Studio Week 4
Free-body Diagrams

Picture credit: The Mandalorian (2019)

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Make sense of everything.**

A note about forces

- When you identify and describe a force:
  - Say what kind of force it is.
  - Determine the object the force is being acted on.
  - Determine the object that is exerting the force.
  - Write a symbolic version of the force that includes the information above.
  - Represent all the forces acting on a single object or system using a ***free-body diagram***.

$$\vec{F}^{type}_{on,by}$$

Example

"The normal force on object A exerted by object B."

$\vec{F}^{N}_{AB}$

N – type of force

B – what source exerts the force

A – what the force acts on

All forces on the same free body diagram should act on the same thing (same first subscript).

(Newton's) Laws of Motion

1. An object in motion (or at rest) stays in motion (or at rest) unless a net external force acts on it.
2. The net force on an object is equal to the object's mass times its acceleration.

$$\vec{F}_{\text{net}} = m\vec{a}$$

Activity 4-1: Moving a Box

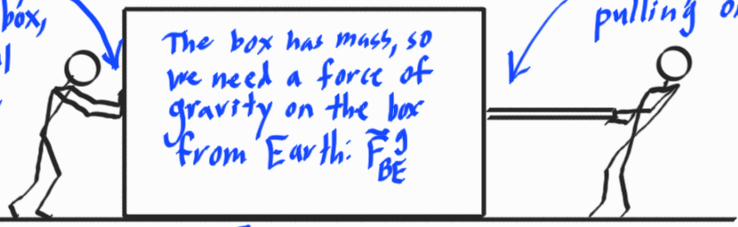
- Mike and Lucas are attempting to move a box, which does not move.
  - Identify all forces acting on the box.
  - Draw a free-body diagram for the box.
  - Indicate the acceleration.

4-1 Moving a Box

Identify all Forces

Mike is pushing directly into the side of the box, so we need a normal force on the box by Mike: $\vec{F}^{N}_{BM}$

The rope held by Lucas is pulling on the box, so we need a force of tension on the box by the rope: $\vec{F}^{T}_{BR}$

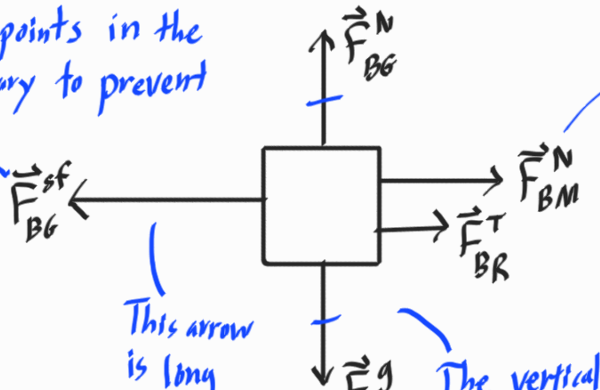


The box has mass, so we need a force of gravity on the box from Earth: $\vec{F}^{g}_{BE}$

The box is sitting on the ground and not budging, so we need a normal force on the box from the ground, and a force of static friction on the box from the ground: $\vec{F}^{N}_{BG}$ & $\vec{F}^{sf}_{BG}$

Free Body Diagram

Static friction points in the direction necessary to prevent slipping.



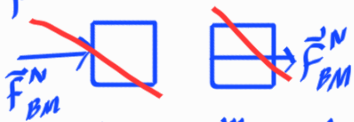
In a basic FBD, we don't care about where the force acts. We put the tail on the box and the arrow points out.

$\vec{F}^{net} = \vec{0}$

$\vec{a} = \vec{0}$

This arrow is long enough to cancel both of the other horizontal forces.

The vertical forces cancel each other.



Physicists don't do this.

We won't do anything like this until we get to rigid body diagrams in PH 212.

Activity 4-2: Moving a Box II

- El pushes the box with her mind, and it begins to speed up.
  - Modify your free-body diagram.
  - Indicate the acceleration.
  - Write a symbolic expression for the acceleration.

4-2 Moving a Box II

Identify all Forces

We still have most of the old forces: $\vec{F}^{N}_{BM}$, $\vec{F}^{g}_{BE}$, $\vec{F}^{T}_{BR}$, $\vec{F}^{N}_{BG}$ $\vec{F}^{sf}_{BG}$

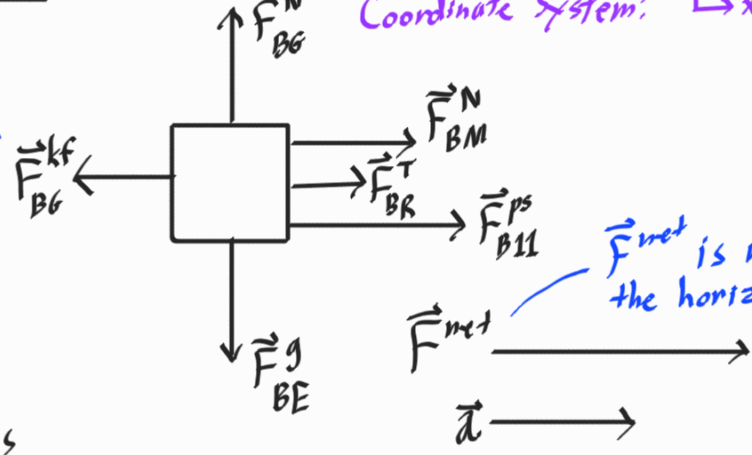
The box is sliding, so we have kinetic friction instead of static: $\vec{F}^{kf}_{BG}$

We need to add in the psychic force exerted on the box by Eleven: $\vec{F}^{ps}_{B11}$

Free Body Diagram

Coordinate System: $\hat{y}$ $\hat{x}$

Kinetic friction is weaker than static friction; it is harder to start an object sliding than to keep it sliding.



$\vec{F}^{net}$ is nonzero, as the horizontal forces are no longer balanced.

Symbolic Expressions

The net force is the vector sum of all forces on the object. Each $\vec{F}$ contains direction information, so you should not put in minus signs at this point.

$$\vec{F}^{net} = \vec{F}^{N}_{BG} + \vec{F}^{N}_{BM} + \vec{F}^{T}_{BR} + \vec{F}^{ps}_{B11} + \vec{F}^{g}_{BE} + \vec{F}^{kf}_{BG}$$

The components of a force are scalars with signs, so they also contain direction information specific to their particular coordinate axes. There is no need to add minus signs when breaking Newton's 2nd law into components.

$$F^{net}_{x} = F^{N}_{BM,x} + F^{T}_{BR,x} + F^{ps}_{B11,x} + F^{kf}_{BG,x}$$

I only included forces with nonzero x-components.

$$F^{net}_{y} = F^{N}_{BG,y} + F^{g}_{BE,y}$$

I only included forces with nonzero y-components.

When I write the components in terms of the magnitudes of the vectors, I have to add signs, as magnitudes are always positive

$$m a_x = F^{net}_{x} = F^{N}_{BM} + F^{T}_{BR} + F^{ps}_{B11} - F^{kf}_{BG}$$

$F^{kf}_{BG,x} = -F^{kf}_{BG}$

We know the box slides horizontally, so $a_y = 0$.

$$0 = m a_y = F^{net}_{y} = F^{N}_{BG} - F^{g}_{BE}$$

$F^{g}_{BE,y} = -F^{g}_{BE}$

Therefore: $a_x = \dfrac{F^{N}_{BM} + F^{T}_{BR} + F^{ps}_{B11} - F^{kf}_{BG}}{m}$

Activity 4-3: Book on a Ramp

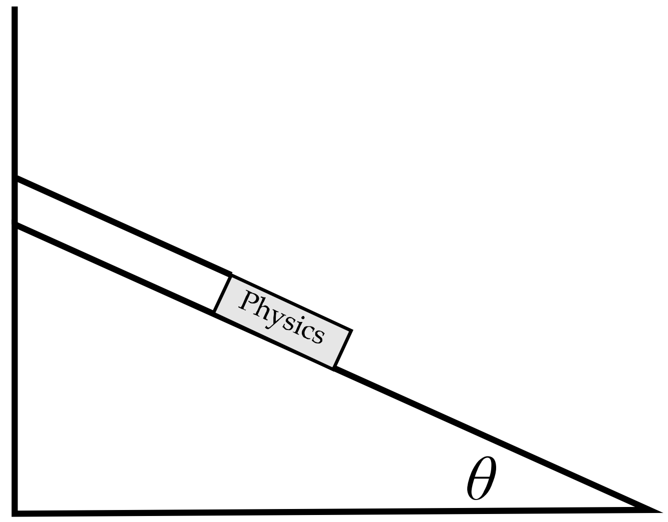
- A physics textbook is on a tilted, frictionless table, supported by a string.

A. Sketch a free-body diagram for the system.

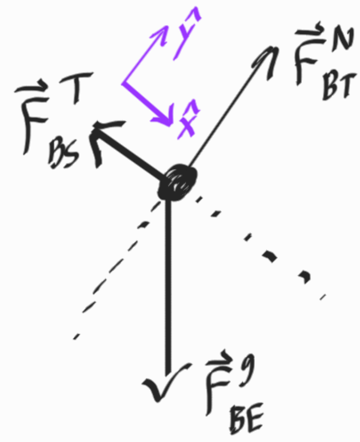
B. What coordinate system do you think will make analyzing this situation easiest?

C. Should the net force on the book be *zero* or *not zero*?

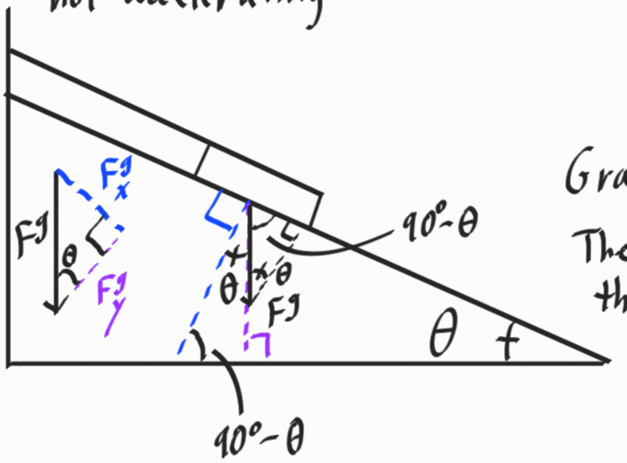
D. Write an expression for the magnitude of each force acting on the system in terms of the gravitational force (F^g).



4-3 Textbook on Tilted Table



$\vec{F}^{net} = \vec{0}$, as the book is stationary, and therefore not accelerating.



When choosing whether to tilt your coordinate system, it pays to pick an angle which either doesn't require you to break the acceleration into multiple components, or to pick an angle which requires you to break down the fewest forces. Here, there is no acceleration, but tilting the coordinate system to put $\hat{x}$ down the slope means the tension and normal force each have only one component:

$$\vec{F}^T_{BS} = -F^T_{BS}\hat{x} \qquad \vec{F}^N_{BT} = F^N_{BT}\hat{y}$$

$$F^T_{BS,x} = -F^T_{BS} \qquad F^N_{BT,y} = F^N_{BT}$$

Gravity still has both components: $\vec{F}^g_{BE} = F^g_{BE,x}\hat{x} + F^g_{BE,y}\hat{y}$

The tricky part is using geometry to figure out these components. In our tilted coordinate system, $\vec{F}^g_{BE}$ makes an angle θ with the y-axis, not the x-axis, so cosine does not go with x and sine does not go with y, as we usually see.

$$F^g_{BE,x} = F^g_{BE}\sin\theta$$

$$F^g_{BE,y} = -F^g_{BE}\cos\theta$$

Note that we need to add this minus sign.

$$\vec{0} = \vec{F}^{net}$$

$$0 = F^{net}_x = F^g_{BE,x} + F^T_{BS,x}$$

$$= F^g_{BE}\sin\theta - F^T_{BS}$$

$$\Rightarrow \boxed{F^T_{BS} = F^g_{BE}\sin\theta}$$

$$0 = F^{net}_y = F^g_{BE,y} + F^N_{BT,y}$$

$$= -F^g_{BE}\sin\theta + F^N_{BT}$$

$$\Rightarrow \boxed{F^N_{BT} = F^g_{BE}\cos\theta}$$

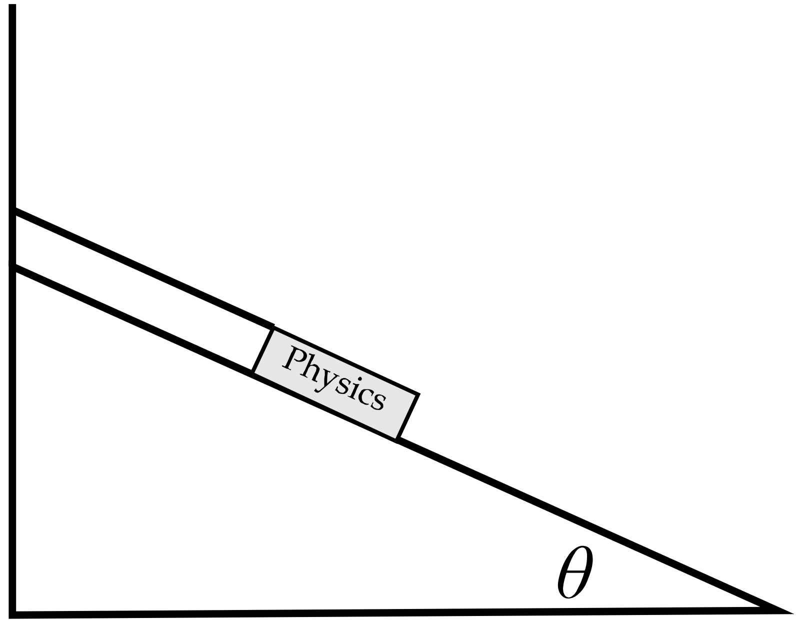
Special-case Analysis

- After you solve for a quantity:
  - Choose a case that is special, not arbitrary.
  - Figure out what your quantity **should** be in the case you chose.
  - Identify the value of one or more other quantities that corresponds to your **case**.
  - Evaluate your answer in the special case.
  - Check whether or not your symbolic answer for the case matches what you expected the answer should be.

Activity 4-4: Book on a Ramp (Special Case)

A physics textbook is on a tilted, frictionless table, supported by a string.

- Suppose the table is slanted so that it becomes *steeper*.
  - Does the magnitude of the normal force *increase, decrease,* or *stay the same*?
  - Does the magnitude of the tension force *increase, decrease,* or *stay the same*?
- What if the table were horizontal?
  - How big **should** each force be?
  - What angle corresponds to this **case**?
  - Does our symbolic answer for the case match what the answer should be?
- What if the table were vertical?
  - How big **should** each force be?
  - What angle corresponds to this **case**?
  - Does our symbolic answer for the case match what the answer should be?



4-4 Tilted Table Sensemaking

Increased Verticality

As the table tilts more, the string has to support the book more and more to keep it from sliding, and the surface of the table correspondingly needs to support the book less and less. Thus, the magnitude of the normal force will decrease, and the magnitude of the tension force will increase.

Note that $\cos\theta$ decreases and $\sin\theta$ increases as θ increases, so our symbolic answers agree with our physical prediction.

Horizontal Case

When the table is flat ($\theta = 0^0$), the string does not need to pull on the book to keep it from sliding. The tension will be zero newtons, and the normal force will be equal in magnitude to the force of gravity.

Our symbolic answers agree with this expectation: $\begin{cases} F^T_{BS} = F^g_{BE}\sin(0^0) = 0\text{ N} \\ F^N_{BT} = F^g_{BE}\cos(0^0) = F^g_{BE} \end{cases}$

Vertical Case

When the table is vertical ($\theta = 90^0$), the string is supporting the entire weight of the book, and the surface of the table does not support the book at all (the book just hangs beside it from the string). The tension will be equal in magnitude to the force of gravity, and the normal force will be zero newtons.

Our symbolic answers agree with this expectation: $\begin{cases} F^T_{BS} = F^g_{BE}\sin(90^0) = F^g_{BE} \\ F^N_{BT} = F^g_{BE}\cos(90^0) = 0\text{ N} \end{cases}$