PH 221 Week 8

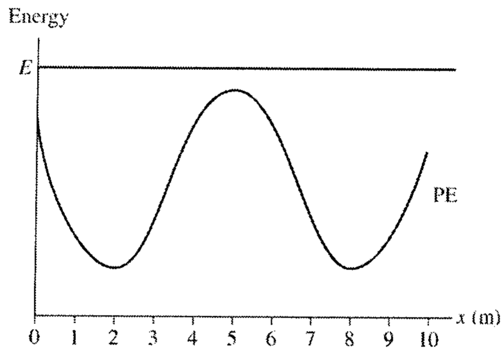
Benjamin Bauml

Spring 2024

This material is borrowed/adapted from Chapter 10 of the *Student Workbook* for *Physics for Scientists and Engineers.*

R8-1: Interpreting a Potential Energy Graph

A particle with the potential energy shown in the graph is moving to the right at $x = 0$ m with total energy E.



(a) At what value or values of x is the particle's speed a maximum?

At 2 m and 8 m, where the potential energy is smallest.

(b) At what value or values of x is the particle's speed a minimum?

At 5 m, where the potential energy is largest.

(c) At what value or values of x is the potential energy a maximum?

At 5 m.

(d) Does this particle have a turning point in the range of x covered by the graph? If so, where?

There are no turning points, as the potential energy never crosses the total energy line.

(e) In which intervals of x is the force on the particle to the right?

Since $F_x = -\frac{dU}{dx}$ (negative of slope), we know that the force is to the right (F_x is positive) from 0 m to 2 m and from 5 m to 8 m.

(f) In which intervals of x is the force on the particle to the left?

The force is to the left (F_x is negative) from 2 m to 5 m and from 8 m to 10 m.

(g) At what value or values of x is the magnitude of the force a maximum?

When the slope of U versus x is steepest: $x = 0$ m, 3.5 m, 6.5 m, and 10 m.

(h) At what value or values of x are positions of stable equilibrium?

Equilibrium occurs when $F_x = 0$ N. Stable equilibrium occurs when the force felt on either side of the point pushes the particle back toward the point. This happens at the minima of potential energy: $x = 2$ m and 8 m.

(i) At what value or values of x are positions of unstable equilibrium?

Unstable equilibrium occurs when the force felt on either side of the equilibrium point pushes the particle away from the point. This happens at the maximum of potential energy: $x = 5$ m.

(j) If the particle is released from rest at $x = 0$ m, will it reach $x = 10m$? Explain.

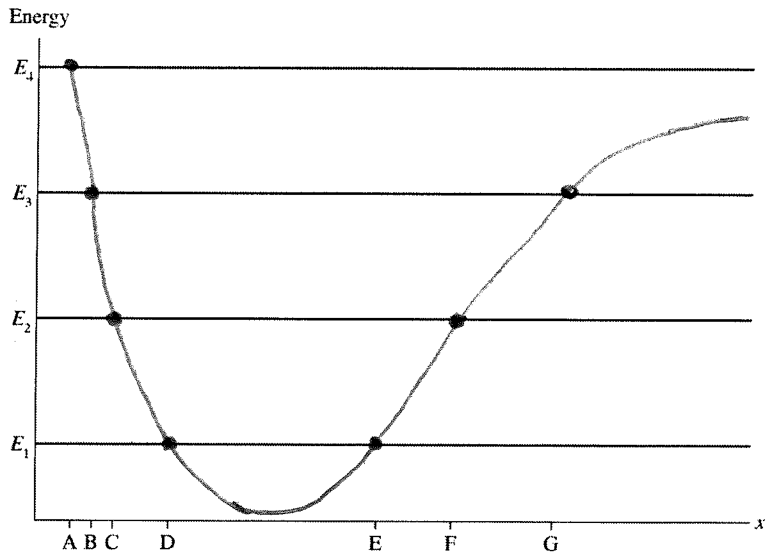
No. If released from rest, the particle's total energy is the initial potential energy and can never exceed this value. The particle will turn around at about $x = 3.5$ m.

R8-2: Producing a Potential Energy Graph

Below is a set of axes on which you are going to draw a potential energy curve. By doing experiments, you find the following information:

- A particle with energy E_1 oscillates between positions D and E.
- A particle with energy E_2 oscillates between positions C and F.
- A particle with energy E_3 oscillates between positions B and G.
- A particle with energy E_4 enters from the right, bounces at A, then never returns.

Draw a potential energy curve that is consistent with this information.



This is just one realization of the graph. Other solutions are possible with extra wiggles. Wherever you are at on the graph, as long as the wiggle you add does not cross above the next energy level above and create new turning points, the graph will still be correct. You may even create wiggles that cross energy levels lower than where you are at on the graph.

R8-3: Potential versus Change in Potential

(a) If the force on a particle at some point in space is zero, must its potential energy also be zero at that point? Explain.

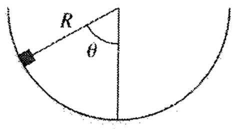
No. The rate of change of potential energy with respect to position will be zero at this point. For example, the potential energy can be a nonzero constant in the region that contains this point.

(b) If the potential energy of a particle at some point in space is zero, must the force on it also be zero at that point? Explain.

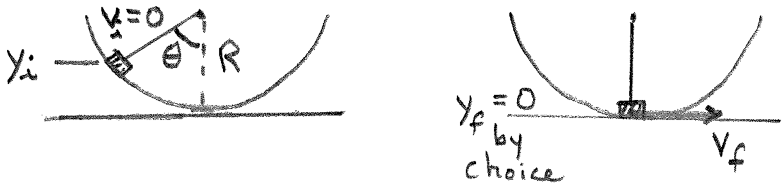
No. If the potential energy is changing along a line that passes through zero potential at this point, then the force will not be zero.

R8-4: Sliding Cube in a Bowl

A small cube of mass m slides back and forth in a frictionless, hemispherical bowl of radius R. Suppose the cube is released at angle θ. What is the cube's speed at the bottom of the bowl?



(a) Begin by drawing a before-and-after pictorial representation. Let the cube's initial position and speed be y_i and v_i. Use a similar notation for the final position and speed.



(b) At the initial position, are either K_i or U_{Gi} zero? If so, which?

The cube is released from rest at the initial position, so $K_i = 0$ J.

(c) At the final position, are either K_f or U_{Gf} zero? If so, which?

We can choose the zero of gravitational potential energy to be at the bottom of the bowl, so $U_{Gf} = 0$ J.

(d) Does thermal energy need to be considered in this situation? Why or why not?

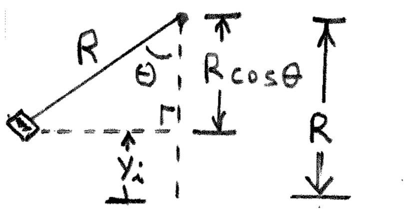
No, because the bowl is frictionless. There are no dissipative forces stealing energy from the system.

(e) Write the conservation of energy equation in terms of position and speed variables, omitting any terms that are zero.

$$\cancel{K_i} + U_{Gi} = K_f + \cancel{U_{Gf}}$$

$$mgy_i = \frac{1}{2}mv_f^2$$

(f) You're given not the initial position but the initial angle. Do the geometry and trigonometry to find y_i in terms of R and θ.



$y_i = R(1 - \cos\theta)$

(g) Use your result of part (f) in the energy conservation equation, and then finish solving the problem.

$$
\begin{aligned}
mgy_i &= \frac{1}{2}mv_f^2 \\
\cancel{m}gR(1-\cos\theta) &= \frac{1}{2}\cancel{m}v_f^2 \\
\implies v_f &= \sqrt{2gR(1-\cos\theta)}
\end{aligned}
$$