Lecture 20: Systems and Momentum

Project Final Drafts

- Include a reflection addressing all feedback (from me and your peers).
- **Cite your sources.**
  - I didn't give this much attention in my feedback (a significant oversight on my part), but it is extremely important to cite your sources.
  - Any information from outside of the course (numerical values for problems, concepts, models, equations) should be cited.
- Currently due on Friday, along with Homework 7.
  - Would you rather...
    (A) Turn them both in that day.
    (B) Postpone the homework due date to Sunday to focus on the project and get it done sooner.
    (C) Postpone the project due date to Sunday to get the homework out of the way sooner and have more time for the project.
  - Whatever you choose, I can still potentially negotiate small extensions with individual groups. However, we need things turned in early so we can give feedback and return them to you quickly before the end of the term.

Impulse and Momentum

Definitions:

- Impulse $\vec{J}_{net} = \int_{t_i}^{t_f} \vec{F}^{net} dt$
- Momentum $\vec{p} = m\vec{v}$
- Impulse-Momentum Theorem $\vec{J}_{net} = \Delta\vec{p}$

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(1) System: Both Cars
For the short duration and large forces involved in a crash, the external impulse (for example, from friction with the road) would be insignificant. We can approximate conservation of momentum.

(2) System: Baseball, Catcher, 2nd Base, Earth.
The forces between the catcher and 2nd base (which is firmly rooted in the Earth) probably cannot reasonably be ignored.

(3) System: Block, Spring, Table Earth.
Unless the table is on frictionless wheels (which would allow it to oscillate too), it is interacting with the Earth in a significant way.

(4) System: Firework.
The strong forces in a fast explosion should vastly outmatch any impulse from gravity.
There is a problem in the online textbook with a firecracker that explodes over a non-negligible amount of time. Impulse cannot be ignored in that problem.

(5) System: Bird, Air, Earth.
Gravity is changing the bird's momentum, and air resistance may have an appreciable effect.

Putting Earth in your system is not conducive to solving a momentum problem. Usually, it is better to leave it out to exert an impulse.

L20-1: Systems and Momentum Conservation

In each scenario below, if possible, identify a system for which momentum is conserved. If not possible, explain why not.

(1) Two cars crash into each other in an intersection.

(2) A baseball player catches a baseball while standing at second base.

(3) A block attached to a spring on a horizontal table oscillates back and forth.

(4) A firework bursts into pieces in the sky.

(5) A bird dives through the air, speeding up.

Case 1

To set up a momentum vector diagram for case 1, I began with entering the initial momenta for the two cars, which told me the initial momentum for the whole system. I also assumed that momentum was conserved (by the reasoning listed in L20-1), which allowed me to fill in the 1 & 2 column. Since the cars get tangled at the end, I know they have the same speed, and I know Car 2 is twice as big as Car 1, so it should have twice the momentum (which means it will have two thirds of the momentum of the whole system). This allowed me to fill in the rest of the final momentum row. The last two entries in the change in momentum row are now determined by the other entries in their columns.

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	Car 1	Car 2	1 & 2
$\vec{p}_i$	↑	↓	↓
$\Delta\vec{p}$	↓	↑	·
$\vec{p}_f$	↓	↓	↓

Already, I know that my cars should go south after the collision. Let us calculate the velocity.

$$m_1 = 100\text{ kg} \qquad m_2 = 200\text{ kg} \qquad m_f = m_1 + m_2$$
$$\vec{v}_1 = (4\text{ m/s})\hat{y} \qquad \vec{v}_2 = -(3\text{ m/s})\hat{y} \qquad \vec{v}_f = v_f\hat{y} = ?$$

$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 v_1 - m_2 v_2)\hat{y}$$
$$\vec{p}_f = (m_1 + m_2)\vec{v}_f = (m_1 + m_2)v_f\hat{y}$$

$$\vec{p}_i = \vec{p}_f \qquad v_f = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$$
$$m_1 v_1 - m_2 v_2 = (m_1 + m_2)v_f \qquad = \frac{(100\text{ kg})(4\text{ m/s}) - (200\text{ kg})(3\text{ m/s})}{100\text{ kg} + 200\text{ kg}}$$
$$= \frac{400 - 600}{300}\text{ m/s} = -\frac{2}{3}\text{ m/s}$$

$$K_i = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$
$$= \frac{1}{2}(100\text{ kg})(4\text{ m/s})^2 + \frac{1}{2}(200\text{ kg})(3\text{ m/s})^2 = 800\text{ J} + 900\text{ J} = 1700\text{ J}$$
$$K_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(300\text{ kg})\left(\frac{2}{3}\text{ m/s}\right)^2 = \frac{200}{3}\text{ J} \approx 66.7\text{ J}$$

A lot of energy is lost to heat and deformation of metal.

L20-2: Bumper Cars

- Two bumper cars collide with each other and get tangled together.
- Car 1 (m_1) moves north at v_1. Car 2 (m_2) moves south at v_2.
- Case 1
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south at 3 m/s.
  - Find the final velocity of the cars.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.
- Case 2
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south.
  - Find the initial velocity of Car 2 assuming they both end at rest.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.

Case 2

To set up a momentum vector diagram for case 2, I began with entering the initial momentum for Car 1, along with the entire final momentum row (which is all zero). for the two cars, which told me the initial momentum for the whole system. I also assumed that momentum was conserved (by the reasoning listed in L20-1), which allowed me to fill in the 1 & 2 column with more zeros. The zero final momentum in the Car 1 column let me fill in the first car's change in momentum, then the zeros in the 1 & 2 column helped me to fill in the momenta for Car 2.

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	Car 1	Car 2	1 & 2
$\vec{p}_i$	$\uparrow$	$\downarrow$	$\cdot$
$\Delta\vec{p}$	$\downarrow$	$\uparrow$	$\cdot$
$\vec{p}_f$	$\cdot$	$\cdot$	$\cdot$

Already, I know that the second car must have equal and opposite initial momentum to the first car.

$$m_1 = 100 \text{ kg} \qquad m_2 = 200 \text{ kg} \qquad m_f = m_1 + m_2$$
$$\vec{v}_1 = (4 \text{ m/s})\hat{y} \qquad \vec{v}_2 = -v_2\hat{y} = ? \qquad \vec{v}_f = \vec{0}$$

$$\vec{p}_i = m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1v_1 - m_2v_2)\hat{y}$$
$$\vec{p}_f = (m_1 + m_2)\vec{v}_f = \vec{0}$$

$$\vec{p}_i = \vec{p}_f \qquad v_2 = \frac{m_1}{m_2}v_1$$
$$m_1v_1 - m_2v_2 = 0 \qquad = \frac{1}{2}v_1$$
$$= 2 \text{ m/s}$$

Car 2 was going 2 m/s (to the south). They both end at rest, so $K_f = 0$ J.

$$K_i = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
$$= \frac{1}{2}(100 \text{ kg})(4 \text{ m/s})^2 + \frac{1}{2}(200 \text{ kg})(2 \text{ m/s})^2 = 800 \text{ J} + 400 \text{ J} = 1200 \text{ J}$$

All of the energy was lost to heat and deformation of metal.

L20-2: Bumper Cars

- Two bumper cars collide with each other and get tangled together.
- Car 1 (m_1) moves north at v_1. Car 2 (m_2) moves south at v_2.
- Case 1
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south at 3 m/s.
  - Find the final velocity of the cars.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.
- Case 2
  - Car 1 (100 kg) moves north at 4 m/s.
  - Car 2 (200 kg) moves south.
  - Find the initial velocity of Car 2 assuming they both end at rest.
  - Determine the initial and final kinetic energies of the cars.
  - Compare the total kinetic energy before and after the collision.

Main Ideas

- Momentum and impulse are useful quantities for solving dynamics problems.
- The impulse is always equal to the change in momentum for a system.
- When the impulse is zero (because the net force is zero), the momentum of the system is constant—it is *conserved*.