Net Zero Vectors

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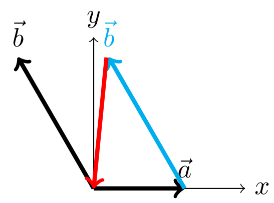
This material is borrowed/adapted from the *Learning Introductory Physics with Activities* textbook.

Activity

Three vectors add together to equal 0. One vector has magnitude 3 and points in the positive x-direction; a second vector has magnitude 5 and points at $120°$ from the positive x-axis. Determine the third vector as a magnitude and direction.

Below, I have drawn the two vectors—I'm calling the first vector along the x-axis $\vec{a}$, and the angled vector $\vec{b}$. A cyan copy of $\vec{b}$ has been placed at the tip of $\vec{a}$ to illustrate the sum $\vec{a}+\vec{b}$. The red vector is what the third vector needs to be for the three to sum to zero. Based on this



representation, we can see that the third vector must be a little shorter than $\vec{b}$, and it will point a little more than $90°$ clockwise from the positive x-axis.

Let's break $\vec{b}$ into components:

$$b_x = 5\cos(120^0) = -2.5,$$
$$b_y = 5\sin(120^0) \approx 4.33.$$

Adding $\vec{a}$ and $\vec{b}$ componentwise gives us

$$\vec{a}+\vec{b} \approx 3\hat{x} + (-2.5\hat{x} + 4.33\hat{y}) = 0.5\hat{x} + 4.33\hat{y},$$

so the third vector (let's call it $\vec{c}$) will be the negative of this:

$$\vec{c} = -\left(\vec{a}+\vec{b}\right) = -0.5\hat{x} - 4.33\hat{y}.$$

We can get the magnitude via the Pythagorean Theorem:

$$|\vec{c}| \approx \sqrt{0.5^2 + 4.33^2} \approx 4.36.$$

The angle can be determined trigonometrically, but most calculators will give you the wrong answer first:

$$\theta^* \approx \arctan\left(\frac{-4.33}{-0.5}\right) \approx 83.4.$$

This would be the angle for $-\vec{c}$. The tangent function has a period of $180°$, so we can subtract this to get another valid angle in the correct quadrant:

$$\theta = \theta^* - 180° \approx -96.6°.$$