Studio Week 6 – Waves and Superposition

Picture credit: *The Great Wave off Kanagawa.*

Principles for Success

- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

Activity 6-1 – Leaf on a Lake

- You are watching a leaf sitting on the surface of a lake. You observe that the leaf is bobbing up and down (both above and below the surface of the water), and you measure the leaf's highest point above the surface of the water to be 12 cm.
- After 3 s, the leaf returns to its highest point above the surface of the water.
  - Write an equation for the position of the leaf as a function of time.
  - What is the leaf's highest speed during its motion?
  - Find all the instants in time when the leaf has this speed.

6-1 Leaf on a Lake

Wrap 2:20 10:20

Model position of leaf as simple harmonic motion (SHM):

$\omega = 2\pi f = \frac{2\pi}{T}$

$$y(t) = y_{max}\cos\left(\frac{2\pi}{T}t\right)$$

$\phi_0 = 0$ by choice; I chose to set $t=0s$ when the leaf was at its peak height.

$y_{max} = 12cm$

$$= (12\,cm)\cos\left(\frac{2\pi}{3s}t\right)$$

$T = 3s$

$$D(x,t) = A\sin(kx \pm \omega t + \phi_0)$$

$\cos(x) = \sin(x + \frac{\pi}{2})$

At a fixed position x, the wave the wave equation reduces to SHM for a single oscillator with phase $kx + \phi_0$. A wave can be thought of like a chain of tiny oscillators, each with their own phase (offset by $k\Delta x$ for oscillators spaced Δx apart).

See also 6-3 demonstration and 6-4 simulation.

Max speed? Take a derivative:

$$V_y(t) = \frac{dy}{dt} = -\frac{2\pi}{T}y_{max}\sin\left(\frac{2\pi}{T}t\right)$$

$$= -\underbrace{\left(8\pi\frac{cm}{s}\right)}_{V_{max}\approx 25\frac{cm}{s}}\sin\left(\frac{2\pi}{3s}t\right)$$

When does max speed happen?

$$\text{Need } \sin\left(\frac{2\pi}{T}t\right) = \pm 1 \Rightarrow \frac{2\pi}{T}t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

$$\Rightarrow t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}, \frac{7T}{4}, \ldots$$

$$= 0.75s, 2.25s, 3.75s, 5.25s, \ldots$$

$$= 0.75s + n(1.5s)$$

for $n = 0, 1, 2, 3, \ldots$

The leaf attains its max speed every 1.5 seconds, starting 0.75 seconds after beginning at maximum height.

Activity 6-2 – Water Tank

- Start the following simulation: https://phet.colorado.edu/sims/html/waves-intro/latest/waves-intro_en.html
  - Explore each of the three tabs. Make a list of your observations as you go.
  - In the water tab, what do you think would happen if you placed a leaf on the surface of the water near the right edge of the tank? Draw a displacement *vs.* time graph (like those provided in the simulation) for the leaf.
  - How does each experiment change if you change the frequency? Explain briefly.

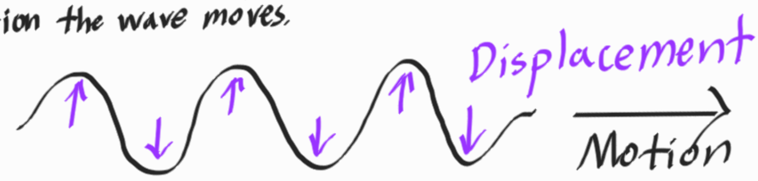
6-2 Water Tank

Start 2:30 10:30 Wrap 2:45 10:45

Observations

These are just mine; there could be others.

- Water waves spread in a circle; light/sound spread in a sphere.
- Amplitude decreases as the wave spreads — not damping, but dispersion; as energy is spread over a wider region (rim of circle or surface of sphere), it becomes less concentrated.
- Water wave is transverse*, as the surface moves perpendicular to the direction the wave moves.



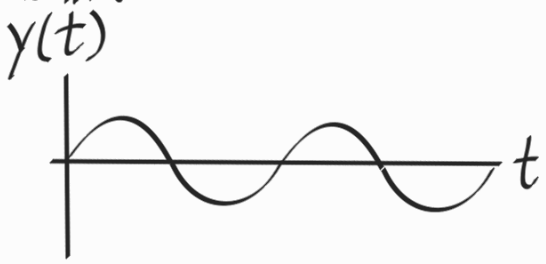
*Techically, a real water wave is more complicated, with surface motion and motions beneath the surface.

- Sound wave is longitudinal (compression), as the air molecules displace parallel to the direction the wave moves.

- Light wave is transverse, as electric field oscillates perpendicular to direction the wave moves.

Leaf on the surface at right edge?

The leaf would oscillate up and down with the surface of the water, just like in 6-1.



Note that simulation is graphing with respect to x! It is showing a movie made from a sequence of snapshot graphs. This is a history graph for a single point in space.

Changing frequency?

Water — changes distance between peaks

Wave speed is not changing, so when the oscillations are more frequent (generating more peaks more rapidly), the peaks are created closer together.

Sound — changes tone

Light — changes color

Activity 6-3 – Wave Demonstration

- The instructor will lead the class in demonstrating wave properties.
  - What does it look like when a wave pulse is sent through a line of bodies?
  - What does it look like when a continuous wave is sent though a line of bodies?
  - What do waves of differing speeds look like?
  - What causes the speeds to be different?
  - What do waves of differing frequencies look like?
  - What causes the frequencies to be different?
  - What was the difference between the two different kinds of waves (transverse and longitudinal)?

- Afterward, the instructor will demonstrate wave reflection.

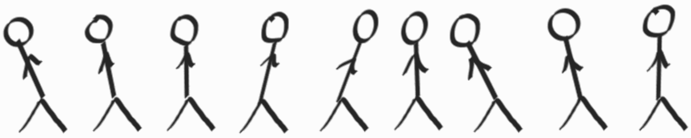
6-3 Wave Demonstration

Human Wave Demo 3:05 11:05 Reflection Demo 3:15 11:15

Human Wave

Get at least nine students to participate.

Transverse:  Displacement perpendicular to wave's direction of travel.

Longitudinal:  Displacement parallel to wave's direction of travel.

- For a single pulse, the wave source (a person probably at one of the ends) will oscillate once (raising and lowering arms or swaying torso), and their neighbors will mimic the motion, causing the disturbance to propagate to the other end, leaving the chain still afterward.
- For a continuous wave, the source will oscillate repeatedly, sending a continuing, periodic disturbance down the line.
- Different wave speed: neighbors react faster, raising hands (or swaying) sooner after the person ahead of them starts to move. Wave speed is a property of the medium the wave travels through.
- Different frequency: the source moves their hands up and down (or sways) faster (and the neighbors copy). Frequency is a property of the source.

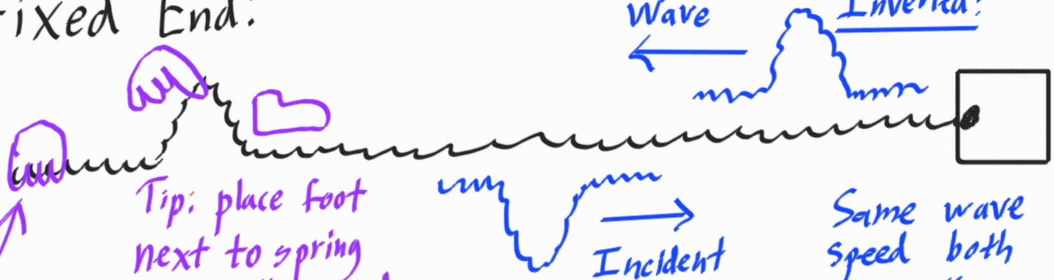
6-3 Wave Demonstration

Human Wave Demo 3:05 11:05 Reflection Demo 3:15 11:15

Reflection

Fixed End:

Stretch demo across low friction floor.



Reflected Wave

Inverted!

Have assistant hold with foot.

Tip: place foot next to spring and pull behind to set up pulse.

Incident Wave

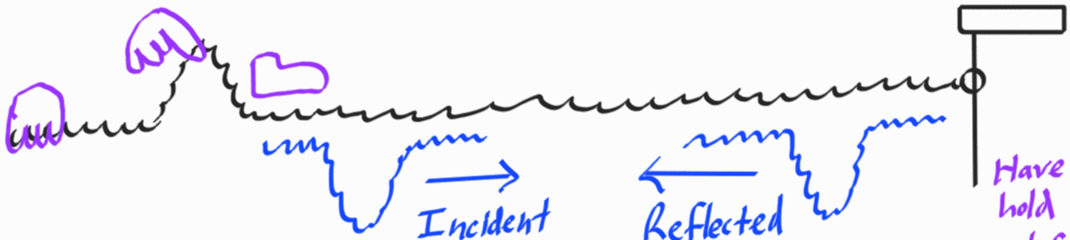
Same wave speed both ways.

Can adjust tension w/ other hand.

Can pull along spring to get longitudinal wave.

Higher $F^T \Rightarrow$ Faster Waves

Free End:



Incident Wave

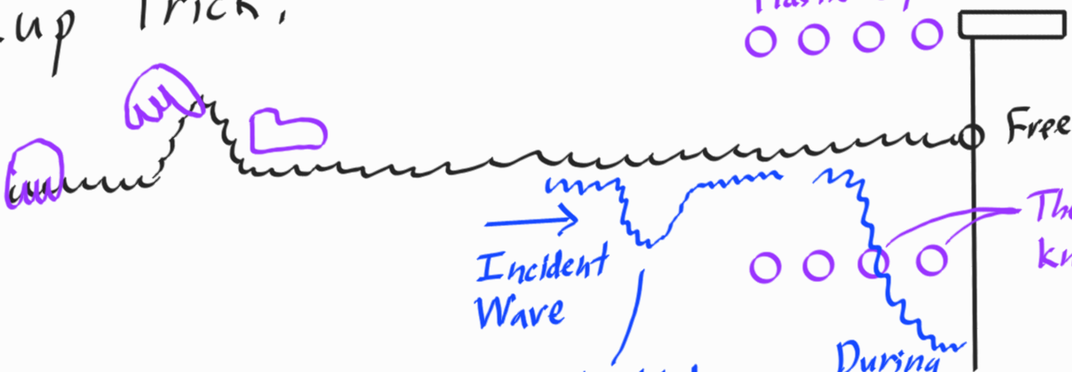
Reflected Wave

Not inverted!

Have assistant hold this end w/ foot (fingers not safe).

Cup Trick:

Plastic Cups



Free end

These two get knocked away!

Incident Wave

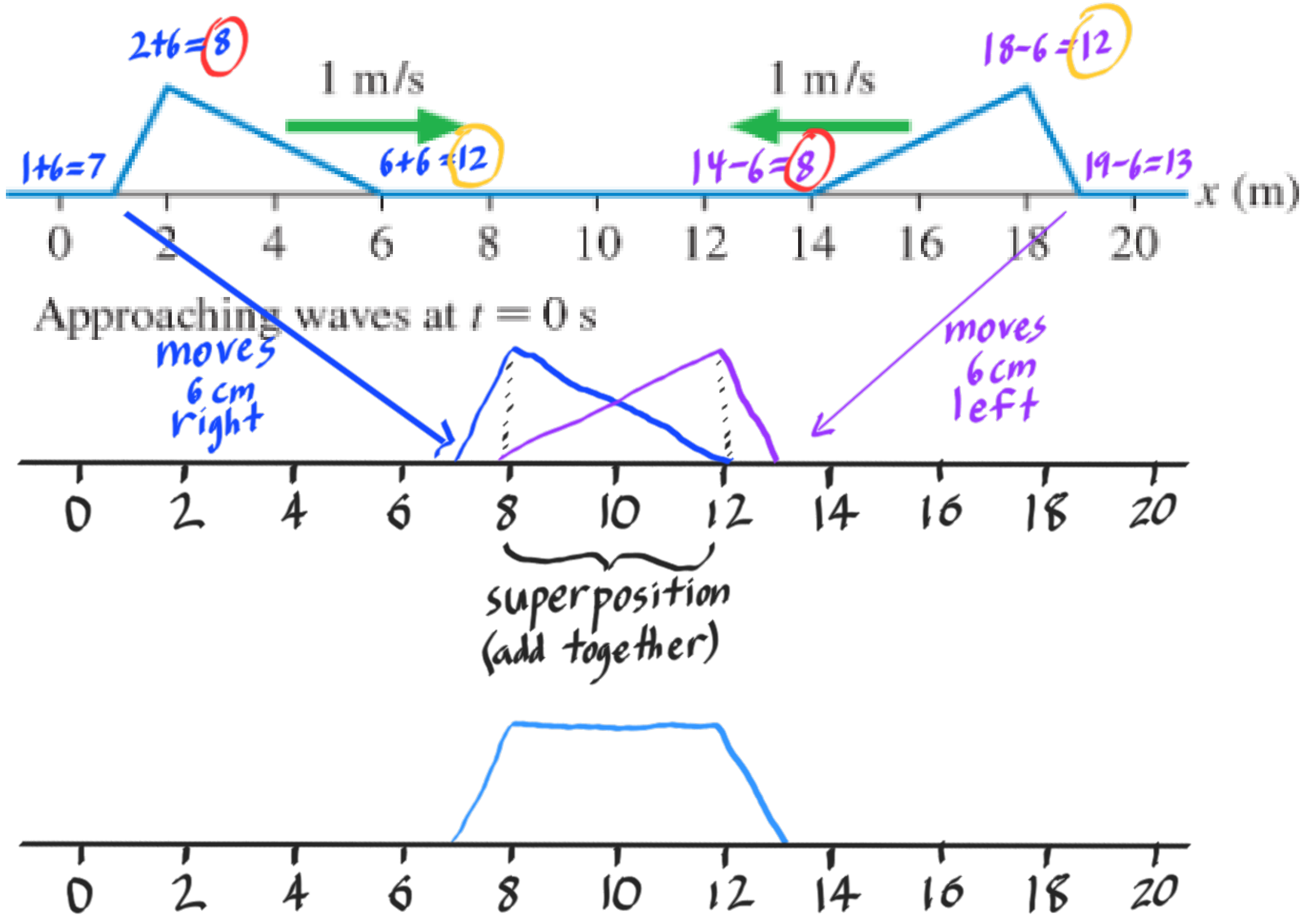
Amplitude not large enough to hit cups

During reflection Bigger?!

For an explanation of why this happens, see 6-4.

Stop to Think 17.1

Two pulses on a string approach each other at speeds of 1 m/s. What is the shape of the string at $t = 6$ s?



Activity 6-4 – Boundaries

- Go to the book and complete the Activities in the section on Boundaries and Discontinuities:
  - https://lipa.physics.oregonstate.edu/sec_boundary.html

6-4 Boundaries

Start 3:25 / 11:25 Wrap 3:40 / 11:40

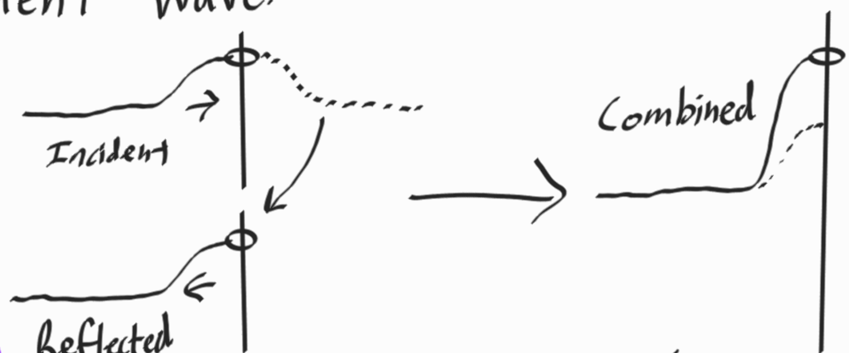
Observations

These are just mine; there could be others.

- Lowering the tension in the string makes the wave travel more slowly.
- When the wave reaches a fixed end, it reflects back and inverts.

- When the wave reaches a free end, it reflects back without inverting.

- While the wave is reflecting from the free end, it briefly gets larger. This is because the uninverted reflected wave interferes constructively with the incident wave.



High Density String

Low Density String

Before After

Wave speed is higher in the lighter string.

Tension across the boundary must be the same, but it won't accelerate strings of different mass densities the same (recall $F^{net} = ma$).

Reflection is needed to conserve energy.

If the density of the LDS is much less than the density of the HDS, the situation is approximately the same as the free end, so the uninverted reflection makes sense.

Before After

If the density of the HDS is much more than the density of the LDS, then this becomes like the fixed end, so an inverted reflection makes sense.