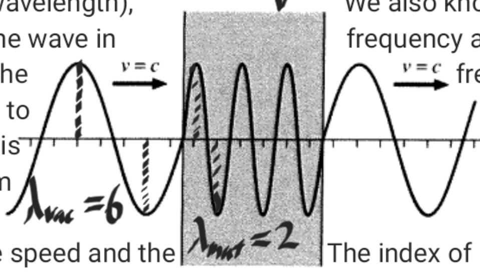
Refraction of Light

(a) A light wave travels from vacuum, through a transparent material, and back to vacuum. What is the index of refraction of this material? Explain.

$n = \frac{c}{v}$

Note that from peak to trough (half the wavelength), the wave in vacuum spans 3 units, and the wave in the transparent material spans 1. Thus, the wavelength goes from 6 units in vacuum to 2 in the material. The index of refraction is the ratio of the speed of light in a vacuum to the speed of light in a material, so we know that $vn = c$, so the product of the speed and the index of refraction will be constant between media.



We also know that the wave speed is related to the frequency and the wavelength by $v = f\lambda$, and frequency will not change across the boundary. As such,

$v_{vac} n_{vac} = v_{mat} n_{mat}$

$\cancel{f}\lambda_{vac} n_{vac} = \cancel{f}\lambda_{mat} n_{mat}$

$6 \cdot 1 = 2\, n_{mat} \Rightarrow n_{mat} = 3$

The index of refraction in the transparent material is 3.

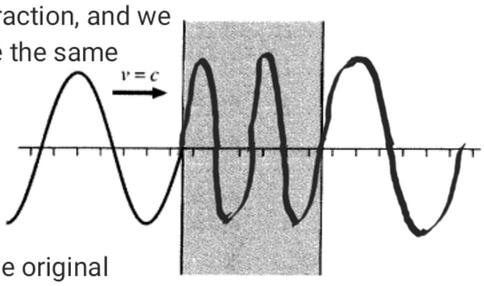
(b) A light wave travels from vacuum, through a transparent material whose index of refraction is $n = 2.0$, and back to vacuum. Finish drawing the snapshot graph of the light wave at this instant.

We know the vacuum wavelength is still 6 units, but now we know the material's index of refraction, and we want to find the wavelength. We can use the same relationship as above:

$\lambda_{vac} n_{vac} = \lambda_{mat} n_{mat}$

$6 \cdot 1 = \lambda_{mat} \cdot 2$

$\Rightarrow \lambda_{mat} = 3$



Once the wave returns to the vacuum, the original wavelength will resume.

Flute A flutist assembles her flute in a room where the speed of sound is 343 m/s. When she plays the note A, it is in perfect tune with a 440 Hz tuning fork. After a few minutes, the air inside her flute has warmed to where the speed of sound is 346 m/s.

(a) What will the new frequency of the flute be?

The possible wavelengths for standing waves in the flute (which we shall model as an open-open tube), are limited by its length. As such, the wavelength does not change when the speed of sound changes.

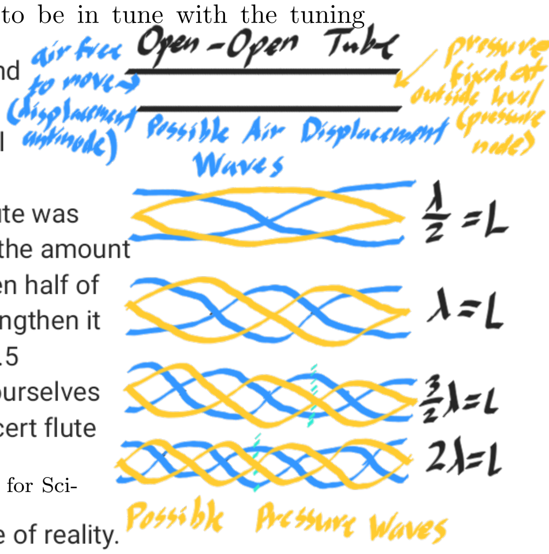
$\frac{v_1}{v_2} = \frac{\cancel{\lambda} f_1}{\cancel{\lambda} f_2} \Rightarrow f_2 = \frac{v_2}{v_1} f_1 = \frac{346\ m/s}{343\ m/s}\ 440\ Hz \approx 443.8\ Hz$

(b) How far does she need to extend the "tuning joint" of her flute to be in tune with the tuning fork?

Since $v = \lambda f$, we can find the original wavelength using the original speed of sound and the original frequency: $343\ m/s = \lambda (440\ Hz) \Rightarrow \lambda \approx 0.780\ m.$

We can find the required wavelength in the same way, by fixing the desired original frequency at the new speed of sound: $346\ m/s = \lambda(440\ Hz) \Rightarrow \lambda \approx 0.786\ m.$

As such, we need to increase the wavelength in the flute by 6 millimeters. If the flute was 0.780 meters long (thus the full wavelength fits in it), then 6 millimeters would be the amount the flautist needs to lengthen it. However, if the flute is only 0.390 meters long, then half of the wavelength fits in it (while producing the same sound), and so we need only lengthen it by 3 millimeters. Hypothetically, a 1.17 meter flute could also play this note with 1.5 wavelengths in it, but that size verges on the unrealistic, so we need not concern ourselves with that. Admittedly, however, a quick Google search tells us that a standard concert flute is about 67 cm long, and a piccolo is about 33 cm long, so this problem is not quite representative of reality.



[0] Select problems may be modified from PH 212 course textbook; Knight Physics for Scientists and Engineers

Yelling on a train You are walking towards your friend at 2 m/s while on opposite sides of a train car when you shout to get their attention at approximately 165 Hz.

(a) Predict: Rank the frequency of the sound when it reaches your friend if the train is:

Same

(i) Fully enclosed, and at rest

(ii) Fully enclosed, and moving 8 m/s

(iii) Open-air (as if you were standing on top of the train), and at rest

In all of these first three cases, your friend is at rest relative to the air. Even when the train is moving, the enclosed car moves the contained air with it. As such, we can expect the first three cases to be the same.

(iv) Open-air, and moving at 8/s

In the open air case with a moving train, the answer depends on whether you are moving in the same direction as the train, or against it. In the former case, it is as though your friend is receding from you at 8 m/s, but you are pursuing at 10 m/s. In the latter case, it is as though your friend is pursuing at 8 m/s, and you are receding at 6 m/s.

$$f_{IV} = f_{em}\frac{(V_s - V_T)}{(V_s - V_T) - V_{em}} \approx 165.99\ Hz$$

$$165.95\ Hz$$

(b) Calculate the observed frequency in each scenario described above.

We begin with a general form of the Doppler effect equation:

$$f_{ob} = f_{em}\frac{V_s \pm V_o}{V_s \pm V_{em}},$$

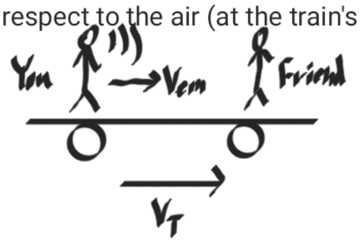
We start here to figure out the proper signs via sensemaking. In particular, we expect the observed frequency to increase if the emitter is moving toward the observer, so to make the fraction larger, we shrink the denominator by subtracting the emitter speed from the speed of sound in the medium. Similarly, we expect the observed frequency to decrease if the observer is moving away from the emitter, so the make the fraction smaller, we shrink the numerator by also subtracting the observer speed from the speed of sound. This gives us

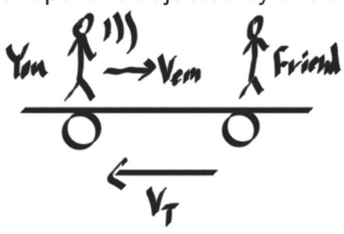
$$f_{ob} = f_{em}\frac{V_s - V_o}{V_s - V_{em}}.$$

In the first three cases, your friend is not moving relative to the medium, so the speed of the observer is zero, giving us

$$f_{ob} = f_{em}\frac{V_s}{V_s - V_{em}} = (165\ Hz)\frac{343\ m/s}{\underbrace{343\ m/s}_{\text{assumed as in prior problem}} - 2\ m/s} \approx 165.97\ Hz$$

In the third case, we shall break it into the cases of moving with the train and moving against the train. Now, the observer is moving with respect to the air (at the train's speed $V_T = 8\ m/s$), and your speed is adjusted by this amount.





$$f_{ob} = f_{em}\frac{V_s - V_T}{V_s - V_T - V_{em}} \approx 165.99\ Hz \qquad f_{ob} = f_{em}\frac{V_s + V_T}{V_s + V_T - V_{em}} \approx 165.95$$

If you are moving with the train, then your friend's observed frequency is higher than in cases (i-iii). If you are moving against the train, then your friend's observed frequency is lower than in the prior cases. Also, it is worth noting that you could group the train speed with the speed of sound to create the quantity $V_s \pm V_T$ in both the numerator and the denominator, which acts as an effective speed of sound adjusted by wind.