Two Blocks on a Frictionless Half-Ramp

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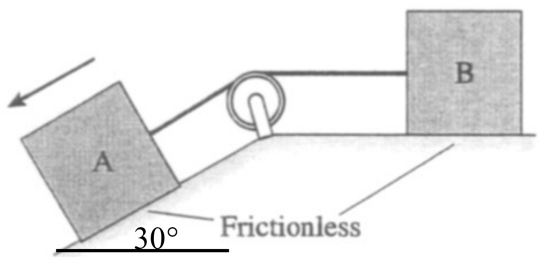
This material is borrowed/adapted from Chapter 7 of the *Student Workbook* for *Physics for Scientists and Engineers.*

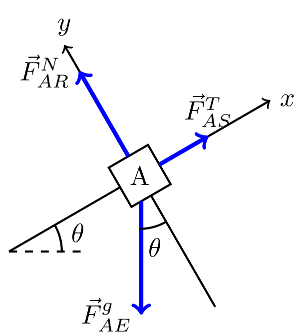
XX-1: Two Blocks on a Frictionless Half-Ramp

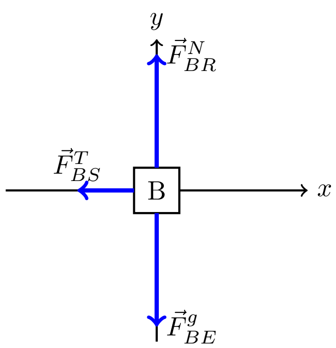
Consider the situation depicted below. Friction is negligible for the blocks and surface, and we will assume the strings and pulley are ideal (massless, frictionless, etc.).



(a) Draw a free-body diagram for each block.

I will indicate the half-ramp with the subscript R and the string with the subscript S.





This is one problem where it is imperative not to use G for "ground" to indicate the surface beneath the blocks. If you do, the force on block A by the ground will have a very unfortunate symbol: $\vec{F}_{AG}$. It may be rather uncomfortable to accidentally write something that looks like a slur in front of your students.

(b) Indicate the Newton's third law pairs.

There are no third law pairs. $\vec{F}^T_{AS}$ and $\vec{F}^T_{BS}$ are equal in magnitude, but they are not opposite in direction, and they are not directly between boxes A and B, instead going through the string and pulley between them.

However, since these two forces are equal in magnitude (due to the ideality of the pulley and the string), I will simplify my notation by giving their magnitudes a single symbol: $F^T = F^T_{AS} = F^T_{BS}$.

(c) Find the normal force on each block.

In order to do this, I will write out Newton's second law in the y-direction for both blocks. Neither is accelerating perpendicular to the surface, so the net force in this direction is zero.

$$\begin{aligned} F^{net}_{A,y} &= F^N_{AR,y} + F^g_{AE,y} \\ 0 &= F^N_{AR} - m_A g\cos\theta \\ F^N_{AR} &= m_A g\cos\theta \end{aligned} \qquad\qquad \begin{aligned} F^{net}_{B,y} &= F^N_{BR,y} + F^g_{BE,y} \\ 0 &= F^N_{BR} - m_B g \\ F^N_{BR} &= m_B g \end{aligned}$$

(d) Find the acceleration of the two blocks.

Both blocks accelerate in the x-direction, and since they are attached, they must have the same magnitude of acceleration. Also, since their coordinate systems agree on which direction along the surface is $+x$, the accelerations will also share the same sign. As such, I will use a single symbol for both accelerations: $a = a_{A,x} = a_{B,x}$.

$$\begin{aligned} F^{net}_{A,x} &= F^T_{AS,x} + F^g_{AE,x} \\ m_A a_{A,x} &= F^T - m_A g\sin\theta \\ m_A a &= F^T - m_A g\sin\theta \end{aligned} \qquad\qquad \begin{aligned} F^{net}_{B,x} &= F^T_{BS,x} \\ m_B a_{B,x} &= -F^T \\ m_B a &= -F^T \end{aligned}$$

Adding these two equations together gives us

$$\begin{aligned} (m_A + m_B)a &= F^T - m_A g\sin\theta + (-F^T) = -m_A g\sin\theta \\ a &= -\frac{m_A}{m_A + m_B} g\sin\theta \end{aligned}$$

The negative sign lets us know that these blocks will accelerate to the left.

In the special case where $m_B \ll m_A$, the fraction $\frac{m_A}{m_A+m_B}$ approaches 1, and we get $a = g\sin\theta$, which is the acceleration we expect from an object freely sliding down a ramp (which we expect to be the limiting case when there is no additional mass attached to A). Conversely, if $m_B \gg m_A$, then $\frac{m_A}{m_A+m_B} \approx \frac{m_A}{m_B} \approx 0$, and the blocks don't move. This we also expect, as a massive enough block B should not allow block A to move it, thus preventing any sliding.

(e) Find the tension in the string.

We already know from part (d) that $F^T = -m_B a$, so we can substitute our answer for the acceleration into this to obtain

$$F^T = \frac{m_A m_B}{m_A + m_B} g\sin\theta.$$

If we wanted to do more algebra, we could also start with both equations from (d) and solve them for F^T instead of for a:

$$\begin{aligned} m_A a &= F^T - m_A g\sin\theta \\ m_A m_B a &= m_B F^T - m_A m_B g\sin\theta \end{aligned} \qquad\qquad \begin{aligned} m_B a &= -F^T \\ m_A m_B a &= -m_A F^T \end{aligned}$$

Subtracting the right hand equation from the left hand equation gives us

$$\begin{aligned} m_A m_B a - m_A m_B a &= m_B F^T - m_A m_B g\sin\theta - (-m_A F^T) \\ 0 &= (m_A + m_B)F^T - m_A m_B g\sin\theta \\ F^T &= \frac{m_A m_B}{m_A + m_B} g\sin\theta. \end{aligned}$$

Our answer is positive (for all sensible angles from 0° to 90°), which should be the case for a magnitude of a vector. Sometimes, when we do force analysis and get a negative number, it just tells us that the direction we assumed for a force is backward from what it actually is, but in this case, getting a negative magnitude would tell us that something was wrong. After all, tension cannot push.

For $m_A \gg m_B$, we obtain $F^T \approx \frac{m_A m_B}{m_A} g \sin\theta = m_B g \sin\theta$. In this situation (as I mentioned in part (d)), block A is accelerating at $g \sin\theta$ down the ramp, so the force on block B needs to be just perfect to get it accelerating at $g \sin\theta$ to keep up.

Conversely, for $m_B \gg m_A$, we obtain $F^T \approx \frac{m_A m_B}{m_B} g \sin\theta = m_A g \sin\theta$. In this situation, both blocks are stationary (B is too massive to move), so the tension needs to be strong enough to counteract the x-component of the force of gravity on A and hold it in place.