PH 222 Activity 1

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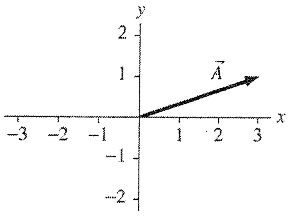
Winter 2021

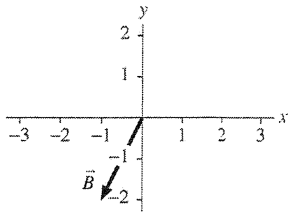
This material is borrowed/adapted from Chapter 3 of the *Student Workbook* for *Physics for Scientists and Engineers*, as well as from the prior term's material.

Activity 1

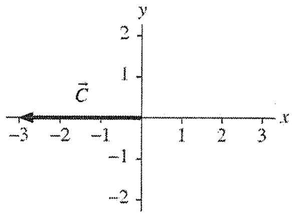
Write the vector in component form (e.g. $3\hat{\imath} + 2\hat{\jmath}$).



$\vec{A} =$ $3\hat{\imath} + \hat{\jmath}$



$\vec{B} =$ $-\hat{\imath} - 2\hat{\jmath}$



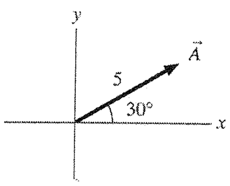
$\vec{C} =$ $-3\hat{\imath}$

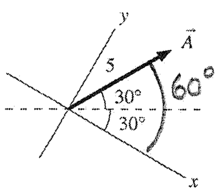
What is the vector sum $\vec{D} = \vec{A} + \vec{B} + \vec{C}$ of the three aforementioned vectors? Write your answer in component form.

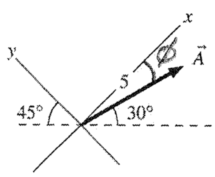
$$\vec{D} = (3 - 1 - 3)\hat{\imath} + (1 - 2 + 0)\hat{\jmath} = -\hat{\imath} - \hat{\jmath}$$

Activity 2

Define vector $\vec{A} = (5,\ 30^\circ$ above the horizontal). Determine the components A_x and A_y in the three coordinate systems shown below. Show your work below the figure.







$\phi = 45^\circ - 30^\circ$
$= 15^\circ$

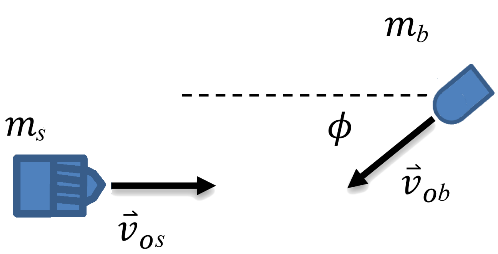
$A_x =$ $5\cos 30^\circ \simeq 4.33$
$A_y =$ $5\sin 30^\circ = 2.50$

$A_x =$ $5\cos 60^\circ = 2.50$
$A_y =$ $5\sin 60^\circ \simeq 4.33$

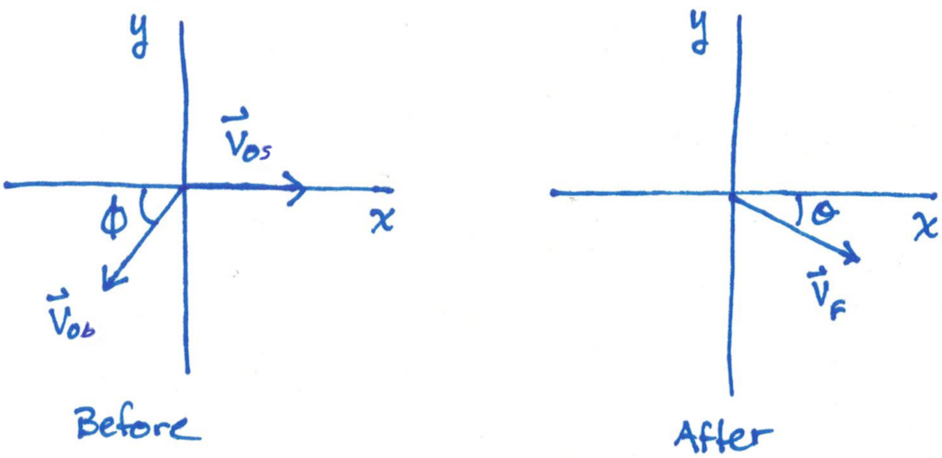
$A_x =$ $5\cos 15^\circ \simeq 4.83$
$A_y =$ $-5\sin 15^\circ \simeq -1.29$

Activity 3

The Fabulous Topperweins, Elizabeth and her husband Ad, are attempting a truly daring trick shot. Ad fires his shotgun due east, with the slug having speed v_{0s} and mass m_s. Simultaneously, Elizabeth fires her pistol from the northeast, at an angle ϕ south of west. Her bullet (with mass m_b) strikes Ad's slug with speed v_{0b}. The impact releases enough heat to melt the two projectiles together. After the collision, the bullet and slug fly as one body at some unknown velocity. Find the final speed and direction angle of the two projectiles after the collision.



(A) Establish a coordinate system and clearly label the axes. Use this coordinate system to draw before and after pictures of the collision, clearly showing/labeling vectors and angles. Write down what is given (known) from the problem statement and what you want to find (unknowns).



Known: $m_s,\ m_b,\ v_{0s},\ v_{0b},\ \phi$

Unknown: $v_f = \sqrt{v_{fx}^2 + v_{fy}^2},\ \theta = \arctan(v_{fy}/v_{fx})$

(B) Write down the conservation of momentum equation(s) in symbolic form for the collision using your coordinate system from part (A). Do not solve or simplify.

When momentum is conserved, we know that the sum of all initial momenta is equal to the sum of all final momenta. In this particular case, that means

$$m_s\vec{v}_{0s} + m_b\vec{v}_{0b} = (m_s + m_b)\vec{v}_f.$$

Breaking this expression into components, we obtain

$$\begin{aligned} m_s v_{0s} - m_b v_{0b}\cos\phi &= (m_s + m_b)v_{fx}; \\ -m_b v_{0b}\sin\phi &= (m_s + m_b)v_{fy}. \end{aligned}$$

(C) Suppose that $v_{0s} = 500$ m/s, $m_s = 0.025$ kg, $v_{0b} = 600$ m/s, $m_b = 0.01$ kg, and $\phi = 30°$. Determine the final speed of the two projectiles after the collision. Check units.

$$\begin{aligned}
v_f &= \sqrt{v_{fx}^2 + v_{fy}^2} \\
&= \sqrt{\left(\frac{m_s v_{0s} - m_b v_{0b}\cos\phi}{m_s + m_b}\right)^2 + \left(\frac{-m_b v_{0b}\sin\phi}{m_s + m_b}\right)^2} \\
&= \frac{1}{m_s + m_b}\sqrt{m_s^2 v_{0s}^2 - 2 m_s m_b v_{0s} v_{0b}\cos\phi + m_b^2 v_{0b}^2 \cos^2\phi + m_b^2 v_{0b}^2 \sin^2\phi} \\
&= \frac{1}{m_s + m_b}\sqrt{m_s^2 v_{0s}^2 - 2 m_s m_b v_{0s} v_{0b}\cos\phi + m_b^2 v_{0b}^2} \\
&\approx \frac{1}{0.035\ \text{kg}}\sqrt{156.25\ \text{kg}^2\text{m}^2/\text{s}^2 - 129.9\ \text{kg}^2\text{m}^2/\text{s}^2 + 36\ \text{kg}^2\text{m}^2/\text{s}^2} \\
&= \frac{1}{0.035\ \text{kg}}\sqrt{62.35\ \text{kg}^2\text{m}^2/\text{s}^2} \\
&\approx 225\ \text{m/s}
\end{aligned}$$

(D) Suppose that $v_{0s} = 500$ m/s, $m_s = 0.025$ kg, $v_{0b} = 600$ m/s, $m_b = 0.01$ kg, and $\phi = 30°$. Determine the direction angle of the two projectiles after the collision. Check units.

$$\begin{aligned}
\theta &= \arctan\left(\frac{v_{fy}}{v_{fx}}\right) \\
&= \arctan\left(\frac{-m_b v_{0b}\sin\phi}{m_s v_{0s} - m_b v_{0b}\cos\phi}\right) \\
&= \arctan\left(\frac{-(6\ \text{kg m/s})\sin(30°)}{12.5\ \text{kg m/s} - (6\ \text{kg m/s})\cos(30°)}\right) \\
&\approx \arctan\left(\frac{-3\ \text{kg m/s}}{7.3\ \text{kg m/s}}\right) \\
&\approx -22.3°
\end{aligned}$$

It is important to check whether or not this angle makes sense. Since v_{fx} is positive and v_{fy} is negative, we know that our vector should be in the fourth quadrant. Pointing 22.3 degrees below the horizontal is consistent with our expectations.