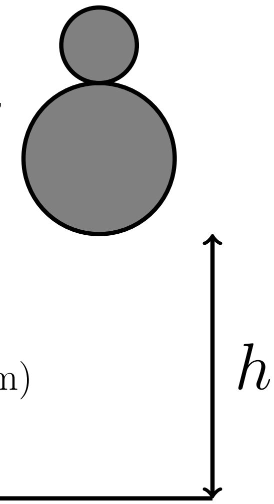
Energy is (C)
The work-energy theorem ($W_{\text{net,ext}} = \Delta E_{\text{total}}$) determines whether energy is conserved. Everything that can do significant amounts of work is in our system.

Momentum is (B)
The impulse-momentum theorem ($\int_{t_i}^{t_f} \vec{F}^{net} dt = \vec{J}_{net} = \Delta \vec{p}$) determines whether momentum is conserved. Everything that exerts a significant force is in our system.

Studio 8: Combining Physics Concepts

Warm-Up Activity



- A tennis ball and a basketball are dropped from a height h as shown.
- Our system is the tennis ball, the basketball, and the Earth.

Energy/Momentum is ...
(answer twice; once for energy and once for momentum)

(A) ... conserved, because energy/momentum is always conserved.

(B) ... conserved, because the net force on the system is zero.

(C) ... conserved, because the work done on the system is zero.

(D) ... not conserved, because energy is never conserved.

(E) ... not conserved, because the net force on the system is not zero.

(F) ... not conserved, because the work done on the system is not zero.

S8-1: Ball Drop – Height

- A tennis ball and a basketball are dropped from a height h as shown.
- Our system is the tennis ball, the basketball, and the Earth.
- If the basketball does not bounce at all, how high does the tennis ball go?
  - Basketball Mass: $M = 0.6$ kg
  - Basketball Radius: $R \approx 11.5$ cm
  - Tennis Ball Mass: $m = 0.04$ kg
  - Drop Height: $h = 0.5$ m

	U_{gTB}	K_{TB}	U_{gBB}	K_{BB}	E_{total}
t_i	$mg(h+2R)$	0	Mgh	0	$mg(h+2R)+Mgh$
t_f	mgh_{max}	0	0	0	mgh_{max}

$$\begin{aligned} mgh_{max} &= mg(h+2R)+Mgh \\ h_{max} &= h+2R+\frac{M}{m}h \\ &= \left(\frac{M}{m}+1\right)h+2R \end{aligned}$$

You don't necessarily need to include the radius of the basketball, as the tennis ball never gets lower than $2R$. If you leave off $2R$, you are effectively setting the zero of potential for the tennis ball at $2R$ above the ground, and so the h_{max} you would find would actually be measured from this higher point.

Plugging in the given numbers, note that $\frac{M}{m} = 15$, so $h_{max} = 8.23$ m. Now, this would exceed the ceiling height in our classroom, but when we actually do the demonstration, the non-negligible bounce of the basketball leads to this being a large overestimate; the tennis ball won't hit the ceiling after being dropped from this height.

However, if you wanted to find out the maximum drop height in the ideal case, it would only take a small rearrangement of the equation:

$$h = \frac{h_{max}-2R}{\frac{M}{m}+1}.$$

Assuming a ceiling height of 3 m, we should drop the ball from less than $h \approx 0.17$ m. In practice, this will be nowhere near the ceiling.

Let us add two times to our table: t_{bb} (right before the bounce) and t_{ab} (right after the bounce).

	U_{gTB}	K_{TB}	U_{gBB}	K_{BB}	E_{total}
t_i	$mg(h+2R)$	0	Mgh	0	$mg(h+2R)+Mgh$
t_{bb}	$mg(2R)$	$\frac{1}{2}mv_{bb}^2$	0	$\frac{1}{2}Mv_{bb}^2$	$mg(2R)+\frac{1}{2}(m+M)v_{bb}^2$
t_{ab}	$mg(2R)$	$\frac{1}{2}mv_{ab}^2$	0	0	$mg(2R)+\frac{1}{2}mv_{ab}^2$
t_f	mgh_{max}	0	0	0	mgh_{max}

Note that the extra starting height of the tennis ball really doesn't matter in some of these calculations. It drops the same height as the basketball, and that change in height is what provides the kinetic energy; the extra $2R$ that remains does nothing. This can be seen in how it will cancel out in equations relating E_i to E_{bb} or E_{ab}.

To find the pre-collision speed, I will choose to set $E_i = E_{bb}$ (which isn't the only choice I could make), which gives

$$\begin{aligned} mg(h+2R)+Mgh &= mg(2R)+\frac{1}{2}(m+M)v_{bb}^2 \\ (m+M)gh &= \frac{1}{2}(m+M)v_{bb}^2 \\ v_{bb} &= \sqrt{2gh}. \end{aligned}$$

To find the post-collision speed, I will choose to set $E_f = E_{ac}$ (which isn't the only choice I could make), which gives

$$\begin{aligned} mgh_{max} &= mg(2R)+\frac{1}{2}mv_{ab}^2 \\ v_{ab} &= \sqrt{2g(h_{max}-2R)} = \sqrt{2gh\left(\frac{M}{m}+1\right)} \end{aligned}$$

Now, from the impulse-momentum theorem (simplifying the impulse as $\vec{J}_{net} = \vec{F}_{avg}^{net}\Delta t$, and choosing a coordinate system with $\hat{y}$ pointing upward), we have

$$\begin{aligned} \vec{F}_{avg}^{net}\Delta t &= \vec{p}_{ab} - \vec{p}_{bb} \\ F_{avg}^{net}\Delta t &= mv_{ab} - (-(m+M)v_{bb}) \\ &= m\sqrt{\frac{M}{m}+1}\sqrt{2gh} + (m+M)\sqrt{2gh} \\ F_{avg}^{net} &= \frac{\sqrt{\frac{M}{m}+1}+(m+M)}{\Delta t}m\sqrt{2gh}. \end{aligned}$$

If we want to get just the normal force from the ground, we need to know the force of gravity (which is constant, and therefore equal to its average). Since our system is both balls, the force should be on both, meaning

$$\begin{aligned} F_{avg}^{net} &= F_{avg}^{N} - F^{g} \\ &= F_{avg}^{N} - (m+M)g \\ F_{avg}^{N} &= F_{avg}^{net} + (m+M)g \\ &= \frac{\sqrt{\frac{M}{m}+1}+(m+M)}{\Delta t}m\sqrt{2gh} + (m+M)g. \end{aligned}$$

S8-2: Ball Drop – Force

- A tennis ball and a basketball are dropped from a height h as shown.
- During the collision with the ground, which takes 0.2 s, you may want to choose a different system.
  - What is the speed of each ball right before the basketball hits the ground?
  - What is the speed of the tennis ball right after it leaves the basketball?
  - What force did the ground exert on the basketball?

S8-3: Frictionless Track

S8-4: Carts on a Track

Main Ideas

- The work-energy and impulse-momentum theorems can be used to solve a broad array of problems.