Spring Reasoning

Benjamin Bauml

Spring 2024

This material is borrowed/adapted from Chapter 9 of the *Student Workbook* for *Physics for Scientists and Engineers.*

XX-1: Spring Reasoning

A spring has an unstretched length of 10 cm. It exerts a restoring force F when stretched to a length of 11 cm.

(a) For what length of the spring is its restoring force $3F$?

Let us keep this very general for now. An ideal spring obeys Hooke's law:

$$\vec{F}_{sp} = -k\Delta\vec{s},$$

At this point, it is probably worth pointing out the negative sign explicitly and going over the meaning with the students.

where k is the spring constant, and $\Delta\vec{s}$ is the displacement of the end of the spring from its equilibrium position. Taking components along the s-direction, we maintain the sign relationship between the force and the displacement, but remove the vector notation:

$$F_{sp,s} = -k\Delta s.$$

Let us have two situations: Case A, where we have some displacement Δs_A and a restoring force $F_{sp,s,A} = -k\Delta s_A$, and Case B, where we have another displacement Δs_B and a restoring force $F_{sp,s,B} = -k\Delta s_B$. Consider what happens when we divide one equation by the other:

$$\begin{aligned} F_{sp,s,A} &= -k\Delta s_A \\ \div(F_{sp,s,B} &= -k\Delta s_B) \\ \implies \frac{F_{sp,s,A}}{F_{sp,s,B}} &= \frac{-k\Delta s_A}{-k\Delta s_B} = \frac{\Delta s_A}{\Delta s_B}. \end{aligned}$$

This is a great opportunity to go over the different ways of solving systems of equations with the students. Many of them seem to rely on solving for a variable and substituting it, but they could be dividing one equation by another, or adding whole equations together to cancel out terms.

From this, we can see that if we triple the force from Case A to Case B, we must also triple the displacement. For our particular situation, we have a positive displacement $\Delta s = 11\text{ cm} - 10\text{ cm} = 1\text{ cm}$ and therefore a negative restoring force $-F$. If we desire triple the restoring force ($-3F$), we get

$$\frac{-F}{-3F} = \frac{1\text{ cm}}{\Delta s_B} \implies \Delta s_B = 3\text{ cm}.$$

Thus, the spring's total length must be 13 cm.

(b) At what compressed length is the restoring force $2F$?

For a compressed spring, the displacement is negative and the restoring force is positive. We have doubled the magnitude of the force, so the displacement must also be doubled in magnitude:

$$\frac{-F}{2F} = \frac{1 \text{ cm}}{\Delta s_B} \implies \Delta s_B = -2 \text{ cm}.$$

Thus, the spring must be 2 cm shorter than its equilibrium length, or 8 cm long.