Lecture 2: Motion

THREE IMPORTANT STEPS:

1. Grab a small whiteboard for yourself! (These are by the doors.)
2. Sit at tables 2, 3, 6, and 7 (the four closest to the center).
3. Grab three large whiteboards for your table!

I will be randomizing your groups today.

A Model for Motion

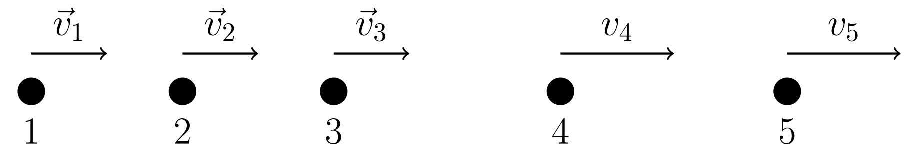
Quantities

- Position: $\vec{r}$
- Velocity: $\vec{v} = \frac{d\vec{r}}{dt}$
- Acceleration: $\vec{a} = \frac{d\vec{v}}{dt}$

Assumptions

- Use the Particle Model

Motion Diagram



Explanations

L2-1: Comparing Motion Diagrams

Principles

- $\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}$
- For a motion diagram, the time intervals are the same.

Reasoning

Method 1:

Let us look at the beginning and end of the motion. Each object starts and ends at the same spot:

$$\Delta \vec{r}_1 = \Delta \vec{r}_2 = \Delta \vec{r}.$$

Object 2 takes more time to get to the final position:

$$\Delta t_2 > \Delta t_1.$$

Now we have

$$\vec{v}_{avg,1} = \frac{\Delta \vec{r}_1}{\Delta t_1}, \qquad \vec{v}_{avg,2} = \frac{\Delta \vec{r}_2}{\Delta t_2}.$$

Since $\Delta \vec{r}$ is the same for both, a larger number in the denominator gives a smaller overall number. Therefore

$$|\vec{v}_{avg,1}| > |\vec{v}_{avg,2}|.$$

Method 2:

Since the velocity appears to be constant (that is, $\Delta \vec{r}$ between two adjacent spots is the same), we can also look at individual positions.

For object 1, $\Delta \vec{r}$ between times 1 and 2 is greater than for object 2:

$$\Delta \vec{r}_1 > \Delta \vec{r}_2.$$

The time between these two points is the same for both objects:

$$\Delta t_1 = \Delta t_2 = \Delta_t.$$

Now we have

$$\vec{v}_{avg,1} = \frac{\Delta \vec{r}_1}{\Delta t}, \qquad \vec{v}_{avg,2} = \frac{\Delta \vec{r}_2}{\Delta t}.$$

The denominators are the same, so having a larger number in the numerator gives a larger overall number. Therefore

$$|\vec{v}_{avg,1}| > |\vec{v}_{avg,2}|.$$

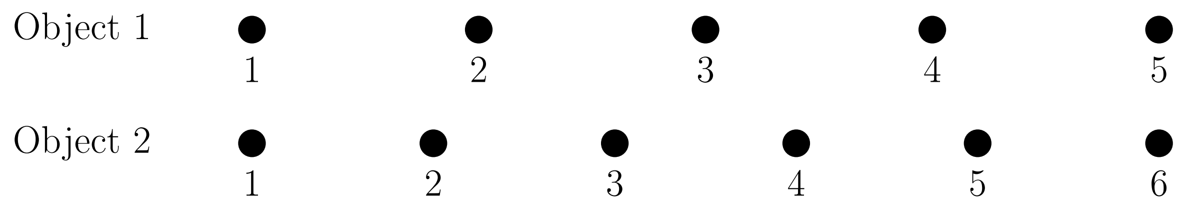
Conclusion

The average speed of object 1 is greater than the average speed of object 2:

$$|\vec{v}_{avg,1}| > |\vec{v}_{avg,2}|.$$

L2-1: Comparing Motion Diagrams

The diagrams below show the motion of two different objects. Is the average velocity of the upper object greater than, less than, or equal to the average velocity of the lower object? Explain your reasoning.



Explanations in Physics

1. **Principles** – what fundamental physics concepts, laws, or definitions did you start with?
2. **Reasoning** – explain all the reasoning steps to go from your starting point to your conclusion.
3. **Conclusion** – state your conclusion clearly.

L2-2 Thrown Ball

The ball goes up, slows down, comes to a brief stop, turns around, and falls back down, increasing in speed.

Analyze and Represent

1a: Understand the Problem

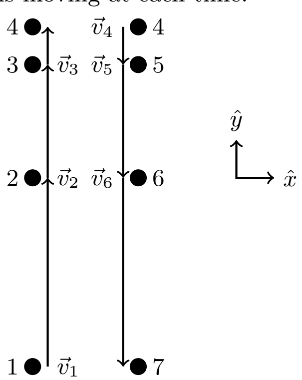
- $\vec{r}$: position of the ball (subscripts will denote which point in time the position corresponds to)
- $\vec{v}$: velocity of the ball (subscripts will denote which point in time the position corresponds to)
- $\vec{r}_1$: initial position
- $\vec{v}_1$: initial velocity
- $\vec{r}_7$: final position
- $\vec{v}_7$: final velocity
- t: time

1b: Identify Assumptions

- The ball is thrown in a straight line.
  - This simplification allows us to consider motion in only one dimension.
- The ball is released and caught at the same spot.
  - An actual toss and catch isn't going to be perfect, but small, random differences in starting and ending height are a minor detail that we don't need to bother with when looking at this general sort of motion.
- Particle model.
  - This assumption allows us to ignore any ambiguity in the object's position (we don't have to decide to measure from the center, top, or bottom), and to ignore any rotational motion the object may have about its own axis.

1c: Represent Physically

A motion diagram can help us visualize the motion of the ball. The velocity vectors (based on forward approximation) give us a sense of how fast the ball is moving at each time.



(Always establish a coordinate system!)

L2-2: Thrown Ball

A ball is thrown straight into the air.

- Describe the motion in words (use complete sentences).
- Identify any quantities of interest with a symbol.
- Draw a motion diagram for the ball.
- Discuss any assumptions or idealizations you want to make.

1. Analyze and Represent

1a. **Understand the problem** – identify quantities by symbol and number.
1b. **Identify Assumptions** – identify important simplifications and assumptions.
1c. **Represent physically** – draw and label one or more appropriate diagrams and/or graphs that might help you solve the problem.

Forward Approximation

The velocity at time t_j is approximated as the average velocity from t_j to t_{j+1}:

$$\vec{v}_j \approx \frac{\vec{r}_{j+1} - \vec{r}_j}{\Delta t}$$

Note that this means we can't give a velocity to the final dot, as there is no next dot to get a final position from.

Backward Approximation

The velocity at time t_j is approximated as the average velocity from t_{j-1} to t_j:

$$\vec{v}_j \approx \frac{\vec{r}_j - \vec{r}_{j-1}}{\Delta t}$$

This time, we can't give a velocity to the first dot, as there is no previous dot to get an initial position from.

Balanced Approximation

The velocity at time t_j is approximated as the average velocity from t_{j-1} to t_{j+1}:

$$\vec{v}_j \approx \frac{\vec{r}_{j+1} - \vec{r}_{j-1}}{2\Delta t}$$

Now, we can't give a velocity to either the first or last dot. Note that we are looking two dots apart, so the time interval is doubled.

Instantaneous

Here, the velocity is actually calculated as the derivative of a known position equation. Note that the velocities from the balanced approximation is extremely close to the instantaneous velocities in this situation.

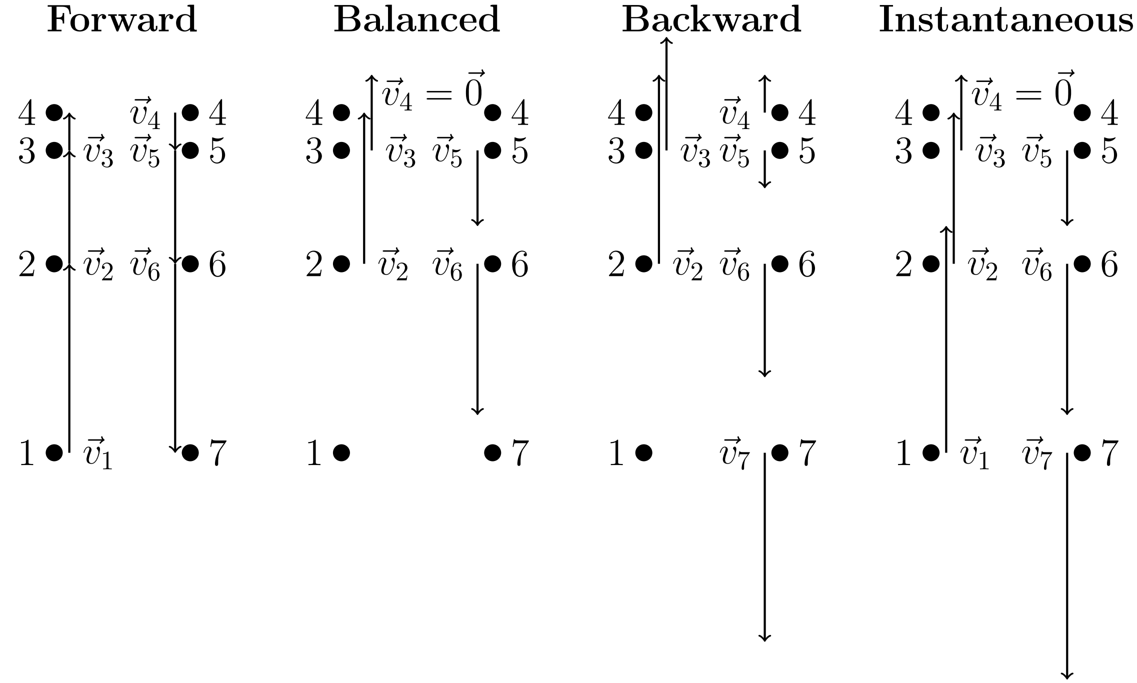
Motion Graph

A motion diagram overlays snapshots of the object at different times into a single picture. A motion graph gives time an axis of its own. Each point on the graph has the same y separation as the corresponding points in the motion diagram, but they now also have a separation in t that clarifies at what point they happen in time.

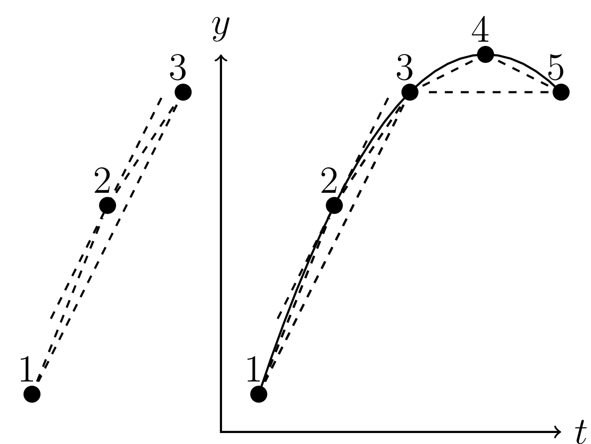
The forward and backward approximations are a bit rough, whereas the balanced approximation is close to the instantaneous velocity. It is still an approximation, though, so it is not perfect.

The approximations of $\vec{v}_2$ were brought out from the graph for clarity. The backward approximation is too steep (giving too large of a velocity), and the forward approximation is not steep enough (giving too small of a velocity).

Motion Diagrams



Motion Graph



L2-2 Thrown Ball
Calculate
2a: Represent Principles

Instantaneous velocity at a particular time can be approximated by the average velocity over a span of time containing that particular time:

$$\vec{v} = \frac{d\vec{r}}{dt} \approx \vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t}.$$

2b: Find unknown(s) Symbolically

For the entire motion, the average velocity from start to finish is

$$\vec{v}_{avg} = \frac{\vec{r}_7 - \vec{r}_1}{t_7 - t_1} = 0,$$

because it starts and ends at the same position ($\vec{r}_7 = \vec{r}_1$)!

For the first half, we can calculate the average velocity from the launch to the peak:

$$\vec{v}_{avg} = \frac{\vec{r}_4 - \vec{r}_1}{t_4 - t_1}.$$

For the second half, we can calculate the average velocity from the peak to the catch:

$$\vec{v}_{avg} = \frac{\vec{r}_7 - \vec{r}_4}{t_7 - t_4}.$$

At this point, we can already tell that the velocity is not constant. Since $\vec{r}_1 = \vec{r}_7$ and $t_4 - t_1 = t_7 - t_4$, we can see that these two velocities are in opposite directions (and even if the vector stays the same magnitude, it is not constant if its direction changes).

2c: Plug in Numbers

We are not given numbers, so we should estimate some. If I toss an object such that it lands back in my hand after half of a second (so it reaches its maximum height after a quarter of a second), it looks like it gets maybe a foot (so a third of a meter) up in the air.

For the first half of the motion, this gives us

$$\Delta \vec{r} \approx \left(\frac{1}{3}\mathrm{m}\right)\hat{y}, \qquad \Delta t \approx 0.5\mathrm{s},$$

which means $\vec{v}_{avg} \approx \frac{1}{6}\frac{\mathrm{m}}{\mathrm{s}}\hat{y}$. For the second half, we get

$$\Delta \vec{r} \approx \left(\frac{1}{3}\mathrm{m}\right)(-\hat{y}), \qquad \Delta t \approx 0.5\mathrm{s},$$

which means $\vec{v}_{avg} \approx \frac{1}{6}\frac{\mathrm{m}}{\mathrm{s}}(-\hat{y})$.

Note that the direction information is all in the unit vector, as seen by the negative sign. Also, the magnitude carries units, whereas the unit vector does not (which is admittedly a confusing naming issue).

L2-2: Thrown Ball

A ball is thrown straight into the air.

- Estimate the average velocity of the ball:
  - During the entire motion.
  - During the first half of the motion.
  - During the second half of the motion.
- Is the velocity constant?

2. Calculate

2a. **Represent principles** – identify relevant concepts, laws, or definitions.
2b. **Find unknown(s) symbolically** – without numbers, find any unknown(s) in terms of symbols representing known quantities.
2c. **Plug in numbers** – plug numbers (with units) into your symbolic answer!

If you want to calculate an estimate of instantaneous acceleration by finding the average acceleration, it works just like calculating average velocity, except it is the difference in velocity that we divide by elapsed time. However, if you are calculating average acceleration from estimates of velocity, your approximation will get worse, since there is error associated wit both calculation results.

Acceleration

- An object that changes in velocity is said to be accelerating.
- Acceleration is defined as the change in velocity divided by the change in time.
- Average:

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t}$$

- Instantaneous:

$$\vec{a} = \frac{d\vec{v}}{dt}$$

Main Ideas

- The motion of an object can be characterized by quantities like position, velocity, and acceleration.
- Velocity is defined as the time rate of change of position.
- Acceleration is defined as the time rate of change of velocity.