Lecture 19: Energy Conservation

Warm-Up Activity

When can we say that energy is conserved?

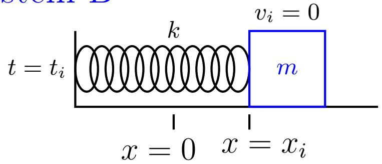
It is often asserted that energy is always conserved, meaning that energy cannot be created or destroyed within the universe as a whole.

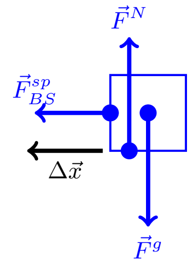
To be more exact and use the idea of energy conservation more practically, we should pick a system and look to the work energy theorem ($W_{\text{net,ext}} = \Delta E_{\text{total}}$) to tell us if energy is conserved for that system. If $W_{\text{net,ext}} = 0$, then the energy of the system is conserved, and we can use energy conservation methods to analyze the system.

Assuming that the universe is a closed system, interacting with nothing outside of it, then our more exact idea of energy conservation implies that energy is indeed always conserved on a universal scale.

For this activity, I will identify the points of contact for forces on my free-body diagrams, which will violate our normal FBD conventions.

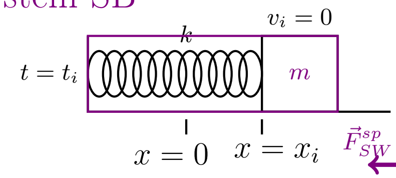
System B

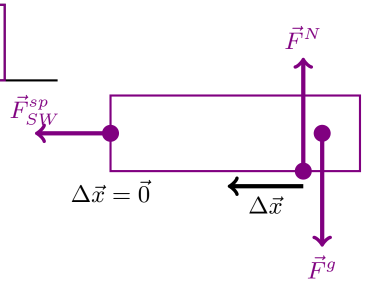




- $W^g = 0$ and $W^N = 0$
  - The points of contact for F^g and F^N are moving, but the forces are perpendicular to the displacement, so the work is zero.
- $W^{sp} > 0$
  - The spring pulls to the left, in the direction of the displacement of its point of contact.

System SB



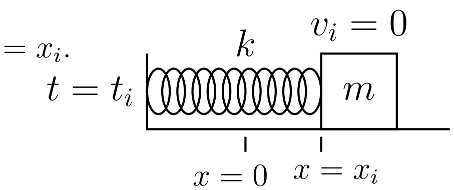


- $W^g = 0$ and $W^N = 0$
  - Same as before.
- $W^{sp} = 0$
  - The work done on the spring by the wall is zero, as the point of contact at the stationary wall undergoes no displacement.

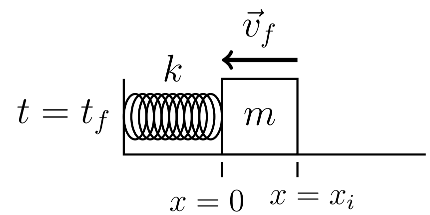
System	B			SB		
Forces	$\vec{F}^g$	$\vec{F}^N$	$\vec{F}^{sp}$	$\vec{F}^g$	$\vec{F}^N$	$\vec{F}^{sp}$
Displacements	$\Delta\vec{x}$	$\Delta\vec{x}$	$\Delta\vec{x}$	$\Delta\vec{x}$	$\Delta\vec{x}$	0
Works	0	0	W^{sp}	0	0	0

L19-1: Block on a Spring – Forces

- A block of mass m on a level, frictionless surface is attached to an ideal, massless spring of constant k that is initially stretched.
- At time t_i, the block is **released from rest** at $x = x_i$.
- At time t_f, the block reaches $x = 0$ moving to the left with speed v_f.



- System B consists of the block alone.
- System SB consists of the spring and the block.
- For the interval from t_i to t_f (for each system):



  (1) List all external forces acting on the system.
  (2) Identify the point of contact associated with each force.
  (3) Draw a vector to indicate the **displacement** associated with each force.
  (4) Determine if the work done by **each force** is positive, negative, or zero.
- What do you want to remember about this situation for future problems?

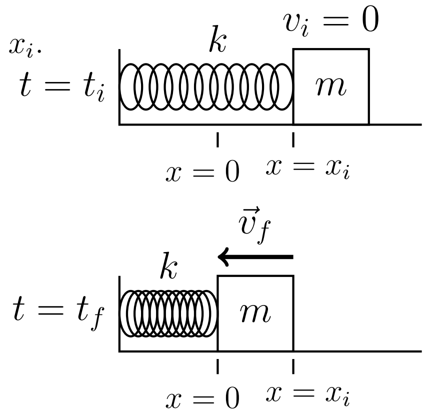
System	B	SB	
$W_{\text{net,ext}}$	$+$	0	For B, there is net external work, but for SB, there is not. This follows from the previous activity.
ΔE_{total}	$+$	0	By the work-energy theorem, $\Delta E_{\text{total}} = W_{\text{net,ext}}$.
ΔK	$+$	$+$	The block starts at rest and ends with nonzero speed, so it must have gained kinetic energy, regardless of our choice of system.
ΔU	0	$-$	

A block by itself that cannot change elevation can only have kinetic energy. It has no way of storing energy (no potential).

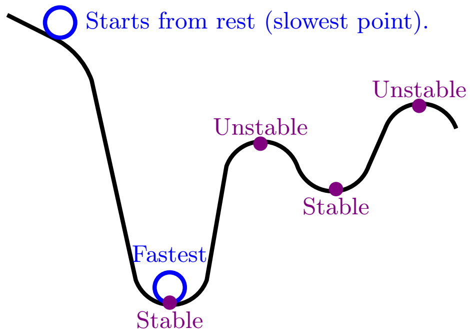
Without external work, the ΔK in SB must come from somewhere, and $\Delta E_{\text{total}} = 0$, so there must be energy stored in the system that is transformed into kinetic energy. By putting the spring in the system, the system can now store potential energy.

L19-2: Block on a Spring – Energy

- A block of mass m on a level, frictionless surface is attached to an ideal, massless spring of constant k that is initially stretched.
- At time t_i, the block is **released from rest** at $x = x_i$.
- At time t_f, the block reaches $x = 0$ moving to the left with speed v_f.
- System B consists of the block alone.
- System SB consists of the spring and the block.
- For the interval from t_i to t_f (for each system):
  - (1) Determine if the **net external work** is positive, negative, or zero (based on L19-1).
  - (2) Determine if the **change in total energy** is positive, negative, or zero.
  - (3) Determine if the **change in kinetic energy** is positive, negative, or zero.
  - (4) Determine if the **change in potential energy** is positive, negative, or zero.



- What is different about the two systems?



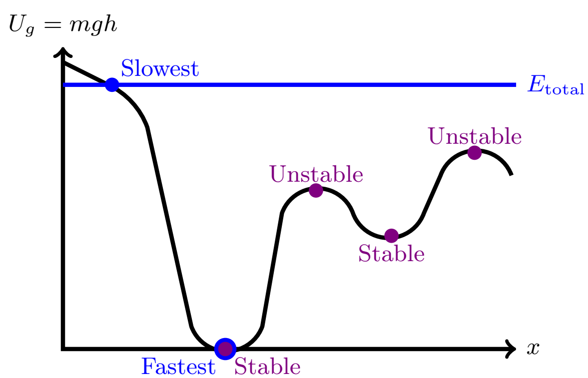
(1) The ball will be moving fastest at the lowest point on the track, where the most potential energy has been transformed into kinetic energy.

(2) The ball will be moving slowest at the top of the track, where it starts with zero speed.
It won't be going slowest at the two small peaks, but those will be local minima for its speed, as its kinetic energy will be at a local minimum in these places.

(3) There are stable equilibrium points at the bottoms of the two dips in the track. If the ball is placed slightly off of one of these points, it will want to fall downhill toward it.

(4) There are unstable equilibrium points at the tops of the two peaks on the track. If the ball is placed perfectly at one of these points, it will stay, but if it is placed slightly off of one of these points, it will want to fall downhill away from it.

(5) Since $U_g = mgh$, the graph will have the same shape as the physical track. We can choose the put the origin anywhere, but for simplicity, let us set $x = 0$ at the left side of the track to make all positions on the track positive, and let us set $U_g = 0$ at the bottom of the lowest point of the track to make all energy on the track positive.



(8) Equilibrium occurs where the slope is zero: $\frac{\partial U_g}{\partial x} = 0$ (which is where the force from that potential is zero).
Stable equilibrium occurs where the graph is concave up: $\frac{\partial^2 U_g}{\partial x^2} > 0$.
Unstable equilibrium occurs where it is concave down: $\frac{\partial^2 U_g}{\partial x^2} < 0$.

L19-3: Undulant Track

Consider the ball that can roll on the track in the center of the room. Ignore friction and air resistance. The ball starts from rest at the top of the track. Use the apparatus to see how the ball moves.

(1) At what point will the ball be moving the fastest? How do you know?

(2) At what point will the ball be moving the slowest? How do you know?

(3) Are there any stable equilibrium points? If so, then where are they located?

(4) Are there any unstable equilibrium points? If so, then where are they located?

(5) Make a graph of the gravitational potential energy of the ball vs. the horizontal position. Remember to define the origin.

(6) On the same set of axes, make a graph of the total mechanical energy of the ball vs. horizontal position. Assume that the ball starts from rest.

(7) Where are the points determined previously for questions 1 through 4 above located on the graph?

(8) How can stable and unstable equilibrium points be defined in terms of the graph of potential energy vs. horizontal position?