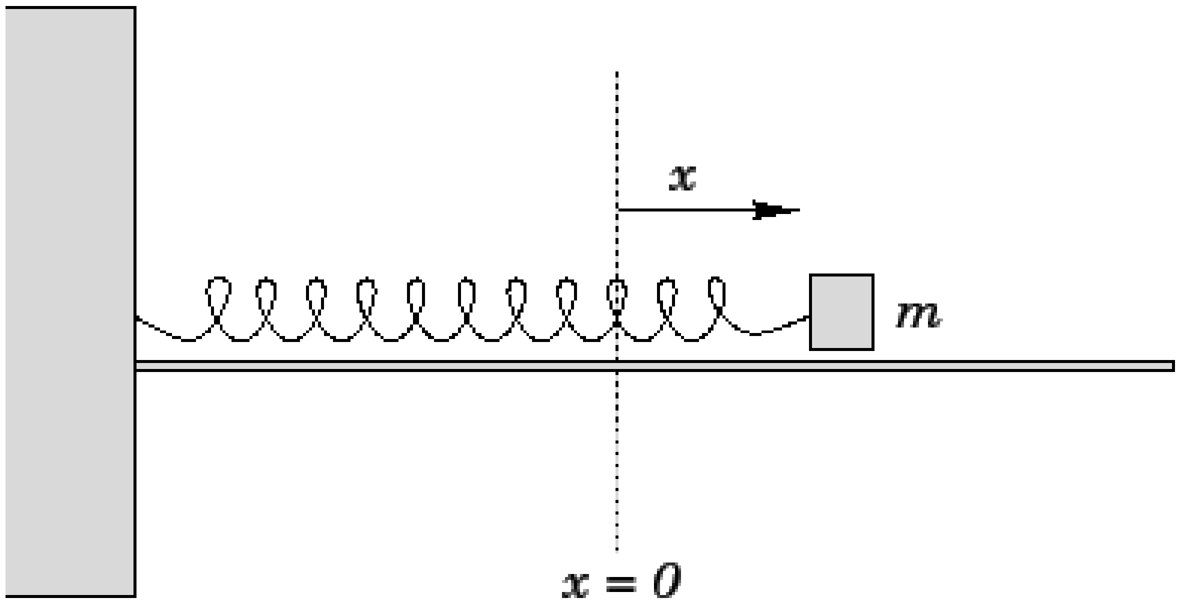
Studio Week 5
Simple Harmonic Motion



Picture credit: http://farside.ph.utexas.edu.

Principles for Success

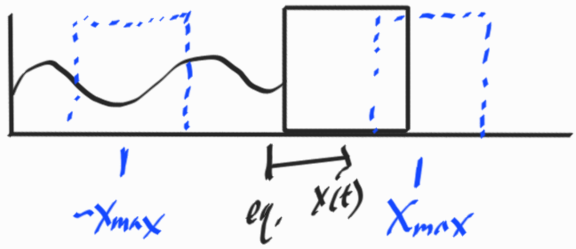
- **Treat everyone with respect.**
- **Learn by doing and questioning.**
- **Everything should make sense.**

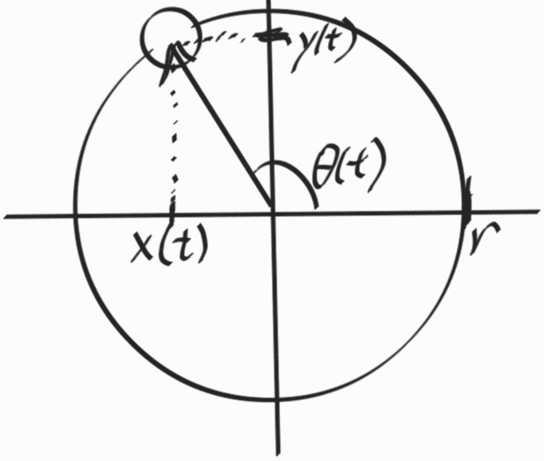
Activity 5-1 – Rotation vs. Oscillation

- Start the following simulation: https://phet.colorado.edu/en/simulation/legacy/rotation
- When the page is displayed, click on "Rotation". Spend ten minutes or so getting familiar with the simulation and trying different settings. Make a list of your observations as you go.
- Now make a table where the first column has the parameters that are important for analyzing oscillatory motion, the second column has the corresponding parameters for uniform circular motion, and the third column has a qualitative explanation of the relationship between uniform circular motion and oscillatory motion.

5-1 Rotation vs. Oscillation

wrap 2:20 10:20





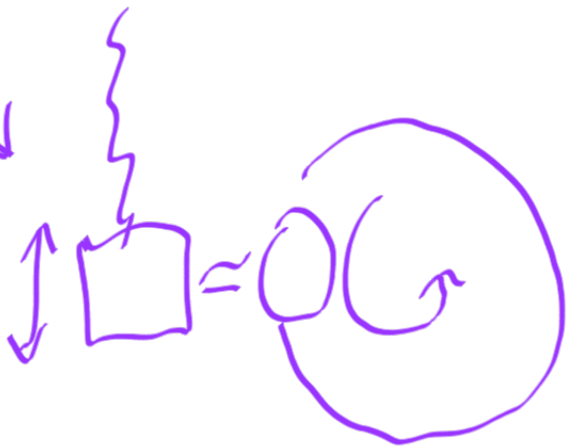
Oscillatory Motion	Rotational Motion	Relationship
$X(t)$ $= X_{max}\cos(\omega t + \phi_0)$	$x(t),\ y(t)$ $r\cos(\theta(t)),\ r\sin(\theta(t))$ $\theta(t)$ $= \omega t + \theta_0$	All describe how far the object ranges from the origin along a particular axis. Tied more strongly to the periodic part of the motion.
$V(t)$	$V_x(t),\ V_y(t)$	All describe the rate and direction of the object's motion along a particular axis.
$a(t)$	$a_x(t),\ a_y(t)$	All describe the change in the rate and direction of the object's motion along a particular axis.
amplitude X_{max}	r	Both describe the limits of the object's freedom of motion.
angular frequency ω	angular speed ω	Both describe how quickly the object goes through periodic motion.
phase constant ϕ_0	initial angular position θ_0	Both describe where the object starts in periodic motion.

This list is not exhaustive, and many different, interesting perspectives could be added!

Videos

https://www.youtube.com/watch?v=9roHexjGRE4

https://www.youtube.com/watch?v=gvPNrdfNo9g



Cart

x_{max} (cm): ~0.5 ~1 ~2 ~2.5

$$\theta(t) = \theta_0 + \omega t$$

$$x(t) = R\cos(\theta(t)) = R\cos(\omega t + \theta_0)$$

https://www.youtube.com/watch?v=gvPNrdfNo9g&feature=youtu.be

Activity 5-2 – Oscillating Cart

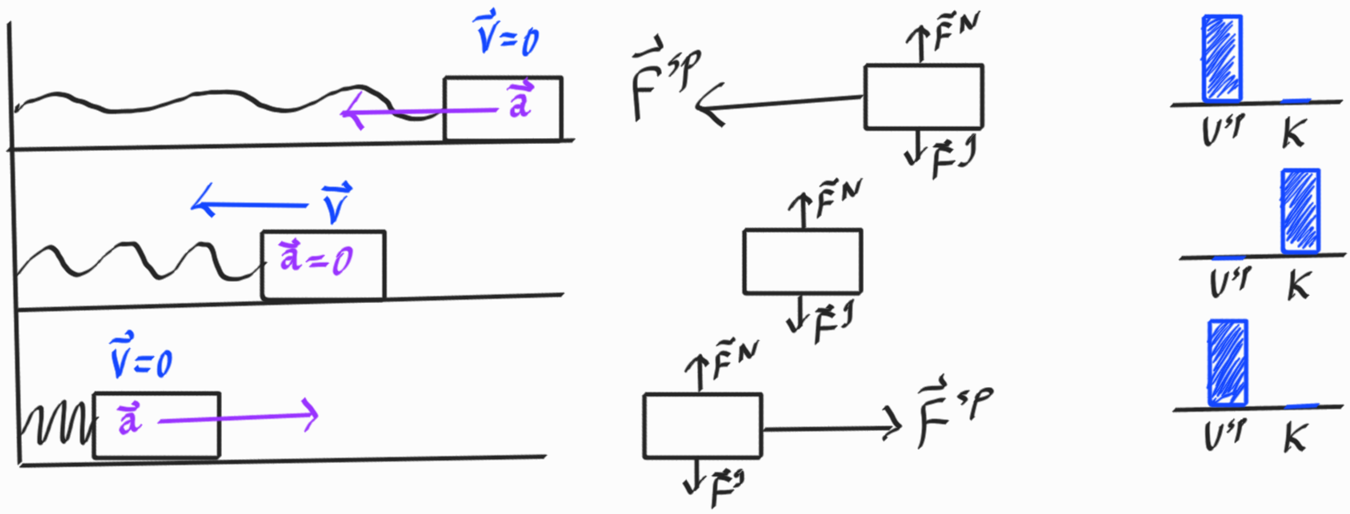
Period is defined as the amount of time it takes for an object to go through its motion once. For an object moving around a circle, it is the time to go around the circle once. For an object oscillating, it's the amount of time to move back and forth once. Amplitude is defined as the maximum distance from the object to the equilibrium position.

A. Draw the velocity and acceleration of the cart at the right-most location, exactly in the middle, and at the left-most location.
B. Draw a free-body diagram for the cart at the same three locations.
C. Describe how much kinetic energy and potential energy the system has at the same three locations.
D. Why did the cart go back to the original position after it was pulled away?
E. Why didn't it stop exactly in the middle when the forces exerted on it by the springs were both zero?
F. Where and when is the cart's velocity and the forces exerted on it by the springs in opposite directions?
G. Did it take the cart the same time to complete one full cycle in all experiments? Why or why not?

https://www.youtube.com/watch?v=gvPNrdfNo9g&feature=youtu.be

5-2 Oscillating Cart

start 2:30 10:30 Wrap 2:50 10:50



D) The cart returns to equilibrium because the spring was stretched, causing it to pull back toward the unstretched length. This is similarly true for compression.

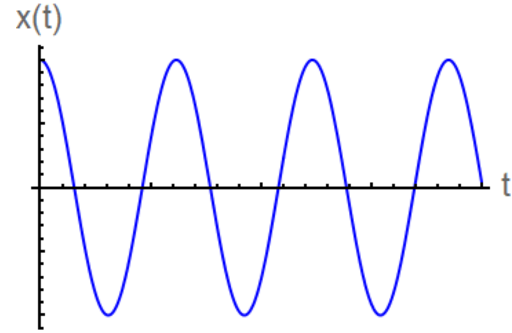
E) $\vec{F}^{net}=\vec{0} \Rightarrow \vec{a}=\vec{0}$, but it has still accumulated velocity returning to the equilibrium point, so it keeps moving through.

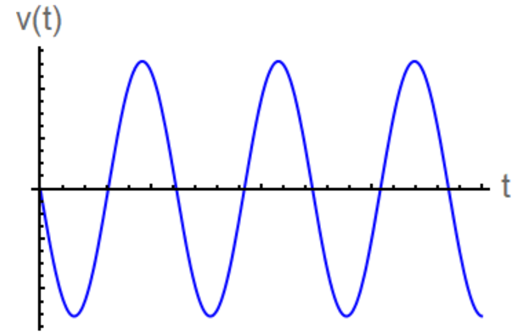
F) $\vec{v}$ points opposite $\vec{F}^{sp}$ (and thus opposite $\vec{a}$) as the cart moves from equilibrium toward the right- or leftmost locations. The spring slows it down so that it stops momentarily at these turning points.

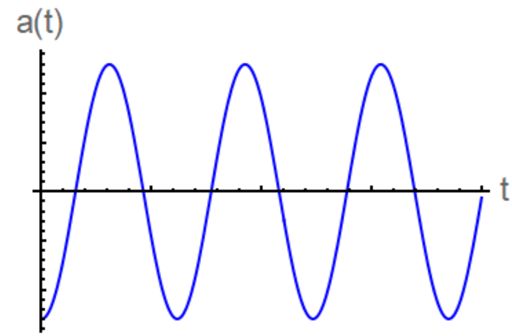
G) Yes, the cart takes the same time to make a cycle no matter how far it is pulled from equilibrium. It has more distance to cover in a cycle, but greater displacement means greater force from the spring, which causes greater acceleration. This allows the cart to cover the distance faster.

Activity 5-3 – Graphing Oscillations

Suppose you have a cart attached to a spring on a lab table. This spring is tied down at the other end to a table clamp. The cart rolls freely back and forth due to the spring. Assume a motion sensor is placed in such a way that it records the cart's motion. Graphs of position-versus-time, velocity-versus-time, and acceleration-versus-time are shown here.







A. Are these three graphs consistent with the motion of the cart in the previous activity?

B. Recall that $v = dx/dt$. Is the shape of the velocity-versus-time graph consistent with this mathematical definition and with the position-versus-time graph shown? Compare the slope of $x(t)$ with the value of v at the maximum, minimum, and zero points. Explain briefly.

C. Recall that $a = dv/dt$. Is the shape of the acceleration-versus-time graph consistent with this mathematical definition and with the velocity-versus-time graph shown? Compare the slope of $v(t)$ with the value of a at the maximum, minimum, and zero points. Explain briefly.

D. Are the direction and magnitude of the acceleration consistent with the direction and magnitude of the restoring force? Remember the restoring force is proportional to $-x$. Explain briefly.

E. Describe the relationship between the position-versus-time graph and the acceleration-versus-time graph. Explain why they mirror each other. Think about Newton's Second Law and the expression for the force of the spring on the cart:

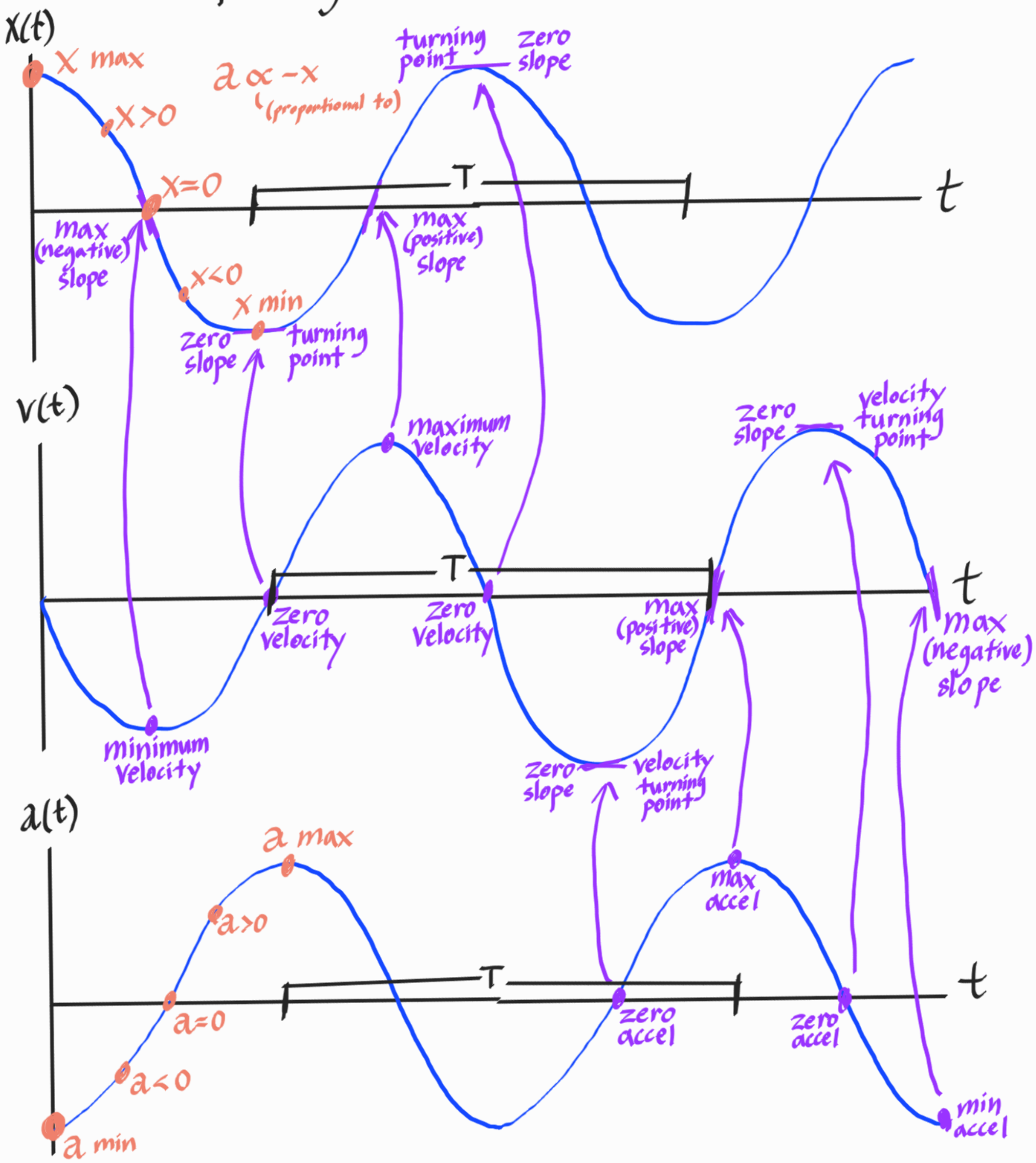
F. Indicate the period (T) on each graph.

G. What mathematical function can be used to describe the position of the cart as a function of time? What functions can be used to represent velocity and acceleration?

H. Write symbolic expressions for $x(t)$, $v(t)$, and $a(t)$. How are the amplitudes related to each other? Check that the expressions you wrote make sense using special cases like $t = 0$ and $t = T$.

5-3 Graphing Oscillations

Start 3:10 11:10 Wrap 3:25 11:25



Newton's 2nd Law

$$F^{net} = F^{sp}$$

$$ma = -kx$$

scale and units are different for x(t) and a(t)

$$a = -\frac{k}{m}x \Rightarrow \text{"mirroring" of } x(t) \text{ and } a(t)$$

$$\frac{d^2x}{dt^2} = -\underbrace{\frac{k}{m}}_{\omega_{sp}^2}x \Rightarrow x(t) = x_{max}\cos(\omega_{sp}t + \phi_0)$$

$\phi_0 = 0$, $\omega_{sp} = \frac{2\pi}{T}$ for these graphs

$$v(t) = \frac{dx}{dt} = -\underbrace{\omega_{sp}x_{max}}_{v_{max}}\sin(\omega_{sp}t + \phi_0)$$

$$a(t) = \frac{dv}{dt} = -\underbrace{\omega_{sp}^2 \overbrace{x_{max}\cos(\omega_{sp}t + \phi_0)}^{x(t)}}_{a_{max}}$$