Lecture 4: Using Integrals in Physics

Office Hours

- Drop-In: 4:00 p.m. – 5:00 p.m. Wednesdays & Thursdays
- Appointments: 11:00 a.m. – 11:45 a.m. (W)/12:00 p.m. (Th)

Warm-Up Activity

How is acceleration symbolically related to velocity? WRITE BIG!

(A) Velocity is acceleration times t.

(B) Acceleration is velocity times t.

(C) Acceleration is the derivative of velocity.

(D) Velocity is the derivative of acceleration.

L4-1: Vax'ildan's Acceleration

- Vax'ildan Vessar is initially located at position x_i, running to the right with initial speed v_i.
- At $t = 0$, Vax clicks his *boots of haste*, which provide an acceleration:
$$\vec{a}(t) = a_0 \left(1 - \frac{t}{T}\right) \hat{x}$$
- Our goals are:
  - Find how much time it takes for Vax to return to his initial velocity.
  - Find Vax's position at this time.

Solving an ARCS Problem

1. Analyze and Represent

1a. **Understand the problem** – identify quantities by symbol and number.
1b. **Identify Assumptions** – identify important simplifications and assumptions.
1c. **Represent physically** – draw and label one or more appropriate diagrams and/or graphs that might help you solve the problem.

2. Calculate

2a. **Represent principles** – identify relevant concepts, laws, or definitions.
2b. **Find unknown(s) symbolically** – without numbers, find any unknown(s) in terms of symbols representing known quantities.
2c. **Plug in numbers** – plug numbers (with units) into your symbolic answer!

3. Sensemake

3a. **Units** – check that the units of your answer agree with the units you expect
3b. **Numbers** – compare your answer to other numbers in the problem or in the everyday world; if relevant, check the sign or direction.
3c. **Symbols** – use a strategy like covariation or special cases to check that your answer makes physical sense.

L4-1: Vax'ildan's Acceleration – Calculate

- At $t = 0$, Vax clicks his *boots of haste*, which provide an acceleration:

$$\vec{a}(t) = a_0 \left(1 - \frac{t}{T}\right) \hat{x}$$

- **Represent Principles**
  - Identify relevant concepts, laws, or definitions.
- **Find Unknown(s) Symbolically**
  - First find a symbolic expression for Vax's velocity as a function of time.
  - Use your expression to find when Vax's velocity is equal to v_i.
- **Plug in Numbers**
  - Estimate any quantities to find numerical answers.

L4-1: Vax'ildan's Acceleration – Calculate

- At $t = 0$, Vax clicks his *boots of haste*, which provide an acceleration:

$$\vec{a}(t) = a_0 \left(1 - \frac{t}{T}\right) \hat{x}$$

- His velocity as a function of time is

$$\vec{v}(t) = \left[v_i + a_0 \left(t - \frac{t^2}{2T}\right)\right] \hat{x},$$

  and he returns to his initial velocity at $t_f = 2T$.

- Now, find a symbolic expression for Vax's position as a function of time and use it to find Vax's position at t_f.

L4-1: Vax'ildan's Acceleration – Sensemake

- At $t = 0$, Vax clicks his *boots of haste*, which provide an acceleration:

$$\vec{a}(t) = a_0 \left(1 - \frac{t}{T}\right) \hat{x}$$

- How can we make sense of these equations?

$$\vec{v}(t) = \left[v_i + a_0 \left(t - \frac{t^2}{2T}\right)\right] \hat{x} \quad \vec{x}(t) = \left[x_i + v_i t + a_0 \left(\frac{t^2}{2} - \frac{t^3}{6T}\right)\right] \hat{x}$$

$$t_f = 2T \qquad \vec{x}_f = \left[x_i + 2v_i T + \frac{2}{3} a_0 T^2\right] \hat{x}$$

- **Units**
- **Numbers**
  - Which things are vectors?
  - Try plugging in some reasonable numbers.
- **Symbols**
  - What happens if you change a_0 or T?
- What do the graphs of $\vec{v}(t)$ and $\vec{a}(t)$ look like?

t (s)

L4-2: Constant Acceleration

- What if Vax's acceleration had been constant?

$$\vec{a}(t) = a\hat{x}$$

Main Ideas

- If we know the acceleration of an object as a function of time, we can determine the velocity as a function of time.
- If we know the velocity as a function of time, we can determine the position as a function of time.