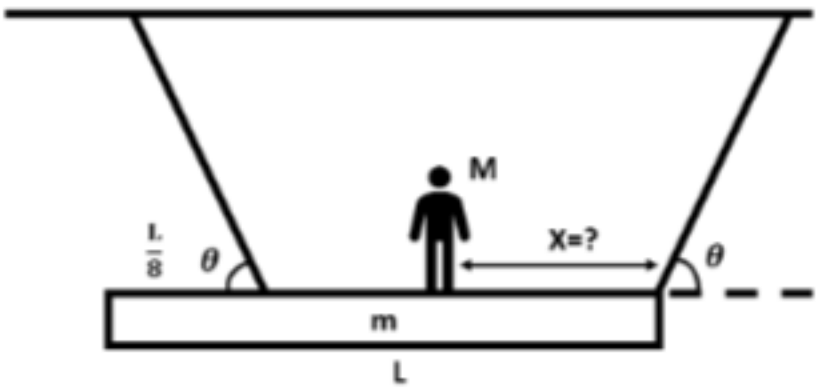
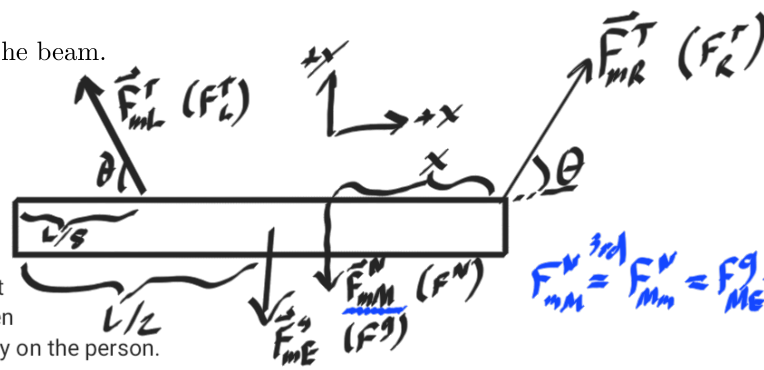
In groups of 3, do the following problems together. Be prepared to share your solutions (not just answers!) with the class.

Balanced Beam A horizontal beam of mass m and length L is hanging from 2 massless wires attached to the ceiling, as shown in the figure below. The wires both come out at angle θ. The right wire is attached to the end of the beam while the left wire is attached a distance $\frac{L}{8}$ in from the left end of the beam. A person, of mass M is somewhere on top of the beam. Find an expression for the distance the person must be standing from the right edge of the beam in order for the beam to be in static equilibrium.



1. Draw an Extended Free Body Diagram for the beam.

In the extended free body diagram, we not only track the size and direction of forces on the beam, but we also mark the relative locations of their points of application. Each force has been labeled with a vector, and with a parenthesized, simplified label for its magnitude. Note that the person exerts a normal force on the beam. There is temptation to mark this as a force of gravity from the person, but we can't be that direct. We must use Newton's third law to connect this normal force to its equal and opposite reaction on the person, then use the person's static equilibrium to equate that to the force of gravity on the person.



$F^N_{mM} \overset{3rd}{=} F^N_{Mm} = F^g_{ME} = Mg$

2. Write all 3 (x, y, rotational) Newton's 2nd Law Equations for the beam.

Since the angles are the same and the tensions are the only two forces exerting any horizontal pull, the x-direction equation tells us that they must be equal in magnitude (allowing us to label them more simply yet again).

$$x:\ F^T_R\cos\theta - F^T_L\cos\theta = 0 \Rightarrow F^T_R = F^T_L = F^T$$

The y-direction equation links the tensions to the weights of the person and the beam, and will allow a substitution later.

$$y:\ F^T_R\sin\theta + F^T_L\sin\theta - mg - Mg = 0 \Rightarrow 2F^T\sin\theta = (m+M)g$$

We could choose any axis for the rotational equation, but the advantage of choosing the right end of the beam is that we can ignore one of the tensions, and write the distance from the axis to the application of the person's weight as the given variable X. Recall that counterclockwise torques are positive by convention.

[0]Select problems may be modified from PH 212 course textbook; Knight Physics for Scientists and Engineers

$$\text{rot}:\ -F^T_L\frac{7L}{8}\sin\theta + mg\frac{L}{2} + MgX = 0$$

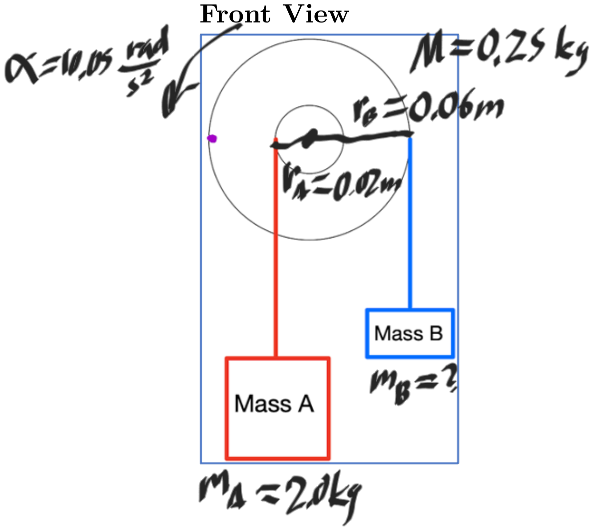
We can solve this for X, then substitute to eliminate the tension.

$$X = \frac{F^T\sin\theta\,\frac{7L}{8} - mg\frac{L}{2}}{Mg} = \frac{\frac{m+M}{2}\cancel{g}\frac{7L}{8} - m\cancel{g}\frac{L}{2}}{M\cancel{g}} = \frac{7M-m}{16M}L$$

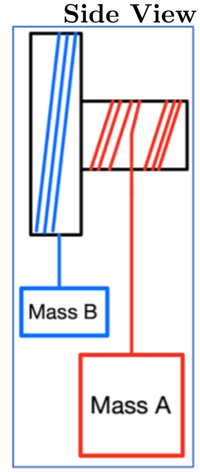
Wheel and Axle

Two masses, Mass A and Mass B, are attached with string to a wheel and axle as shown in the figure below. A purple dot is marked on the wheel to provide a reference point for observing the rotation.

Front View



Side View



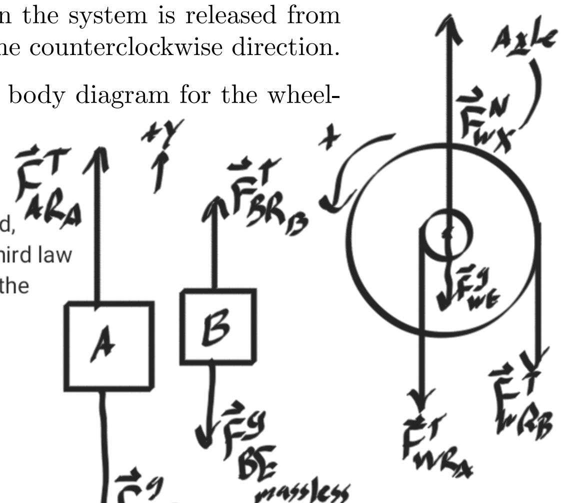
Mass A is attached at a radius of 0.02 m. Mass B is attached at a radius of 0.06 m. Assume that each is connected to the wheel and axle with enough string that they can rise or fall without hitting the wheel or ground.

We will approximate the moment of inertia of the wheel and axle as the same as the moment of inertia of a disk, $I = \frac{1}{2}mR^2$. Obviously not a perfect model, but good enough for now.

Mass A is 2.0 kg, and the wheel-and-axle has a mass of 0.25 kg. When the system is released from rest, the angular acceleration of the wheel-and-axle is 10.05 rad/s^2 in the counterclockwise direction.

1. Draw a Free Body Diagram for each block, and an extended free body diagram for the wheel-and-axle.

There are two particularly interesting things about these diagrams. First, we have the a normal force from the axle, and a force of gravity on the wheel. These are required to balance its forces, but not relevant to the problem, as the axis of rotation is through their point of application. Second, we cannot directly use Newton's 3rd law to link the tensions from the masses to the wheel. The third law links the tensions on the masses and wheel to the tensions on the ropes by those masses, while the massless string assumption tells us that the tensions are the same throughout each rope.



$$F^T_{AR_A} \overset{3rd}{=} F^T_{R_A A} \overset{\text{massless rope}}{=} F^T_{R_A W} \overset{3rd}{=} F^T_{WR_A} = F^T_A$$

$$F^T_{BR_B} \overset{3rd}{=} F^T_{R_B B} = F^T_{R_B W} = F^T_{WR_B} = F^T_B$$

simpler notation

2. Write Newton's Second Law (y-direction) for Mass A

$$\sum F_{Ay} = m_A a_A = F^T_A - m_A g$$

3. Write Newton's Second Law (y-direction) for Mass B

$$\sum F_{By} = m_B a_B = F^T_B - m_B g$$

4. Write Newton's Second Law (rotational) for the wheel-and-axle

Disk Assumption:

$$I \approx \frac{1}{2} M r_B^2$$

ignoring smaller hub

$$\sum \tau_w = I\alpha = F^T_A r_A - F^T_B r_B$$

5. How are the acceleration of mass A, acceleration of mass B, and and the angular acceleration of the wheel-and-axle related?

We know that the string connections force the masses to accelerate with the same tangential acceleration as the wheel at the points they are attached, which provides a natural link to the angular acceleration. Care must be taken with the sign; since mass A accelerates negatively when it falls, which corresponds to the wheel accelerating counterclockwise (positively), the mass' acceleration is equal to the negative of the tangential acceleration.

$$a_A = -\alpha r_A \qquad a_B = \alpha r_B$$

Now, we can substitute these accelerations into our previous equations, and link them all together to solve for the mass of B.

$$-m_A \alpha r_A = F_A^T - m_A g \Rightarrow F_A^T = m_A(g - \alpha r_A)$$

$$m_B \alpha r_B = F_B^T - m_B g \Rightarrow F_B^T = m_B(g + \alpha r_B)$$

$$\frac{1}{2} M r_B^2 \alpha = F_A^T r_A - F_B^T r_B$$

$$= m_A r_A (g - \alpha r_A) - m_B r_B (g + \alpha r_B)$$

$$\Rightarrow m_B = \frac{m_A r_A (g - \alpha r_A) - \frac{1}{2} M r_B^2 \alpha}{r_B (g + \alpha r_B)}$$

$$\approx 0.61\ \text{kg}$$

$m_A = 2.0\,\text{kg}$
$r_A = 0.02\,\text{m}$
$r_B = 0.06\,\text{m}$
$M = 0.25\,\text{kg}$
$\alpha = 10.05\ \frac{\text{rad}}{\text{s}^2}$
$g = 9.8\,\text{m/s}^2$