Lecture 14: Dynamics of Related Systems

Warm-Up Activity

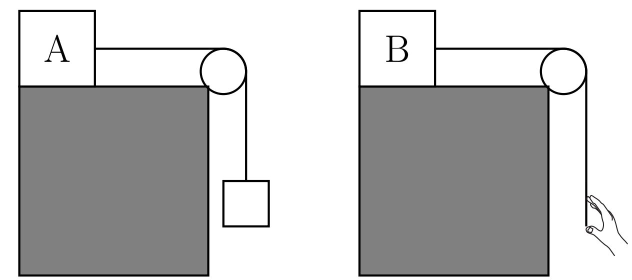
Is the acceleration of block B greater than, less than, or equal to the acceleration of block A?

(A) Greater than

(B) Less than

(C) Equal to

(D) Not enough information



Weight = 10 N

Tension = 10 N

A Model for Interactions

- Quantities
  - Mass m
  - Force $\vec{F}$
- Laws
  - Net force is proportional to acceleration: $\vec{F}^{net} = m\vec{a}$
  - Forces come in pairs: $\vec{F}_{AB} = -\vec{F}_{BA}$
- Assumptions
  - We can treat multiple objects as a system.
  - All forces act as if on the center of the system.

Types of Forces

- Gravity $\vec{F}^g_{AB} = m_A \vec{g}_B$
  - Newtonian $\vec{g}_B = G\frac{M_B}{r^2}(-\hat{r})$, $G = 6.67408 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
  - Near-Earth $\vec{g}_E = g(-\hat{y}),\ g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2} \approx 10\frac{\mathrm{m}}{\mathrm{s}^2}$
- Normal $\vec{F}^N$ always $\perp$; varies in magnitude
- Tension $\vec{F}^T$ uniform (massless, inextensible rope)
- Spring $\vec{F}^S = -k(\vec{x} - \vec{x}_{eq})$
- Friction
  - Static Friction $F^{sf} \leq \mu_s |\vec{F}^N|$
  - Kinetic Friction $F^{kf} = \mu_k |\vec{F}^N|$

Newton's 3rd Law of Motion

- If A exerts a force on B, then B exerts a force of the same magnitude on A in the opposite direction:

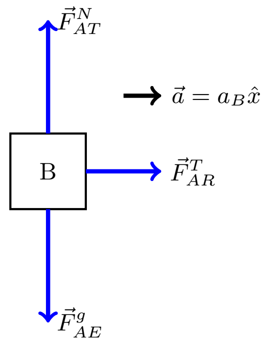
$$\vec{F}^t_{AB} = -\vec{F}^t_{BA}$$

- These two forces make a *Newton's 3rd law pair*, or an *action-reaction pair*.

- 3rd law pair forces...
  - are the same type of force;
  - appear on different free body diagrams.

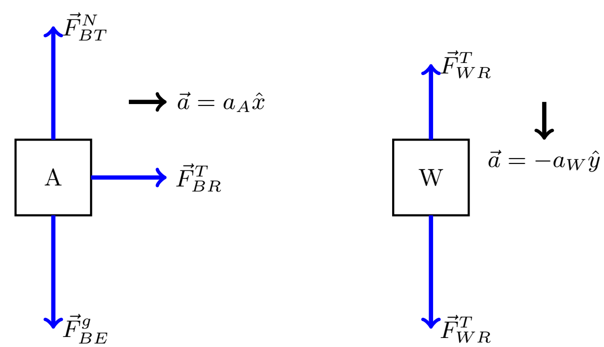
First, we know from the ideal rope and ideal pulley assumptions that the tension is uniform throughout the rope, so we can define $F^T_{AR} = F^T_{WR} = F^T$.

Case B is fairly straightforward.



We can see that $\vec{F}^{net} = \vec{F}^T_{BR}$, therefore $a_B = \frac{F^T_{BR}}{m_B}$.

In case A, we have two free-body diagrams, one for B and one for the weight W:



We still have $F^T = m_A a_A$, just like before. On the weight, Newton's second law tells us that $F^T = m_W g - m_W a_A$ (both have the same acceleration, as they are attached). Combining these, we have

$$\begin{aligned} m_W g - m_W a_A &= m_A a_A \\ m_W g &= a_A(m_A + m_W) \\ a_A &= \frac{m_W g}{m_A + m_W} \end{aligned}$$

We are given that $m_W g = 10$ N in case A, and $F^T_{BR} = 10$ N in case B. As such, it can be seen that $a_A < a_B$.

Conceptually, the only way the blocks could tie is if they had the same acceleration, which would mean having the same tension in cases A and B. However, if the rope in A had 10 N of tension, then the forces on the weight would be balanced (10 N tension versus 10 N gravity) and the weight itself would not accelerate. We need the forces on the weight to be unbalanced, which means the tension must be smaller than the force of gravity, and A won't be pulled as hard.

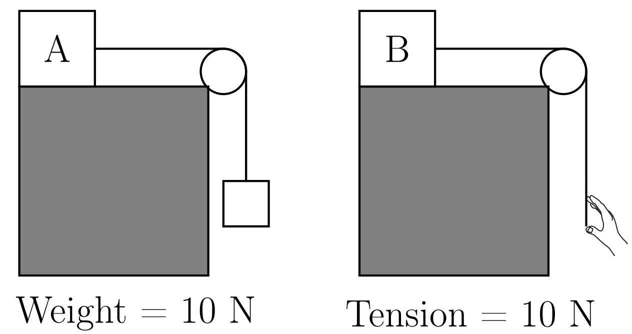
L14-1: The Block Race

Below left, block A is accelerated across a frictionless table by a hanging 10 N weight. Below right, an identical block B is accelerated by a constant 10 N tension in the string. Neglect friction in both cases.

(A) Draw free-body diagrams for each situation.

(B) Indicate the direction of the acceleration for each object.

(C) Solve for the acceleration of each block.



Main Ideas

- The magnitude of kinetic friction can be modeled as directly proportional to the magnitude of the normal force.
- Newton's 3rd law of motion can be used to relate the forces acting on *different* systems.