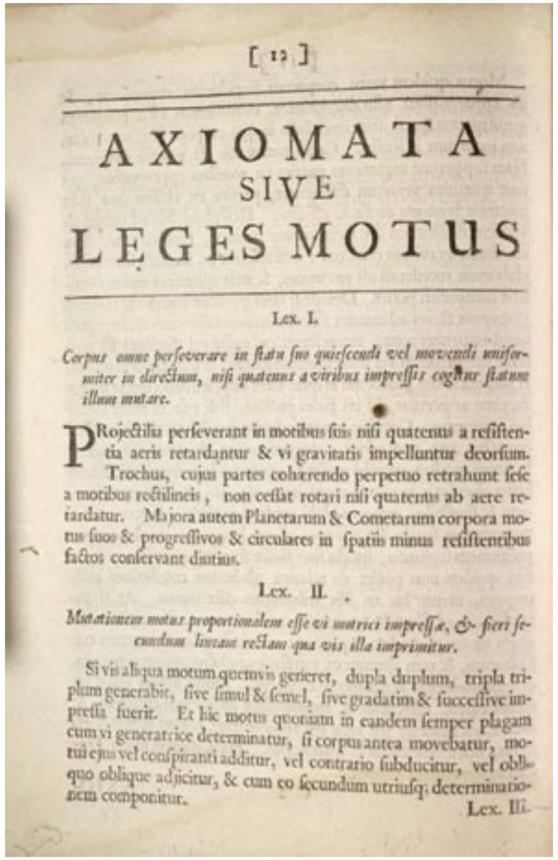
Studio Week 5
Laws of Motion


[12]

AXIOMATA
SIVE
LEGES MOTUS

Lex. I.

Corpus omne perseverare in statu suo quiescendi vel movendi uniformiter in directum, nisi quatenus a viribus impressis cogitur statum illum mutare.

Projectilia perseverant in motibus suis nisi quatenus a resistentia aeris retardantur & vi gravitatis impelluntur deorsum. Trochus, cujus partes cohaerendo perpetuo retrahunt sese a motibus rectilineis, non cessat rotari nisi quatenus ab aere retardatur. Majora autem Planetarum & Cometarum corpora motus suos & progressivos & circulares in spatiis minus resistentibus factos conservant diutius.

Lex. II.

Mutationem motus proportionalem esse vi motrici impressae, & fieri secundum lineam rectam qua vis illa imprimitur.

Si vis aliqua motum quemvis generet, dupla duplum, tripla triplum generabit, sive simul & semel, sive gradatim & successive impressa fuerit. Et hic motus quoniam in eandem semper plagam cum vi generatrice determinatur, si corpus antea movebatur, motui ejus vel conspiranti additur, vel contrario subducitur, vel obliquo oblique adjicitur, & cum eo secundum utriusq; determinationem componitur.

Lex. III.

Picture credit: Wikipedia.com.

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Make sense of everything.**

Activity 5-2: Book on the Wall

- You push a book against a vertical wall so the book does not move.
  - Sketch a picture of the book and the wall.
  - Identify and describe each force acting on the book.
  - Draw and label a free-body diagram for the book.
- What do you know about the magnitudes of the forces acting on the book?
- What happens to each force if you push harder?

5-2 Book and Wall

$\vec{F}^{g}_{BE}$: The force of gravity on the book by the Earth. (Points straight down.)

Normal Forces

Normal forces are always perpendicular to the surface of contact, and always push.

$\vec{F}^{N}_{BH}$: The normal force on the book by the hand. (Points to the right.)

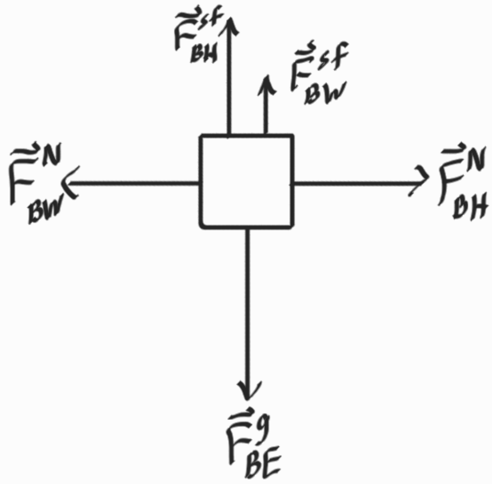
$\vec{F}^{N}_{BW}$: The normal force on the book by the wall. (Points to the left.)

Static Friction

Frictional forces are always parallel to the surface of contact. Static friction points in whatever direction is needed to prevent motion. To keep the book from falling down, both static frictions below must point straight up.

$\vec{F}^{sf}_{BH}$: The force of static friction on the book by the hand.

$\vec{F}^{sf}_{BW}$: The force of static friction on the book by the wall.



To keep $F^{net}_x = 0$, we must have $F^{N}_{BH} = F^{N}_{BW}$.

To keep $F^{net}_y = 0$, we need $F^{g}_{BE} = F^{sf}_{BH} + F^{sf}_{BW}$.

For static friction, we know $F^{sf} \leq \mu_s F^{N}$, so we can find the maximum possible static frictions, but that does not tell us what F^{sf}_{BH} and F^{sf}_{BW} are exactly. We don't even know if they are equal to each other.

Pushing Harder

$F^{g} = mg$, so it will remain constant.

F^{N}_{BH} is the push, and $F^{N}_{BW} = F^{N}_{BH}$, so both must increase.

$F^{sf}_{BH} + F^{sf}_{BW} = F^{g}_{BE}$, so $F^{sf}_{BH} + F^{sf}_{BW}$ is constant, but we don't really know about F^{sf}_{BH} and F^{sf}_{BW} individually.

Activity 5-3: Book on the Wall II

- You push a different book horizontally against a vertical wall so that the book slides downward at constant speed.
- You know the coefficients of friction are μ_b (between the book and the wall) and *zero* (between the book and your hand). You also know the mass m of the book.
  - Draw a free-body diagram for this situation.
  - Determine the magnitude of the force your hand must exert on the book.
  - Make sense of your answer in at least three different ways.

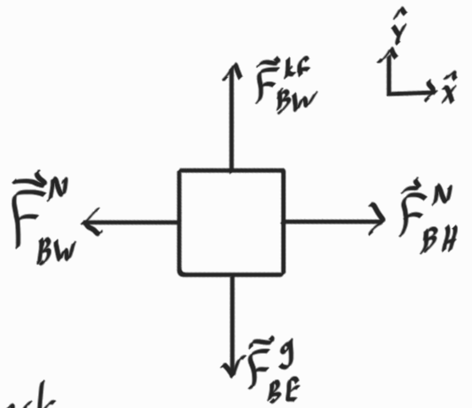
5-3 Book and Wall II

We know the mass m of the book, the coefficient of kinetic friction μ_{bw} between the book and the wall, and the coefficient of friction between the book and the hand $\mu_{BH}=0$.

As such, we have one less force (no F^{sf}_{BH}), and we are now working with kinetic friction: F^{kf}_{BW}.

Kinetic friction opposes motion, so if the book is sliding down the wall, $\vec{F}^{kf}_{BW}$ points up. Also, we have a formula that exactly tells us the magnitude of kinetic friction: $F^{kf}_{BW} = \mu_{bw} F^{N}_{BW}$.

Speed is constant, so $\vec{F}^{net} = \vec{0}$ still.



$$F^{net}_x = F^N_{BH} - F^N_{BW} = 0$$

$$\Rightarrow F^N_{BH} = F^N_{BW}$$

$$F^{net}_y = F^{kf}_{BW} - F^g_{BE} = 0$$

$$\Rightarrow F^{kf}_{BW} = F^g_{BE}$$

$$\Rightarrow \mu_{bw} F^N_{BW} = mg$$

$$\Rightarrow F^N_{BH} = \frac{mg}{\mu_{bw}}$$

The hand must exert a force of magnitude $F^N_{BH} = \dfrac{mg}{\mu_{bw}}$.

Unit Check

$$F^N_{BH} = \frac{mg}{\mu_{bw}}$$

F^N_{BH} is a force, so the units are newtons.

mg is a force, so the units are newtons.

Coefficients of friction are unitless.

We have newtons on both sides of the equal sign.

Covariation

If the book gets heavier (m increases, or we do this on a planet with larger g than Earth), then we should expect it to be harder to support.

$F^N_{BH} = \dfrac{mg}{\mu_{bw}}$, so if mg increases, F^N_{BH} increases, as we expect.

If the wall gets rougher or grippier (μ_{bw} increases), then it should be easier to keep the book from falling faster.

$F^N_{BH} = \dfrac{mg}{\mu_{bw}}$, so if μ_{bw} increases, F^N_{BH} decreases, as we expect.

Special Case

If the wall is frictionless ($\mu_{bw}=0$), then nothing will keep the book from accelerating downward, so it is impossible to push it hard enough.

$F^N_{BH} = \dfrac{mg}{0} = \infty$ Infinite normal force is impossible.

Activity 5-4: Suitcase on a Ramp

- A suitcase of mass m is at rest on a ramp that makes an angle θ with the horizontal. You know the coefficients of friction (μ_s and μ_k) between the suitcase and the ramp.
  - Following the ARCS format, determine the possible angles for which the suitcase remains at rest.

5-4 Suitcase on a Ramp

Analyze & Represent

1a) Understand the Problem

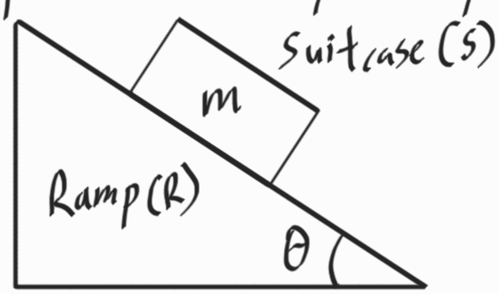
Known: m (mass of suitcase), μ_s & μ_k (coefficients of static & kinetic friction between the suitcase and the ramp)

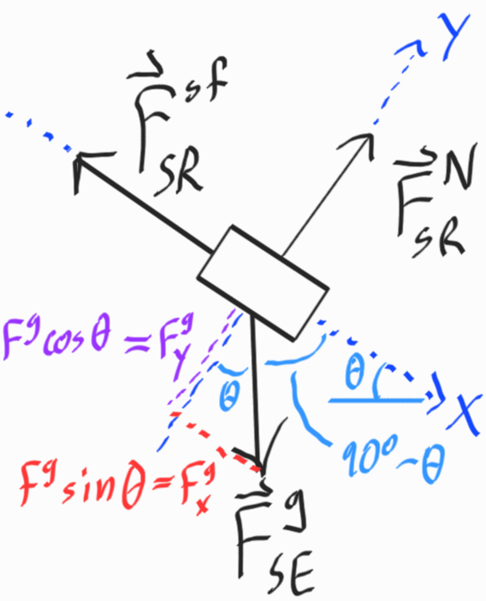
Unknown: θ (allowed angles of ramp without the suitcase slipping)

1b) Identify Assumptions

- μ_s & μ_k are constant
  - Real surfaces are not uniformly perfect, so μ could vary depending on where the objects are in contact. We will not worry about this to make the problem tractable.
- The suitcase is a particle.
  - A real suitcase has mass distributed unevenly inside of it, and objects inside could shift if not packed tightly. We need to ignore these considerations to solve the problem
- The suitcase is near the surface of Earth.
  - This lets us use $F^g = mg$ with $g \approx 9.8\,m/s^2$.

1c) Represent Physically





Since there is no possibility of mixing up these forces, let's simplify the notation: $\vec{F}^{sf} = \vec{F}^{sf}_{SR}$, $\vec{F}^{N} = \vec{F}^{N}_{SR}$, $\vec{F}^{g} = \vec{F}^{g}_{SE}$

Calculate

2a) Represent Principles

Known Unknown Wanted

$$\vec{F}^{net} = \sum_i \vec{F}_i$$

$$\vec{F}^{net} = m\vec{a} \quad \text{(Newton's 2nd law)}$$

$$F^{sf} \leq \mu_s F^N$$

$$\cancel{F^{kf} = \mu_k F^N}$$

The suitcase is not slipping, so we don't need this one.

$$\vec{F}^g = m\vec{g}$$

Trigonometry:

$$F^g_x = F^g \sin\theta$$

$$F^g_y = -F^g \cos\theta$$

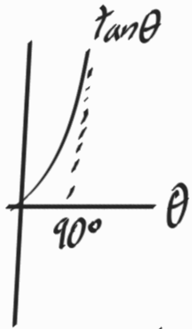
2b) Find Unknown(s) Symbolically

$$0 = ma_x = F^{net}_x = F^g_x - F^{sf} = F^g \sin\theta - F^{sf}$$

$$\Rightarrow F^g \sin\theta = F^{sf}$$

$$0 = ma_y = F^{net}_y = F^N + F^g_y = F^N - F^g \cos\theta$$

$$\Rightarrow F^N = F^g \cos\theta$$



$$F^{sf} \leq \mu_s F^N \Rightarrow \cancel{F^g}\sin\theta \leq \mu_s \cancel{F^g}\cos\theta$$

$$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta} \leq \mu_s$$

$$\Rightarrow \boxed{\theta \leq \tan^{-1}(\mu_s)}$$

Sensemake

$\tan\theta$ is monotone increasing between its asymptotes, so $\tan\theta \leq \mu_s$ if and only if $\theta \leq \tan^{-1}(\mu_s)$

2c) Plug in Numbers

The coefficients of friction used here were taken from the back of Physics for Scientists and Engineers by R. D. Knight.

This setup is more general than a suitcase on a ramp.
Suppose we had a wood block on a wood ramp ($\mu_s = 0.50$):

$$\theta \leq \tan^{-1}(0.50) \approx 27^\circ$$

Suppose we had a rubber-soled shoe on a dry concrete ramp ($\mu_s = 1.0$):

$$\theta \leq \tan^{-1}(1.00) = 45^\circ$$

Sensemake

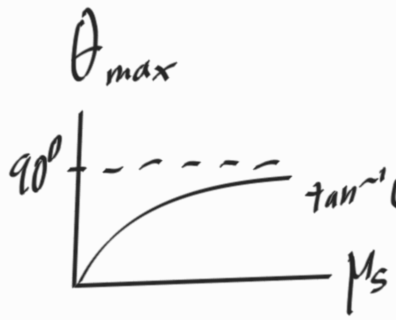
3a) Units

The tangent function takes an angle and outputs a unitless quantity; μ_s is unitless, so $\tan\theta \leq \mu_s$ has the correct units on either side of the inequality.

3b) Numbers

The calculations above suggest that a wood block would slip on a wood ramp at an angle above 27°, while a shoe on a concrete ramp would slip at an angle above 45°. Shoes are designed to grip, so it makes sense that they could hold on to steeper surfaces.

3c) Symbols $\theta_{max} = \tan^{-1}(\mu_s)$



θ increases as μ_s increases, which suggests that a rougher, grippier surface can keep the suitcase from sliding at steeper angles, which is a reasonable expectation.

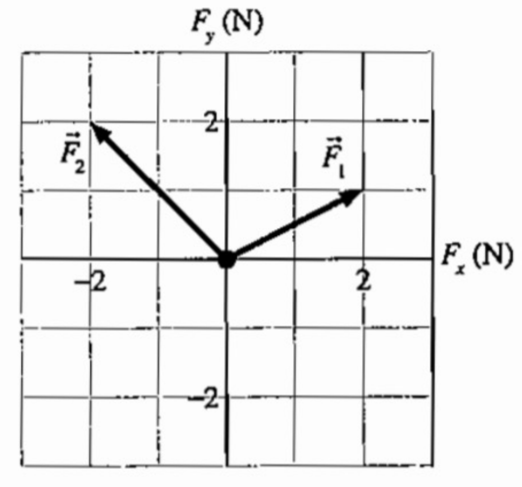
Note that θ_{max} may approach but never be equal to 90°. No matter how much friction there is, a suitcase cannot sit suspended on a vertical wall.

Note that $\theta_{max} = 0$ if $\mu_s = 0$. An object on a frictionless surface only stays put if the surface is flat.

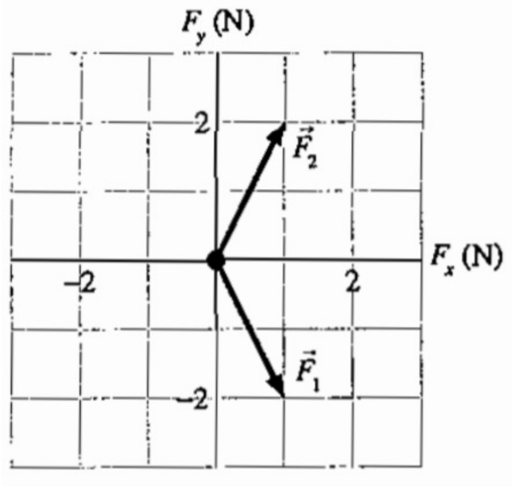
Activity 5-5: Forces

- Three forces cause a 0.5 kg object to accelerate as indicated.
- Two of the forces are shown on the free-body diagrams, but the third is missing.
- For each, draw and label the missing force vector on the grid.
- Challenge: draw and label the missing force if the object instead moves with constant velocity.

a = 2 m/s² to the right



a = 3 m/s² downward



5-5 Missing Forces

Our net force is the sum of three forces, and we can relate the net force to the acceleration:

$$m\vec{a} = \vec{F}^{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3$$

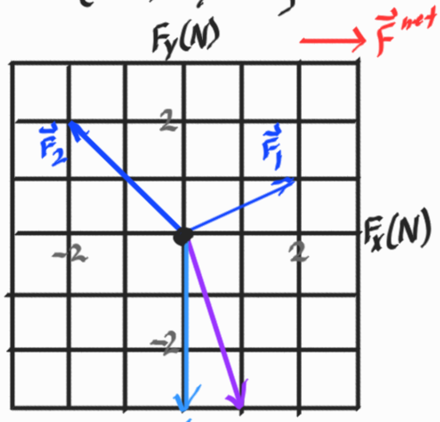
We know $\vec{F}_1$ and $\vec{F}_2$, so we want to find $\vec{F}_3$ such that we get the right $\vec{F}^{net}$. This can be done purely graphically, or numerically, as

$$\vec{F}_3 = m\vec{a} - (\vec{F}_1 + \vec{F}_2)$$

$$\left.\begin{array}{l} m = 0.5\ \text{kg} \\ a = (2\ \text{m/s}^2)\,\hat{x} \end{array}\right\} \vec{F}^{net} = (1\text{N})\,\hat{x}$$

$\vec{F}_1 = (2\text{N})\,\hat{x} + (1\text{N})\,\hat{y}$

$\vec{F}_2 = (-2\text{N})\,\hat{x} + (2\text{N})\,\hat{y}$



$\vec{F}_1 + \vec{F}_2 = (3\text{N})\,\hat{y}$

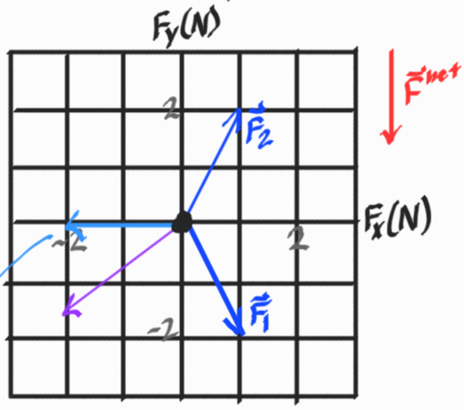
$\vec{F}_3 = \vec{F}^{net} - (\vec{F}_1 + \vec{F}_2) = (1\text{N})\,\hat{x} - (3\text{N})\,\hat{y}$

If we only wanted constant speed, $\vec{F}_3 = (-3\text{N})\,\hat{y}$ would be enough to cancel the other two forces.

$$\left.\begin{array}{l} m = 0.5\ \text{kg} \\ a = (-3\ \text{m/s}^2)\,\hat{y} \end{array}\right\} \vec{F}^{net} = (-1.5\text{N})\,\hat{y}$$

$\vec{F}_1 = (1\text{N})\,\hat{x} + (2\text{N})\,\hat{y}$

$\vec{F}_2 = (1\text{N})\,\hat{x} - (2\text{N})\,\hat{y}$



$\vec{F}_1 + \vec{F}_2 = (2\text{N})\,\hat{x}$

$\vec{F}_3 = \vec{F}^{net} - (\vec{F}_1 + \vec{F}_2) = (-1.5\text{N})\,\hat{y} - (2\text{N})\,\hat{x}$

If we only wanted constant speed, $\vec{F}_3 = (-2\text{N})\,\hat{x}$ would be enough to cancel the other two forces.