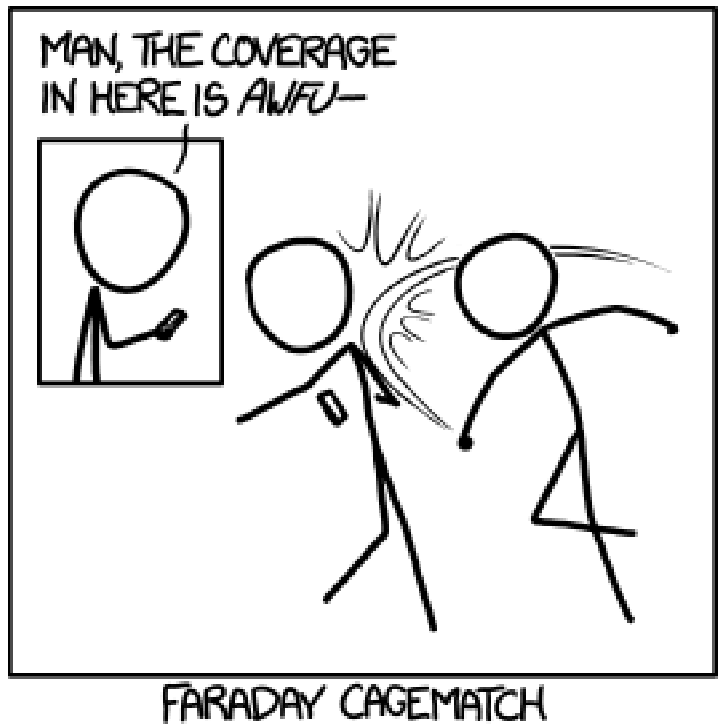
Studio Week 5
Potential and Field



Picture credit: xkcd.com.

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

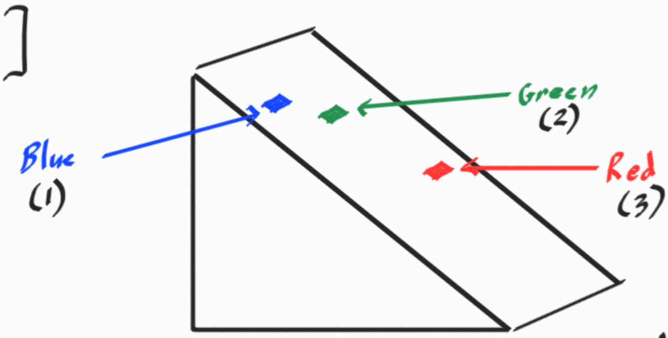
Activity 5-1

- Before you is a plastic surface representing the electric potential between two charged plates. A 1 cm height difference corresponds to an electric potential difference of 1 V.
  1. Rank the three points by the value of the electric potential from greatest to least.
  2. Mark three points on the surface that are separated by equal (non-zero!) changes in potential.
     a. Identify (and mark) all the points with the same value of potential as your three points.
     b. What patterns do you notice?
  3. Align your surface with your contour map. How are you making this alignment? Where is the surface a good approximation of the potential? Where is the approximation less good?
  4. Sketch a graph of the potential (V) *vs.* distance from the negative plate (x).
     a. Describe the relationship between potential and distance from the negative plate.
     b. Propose an equation to describe the electric potential as a function of the distance, x, from the negative plate. Where did choose for the location of $V = 0$?
     c. What do you think the slope of your graph of V *vs.* x represents?

Take a picture!

5-1 Representations of Potential

[1]



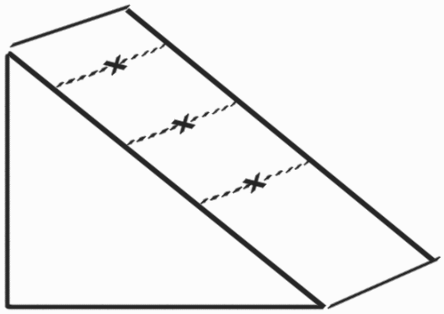
Height on the ramp corresponds to electric potential. Assuming greater height corresponds to higher potential, the three spots have potentials

$$V_1 > V_2 > V_3$$

Greatest Height ⇒ Greatest V

Lowest Height ⇒ Lowest V

[2]

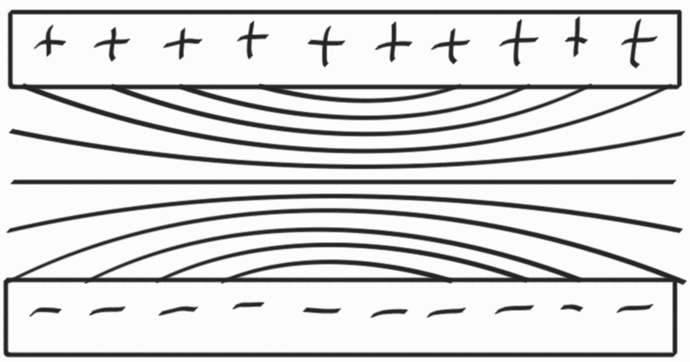


The three "x" marks are equally spaced down the ramp, so the change in height from one to the next is the same. This corresponds to equal change in potential from one x to the next.

The dashed lines are at the same height as the x marks they pass through, so they have the same potential as those x marks.

I could have selected any one point from each of those three lines (not necessarily in alignment with each other, like the x marks I chose) to obtain three x marks of equally spaced potential.

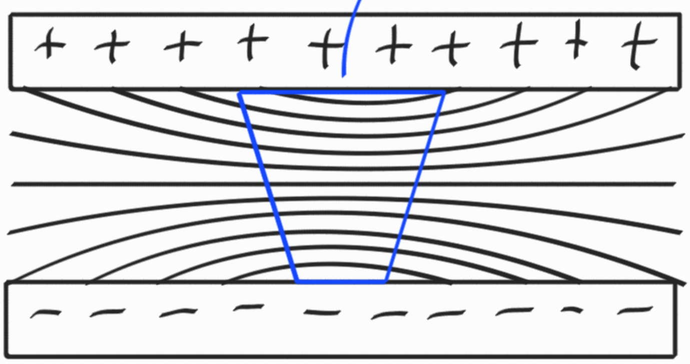
This contour map represents the potential between two finite, uniformly charged plates (not between two plates of uniform potential, since the equipotential lines cross them; thus, these plates cannot be conductors, which must stay at uniform potential).



The surface would not be a good approximation where the contours curve and are not parallel.

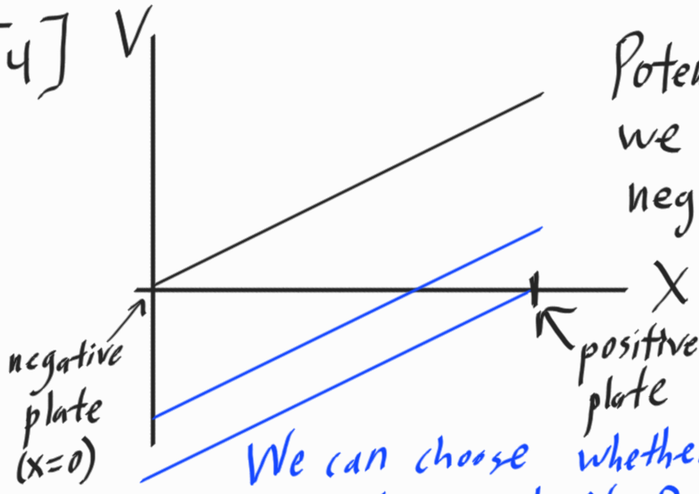
[3] Aligning the surface to the contour map

The taller end goes by the + plate, where potential is higher.



In the middle of the plates, the equipotential lines are nearly parallel. The equipotential lines on the surface (which are lines of equal height) are parallel, so the surface best approximates this center region. We can align the surface's equipotential lines with these contours.

[4]



Potential is increasing linearly as we get farther from the negative plate (assuming I graph near the center of the plates).

We can choose whether or not to set $V=0$ at the negative plate, so the exact vertical intercept of this graph is not of critical importance.

Equation:

$$V(x) = mx + V_0$$

constant slope $m = \frac{\Delta V}{\Delta X}$ ← total potential difference; ← plate separation

distance from negative plate

zero for the black graphed line

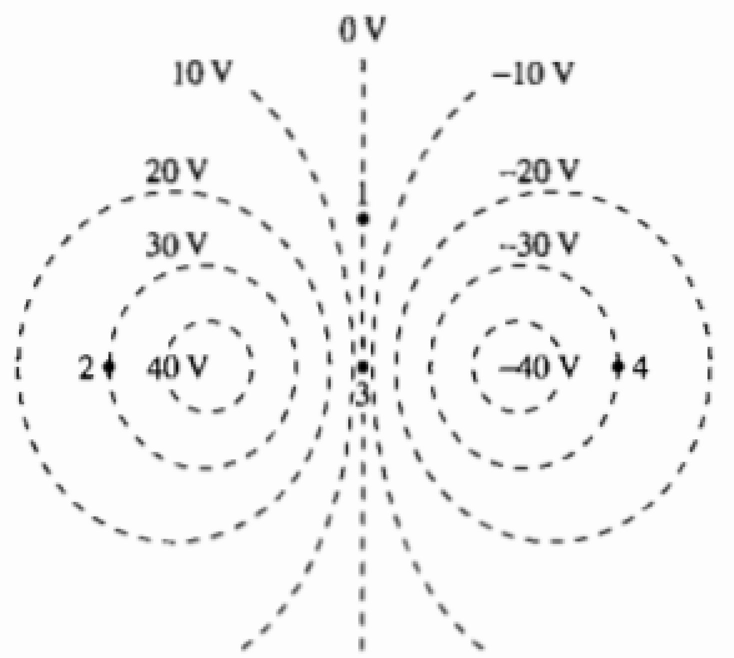
Note that V has units of $\frac{J}{C}$, so the slope of this graph must have units of $\frac{J}{Cm} = \frac{N}{C}$.

Those are the units of $\vec{E}$! As it turns out, the slope of a potential graph at any point is the negative value of the electric field at that point. In other words, $\vec{E} = -\nabla V = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$.

It is negative, as $\vec{E}$ points from higher potential to lower potential.

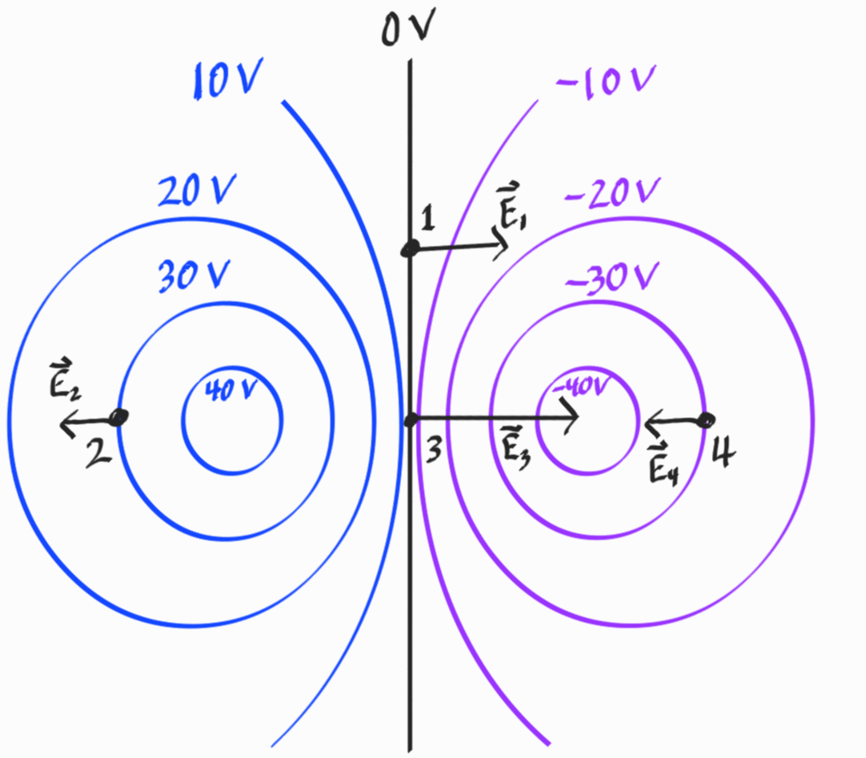
Activity 5-2

- Rank, in order from greatest to smallest, the electric potential at points 1, 2, 3, and 4.
- Sketch qualitatively accurate vectors to represent the electric field at points 1, 2, 3, and 4.
- Rank, in order from greatest to smallest, the magnitude of the electric field at points 1, 2, 3, and 4.



Take a picture!

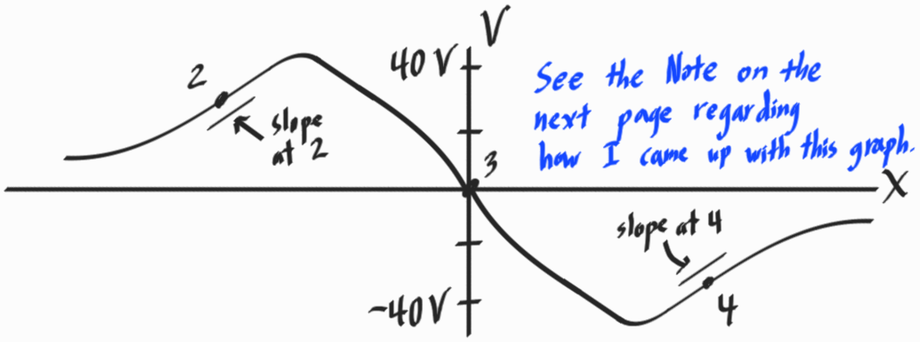
5-2 Almost-a-Dipole Potential



$V_2 > V_1 = V_3 > V_4$

$V_2 = 30V$, $V_1 = V_3 = 0V$, $V_4 = -30V$

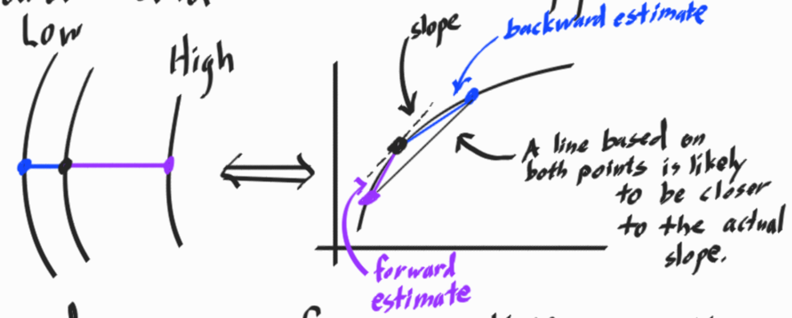
Each $\vec{E}$ vector is perpendicular to the equipotential line going through the given point, and more closely spaced equipotentials around the point indicate a higher magnitude of the electric field.



$E_3 > E_1 > E_2 = E_4$

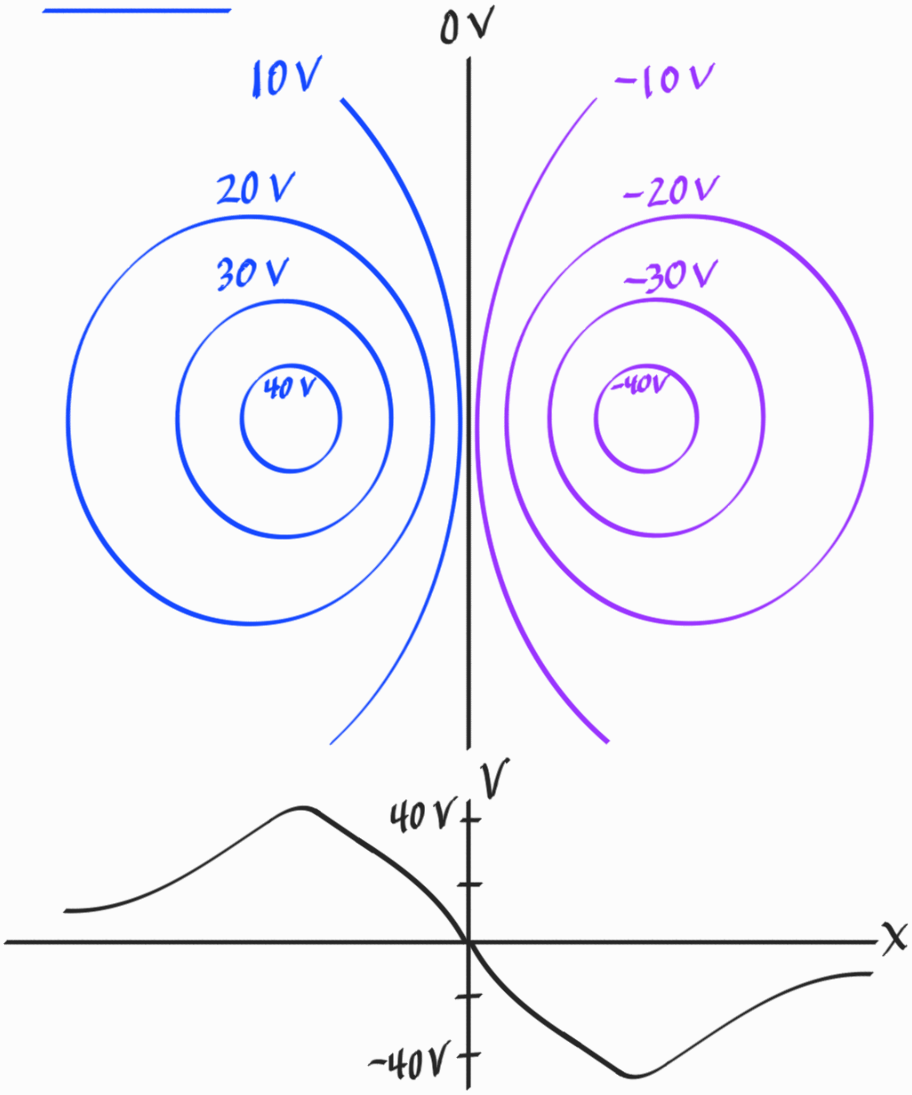
<u>$E_2 = E_4$?</u> Only the spacing of lines around the point matters for determining magnitude. The way the vector points (toward greater or smaller spacing of contours, for example) does not.

In other words, looking at both adjacent lines is important, like looking at forward and backward approximations to get a derivative:

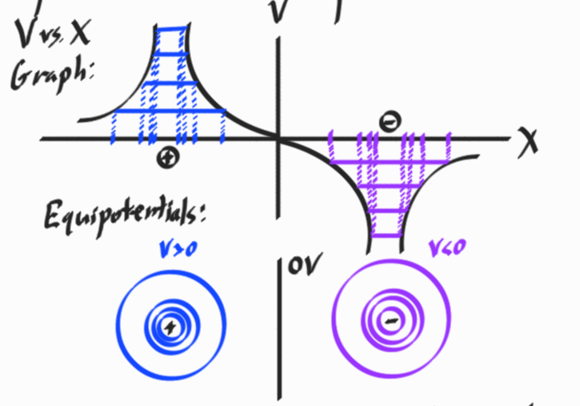


Also, if we swapped negative for positive on the contour map, the slopes would not change magnitudes, just signs (reversing $\vec{E}$ without stretching or shrinking it).

Note



This is not a simple dipole of two point charges.



Between the two charges, I would expect the weakest $\vec{E}$ (the shallowest slope of V, the greatest spacing of equipotentials) right in the middle. The actual equipotentials in this activity are closest together in the middle, and get more spread out as one moves toward ±40V. If there were point charges, I would expect the equipotentials to get denser toward ±40V. Since they don't I chose to assume the potential actually approaches a finite maximum/minimum. That is not completely certain, but it seems like a fair possibility.

Activity 5-3

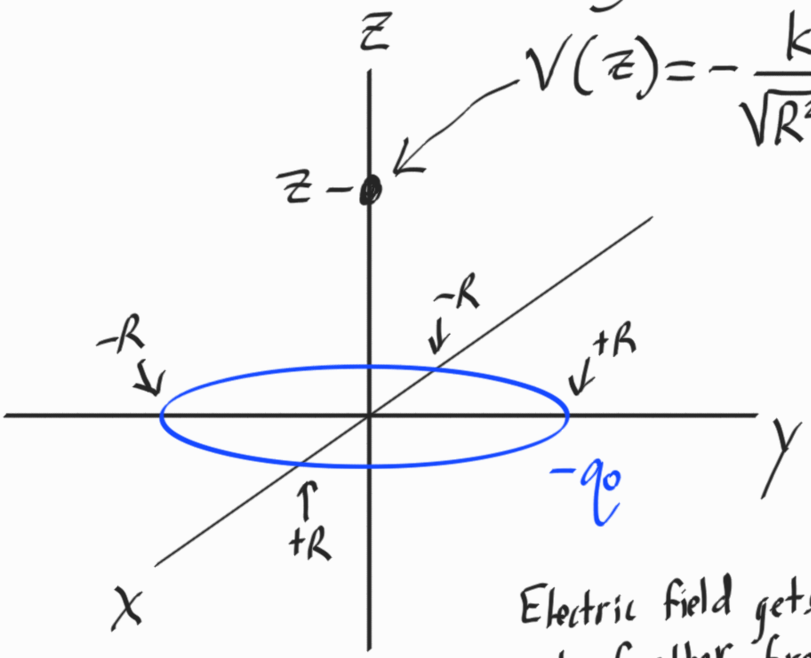
- A uniformly charged ring with total charge $-q_o$ and radius R is located in the xy-plane.
- On a previous homework, you found the electric potential a distance z above the center of the ring: $V(z) = -\frac{kq_o}{\sqrt{R^2+z^2}}$.

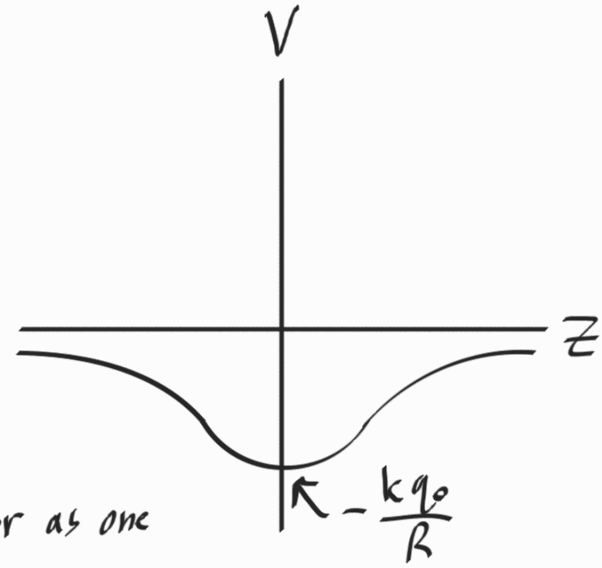
1. Sketch and label a physical picture of the situation.
2. Sketch a graph of the electric potential *vs.* position.
3. Predict how you expect the electric field to change as you get farther from the ring.
4. Find the electric field along the z-axis.
5. Sketch a graph of the z-component of the electric field *vs.* position. Also sketch electric field vectors on your physical picture.
6. Check that your two graphs agree with each other.

Take a picture!

5-3 Uniform Ring

$$V(z) = -\frac{kq_0}{\sqrt{R^2+z^2}}$$





Electric field gets weaker as one gets farther from the source charges, so it will presumably decrease as we get farther from the ring. It will point back toward the center of the ring, since the charge on the ring is negative.

$$\vec{E} = -\nabla\vec{V} = -\left(\frac{\partial V}{\partial x}\hat{x} + \frac{\partial V}{\partial y}\hat{y} + \frac{\partial V}{\partial z}\hat{z}\right)$$

constants out

power rule

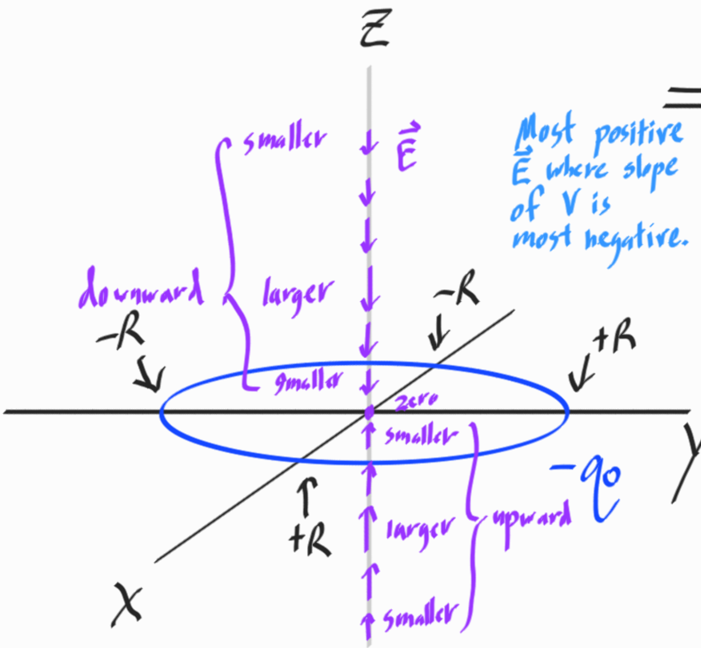
chain rule

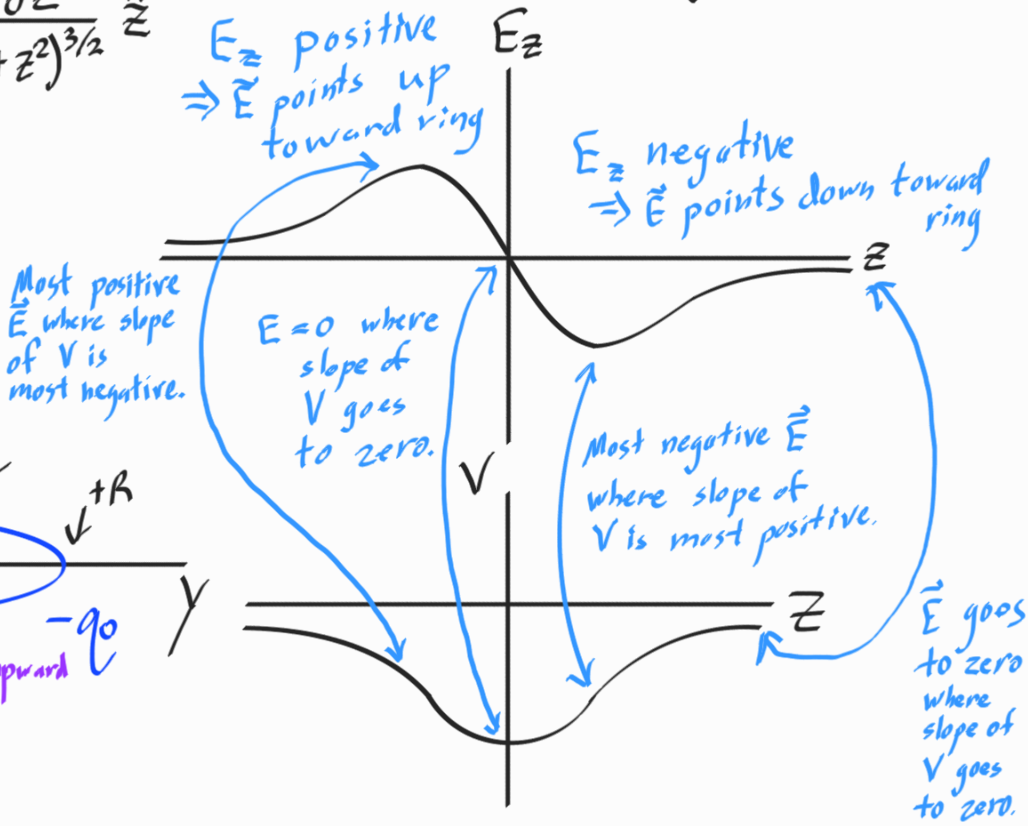
$$E_z = -\frac{\partial}{\partial z}V(z) = -\frac{\partial}{\partial z}\left(-\frac{kq_0}{\sqrt{R^2+z^2}}\right) = kq_0\frac{\partial}{\partial z}(R^2+z^2)^{-\frac{1}{2}} = kq_0\left[-\tfrac{1}{2}(R^2+z^2)^{-\frac{3}{2}}\right][2z]$$

$$= -\frac{kq_0 z}{(R^2+z^2)^{3/2}}$$

(No $\hat{x}$ or $\hat{y}$ components by symmetry)

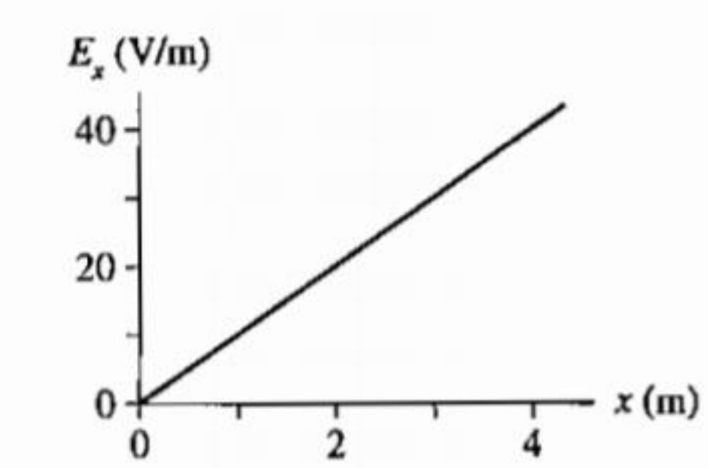
$$\vec{E} = -\frac{kq_0 z}{(R^2+z^2)^{3/2}}\hat{z}$$





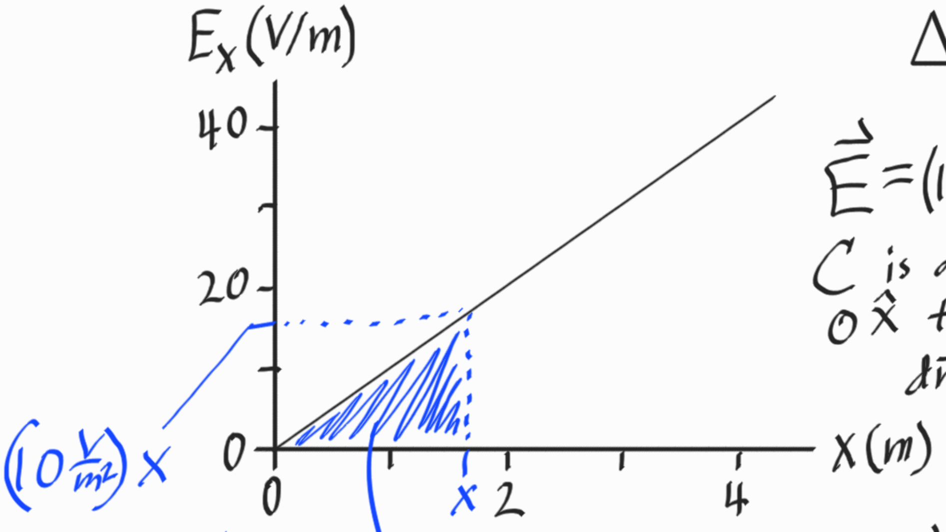
Activity 5-4

- The graph shows E_x *vs.* x for an electric field that is parallel to the x-axis. Assume that $E_y = 0$.
  1. Draw a graph of V *vs.* x, letting $V = 0$ at $x = 0$. Add an appropriate scale to your vertical axis.
  2. Draw an equipotential map for the xy-plane. Space your equipotential lines every 20 Volts and label each equipotential line. You may also find it valuable to draw an electric field vector map!



Take a picture!

5-4 Potential from Electric Field



Calculus

$$\Delta V = -\int_C \vec{E}\cdot d\vec{r}$$

Electric Field

line element of curve

"curve" – path through space from $\vec{r}_i$ to $\vec{r}_f$

$\vec{E} = (10\frac{V}{m^2})x\,\hat{x}$

C is a path from $0\hat{x}$ to $x\hat{x}$, so $d\vec{r} = dx\,\hat{x}$.

$$V(x) - \cancel{V_0}^{\,0} = -\int_0^x (10\frac{V}{m^2})x^*\,dx^*$$

$V(x=0)=0$

$$= -(5\frac{V}{m^2})(x^*)^2\Big|_{x^*=0}^{x}$$

$$= -(5\frac{V}{m^2})x^2$$

Geometry

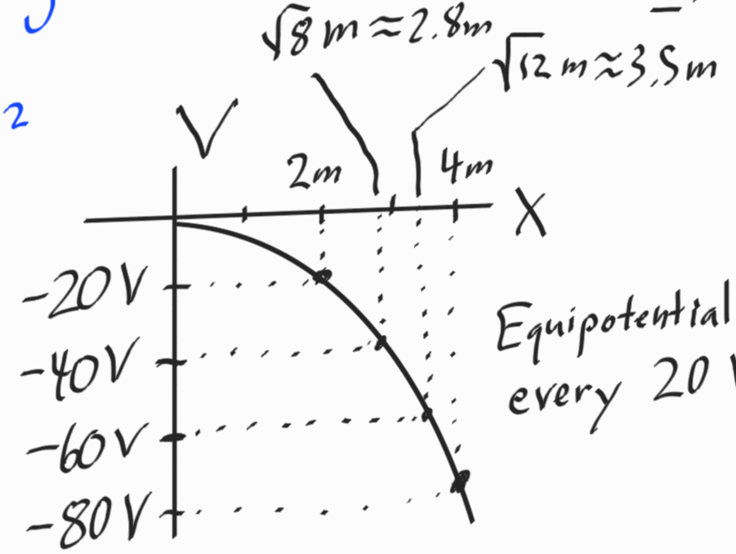
Area of a triangle

$$V(x) = -\frac{1}{2}\times \underbrace{(10\frac{V}{m^2})x}_{\text{height}} + \cancel{V_0}^{\,0}$$

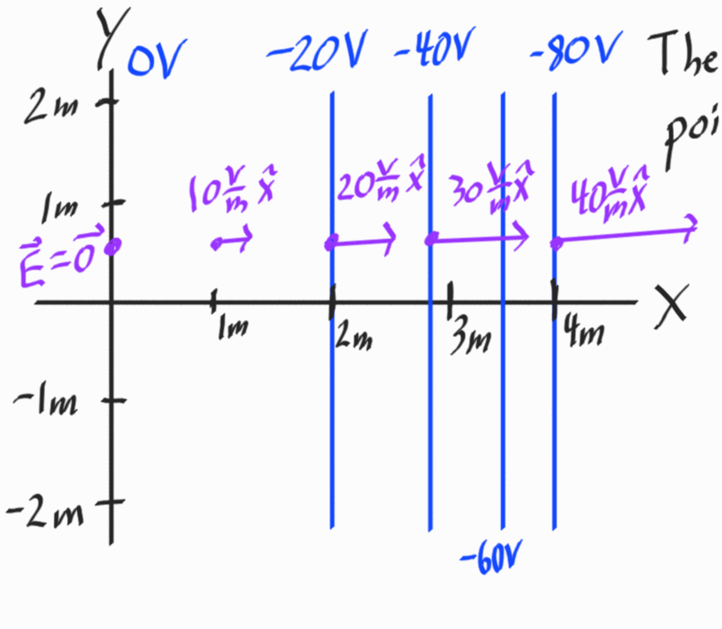
$\vec{E}$ points toward lower potential.

base

$$= -(5\frac{V}{m^2})x^2$$



Equipotential lines occur every 20 V.



The equipotential lines are vertical, so $\vec{E}$ points along the x-axis, and they get closer together as x increases, so $\vec{E}$ increases in magnitude.