Studio 5: Interacting Systems

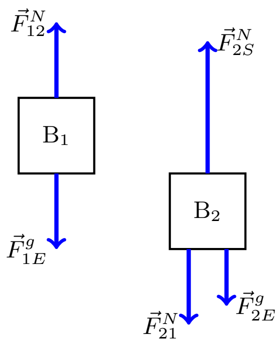
Announcements

- Homework will now be turned in on Gradescope.
- Feedback will be viewable directly on Gradescope.
- Feedback PDFs generated by Gradescope will be attached to the assignment in Canvas (so you will still be able to get feedback the way you did before, if you prefer).
- Starting **next week**, Get-Ready assignments will also be submitted on Gradescope.

Let Book 1 (B_1) be the larger book on top, and let Book 2 (B_2) be the smaller book on the bottom. S will symbolize the surface of the elevator floor.

(A) & (B)

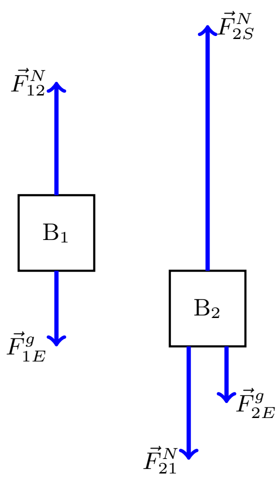
When the elevator is at rest or moving downward at constant velocity, the acceleration is zero, and all forces must cancel. As such, we actually have the same free-body diagrams for both situations:



We know that $F^N_{21} = F^N_{12}$ by virtue of being third-law pairs. Since $\vec{a} = \vec{0}$, we also have that $F^g_{1E} = F^N_{12}$ by Newton's 2nd law. Even though they are equal and opposite, they are not a third-law pair (due to being different types of forces, only on one FBD, and not between the same two objects).

(C)

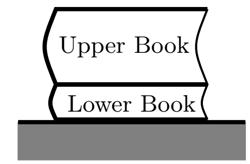
When the elevator accelerates upward, the net force on each book must point upward, so the forces do not balance anymore. We need $\vec{F}^N_{12}$ to get larger (which in turn means $\vec{F}^N_{21}$ gets larger, as they are still third law pairs), and we need F^N_{2S} to increase as well.



As I said, we still have the third law to tell us that $F^N_{21} = F^N_{12}$. However, We need $F^N_{12} > F^g_{1E}$ and $F^N_{2S} > F^N_{21} = F^g_{2E}$ in order for the acceleration to be nonzero.

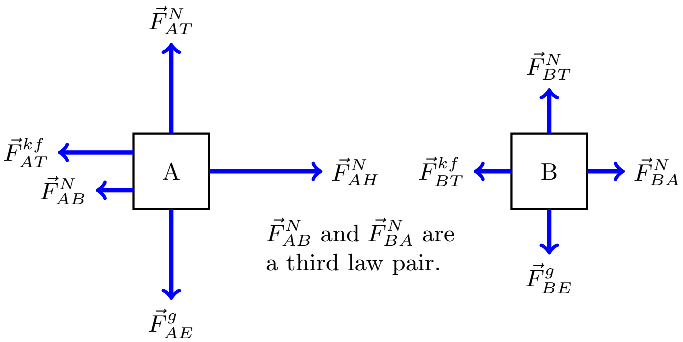
S5-1: Book Stack

- The stack of books below is sitting in an elevator. Consider the following situations:
  - (A) The elevator is at rest.
  - (B) The elevator is moving downward at a constant velocity.
  - (C) The elevator is accelerating upward.
- For each situation:
  - Draw a free-body diagram for each book.
  - Identify all third-law pairs.
  - Determine if any forces are equal in magnitude.



At constant speed, $a_A = a_B = 0$, so $F^{net}_A = F^{net}_B = 0$.

Since the two stacked bricks are together in system A, the normal forces and forces of static friction between them are internal to the forces. We only put external forces on free-body diagrams.



$$\underset{\text{2nd}}{F^N_{AH} >} F^{kf}_{AT} \underset{\text{2nd \& kinetic friction}}{>} F^{kf}_{BT} \underset{\text{2nd}}{=} F^N_{BA} \underset{\text{3rd}}{=} F^N_{AB}$$

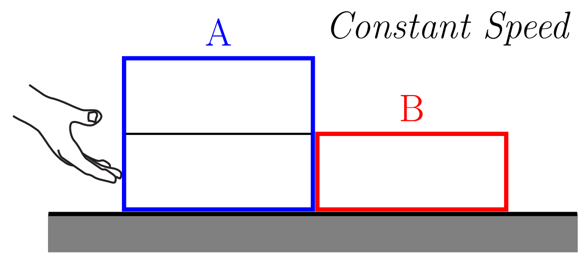
$$\begin{aligned} F^{net}_{Ax} &= F^N_{AH} - F^{kf}_{AT} - F^N_{AB} \\ &= m_A a_{Ax} = 0 \end{aligned}$$

$$\begin{aligned} F^{net}_{Bx} &= F^N_{BA} - F^{kf}_{BT} \\ &= m_B a_{Bx} = 0 \end{aligned}$$

$$F^{kf}_{AT} = \mu_k F^N_{AT} \underset{\substack{\text{2nd law} \\ y\text{-direction} \\ \text{system A}}}{=} \mu_k m_A g \underset{\substack{\text{A has 2} \\ \text{bricks}}}{=} 2\mu_k m_B g \underset{\substack{\text{2nd law} \\ y\text{-direction} \\ \text{system B}}}{=} 2\mu_k F^N_{BT} = 2F^{kf}_{BT}$$

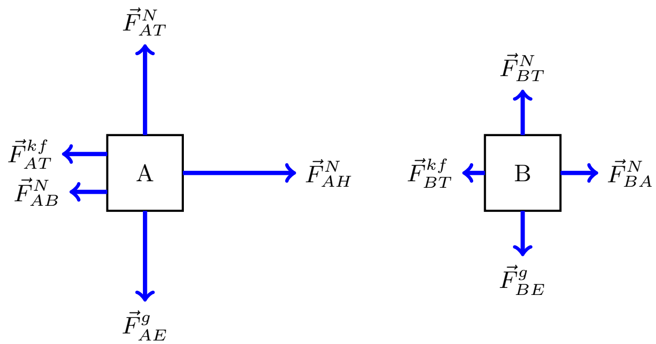
S5-2: Moving Bricks I

- Three identical bricks are pushed across a table at *constant speed* as shown. The hand pushes horizontally. There is friction.



- System A is the left (stacked) bricks, and system B is the right brick.
  - Compare the *net force* on system A to that on system B.
  - Draw separate free-body diagrams for system A and system B.
  - Identify all of the Newton's third law (action-reaction) force pairs.
  - Rank the *horizontal* forces by magnitude, from largest to smallest.
  - Explain how you used Newton's laws.

With μ_k lower, the forces of friction decrease (by the same factor), therefore the boxes will now be accelerating together. Since $\vec{a}_A = \vec{a}_B$ and $m_A > m_B$, we know that $F_A^{net} > F_B^{net}$.



F^N_{AB} and F^N_{BA} do not change when the friction changes. The argument is a bit involved, and requires both the 2nd and 3rd laws.

$$\begin{aligned}
m_B a &= F^{net}_{Bx} \\
&= F^N_{BA} - F^{kf}_{BT} \\
&= F^N_{AB} - \mu_k m_B g \\
m_A a &= F^{net}_{Ax} \\
&= F^N_{AH} - F^{kf}_{AT} - F^N_{AB} \\
&= F^N_{AH} - \mu_k m_A g - F^N_{AB} \\
2 m_B a &= F^N_{AH} - 2\mu_k m_B g - F^N_{AB}
\end{aligned}$$

The third and seventh lines together give us

$$F^N_{AH} - 2\mu_k m_B g - F^N_{AB} = 2(F^N_{AB} - \mu_k m_B g),$$

and rearranging this and canceling some terms gives us $F^N_{AH} = 3F^N_{AB}$, which has no dependence on μ_k.

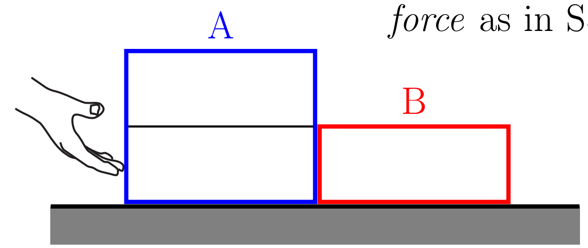
We need to know exactly how much μ_k changes in order to know if F^{kf}_{AT} got smaller than F^N_{AB}, so we cannot completely rank the forces anymore. Other than that, the reasoning is the same as in S5-2:

$$F^N_{AH} \overset{\text{2nd}}{>} F^N_{AB} \overset{\text{3rd}}{=} F^N_{BA} \overset{\text{2nd}}{>} F^{kf}_{BT}, \qquad F^N_{AH} \overset{\text{2nd}}{>} F^{kf}_{AT} \underset{\text{\& kf}}{\overset{\text{2nd}}{>}} F^{kf}_{BT}.$$

S5-3: Moving Bricks II

- The hand still pushes horizontally, but the coefficient of friction is less than it was in S5-2.

Hand pushes with *same force* as in S5-2.

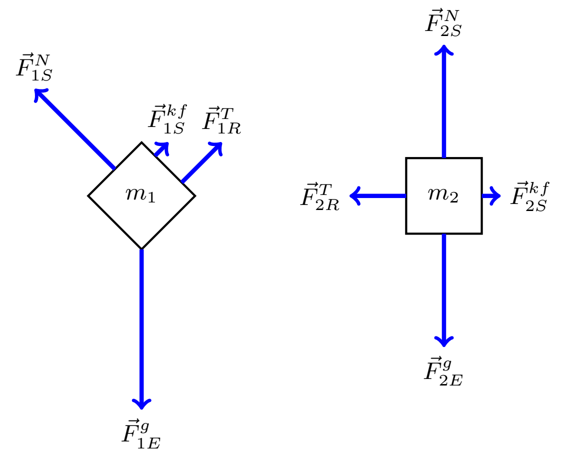


Coefficient of friction *less than* in S5-2.

  - How does the motion of the blocks change, if at all?
  - Compare the *net force* on system A to that on system B.
  - Draw separate free-body diagrams for system A and system B.
  - Rank the *horizontal* forces by magnitude, from largest to smallest. (Is it possible to rank the horizontal forces *completely*?)
  - Explain how you used Newton's laws.

First, you do not need to assume that $m_1 > m_2$ for this setup to work. You can imagine that, in the frictionless case, this system would certainly start to slide, and low friction cases should approach the outcome of the frictionless version.

If the system accelerates from rest, it will start to slide left (sliding right, uphill for m_1, makes no physical sense in this situation), so kinetic friction will point to the right, along the surface.

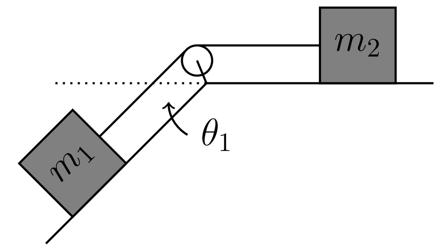


There are no 3rd law pairs!
The two tensions are equal ($F_{1R}^T = F_{2R}^T$), but not directly because of the third law. Third law pairs must be between two directly interacting objects (in the notation, the subscripts should reverse: $\vec{F}_{AB}$ versus $\vec{F}_{BA}$). Third law pairs must also be equal and opposite ($\vec{F}_{AB} = -\vec{F}_{BA}$), but $\vec{F}_{1R}^T$ and $\vec{F}_{2R}^T$ do not point in opposite directions.

- $\vec{F}_{1R}^T$ is in a 3rd law pair with $\vec{F}_{R1}^T$ (the force of tension from m_1 on the rope).
- $\vec{F}_{2R}^T$ is in a 3rd law pair with $\vec{F}_{R2}^T$ (the force of tension from m_2 on the rope).
- $F_{R2}^T = F_{R1}^T$ due to two assumptions: (i) an ideal (massless and inextensible) rope, and (ii) an ideal pulley.
  (i) keeps the tensions constant throughout lengths of free rope
  (ii) keeps the tensions equal across the pulley
- In summary: $F_{1R}^T \overset{\text{3rd}}{=} F_{R1}^T \underset{\text{(ii)}}{\overset{\text{(i)}}{=}} F_{R2}^T \overset{\text{3rd}}{=} F_{2R}^T$.

S5-4: Angled Ramp

- Draw separate free-body diagrams for both objects in the situation below.
  - Assume there is friction between all blocks and surfaces.
  - Assume each system accelerates from rest.
- Identify all of the Newton's third law (action-reaction) force pairs.



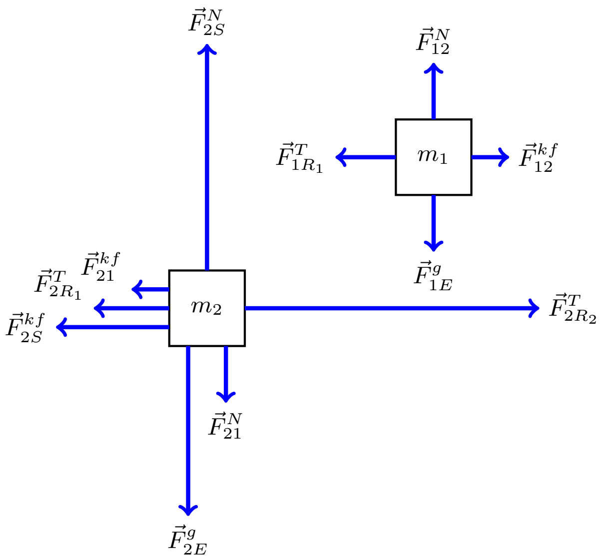
Let us call the rope that goes around the pulley R_1, and the rope pulled by the hand R_2. I am comfortable giving the entire rope around the pulley a single label, as using the ideal string and ideal pulley assumptions allow me to keep the tension constant in the rope. In a more general circumstance, I might give different names to the parts of the rope separated by the pulley.

Counting Forces

- Each block has one long-range force (gravity).
- m_1 has 3 contact forces. The rope R_1 is pulling it (that's one, tension) and it is touching m_2 (that's two, normal and friction).
- m_2 has 6 contact forces. There are two ropes pulling it, and it touches both the surface below (two more forces) and m_1 above (two more forces).

Free-Body Diagrams

If the system accelerates from rest, the top block must be pulled left, and the bottom block must be pulled right. First, this tells us that the net force on m_1 must be to the left, and the net force on m_2 must be to the right. Furthermore, based on the relative motion between the two boxes, the friction on m_1 by m_2 must point right, and the friction on m_2 by m_1 must point left.



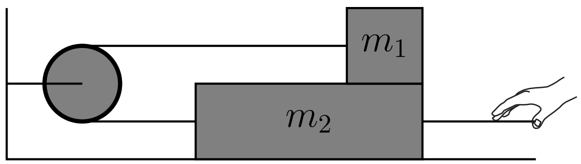
3rd Law Pairs

- $\vec{F}^N_{12} = -\vec{F}^N_{21}$
- $\vec{F}^{kf}_{12} = -\vec{F}^{kf}_{21}$

Even though $\vec{F}^T_{1R_1}$ and $\vec{F}^T_{2R_1}$ are equal in magnitude, they are not a 3rd law pair. They point in the same direction, and these forces are not part of an interaction that is directly between the two masses (the rope acts as an intermediary).

S5-5: Horizontal Pulley

- Draw separate free-body diagrams for both objects in the situation below.
  - Assume there is friction between all blocks and surfaces.
  - Assume the strings and the pulley are ideal.
  - Assume each system accelerates from rest.
- Identify all of the Newton's third law (action-reaction) force pairs.



Main Ideas

- Newton's 3rd law of motion can be used to relate the forces acting on *different* objects or systems.