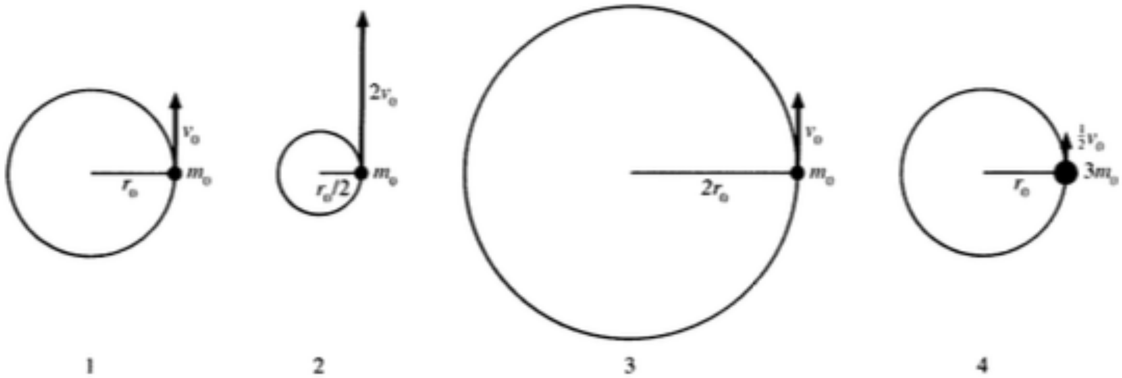
Rank Angular Momenta

Rank in order, from largest to smallest, the angular momenta L_1 to L_4.



$\vec{L} = \vec{r} \times \vec{p} \qquad \vec{p} = m\vec{v}$

$L_1 = r_o m_o v_o$

$L_2 = \frac{r_o}{2} m_o 2v_o = r_o m_o v_o$

$L_3 = 2r_o m_o v_o$

$L_4 = r_o 3m_o \frac{1}{2} v_o = \frac{3}{2} r_o m_o v_o$

$\Rightarrow \quad L_3 > L_4 > L_1 = L_2$

Rotating Rod + Clay

A 75 g, 0.30 m long rod hangs vertically on a frictionless, horizontal axle passing through its center. A 10 g ball of clay traveling horizontally at 2.5 m/s hits and sticks to the very bottom tip of the rod. What angular speed does the rod have immediately after the clay sticks to it?

Before you do any calculations, write down:

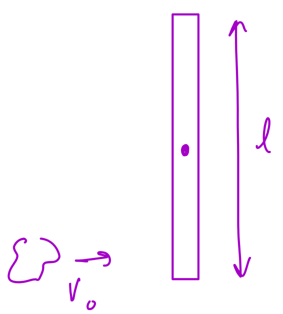
What approach to solving this problem are you going to use?

Conservation of angular momentum!

What units should your answer have?

Angular Speed: ω $[\omega] = \left[\frac{rad}{s}\right]$

Solution:



$$\vec{L}_i = \vec{r} \times \vec{p} = \frac{l}{2} m_c v \odot$$

$$\vec{L}_f = I\vec{\omega}_f = (I_{rod} + I_{clay})\vec{\omega}_f \odot$$

Moment of inertia: $I_{rod} = \frac{1}{12} m_r l^2$

$I_{clay} = m_c \frac{l^2}{4}$

$$\vec{L}_i = \vec{L}_f \rightarrow \frac{l}{2} m_c v = (I_{rod} + I_{clay})\omega_f$$

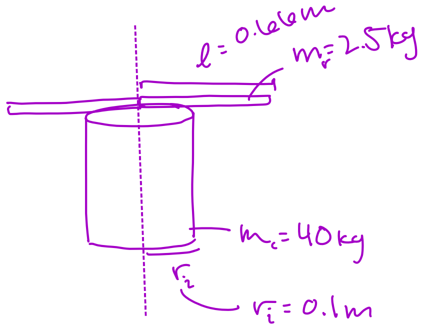
$$\omega_f = \frac{l}{2} m_c v \left(\frac{1}{12} m_r l^2 + m_c \frac{l^2}{4}\right)^{-1} = \frac{v}{2}\left(\frac{1}{12}\frac{m_r}{m_c} + \frac{1}{4}\right)^{-1}$$

$$\boxed{\omega_f = 4.76 \frac{rad}{s}}$$

The Figure Skater

A 45 kg figure skater is spinning on the toes of her skates at 1.0 rev/s. Her arms are outstretched as far as the will go. In this orientation, the skater can be modeled as a cylindrical torso (40 kg, 0.1 m radius) plus two rods (2.5 kg each, 0.66 m long) for the arms. Then she puts her arms at her side, which we will model as a single cylinder of mass 45 kg, and radius 0.2m. What is her new angular velocity (in rad/s)?

Initial



$m_t = 45\,kg$

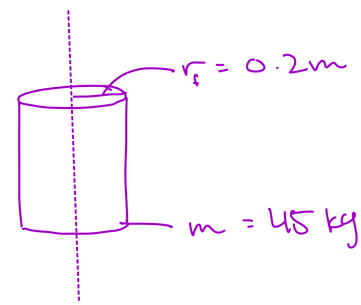
$\omega_i = 1.0\,\frac{rev}{s} = 2\pi\,\frac{rad}{s}$

$\vec{L}_i = I\vec{\omega}$

$I = I_{cylinder} + 2I_{rod}$

$= \frac{1}{2}m_c r_i^2 + 2\left(\frac{1}{3}m_r \ell^2\right)$

Final



$\omega_f = ?$

$\vec{L}_f = I\vec{\omega}_f$

$I = \frac{1}{2}m r_f^2$

$\rightarrow L_i = L_f$

$\left(\frac{1}{2}m_c r_i^2 + \frac{2}{3}m_r \ell^2\right)\omega_i = \left(\frac{1}{2}m r_f^2\right)\omega_f \rightarrow \omega_f = \dfrac{\left(\frac{1}{2}m_c r_i^2 + \frac{2}{3}m_r \ell^2\right)\omega_i}{\frac{1}{2}m r_f^2}$

$\Rightarrow \omega_f = \dfrac{\left[\frac{1}{2}(40\,kg)(0.1\,m)^2 + (2.5\,kg)(0.66\,m)^2\right]\left(2\pi\,\frac{rad}{s}\right)}{\frac{1}{2}(45\,kg)(0.2\,m)^2}$

$\boxed{\omega_f \approx 6.5\,\frac{rad}{s}}$