**Wave on a String 1** A wave on a string is described by

$$y(x,t) = 0.02\sin\left(12.57x - 638t + \frac{\pi}{4}\right)$$

$$y(x,t) = y_{max}\sin(kx - \omega t + \phi_0)$$

where x and y are in meters, t is in seconds, and all constants have appropriate units. The linear density of the string is 0.005 kg/m. $= \mu$

(a) What is the maximum displacement of a point on the string?

Just like with simple harmonic motion, the amplitude of our y function is the maximum displacement from equilibrium for a point on the string. $y_{max} = 0.02\,m$

(b) What is the maximum speed of a point on the string?

The given function tells us the displacement of a point on the string, so if we take the first derivative with respect to time, we will have a function telling us the velocity of a point on the string.

$$V_y(x,t) = \frac{dy}{dt}(x,t) = -638 \times 0.02\cos\left(12.57x - 638t + \frac{\pi}{4}\right) \Rightarrow V_{y\,max} = \omega y_{max} = \left(638\,\frac{rad}{s}\right)(0.02\,m) = 12.76\,\frac{m}{s}$$

(c) What is the wave speed?

Wave speed can be calculated from the distance the wave moves through the string over the course of one period of oscillation; in other words, it is the wavelength divided by the period. Substituting expressions for angular frequency and wave number gives us an expression in terms of given quantities.

$$V_w = \frac{\lambda}{T} = f\lambda = \left(\frac{\omega}{\cancel{2\pi}}\right)\left(\frac{\cancel{2\pi}}{k}\right) = \frac{\omega}{k} = \frac{638\,rad/s}{12.57\,rad/m} \approx 50.8\,\frac{m}{s} \qquad k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T}$$

(d) What is the string tension?

The wave speed is determined by the properties of the medium. In the case of a wave in a string, the wave speed is determined by the tension $F^T$ the string is under and the linear mass density $\mu$ of the string.

$$V_w = \sqrt{\frac{F^T}{\mu}} \Rightarrow F^T = V_w^2\mu \approx \left(50.8\,\frac{m}{s}\right)^2\left(0.005\,\frac{kg}{m}\right) \approx 12.9\,N$$

**Wave on a String 2** A sinusoidal wave travels along a stretched string. A particle on the string has a maximum speed of 0.900 m/s and a maximum acceleration of $270 m/s^2$. Given that the tension of the string is 46 N and the linear mass density is 0.0035 kg/m, construct the function of the wave in terms of time and position.

We are trying to fill out all of the variables in the function

$$y(x,t) = y_{max}\sin(kx \pm \omega t + \phi_0).$$

To get the angular frequency, we can use our knowledge of the maximum speed and acceleration, as they are related to both our maximum displacement and the angular frequency, as seen through taking derivatives:

$$V_y(x,t) = \frac{dy}{dt}(x,t) = \pm\omega\, y_{max}\cos(kx \pm \omega t + \phi_0) \Rightarrow V_{y\,max} = \omega\, y_{max}$$

$$a_y(x,t) = \frac{d^2y}{dt^2}(x,t) = -\omega^2 y_{max}\sin(kx \pm \omega t + \phi_0) \Rightarrow a_{y\,max} = \omega^2 y_{max}$$

$$\Rightarrow \omega = \frac{\omega^2 y_{max}}{\omega\, y_{max}} = \frac{a_{max}}{V_{max}} = \frac{270\,m/s^2}{0.900\,m/s} = 300\,\frac{rad}{s}$$

$$\Rightarrow y_{max} = \frac{V_{y\,max}}{\omega} = \frac{0.900\,m/s}{300\,rad/s} = 0.003\,m$$

We can now get the wave number from its relationship to both angular frequency and wave speed, the latter of which we can calculate from the linear mass density and tension.

$$\frac{\omega}{k} = V_w = \sqrt{\frac{F^T}{\mu}} = \sqrt{\frac{46N}{0.0035\,\frac{kg}{m}}} \approx 115\,m/s$$

$$k = \omega\sqrt{\frac{\mu}{F^T}} = 300\,\frac{rad}{s}\sqrt{\frac{0.0035\,\frac{kg}{m}}{46N}} \approx 2.6\,rad/m$$

[0]Select problems may be modified from PH 212 course textbook; Knight Physics for Scientists and Engineers

$$y(x,t) = (0.003m)\sin\left((2.6\,rad/m)x \pm (300\,rad/s)t + \phi_0\right)$$

We cannot determine whether it is + or -, as we don't know if the wave is moving to the right (+) or to the left (-). We also weren't told whether a point on the string at *x = 0* was at a peak, a trough, or somewhere in between at *t = 0*, so we cannot determine the phase constant.

**Yelling on a train** You are walking towards your friend at 2 m/s while on opposite sides of a train car when you shout to get their attention at approximately 165 Hz.

(a) Predict: Rank the frequency of the sound when it reaches your friend if the train is:

Same

(i) Fully enclosed, and at rest

(ii) Fully enclosed, and moving 8 m/s

(iii) Open-air (as if you were standing on top of the train), and at rest

In all of these first three cases, your friend is at rest relative to the air. Even when the train is moving, the enclosed car moves the contained air with it. As such, we can expect the first three cases to be the same.

(iv) Open-air, and moving at 8/s

In the open air case with a moving train, the answer depends on whether you are moving in the same direction as the train, or against it. In the former case, it is as though your friend is receding from you at 8 m/s, but you are pursuing at 10 m/s. In the latter case, it is as though your friend is pursuing at 8 m/s, and you are receding at 6 m/s.

$$f_{IV} = f_{em}\frac{(V_s - V_T)}{(V_s - V_T) - V_{em}} \approx 165.99\text{ Hz}$$

$$165.95\text{ Hz}$$

(b) Calculate the observed frequency in each scenario described above.

We begin with a general form of the Doppler effect equation:

$$f_{ob} = f_{em}\frac{V_s \pm V_o}{V_s \pm V_{em}},$$

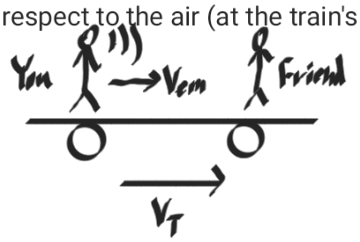
We start here to figure out the proper signs via sensemaking. In particular, we expect the observed frequency to increase if the emitter is moving toward the observer, so to make the fraction larger, we shrink the denominator by subtracting the emitter speed from the speed of sound in the medium. Similarly, we expect the observed frequency to decrease if the observer is moving away from the emitter, so the make the fraction smaller, we shrink the numerator by also subtracting the observer speed from the speed of sound. This gives us

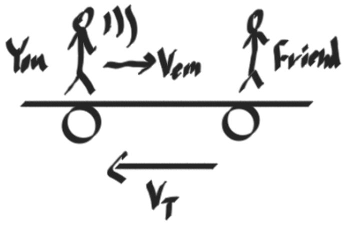
$$f_{ob} = f_{em}\frac{V_s - V_o}{V_s - V_{em}}.$$

In the first three cases, your friend is not moving relative to the medium, so the speed of the observer is zero, giving us

$$f_{ob} = f_{em}\frac{V_s}{V_s - V_{em}} = (165\text{ Hz})\frac{343\text{ m/s}}{\underbrace{343\text{ m/s}}_{\text{assumed as in prior problem}} - 2\text{ m/s}} \approx 165.97\text{ Hz}$$

In the third case, we shall break it into the cases of moving with the train and moving against the train. Now, the observer is moving with respect to the air (at the train's speed $V_T = 8$ m/s), and your speed is adjusted by this amount.




$$f_{ob} = f_{em}\frac{V_s - V_T}{V_s - V_T - V_{em}} \approx 165.99\text{ Hz} \qquad f_{ob} = f_{em}\frac{V_s + V_T}{V_s + V_T - V_{em}} \approx 165.95$$

If you are moving with the train, then your friend's observed frequency is higher than in cases (i-iii). If you are moving against the train, then your friend's observed frequency is lower than in the prior cases. Also, it is worth noting that you could group the train speed with the speed of sound to create the quantity $V_s \pm V_T$ in both the numerator and the denominator, which acts as an effective speed of sound adjusted by wind.