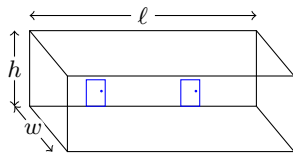
(a) Write a description of how to find the volume of the room in words.

If we measure the dimensions of the room (ceiling height, length, width), and multiply them together, we can get a decent estimate of the volume of the room.

Of course, it won't be perfect. The floor isn't even, the walls have a few pillars embedded in them, the doors lie in slight recesses, and there are air ducts sticking into the room. We could find their volumes and subtract, perhaps, if we need that level of accuracy.

Instead, we will assume for simplicity that the room is rectangular and empty.

(b) Sketch a diagram that would help you find the volume of the room.



I put the doors in to orient the drawing. They are otherwise not critical, and their sizes and positions are not quantitatively accurate.

(c) Write a symbolic expression that would allow you to find the volume of the room. Check the units of your expression.

$$V = \ell wh$$

Length  $\ell$ , width  $w$ , and height  $h$  are all lengths, which would be measured in meters in SI units. Volume is measured in  $\text{m}^3$  in SI units. When we multiply  $\ell wh$ , we are multiplying meters by meters by meters, which gives us  $\text{m}^3$ , so we have the same units on either side of the equal sign:

$$[V] = \text{m}^3, \quad [\ell wh] = [\ell][w][h] = \text{m} \cdot \text{m} \cdot \text{m} = \text{m}^3.$$

(d) Without standing up, estimate the volume of the room as a number. Make sense of the number by comparing it to something.

There are several ways to estimate the dimensions of the room without measuring it directly:

- Guess your local TA's height (or just ask) and compare them to the height of the ceiling, like a living measuring stick.
- Look up the height of a standard interior door (Home Depot says 80 inches, which is about 2.032 meters) and estimate how many doors high the ceiling is.
- The carpet tiles are a regular size. Measure one nearby and count how many you see from one wall to another.

For comparison, an Olympic pool is about  $2500 \text{ m}^3$  (technically, there is no standard depth, so this can vary). Comparing this to the size of the room can give you some idea about how accurate your estimate is (for example, the room is certainly not larger than an Olympic pool).

## S1-1: Representations of Volume

You want to determine the volume of the room you are in.

(a) Write a description of how to find the volume of the room in words.

(b) Sketch a diagram that would help you find the volume of the room.

(c) Write a symbolic expression that would allow you to find the volume of the room. Check the units of your expression.

(d) Without standing up, estimate the volume of the room as a number. Make sense of the number by comparing it to something.

These quotations come from upper division students at the University of Pittsburgh, where they originated this writing and discussion activity. The interviewees discussed their transitions into college and their experiences in an introductory physics course.

## Challenges in Taking Physics

“I remember taking my first Physics class as a freshman. Before coming to college, I didn’t worry much about grades, so I felt unprepared for the increased workload and differences in grading. I remember being surprised after getting burned grade-wise several times, and feeling stressed as a result. But then I got some help from the instructor and the TA, found a study group, and was able to turn things around. Looking back now, **I think my struggles were pretty normal.** Even though people don’t like to admit it, basically everyone has trouble with certain concepts. Although it was a somewhat rocky start, it felt good to learn from my mistakes, and **I am proud of the success I have had.**”

—Nathan, Pitt Bioengineering Senior

“I was one of just a handful of women in one of my intro physics study groups, and sometimes I felt a little embarrassed to ask questions. However, **I quickly learned that other students usually had the same question I did,** and we all benefitted from working with each other and learning from each other. Sometimes I had difficulty with an idea that my classmates understood. Other times, they struggled with concepts that I understood. I remember there wasn’t always an “aha!” moment, where everything clicked. It was usually much more gradual, with some concepts only becoming clear after lots of practice and discussion with my study group. **I realized that everyone struggles some times,** and the important thing is to not give up and help each other out.”

—Allison, Pitt Electrical Engineering Senior

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## Challenges in Taking Physics

“When I first got here, I was worried because I was really struggling with some of the physics concepts. **It felt like everyone else was doing just fine**, but I just wasn’t sure if I was cut out for a physics course. At some point during the first semester, I came to realize that, actually, a lot of other students were struggling, too. And **I started to look at struggling as a positive thing**. After I struggled with a hard problem and then I talked to other classmates and my TA about the solution—I realized that all that effort was worth it because it helped me learn and remember much more.”

—Aniyah, Pitt Chemical Engineering Graduate

“I didn’t go to a very good high school, and **I was worried that my high school courses had not prepared me well for college**. Honestly, when I got here, I thought professors were scary. I thought they were critical and hard in their grading, and sometimes it felt like they put things on the quizzes or exams that we hadn’t discussed in class. But then I realized that the professor wanted me to be able to apply the physics concepts in many different situations. So I started to study in a way that would help me do that, and **I did my best to learn from my mistakes on quizzes and exams**. And I saw that even when the professors’ grading seemed tough, it didn’t mean they looked down on me or that I didn’t belong. It was just their way of motivating high achieving students.”

—Anil, Pitt Civil Engineering Senior

Additional questions to stimulate group discussion:

- Why do you think people wonder at first if they belong?
- Why do you think people often think they're the only one who worries about whether they fit in in college?
- Why and how does people's experience change over time? What do people do that helps them improve their experience with time?
- Ask students to share additional experiences that resonate with what the group has been discussing.

## Challenges in Taking Physics

With your small group, discuss what you wrote about in the first Get-Ready activity and the quotes you have just read. Please answer the following questions as a group:

- What are some common themes across several of the quotes we read?
- Why do you think that sometimes students don't realize that other people are also struggling with the course?

## S1-2: Driving to Portland

Discuss “sensemaking” with your group. Identify several ways of making sense of answers or contexts you have used in math or science courses.

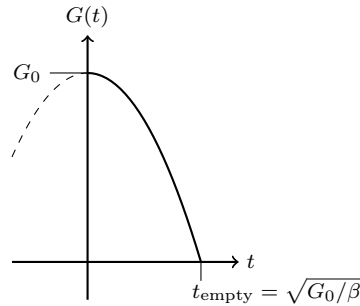
- **Numerical Sensemaking:** Compare a numerical result against a reference number.
- **Unit Check:** Check the units of your expression to make sure they are what you expect.  
*Example:* The units of  $\beta$  must be  $[\beta] = \frac{\text{gal}}{\text{h}^2}$  for  $\beta t^2$  to have units of gallons.
- **Covariation:** See how changing the variables changes the output of your expression. See what the signs in your expression tell you.  
*Example:* The minus sign in  $G_0 - \beta t^2$  tells us that the tank is getting emptier as time progresses (assuming  $\beta > 0$ ), which we expect of a vehicle consuming gas as an energy source.
- **Special Case Analysis:** Choose values for variables which correspond to “special cases,” where the physical expectation is obvious and the math is simpler.  
*Example:* We should have the most gas in the tank when we start, and if we plug in an initial time of  $t = 0$ , we get  $G(0) = G_0$ . This tells us that  $G_0$  is the initial amount of gas in the tank.

**IMPORTANT!** Do not just comment on the behavior of an expression without comparing to physical expectations. For example, do NOT just say “ $G(t)$  is decreasing, which makes sense.” Say **why**: “ $G(t)$  is decreasing, which makes sense, as gas is consumed during travel.”

You are driving from Corvallis to Portland, and you measure how full your gas tank is (in gallons) as a function of time (in hours):

$$G(t) = G_0 - \beta t^2$$

Multiple sensemaking examples are given in part (a). Additionally, we can learn a lot by looking at a visual representation. Let us graph the function:



There is a really important lesson here. Even if we provide you with an equation that is mathematically valid for all values of  $t$ , that does not mean it should be assumed to be an accurate model for all  $t$ .

For instance,  $G(t)$  increases for  $t < 0$ , which does not make sense while driving. The model probably should only be applied to  $t > 0$ .

Furthermore, note that the slope gets steeper as  $t$  increases. This tells us that gas is being consumed faster as time goes on. Should we expect fuel efficiency to vary like this?

Here is another very important question: when do we reach Portland? It is possible that we should have stopped graphing before  $G(t) = 0$  (out of gas). Graphing beyond the point at which the model applies may show us predictions that don’t make sense.

## S1-2: Driving to Portland

Discuss “sensemaking” with your group. Identify several ways of making sense of answers or contexts you have used in math or science courses.

(a) Identify several ways of making sense of answers or contexts you have used in math or science courses.

You are driving from Corvallis to Portland, and you measure how full your gas tank is (in gallons) as a function of time (in hours):

$$G(t) = G_0 - \beta t^2$$

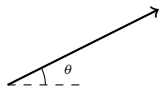
(b) Make sense of this expression with your group in as many different ways as you can, making use of as many different representations as you can.

# S1-3: Representations of Vectors

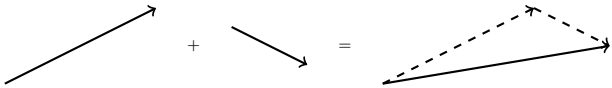
Discuss **vectors** with your group.

The following list only contains my thoughts about vectors. It should not be assumed to be definitive. There are many other things about vectors that could have been included.

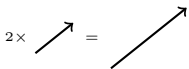
- A vector has direction and magnitude.
- A vector is often represented with an arrow; the length is its magnitude, and the orientation of the arrow (its angle) indicates direction.



- Vectors add “tip-to-tail.”



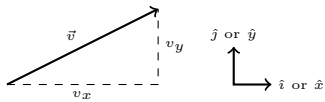
- Multiplying a vector by a scalar changes its length.



- Multiplying a vector by −1 reverses its direction.



- Vectors can be broken into components.



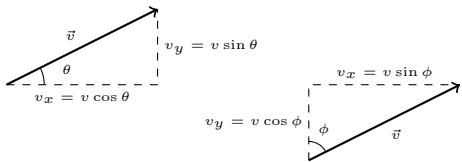
Parenthetical or angle-bracket notation was probably used in your math classes:

$$\vec{v} = (v_x, v_y) = \langle v_x, v_y \rangle.$$

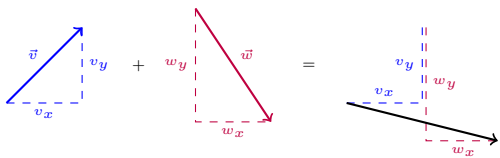
It is okay, but parentheses and angle brackets can mean a lot of things, so it may not always be clear.  
We recommend unit vector notation, as it is much clearer:

$$\vec{v} = v_x \hat{i} + v_y \hat{j} = v_x \hat{x} + v_y \hat{y}.$$

Using  $\hat{x}$ ,  $\hat{y}$ ,  $\hat{z}$  instead of  $\hat{i}$ ,  $\hat{j}$ ,  $\hat{k}$  helps to more clearly associate the Cartesian unit vectors with the coordinate axes.  
**Warning!** Cosine is not always  $x$ ! It depends on where the angle is.



- Vectors add “componentwise.”



$$\vec{v} + \vec{w} = v_x \hat{x} + v_y \hat{y} + w_x \hat{x} + w_y \hat{y} = (v_x + w_x) \hat{x} + (v_y + w_y) \hat{y}$$

# S1-3: Representations of Vectors

Discuss **vectors** with your group.

Write down a list of things about vectors on your shared whiteboard. You can write:

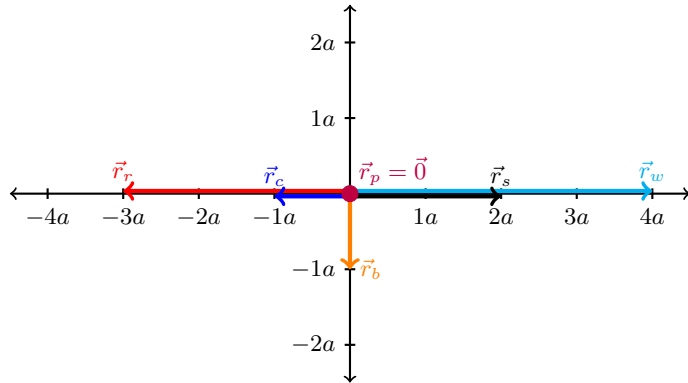
- words or sentences
- numbers or symbols
- pictures or diagrams

Write large!

## S1-4: Vectors in a Garden

You visit a garden with a trail that includes the following landmarks.

- Red roses at  $\vec{r}_r = -3a\hat{x}$
- White roses at  $\vec{r}_w = +4a\hat{x}$
- A pond at  $\vec{r}_p = 0\hat{x}$
- A bench at  $\vec{r}_b = -a\hat{y}$
- A bridge over a creek at  $\vec{r}_c = -a\hat{x}$
- A statue at  $\vec{r}_s = +2a\hat{x}$



(2) Find the following displacement vectors using both symbols and diagrams.

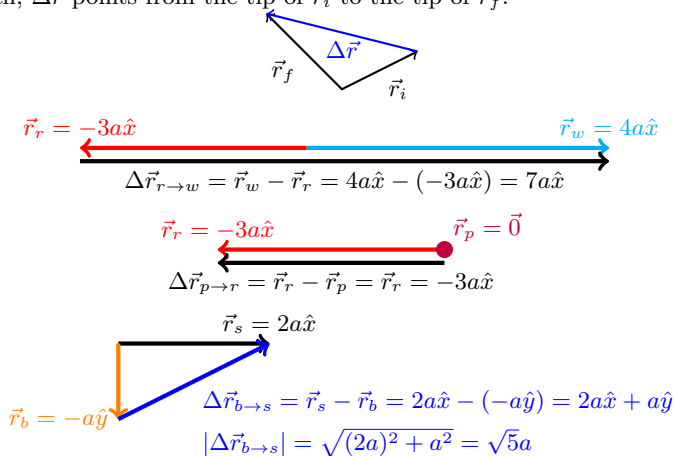
A displacement vector,  $\Delta\vec{r}$ , can be thought of as the difference of the final position vector,  $\vec{r}_f$ , and the initial position vector,  $\vec{r}_i$ :

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i.$$

It can also be thought of as the vector that, when added to  $\vec{r}_i$ , gives you  $\vec{r}_f$ :

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i.$$

As such,  $\Delta\vec{r}$  points from the tip of  $\vec{r}_i$  to the tip of  $\vec{r}_f$ :



## S1-4: Vectors in a Garden

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- A bridge over a creek at  $\vec{r}_c = -a\hat{x}$
- A statue at  $\vec{r}_s = +2a\hat{x}$

(1) Sketch and label the garden and its landmarks.

(2) Find the following displacement vectors using both symbols and diagrams.

(a) From the red roses to the white roses

(b) From the pond to the red roses

(c) From the bench to the statue