Springloaded Sled

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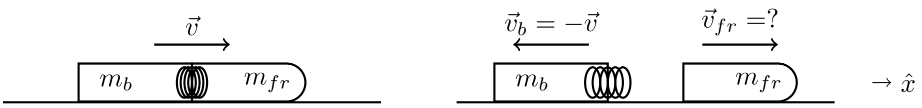
Spring 2024

XX-1: Springloaded Sled

You are designing a sled with a compressed spring inside, which can be released to separate the sled into two pieces of equal mass ($m/2$). You are racing the sled across level snow at speed v when you trigger the separation.

Right after the two halves push apart, the back end of the sled is moving backward with speed v. What is the velocity of the other piece? How much kinetic energy did the system gain?



When setting up a motion vector diagram for this problem, I know the initial momentum of the combined sled system, and I know both parts of the sled have half of this momentum. I also know that the final momentum of the back half is reversed, and the total momentum of the system is unchanged, so I can infer the rest of the table from there.

	Back	Front	Both
$\vec{p}_i$	$\rightarrow$	$\rightarrow$	$\longrightarrow$
$\Delta\vec{p}$	$\longleftarrow$	$\longrightarrow$	$\bullet$
$\vec{p}_f$	$\leftarrow$	$\longrightarrow$	$\longrightarrow$

The initial momentum of the system is $mv\hat{x}$, and the final momentum is $(\frac{m}{2}v_{fr} - \frac{m}{2}v)\hat{x}$. Since the momentum is conserved (no friction, and the normal force and gravitational force are in balance, so no other net impulse), we know that

$$\begin{aligned} mv\hat{x} &= \left(\frac{m}{2}v_{fr} - \frac{m}{2}v\right)\hat{x} \\ mv &= \frac{m}{2}v_{fr} - \frac{m}{2}v \\ 2v &= v_{fr} - v \\ v_{fr} &= 3v. \end{aligned}$$

The front half of the sled gets launched forward at triple its original speed!

As for kinetic energy, the system started with $K_i = \frac{1}{2}mv^2$, and now it has

$$K_f = \frac{1}{2}\frac{m}{2}v^2 + \frac{1}{2}\frac{m}{2}(3v)^2 = \frac{5}{2}mv^2,$$

therefore the change in kinetic energy is

$$\Delta K = K_f - K_i = 2mv^2.$$

This came from the spring. If the spring constant is k and the spring was compressed by a length Δx, then we have

$$\begin{aligned}\frac{1}{2}k\Delta x^2 &= 2mv^2 \\ k\Delta x^2 &= 4mv^2.\end{aligned}$$

This has some interesting design implications. For example, say each half of our sled is 100 kg (say that accounts for the machinery and the load of a single passenger on each half) and its initial speed was a lazy 1 m/s. That would mean the spring has to store 400 J of energy. If $\Delta x = 0.5$ m (which may be too much of a compression for a reasonable use of Hooke's law), then $k = 3200$ N/m (or 32 N/cm), which is a pretty stiff spring. If we cannot get a spring this stiff, then we need more compression, but if we cannot obtain a spring that compresses far enough without permanently deforming, then we need it stiffer. The key will be finding the perfect middle ground (and those of you who are doing experiments with elasticity in your capstone lab might have a better idea than using a single spring).