PH 223 Week 6

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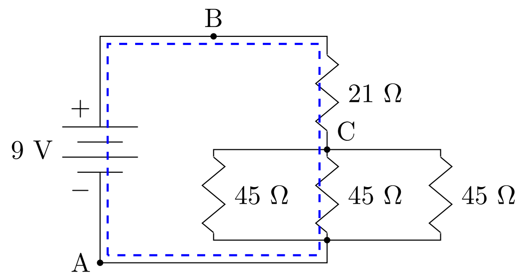
Winter 2024

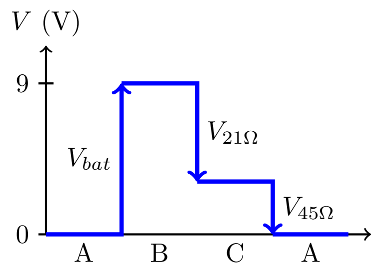
The first two problems are adapted from Grant Sherer. The final problem is adapted from Chapter 28 of the *Student Workbook* for *Physics for Scientists and Engineers*.

Activity 1

Three 45 Ω resistors are in parallel, connected to a 21 Ω resistor in series with a battery of potential difference 9 V.

(a) Draw a diagram of the circuit and a voltage diagram for at least one loop.

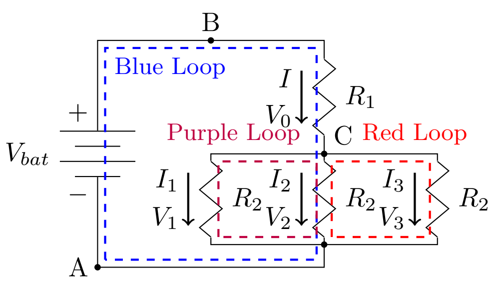




The voltage diagram depicts the voltage around the loop indicated in the circuit diagram by the dashed blue line. This diagram would have been valid through any of the three 45 Ω resistors. I have predicted that the voltage drop across the 21 Ω resistor will be larger, as there will be one third of the total current through each 45 Ω resistor:

$$|V_{21\Omega}| = I(21\ \Omega) > I(15\ \Omega) = \frac{I}{3}(45\ \Omega) = |V_{45\Omega}|.$$

(b) Write loop and junction rules that completely characterize the circuit. (Use each resistor in at least 1 loop rule and 1 junction rule.)



On the above picture, I have replaced each number from our original circuit diagram with a symbolic quantity, where $V_{bat} = 9$ V, $R_1 = 21\ \Omega$ and $R_2 = 45\ \Omega$. The total current from the battery (and thus the current through the solitary R_1) is I, and the voltage drop across the R_1

resistor is V_0. The three R_2 resistors have been given their own currents and voltage drops (we know these should be identical between these resistors, but we will leave this in more generality for now as we set up the loop rules), ordered from left to right. Note that V_0, V_1, V_2, and V_3 are negative quantities, as they are describing the change in voltage across the resistors when following the current flow.

I have also explicitly marked three loops. The blue loop goes through the battery, through R_1, and finally through the middle R_2. The purple loop goes through the left and middle R_2 resistors, while the red loop goes through the center and right R_2 resistors. Let us traverse all of these loops clockwise, which will be important when introducing signs in our loop rules.

We have two junctions (one at C and one unmarked below it), but they are both characterized by the same junction rule:

$$I = I_1 + I_2 + I_3.$$

The blue loop is characterized by the following loop rule:

$$V_{bat} + V_0 + V_2 = 0.$$

The purple loop is characterized by

$$-V_1 + V_2 = 0,$$

where a negative sign has been added to V_1 since we are going against the flow of current (turning the negative quantity V_1 into a positive voltage change). Similarly, the red loop is characterized by

$$-V_2 + V_3 = 0.$$

(c) Find R_{eq}.

For the three resistors in parallel, we have

$$\frac{1}{R_{eq,3}} = \frac{1}{R_2} + \frac{1}{R_2} + \frac{1}{R_2} = \frac{3}{R_2} \implies R_{eq,3} = \frac{R_2}{3}.$$

This trio is in series with R_1, so they add directly to gain the overall equivalent resistance

$$R_{eq} = R_1 + R_{eq,3} = R_1 + \frac{R_2}{3}.$$

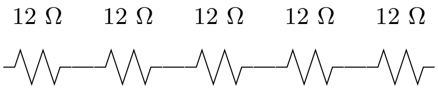
Plugging in our given numbers, we obtain $R_{eq} = 36\ \Omega$.

Activity 2

Sketch a circuit that uses any number of 12 Ω resistors to create an equivalent resistance of:

(a) 60 Ω

To get a larger resistance, we know we will need to be adding resistors in series. In particular, five 12 Ω resistors in series have an equivalent resistance of 60 Ω .



(b) 3 Ω

To get a smaller resistance, we know we will need to be adding resistors in parallel. In general, the equivalent resistance R_{eq} of adding n resistors (the ith resistor having resistance R_i) in parallel is given by

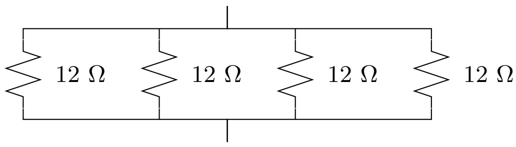
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}.$$

If the resistors are identical ($R_i = R$ for all i), then this simplifies to

$$\frac{1}{R_{eq}} = \frac{n}{R},$$

which means $R_{eq} = \frac{R}{n}$ for n identical resistors in parallel.

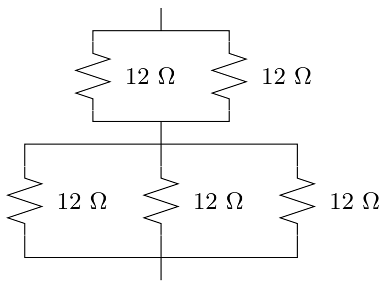
In this particular case, 3 is equal to 12 divided by 4, so we can achieve an equivalent resistance of 3 Ω by putting four 12 Ω resistors in parallel.



(c) 10 Ω

This equivalent resistance is smaller, so we will need to use parallel resistors, but it isn't as simple as part (b), as there is no integer which divides 12 to give 10. Still, we can put a few identical ones in parallel to give ourselves some (resistance-wise) smaller building blocks to work with. Once we have these smaller equivalent resistors, we can try to put some of them in series to increase our total resistance back toward the goal.

In particular, note that two 12 Ω resistors in parallel have an equivalent resistance of 6 Ω , and three of these resistors in parallel have an equivalent resistance of 4 Ω . By putting the two parallel resistors in series with the three parallel resistors, we get an overall equivalent resistance of 10 Ω .



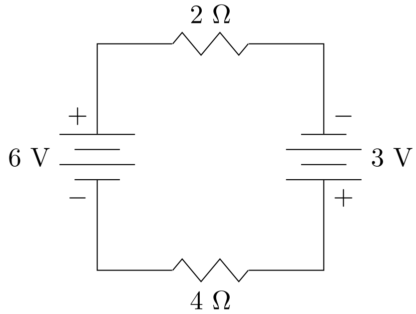
Activity 3

(a) Draw a circuit for which the Kirchhoff's loop law equation is

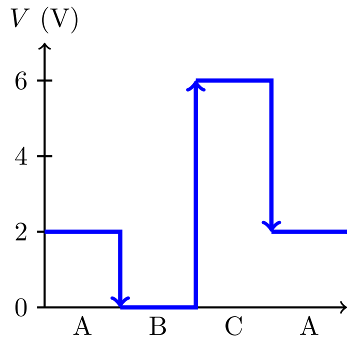
$$6\ \text{V} - I \cdot 2\ \Omega\ + 3\ \text{V} - I \cdot 4\ \Omega\ = 0.$$

Assume that the analysis is done in a clockwise direction.

First, note that $-I \cdot 2\ \Omega$ and $-I \cdot 4\ \Omega$ are voltage drops across resistors. The voltage change across a resistor is negative when we are tracing our loop in agreement with the direction of the current. As such, the current is flowing clockwise. The other two voltage differences appear to be gains from moving from the negative terminal of a battery to the positive terminal of a battery.



(b) A voltage diagram is shown below for a different circuit. The current in the circuit is 2.0 A. Draw the circuit diagram and identify the points on the circuit diagram that are on the voltage diagram.



Moving in agreement with the current, we see that the voltage goes down by 2 V when we cross the first circuit element (the horizontal regions in between the vertical arrows are wires acting as perfect conductors). A 1 Ω resistor would have this voltage drop for 2.0 A of current. The next circuit component increases the potential by six volts, which suggests that we have gone from the negative terminal to the positive terminal of a 6 V battery. Finally we drop 4 V in the last circuit element, and this can be accomplished by a 2 Ω resistor at 2.0 A.