Since the force is constant in magnitude and direction, the work simplifies to

$$W = \vec{F}_0 \cdot \Delta\vec{s}.$$

Case 1
$\vec{F}_0$ is parallel to $\Delta\vec{s}$, so $W_1 = \vec{F}_0 \cdot \Delta\vec{s} = F_0 \Delta s > 0$.
Case 2
$\vec{F}_0$ makes an acute angle with $\Delta\vec{s}$ (let's call this angle, the angle $\Delta\vec{s}$ makes below the horizontal, θ), so $W_2 = \vec{F}_0 \cdot \Delta\vec{s} = F_0 \Delta s \cos\theta$, where $0 < \cos\theta < 1$, so $W_2 > 0$.
Case 3
$\vec{F}_0$ is perpendicular to $\Delta\vec{s}$ so $W_3 = \vec{F}_0 \cdot \Delta\vec{s} = 0$.
In this case, remember that the push from the hand and the motion of the block are not necessarily related. The block is already moving for some reason, and we choose to push down on it. We may push, and our hand may move with it, but we add no energy to the system by pushing in a direction the block isn't moving.

Since all of the works are nonnegative, they are equal to their absolute values, and we can rank them as follows:

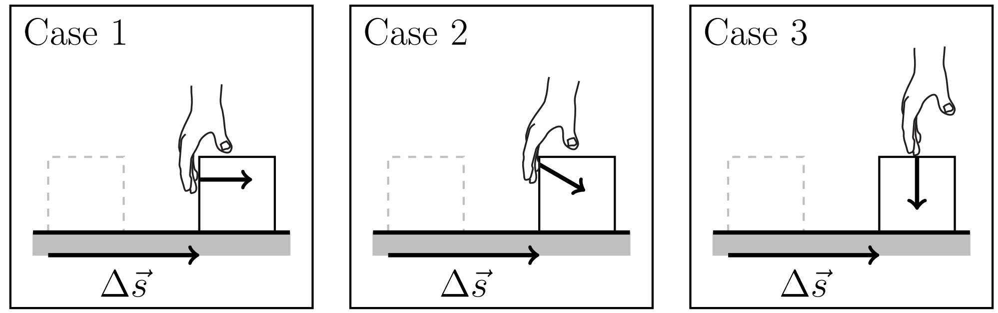
$$W_1 > W_2 > W_3 = 0.$$

Studio 6: Work and Kinetic Energy

Warm-Up Activity

A block is moving to the right (displacement $\Delta\vec{s}$) while a hand exerts a force of magnitude F_0 on the block.

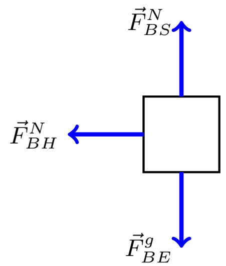
- In each case, is the work done by the hand *positive*, *negative*, or *zero*?
- How does the absolute value of the work compare in the three cases?



A Deeper Model for Interactions

- Quantities
  - Energy E
  - Work $W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$
  - Kinetic Energy $K = \frac{1}{2}mv^2$
- Laws
  - Work-energy theorem $W_{\text{net,ext}} = \Delta E_{\text{total}}$

The free body diagram is the same in both stages:



The work done by the normal force from the surface and work done by the force of gravity are both zero, as these forces are perpendicular to the horizontal displacement. For the force from the hand, let us fill in the table (for two different choices of coordinate system):

$\hat{x}$ $\longrightarrow$	Displacement		Force by Hand		$W_{\text{by hand}}$	$W_{\text{net,ext}}$
	Direction	Sign	Direction	Sign	Sign	Sign
Stage 1	$\longrightarrow$	+	$\longleftarrow$	−	−	−
Stage 2	$\longleftarrow$	−	$\longleftarrow$	−	+	+

$\hat{x}$ $\longleftarrow$	Displacement		Force by Hand		$W_{\text{by hand}}$	$W_{\text{net,ext}}$
	Direction	Sign	Direction	Sign	Sign	Sign
Stage 1	$\longrightarrow$	−	$\longleftarrow$	+	−	−
Stage 2	$\longleftarrow$	+	$\longleftarrow$	+	+	+

The work done by a force does not depend on your choice of coordinate system. There are some signs that are coordinate system dependent (such as those applied to the directions of vectors), but those with physical meaning (such as the sign of work, which indicates whether energy is entering or leaving the system) do not.

The work done is negative in stage 1, where the force is opposite the displacement. Energy is subtracted from the system, and the speed of the block decreases.

The work done is positive in stage 2, where the force is in the same direction as the displacement. Energy is added to the system, and the speed of the block increases.

S6-1: Reversing a Block

- A block is initially moving to the right on a level, frictionless surface. The system consists of only the block.
- A hand exerts a constant horizontal force on the block that causes the block to slow down (stage 1), then move in the opposite direction while speeding up (stage 2).
- For each stage:
  - Draw a free-body diagram for the block.
  - Determine whether the work done by each force is positive, negative, or zero.
  - Fill in the table:

	Displacement		Force by Hand		$W_{\text{by hand}}$	$W_{\text{net,ext}}$
	Direction	Sign	Direction	Sign	Sign	Sign
Stage 1						
Stage 2						

Pushed inward by the two forces, both blocks will accelerate, so they each have a nonzero final speed. They began at rest, so $K_i = 0$ J, and their final kinetic energy is $K_f = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$, there fore the change in kinetic energy of the system is positive ($\Delta K > 0$).

The force of the hand on A points right, and its point of application (where the hand is touching the block) is displaced to the right, so the work done by the hand is positive ($W_A^{\text{hand}} > 0$).
For block B, the force of the hand and the displacement are both to the left, so the work is still positive ($W_B^{\text{hand}} > 0$).
Note that W_B^{hand} is not negative due to being "in the opposite direction." The sign of work is determined by the direction of the force relative to the direction of the dipslacment of its point of contact.

The work done by gravity and the work done by the normal force on both blocks are zero ($W_A^g = W_B^g = W_A^N = W_B^N = 0$). Their points of contact are displacing, but the forces are perpendicular to the displacement.

The net work is the sum of the individual works:

$$\begin{aligned} W_{AB}^{\text{net}} &= W_A^{\text{net}} + W_B^{\text{net}} \\ &= W_A^{\text{hand}} + W_A^g + W_A^N + W_B^{\text{hand}} + W_B^g + W_B^N \\ &= W_A^{\text{hand}} + W_B^{\text{hand}} \\ &> 0. \end{aligned}$$

The net work on the system is positive.
Note that the net work is **not** generally equal to the integral of the net force dotted with the displacement of the center of mass:

$$W^{\text{net}} \neq \int \vec{F}^{net} \cdot d\vec{s}_{CoM}.$$

This would give the wrong answer here, as the net force on AB is zero and the center of mass doesn't move.

Since both $W_{AB}^{\text{net}} > 0$ and $\Delta K > 0$, our results are consistent with the work-energy theorem. Positive work increases the total energy of the system.

S6-2: Pushing Boxes Together

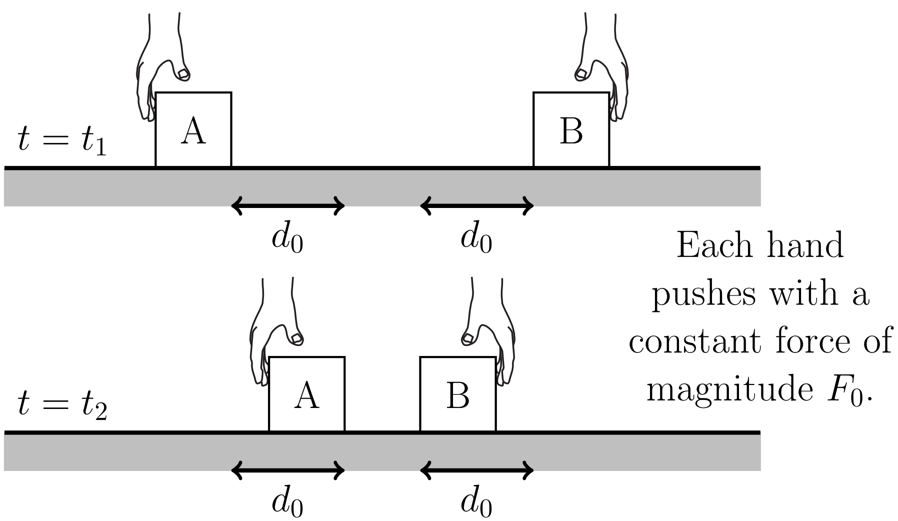
Two identical blocks, A and B, are initially at rest on a level, frictionless surface. System AB consists of **both blocks**.
At time $t = t_1$, hands begin to push the blocks toward each other.
Each hand exerts a constant horizontal force of magnitude F_0.
At time $t = t_2$, each block has moved a distance d_0 from its initial position.

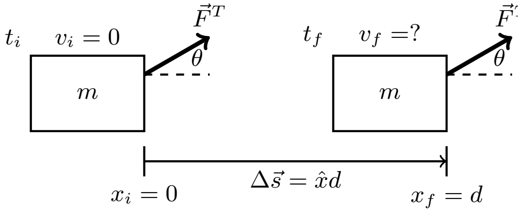
(1) Determine if the **change in kinetic energy** of system AB is *positive*, *negative*, or *zero*.

(2) Determine if the work done by **each force** is *positive*, *negative*, or *zero*.

(3) Determine if the **net work** on system AB is *positive*, *negative*, or *zero*.

(4) Check to see if your answers are consistent with the work-energy theorem.





Gravity and the normal force do no work here, so in general, for a force with constant direction:

$$W^{net} = W^T = \int_0^d \vec{F}^T \cdot d\vec{x} = \int_0^d F^T \cos\theta dx = \cos\theta \int_0^d F^T dx.$$

By the work-energy theorem, $W^{net} = \Delta K$, and since we know $K_i = 0$ and $K_f = \frac{1}{2}mv_f^2$, this means $v_f = \sqrt{\frac{2W^{net}}{m}}$.

Case A
For $F^T = T_0$, we have

$$W_T = T_0 \cos\theta \int_0^d dx = T_0 d \cos\theta, \qquad v_f = \sqrt{2\frac{T_0 d}{m}\cos\theta}.$$

Case B
To get a linear decrease (with respect to x) as described, the following equation should be used for the force:

$$F^T(x) = 3T_0 - \frac{2T_0}{d}x.$$

This gives us

$$\begin{aligned} W_T &= T_0 \cos\theta \int_0^d \left(3 - \frac{2}{d}x\right) dx \\ &= T_0 \cos\theta \left[3x - \frac{1}{d}x^2\right]_{x=0}^d \\ &= T_0 \cos\theta \left(3d - \frac{1}{d}d^2\right) \\ &= 2T_0 d \cos\theta, \\ v_f &= 2\sqrt{\frac{T_0 d}{m}\cos\theta}. \end{aligned}$$

Unit Check
2 and $\cos\theta$ are unitless, and as for the rest,

$$\left[\sqrt{\frac{T_0 d}{m}}\right] = \sqrt{\frac{[T_0][d]}{[m]}} = \sqrt{\frac{\mathrm{kg}\cdot\mathrm{m}\cdot\mathrm{s}^{-2}\cdot\mathrm{m}}{\mathrm{kg}}} = \sqrt{\frac{\mathrm{m}^2}{\mathrm{s}^2}} = \frac{\mathrm{m}}{\mathrm{s}} = [v_f].$$

Covariation
Pushing the block harder or pulling it farther will impart more kinetic energy, causing it to end up going faster. This is reflected in our equations, as increasing T_0 or d causes v_f to increase.
Pulling at a larger angle to the displacement is less efficient for doing work, and a more massive object resists accelerating more. This is reflected in our equations, as increasing θ or m causes v_f to decrease.

S6-3: Dragging a Box

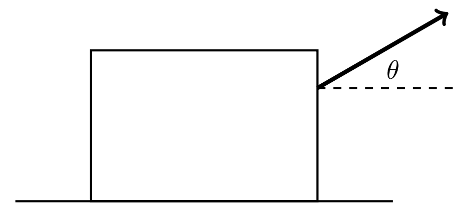
- You are pulling a box with known mass m at the angle shown below. The box moves from $x = 0$ to $x = d$. Find the work done by the rope in each of the following situations:

  (A) The tension is constant.

  (B) The tension decreases linearly from $3T_0$ to T_0.

- If the box began at rest, find the final speed of the box in each of the situations above.

- Don't forget to make sense of your answers!



Main Ideas

- Energy is a powerful, ubiquitous concept that can help us solve a wide array of physics problems.
- Energy is a *scalar*—it is not a vector.
- There are different forms of energy, and energy can be transferred between objects and between forms.