PH 221 Week 6

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This material is borrowed/adapted from Chapter 9 of the *Student Workbook* for *Physics for Scientists and Engineers.*

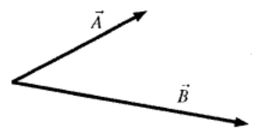
R6-1: Visual Dot Product Practice

(a) If $\vec{A} \cdot \vec{B} = 0$, can you conclude that one of the vectors has zero magnitude? Explain.

No. The vectors $\vec{A}$ and $\vec{B}$ could be perpendicular to each other.

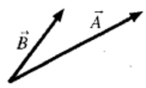
(b) For each pair of vectors, is the sign of $\vec{A} \cdot \vec{B}$ positive (+), negative (−), or zero (0)?

a.



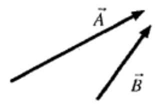
Sign = +

b.



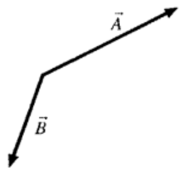
Sign = +

c.



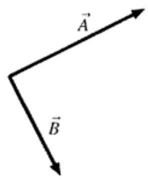
Sign = +

d.



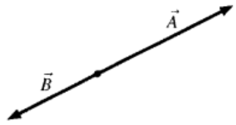
Sign = −

e.



Sign = 0

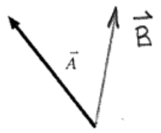
f.



Sign = −

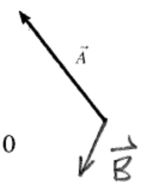
(c) Each of the diagrams below shows a vector $\vec{A}$. Draw and label a vector $\vec{B}$ that will cause $\vec{A} \cdot \vec{B}$ to have the sign indicated.

a.



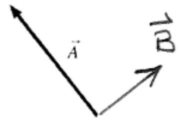
$\vec{A} \cdot \vec{B} > 0$

b.



$\vec{A} \cdot \vec{B} < 0$

c.



$\vec{A} \cdot \vec{B} = 0$

R6-2: Lifting Boxes

Rudy picks up a 5 kg box and lifts it straight up, at constant speed, to a height of 1 m. Beth uses a rope to pull a 5 kg box up a 15° frictionless slope, at constant speed, until it has reached a height of 1 m. Which of the two does more work? Or do they do equal amounts of work? Explain.

The amount of work done is the same for both. To raise an object higher, one must do work against gravity equal to mgh independent of the path taken to reach the height h.

Using a work approach, we know that Rudy must be exerting a force of $mg = (5\text{ kg})(9.8\text{ m/s}^2) = 49$ N up (opposing gravity) to lift the box at constant speed. The displacement is $h = 1$ m, and force and displacement are in the same direction, so the work done is $W = mgh = 49$ J. When Beth pulls the box up the $\theta = 15°$ slope, she must pull with $mg\sin\theta$ to oppose the force of gravity parallel to the slope. We know that the height of the slope is h, and if the length of the slope (the hypotenuse of the triangle it forms with the vertical axis and the ground) is L, then we know that $\sin\theta = \frac{h}{L}$, and thus the displacement of the box $L = \frac{h}{\sin\theta}$. The force and displacement are in the same direction, so the work done by Beth is $W = (mg\sin\theta)L = mgh = 49$ J. Again, both do the same amount of work. Beth just does it over a longer distance with a lesser force.

Make sure that students walk through work in each case. This is also a valuable opportunity to spend more time on relating the angle of the ramp to the angle of the forces in one's coordinate system. There is the exact, geometrical approach of setting up the similar triangles, there is the visualization approach of thinking about how the coordinate system tilts with the ramp, and there is the special cases approach of seeing if the chosen angle makes sense when the ramp is horizontal or vertical (for example, seeing if the component of gravity down the ramp goes to zero when it is horizontal and to mg when it is vertical).

R6-3: Spring Reasoning

A spring has an unstretched length of 10 cm. It exerts a restoring force F when stretched to a length of 11 cm.

(a) For what length of the spring is its restoring force $3F$?

Let us keep this very general for now. An ideal spring obeys Hooke's law:

$$\vec{F}_{sp} = -k\Delta\vec{s},$$

At this point, it is probably worth pointing out the negative sign explicitly and going over the meaning with the students.

where k is the spring constant, and $\Delta\vec{s}$ is the displacement of the end of the spring from its equilibrium position. Taking components along the s-direction, we maintain the sign relationship between the force and the displacement, but remove the vector notation:

$$F_{sp,s} = -k\Delta s.$$

Let us have two situations: Case A, where we have some displacement Δs_A and a restoring force $F_{sp,s,A} = -k\Delta s_A$, and Case B, where we have another displacement Δs_B and a restoring force $F_{sp,s,B} = -k\Delta s_B$. Consider what happens when we divide one equation by the other:

$$\begin{aligned} F_{sp,s,A} &= -k\Delta s_A \\ \div(F_{sp,s,B} &= -k\Delta s_B) \\ \implies \frac{F_{sp,s,A}}{F_{sp,s,B}} &= \frac{-k\Delta s_A}{-k\Delta s_B} = \frac{\Delta s_A}{\Delta s_B}. \end{aligned}$$

This is a great opportunity to go over the different ways of solving systems of equations with the students. Many of them seem to rely on solving for a variable and substituting it, but they could be dividing one equation by another, or adding whole equations together to cancel out terms.

From this, we can see that if we triple the force from Case A to Case B, we must also triple the displacement. For our particular situation, we have a positive displacement $\Delta s = 11\text{ cm} - 10\text{ cm} = 1\text{ cm}$ and therefore a negative restoring force $-F$. If we desire triple the restoring force $(-3F)$, we get

$$\frac{-F}{-3F} = \frac{1\text{ cm}}{\Delta s_B} \implies \Delta s_B = 3\text{ cm}.$$

Thus, the spring's total length must be 13 cm.

(b) At what compressed length is the restoring force $2F$?

For a compressed spring, the displacement is negative and the restoring force is positive. We have doubled the magnitude of the force, so the displacement must also be doubled in magnitude:

$$\frac{-F}{2F} = \frac{1\text{ cm}}{\Delta s_B} \implies \Delta s_B = -2\text{ cm}.$$

Thus, the spring must be 2 cm shorter than its equilibrium length, or 8 cm long.