One thing a teacher of mine liked to remind her students: how we perform on exams has nothing to do with our value as human beings.

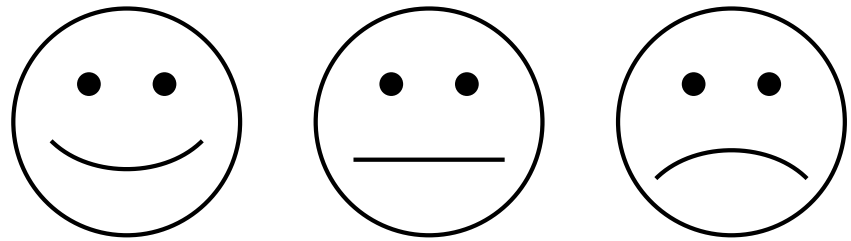
You should discuss the feedback from your quizzes on the ungrading assignments, and you can address how they went and whether you think they were a good measure of your learning.

Studio 4: Gravity and Friction

Warm-Up Activity

How do you feel about the quizzes?

(A) (B) (C) (D) (E)



(Newton's) Laws of Motion

(1) An object in motion (or at rest) stays in motion (or at rest) unless a net external force acts on it.

(2) The net force on an object is equal to the object's mass times its acceleration:
$$\vec{F}^{net} = m\vec{a}.$$

(3) If A exerts a force on B, then B exerts a force of the same magnitude on A in the opposite direction:
$$\vec{F}_{AB} = -\vec{F}_{BA}.$$

Types of Forces

- Gravity $\vec{F}^g_{AB} = m_A \vec{g}_B$
  - Newtonian $\vec{g}_B = G\frac{M_B}{r^2}(-\hat{r})$, $G = 6.67408 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
  - Near-Earth $\vec{g}_E = g(-\hat{y}),\ g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2} \approx 10\frac{\mathrm{m}}{\mathrm{s}^2}$
- Normal $\vec{F}^N$ always $\perp$; varies in magnitude
- Tension $\vec{F}^T$ uniform (massless, inextensible rope)
- Spring
- Friction
  - Static Friction $F^{sf} \leq \mu_s|\vec{F}^N|$
  - Kinetic Friction $F^{kf} = \mu_k|\vec{F}^N|$

S4-1: The Elevator

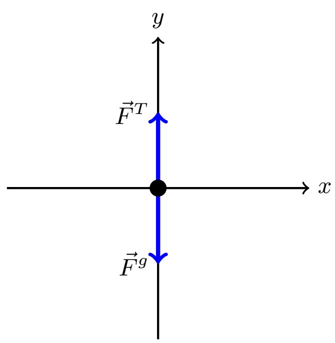
- You attach a volleyball with mass 275 grams to a string and suspend it from the ceiling of an elevator.
- The elevator is moving upward at constant speed 1.5 m/s.
- Our goal is to determine the tension in the string.
  - Choose a system and identify any assumptions you are making.
  - Sketch and label a free body diagram.
  - Determine the tension.
  - Make sense of your answer.

System

We don't want to pick the Earth, the elevator, and the string and ball, as then there are no external forces in that case. Instead, let us choose just the ball, which has tension from the string and a gravitational force.

Assumptions

- Near-Earth gravity: $g \approx 9.81 \text{ m/s}^2$. Even if the elevator goes from the bottom to the highest point in the building, it remains near the surface of the Earth. Variation in gravitational acceleration will be negligible.
- Particle model. The ball will not deform or spin while hanging from a string. In particular, it probably can't spin very much with a string wrapped around it.
- No swinging. The problem ceases to be 1-D if the ball starts to swing side-to-side, and the swinging might affect the tension, so it is best to assume the ball hangs straight down as the elevator goes.
- Ideal (massless, inextensible) string. We need to assume that the mass in the string is negligible in comparison to the volleyball (which is possible for a very light string), otherwise the tension in the string will not be uniform.



The speed is constant, so the net force is zero:

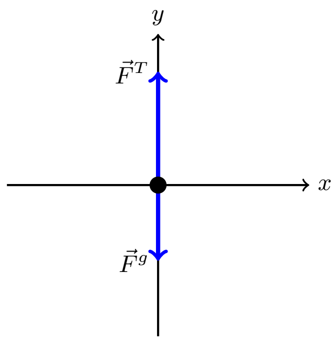
$$\begin{aligned} m\vec{a} &= \vec{F}^{net} \\ \vec{0} &= \vec{F}^T + \vec{F}^g \\ 0 &= F_y^T + F_y^g \\ &= F^T - F^g \\ F^T &= F^g = mg \end{aligned}$$

Plugging in the numbers, we get

$$F^T = (0.275 \text{ kg})(9.8 \text{ m/s}^2) = 2.695 \text{ N}.$$

This number is just the force necessary to support the weight of the volleyball, which seems reasonable; we aren't trying to accelerate it, just to hold it up.

The system and assumptions stay the same, as does the force of gravity, but the force of tension must increase to give a net force upward.

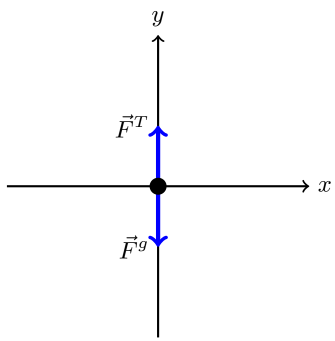


$$
\begin{aligned}
m\vec{a} &= \vec{F}^{net} = \vec{F}^T + \vec{F}^g \\
ma\hat{y} &= F^T\hat{y} - F^g\hat{y} \\
ma &= F^T - F^g \\
F^T &= ma + F^g \\
&= m(a+g) \\
&= (0.275\ \text{kg})(2.5\ \text{m/s}^2 + 9.8\ \text{m/s}^2) \\
&= (0.275\ \text{kg})(12.3\ \text{m/s}^2) \\
&= 3.3825\ \text{N}.
\end{aligned}
$$

S4-2: The Accelerating Elevator

- You attach a volleyball with mass 275 grams to a string and suspend it from the ceiling of an elevator.
- The elevator is moving upward at constant speed 1.5 m/s and accelerating upward at a constant rate of 2.5 m/s^2.
  - Do you want to change your system or assumptions?
  - How (if at all) does your free body diagram change?
  - How (if at all) does the tension change?

Here, our system stays the same, but we have to dispense with the near-Earth assumption and use full Newtonian gravity. The speed is constant, so the tension and force of gravity must be in balance again, though both will be weaker than before, since gravity gets weaker the farther from Earth one gets.



We have $F^T = mg$, but now

$$\vec{g} = \frac{GM_E}{r^2}(-\hat{r}),$$

where $G = 6.675 \times 10^{-11}$ Nm^2/kg^2, the mass of Earth is $M_E = 6 \times 10^{24}$ kg, the radius of Earth is $R_E = 6.38 \times 10^6$ m, and

$$r = R_E + 400 \text{ km} = 6.38 \times 10^6 \text{ m} + 0.4 \times 10^6 \text{ m} \approx 6.8 \times 10^6 \text{ m}.$$

Plugging these in, we find

$$\begin{aligned}\vec{g} &= \frac{GM_E}{r^2}(-\hat{r}) \\ &= \frac{(0.275 \text{ kg})(6.675 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(6 \times 10^{24} \text{ kg})}{(6.8 \times 10^6 \text{m})^2}(-\hat{r})\end{aligned}$$

$$\approx 8.7 \text{m/s}^2(-\hat{r}),$$

and thus $F^T = mg \approx 2.38$.

This is about at the height of the International Space Station, and yet the force of gravity up there isn't much smaller than it is on Earth's surface. How is it that things seem weightless up there? This force of gravity is actually what holds the space station in orbit around Earth; the station is constantly falling toward Earth (and things in free-fall experience the feeling of weightlessness), but it moves so quickly toward the horizon that it never descends. It just keeps changing direction and moving in a circle around Earth.

S4-3: The Space Elevator

- You attach a volleyball with mass 275 grams to a string and suspend it from the ceiling of an elevator.
- The elevator is located 400 km above the surface of the Earth and moving upward at constant speed.
  - Do you want to change your system or assumptions?
  - How (if at all) does your free body diagram change?
  - How (if at all) does the tension change?

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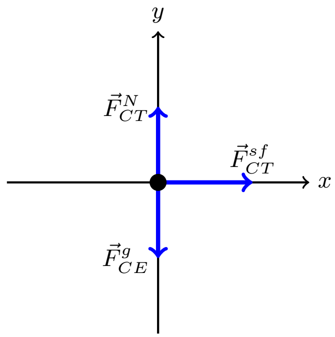


Type of Friction
The crate is not sliding against the roof of the truck (no relative motion), so any friction between it and the truck must be static.

Direction of Friction
For the crate to accelerate to the right, with the truck, the net force on it must also be to the right. Friction is the only horizontal force, so it must point to the right. If there were no friction, then the box would slide toward the back of the truck. Thus, the attempted relative motion would be to the left across the surface of the roof of the truck, and the force of static friction must oppose the attempted relative motion.

Motion of the Truck
If the ground is flat, then $\vec{a} = a\hat{x}$ (no vertical motion, which is assumed). We can see that

$$F^{sf}_{CT} = F^{net}_x = ma_x = ma,$$
$$F^N_{CT} - F^g_{CE} = F^{net}_y = ma_y = 0 \implies F^N_{CT} = F^g_{CE}.$$

Furthermore, we know that

$$F^{sf}_{CT} \le \mu_s F^N_{CT} = \mu_s F^g_{CE} = \mu_s mg.$$

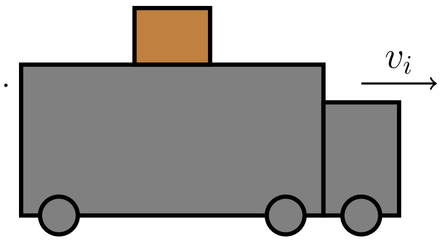
Combined with the above observation that $F^{sf}_{CT} = ma$, we find that

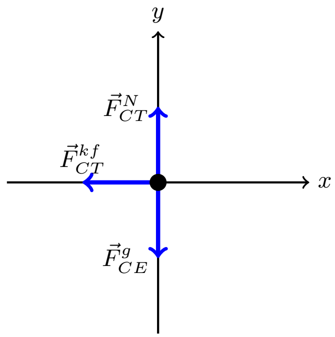
$$ma \le \mu_s mg \implies a \le \mu_s g.$$

The acceleration of the truck must be less than $\mu_s g$ in order for the crate to not slide (the velocities of the crate and the truck are equal) relative to each other as the truck speeds up.

S4-4: The Crate on Top of the Truck I (Gaining Speed)

- A truck is initially moving to the right with speed v_i.
- The driver left a crate on top of the truck. We know the mass of the crate (m_c) and the coefficients of friction (μ_s and μ_k).
- The truck begins *speeding up*, but the driver wants to prevent the crate from sliding.
  - Draw a free-body diagram for the crate.
    - What kind of friction acts on it?
    - What direction is the friction?
  - What do we know about the truck's velocity? What about its acceleration?





Differences
The force of gravity doesn't change, and nor does the normal force that balances it out. The friction has switched directions and is now kinetic, rather than static. As the crate slides toward the front of the truck, its motion relative to the roof of the truck is also to the right, and the new direction of friction opposes this motion.

Calculation We know $\vec{F}^{net} = m\vec{a}$, and therefore

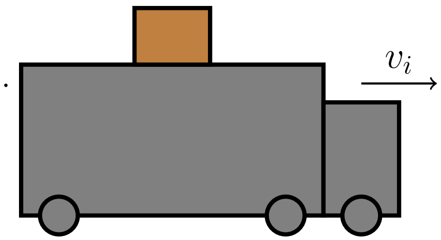
$$ma_A = F^{net}_x = -F^{kf} \qquad\qquad 0 = F^{net}_y = F^N - F^g,$$

which means,

$$a_A = -\frac{F^{kf}}{m} = -\mu_{kf}\frac{F^N}{m} = -\mu_{kf}\frac{F^g}{m} = -\mu_{kf}g.$$

S4-5: The Crate on Top of the Truck II (Slamming on the Brakes)

- A truck is initially moving to the right with speed v_i.
- We know the mass of the crate (m_c) and the coefficients of friction (μ_s and μ_k).
- The driver suddenly has to slam on the brakes, causing the crate to begin sliding.
  - Draw a free-body diagram for the crate.
    - How is it different from before?
  - Determine the acceleration of the crate relative to the ground.



Solving Problems Using Forces

- Identify a system.
- Identify the (external) forces acting on the system.
  - Draw a free-body diagram.
- Identify the acceleration (**not a force**).
  - Static/dynamic equilibrium (acceleration = 0)
  - Dynamics (acceleration not 0)
- Use the laws of motion.

Main Ideas

- Forces arise from interactions between objects.
- There are many different *kinds* of forces that we can analyze differently.
- Objects can only change their motion when acted upon by an external force.
- The net force on an object is equal to its mass times its acceleration.
- Forces are vectors.
- When more than one force acts on an object, we can add all the forces together.
- We can model forces quantitatively.