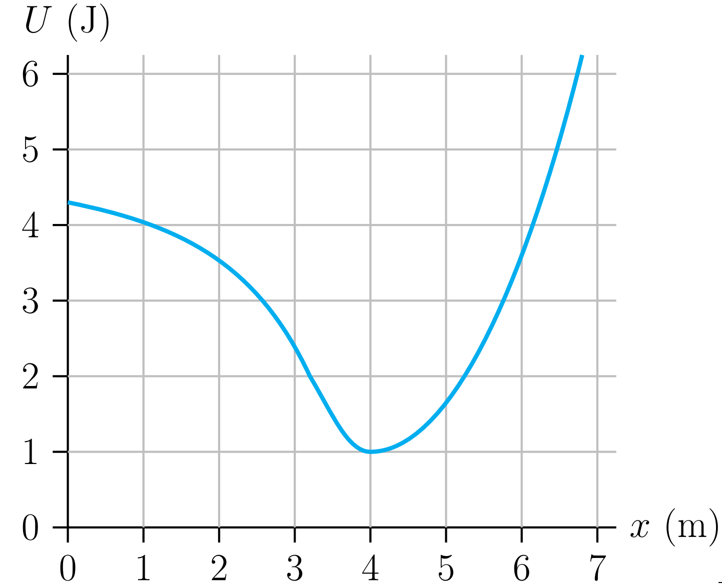
Lecture 18: Potential Energy Diagrams

Warm-Up Activity

A potential energy diagram for some system is shown at right. At what position(s) is the magnitude of the force at a maximum?



Recall that force is the negative derivative of the potential energy:

$$\vec{F} = -\frac{dU}{dx}\hat{x}.$$

As such, the largest force occurs where the slope is steepest. This appears to happen at about $x = 6.8$ m (where the slope is positive, and therefore the force is pointing in the negative direction) and close to $x = 3$ m (where the slope is negative, and therefore the force is pointing in the positive direction).

A Deeper Model for Interactions

- Quantities
  - Energy E
  - Work $W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$
  - Kinetic Energy $K = \frac{1}{2}mv^2$
  - Potential Energy $U =$ depends on interaction
    You have to tell everyone where zero PE is!
    * Gravity $U_g = mgy$
    * Spring $U_{sp} = \frac{1}{2}kx^2$
- Laws
  - Work-energy theorem $W_{\text{net,ext}} = \Delta E_{\text{total}}$

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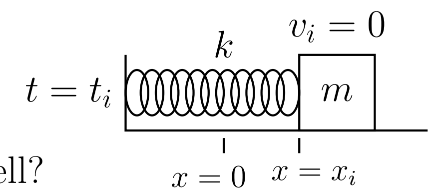
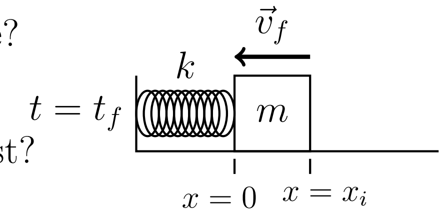
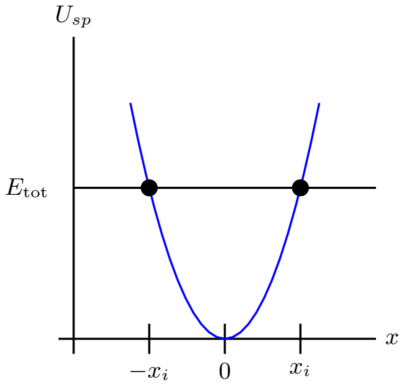
L18-1: Potential Energy Diagrams

The potential energy of the spring is given by $U_{sp} = \frac{1}{2}kx^2$.

- Sketch a potential energy diagram (a graph of U_{sp} vs. x).
- Where is the potential energy largest? Smallest? How can you tell?
- How does the total energy change? How can you tell?
- Where is the kinetic energy largest? Smallest? How can you tell?
- Where is the spring force largest? Smallest? How can you tell?







The spring is stretched when $x > 0$ and compressed when $x < 0$. The potential energy is largest at maximum stretch or compression ($\pm x_i$ in this particular case), and it is smallest (zero) when the spring is at its equilibrium length ($x = 0$). These things can be directly read off of the graph.

The total energy of the block-and-spring system stays the same, as there are no external forces that do work (which we will get into more on Monday). Energy in the system is converted back and forth between kinetic and potential energy without loss or gain. We represent this on the graph with a horizontal line for total energy.

We know $E_{\text{total}} = K + U_{sp}$, so $K = E_{\text{total}} - U_{sp}$. When potential energy is at a minimum, kinetic energy is at a maximum, and vice versa. We have the most kinetic energy at the equilibrium length ($x = 0$), and the least (zero kinetic energy) when the spring is at maximum stretch or compression (which makes sense, as if the spring had any kinetic energy left when it reached one of these end points, it would be able to continue moving and reach a farther point from equilibrium).

The spring force is largest when potential energy is highest (because we are at maximum stretch), and smallest at $x = 0$ (where the spring is unstretched and potential energy is zero). This can be seen from the slope of the graph, as $\vec{F}^{sp} = -\frac{dU_{sp}}{dx}\hat{x}$.

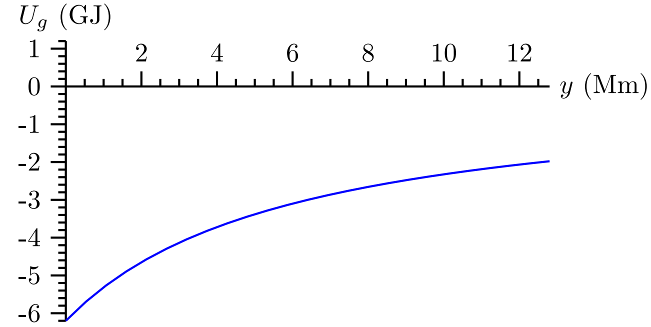
As y increases, the potential energy increases (it becomes less negative). We expect to store more potential energy in the gravitational field when we move farther from the surface (more kinetic energy will be gained in a fall from a greater height).

The strange part is that the energy is entirely negative, and the zero of potential energy is out at $y = \infty$. In orbital mechanics, it is actually convenient to have the energy near a planet be negative, as that tells you how much energy an object needs to escape the planet's gravity. In this case, if we give the astronaut at least 6.2 GJ of kinetic energy, they will be able to get arbitrarily far away from the planet without running out of speed and falling back to the surface.

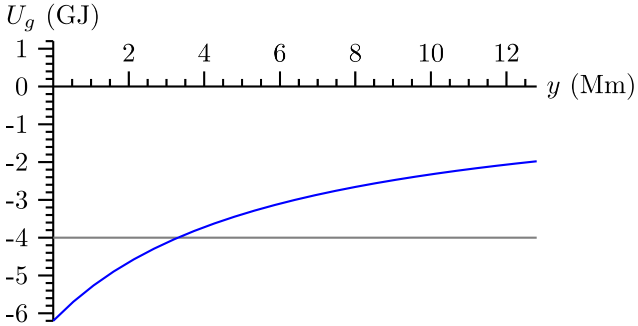
The force is the negative slope of the potential energy: $\vec{F} = -\frac{dU}{dy}\hat{y}$ (the slope is positive everywhere, so the force always points back toward Earth). The slope decreases as we get farther from the surface, indicating that the force of gravity decreases in magnitude. We expect the force of gravity to get weaker as we get farther away from a gravitating body.

L18-2: Astronaut Energy I

We send an astronaut into space, creating the following graph of potential energy U_g vs. distance from the surface of the Earth y.



- Describe how the potential energy of the system changes as y increases. Do you think this behavior makes sense?
- Describe how the gravitational force changes as y increases. Do you think this behavior makes sense? (**Hint:** What feature of the graph tells you about the force?)



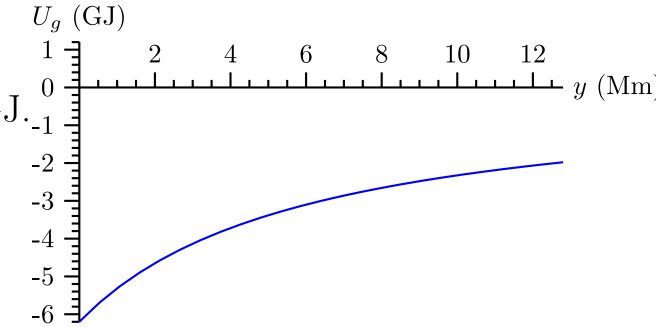
Since $E_{\text{total}} = -4$ GJ, and $E_{\text{total}} = K + U_g$, we know that the astronaut stops when the potential energy is equal to the total energy (meaning there is no kinetic energy, and the astronaut has stopped ascending). This occurs at approximately $y = 3.5$ Mm above the planet's surface.

The largest kinetic energy occurs when the potential energy is smallest, which happens to be at the surface of the Earth. When $y = 0$ Mm, $U_g = -6.2$ GJ, and therefore

$$K = E - U_g = -4 \text{ GJ} - (-6.2 \text{ GJ}) = 2.2 \text{ GJ}.$$

L18-3: Astronaut Energy II

We send an astronaut into space, creating the following graph of potential energy U_g vs. distance from the surface of the Earth y.



Assume the total energy of the Astronaut-Earth system is -4 GJ.

- How high above the surface of the Earth does the astronaut go?
- What is the astronaut's largest kinetic energy? Where does this happen?

Main Ideas

- The work-energy theorem can be used to solve a broad array of problems.
- A variety of representations can be helpful in solving problems using work and energy.