Concluding Lecture (L24): Choosing a Model

Warm-Up Activity
What was the topic you found most interesting in PH 211?

(A) Kinematics

(B) Forces

(C) Energy

(D) Momentum

A Model for Motion

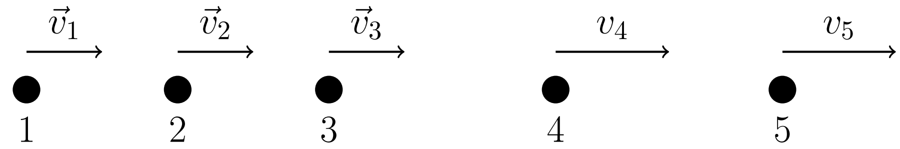
Quantities

- Position: $\vec{r}$
- Velocity: $\vec{v} = \frac{d\vec{r}}{dt}$
- Acceleration: $\vec{a} = \frac{d\vec{v}}{dt}$

Assumptions

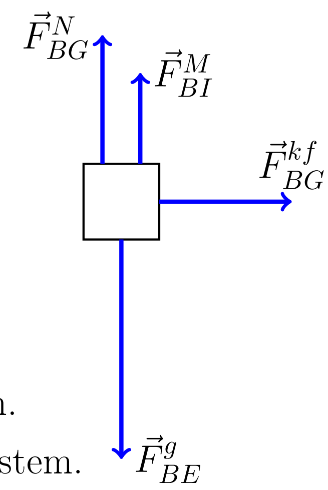
- Use the Particle Model

Motion Diagram



A Model for Interactions

- Quantities
  - Mass m
  - Force $\vec{F}$
- Laws
  - Net force is proportional to acceleration: $\vec{F}^{net} = m\vec{a}$
  - Forces come in pairs: $\vec{F}_{AB} = -\vec{F}_{BA}$
- Asssumptions
  - We can treat multiple objects as a system.
  - All forces act as if on the center of the system.
- Diagram



Types of Forces

- Gravity $\vec{F}^g_{AB} = m_A \vec{g}_B$
  - Newtonian $\vec{g}_B = G\frac{M_B}{r^2}(-\hat{r})$, $G = 6.67408 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$
  - Near-Earth $\vec{g}_E = g(-\hat{y}),\ g = 9.81\frac{\mathrm{m}}{\mathrm{s}^2} \approx 10\frac{\mathrm{m}}{\mathrm{s}^2}$
- Normal $\vec{F}^N$ always $\perp$; varies in magnitude
- Tension $\vec{F}^T$ uniform (massless, inextensible rope)
- Spring $\vec{F}^S = -k(\vec{x} - \vec{x}_{eq})$
- Friction
  - Static Friction $F^{sf} \leq \mu_s |\vec{F}^N|$
  - Kinetic Friction $F^{kf} = \mu_k |\vec{F}^N|$

Not Forces

- Momentum
- Inertia
- Velocity
- Acceleration

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A Deeper Model for Interactions

- Quantities
  - Energy E
  - Work $W = \int_{r_i}^{r_f} \vec{F} \cdot d\vec{r}$
  - Kinetic Energy $K = \frac{1}{2}mv^2$
  - Potential Energy $U =$ depends on interaction
    You have to tell everyone where zero PE is!
    * Gravity $U_g = mgy$
    * Spring $U_{sp} = \frac{1}{2}kx^2$
  - Momentum $\vec{p} = m\vec{v}$
  - Impulse $\vec{J}_{net} = \int_{t_i}^{t_f} \vec{F}^{net} dt$
- Laws
  - Work-energy theorem $W_{\text{net,ext}} = \Delta E_{\text{total}}$
  - Impulse-momentum theorem $\vec{J}_{net} = \Delta\vec{p}$

Putting the Pieces Together

- How do we use problems we've already solved to help us solve new ones?
- For each scenario, which model(s) would you use, and why?
  - Kinematics (motion, projectiles)
  - Forces (friction, springs, multiple objects)
  - Energy (work, power, potential energy, systems)
  - Momentum (impulse, systems)

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We already know that the dart is traveling under the influence of gravity, so this is a projectile motion problem. We have nothing more to prove with forces. Momentum would also be overkill, as solving this problem with impulse (which itself depends on forces) would just reproduce results that are already possible with kinematics. We want to find out how far the dart falls, which depends on how long it is in the air. Our energy equations don't really deal with how long things take, just the state of the total energy at specific times.

Kinematics
We know that

$$\Delta x = v_{ix} t,$$

and we know both how far away the target is ($\Delta x = 2.4$ m) and how fast the dart is moving initially in this direction ($v_{ix} = 10$ m/s). We can solve this for time (obtaining $t = \frac{\Delta x}{v_{ix}} = 0.24$ s) and see how far the dart falls in that time. In the vertical direction, there is no initial velocity ($v_{iy} = 0$), so

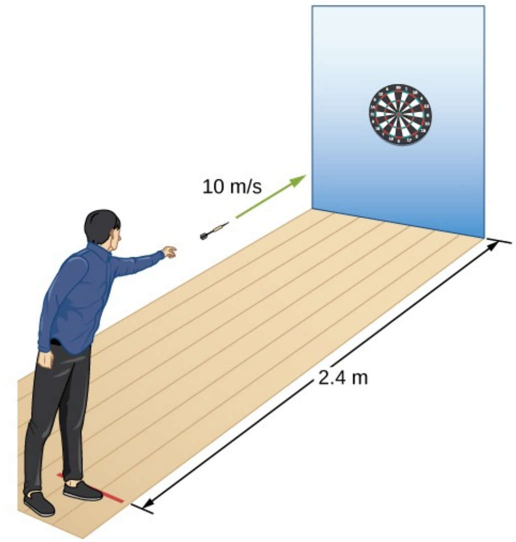
$$\Delta y = v_{iy} t - \frac{1}{2} g t^2 = -\frac{1}{2} g t^2 = -\frac{1}{2} g \left(\frac{\Delta x}{v_{ix}}\right)^2 \approx -0.29 \text{ m}.$$

The dart fell almost an entire foot below the bullseye; based on how big a dartboard typically is, I would wager the dart missed the board entirely.

L24-1: Choosing a Model – Dart

A dart is thrown horizontally at a speed of 10 m/s directly at the bullseye of a dartboard 2.4 meters away from the thrower.

Where does the dart strike the board?



The jump over the cars is projectile motion, so it is probably a kinematics problem. We are just missing the launch speed off of the ramp. The shape of the ramp makes it too complicated for the simplest forms of kinematics, and a force analysis across a curving surface as an object moves would be prohibitively difficult. We have an initial and final height, which means we know the change in potential energy, so energy methods would be natural here.

Energy

Let us set the zero for gravitational potential energy at the height of the launch ramp. Thus, $U_{gi} = mgH_0$ and $U_{gf} = 0$. The cyclist starts at rest, so $K_i = 0$, which leaves $K_f = \frac{1}{2}mv^2$ to be found. The total final energy is equal to the total initial energy if we put the cyclist and the Earth into the system, as there will be no net external work:

$$\begin{aligned} E_f &= E_i \\ U_{gf} + K_f &= U_{gi} + K_i \\ \frac{1}{2}mv^2 &= mgH_0 \\ v &= \sqrt{2mH_0}. \end{aligned}$$

This is the launch speed.

Kinematics

In vector form, at angle θ from the horizontal, we have that the launch velocity is

$$\vec{v}_i = \sqrt{2gH_0}(\cos\theta_0\hat{x} + \sin\theta_0\hat{y}).$$

The jump is complete when the bicycle returns to its original launch height:

$$0 = \Delta y = v_{iy}t - \frac{1}{2}gt^2 \implies v_{iy} = \frac{1}{2}gt \implies t = 2\frac{\sqrt{2gH_0}}{g}\sin\theta_0.$$

At this time, the bicycle has covered horizontal distance X_0:

$$\begin{aligned} X_0 = v_{ix}t &= \sqrt{2gH_0}\cos\theta_0 \cdot 2\frac{\sqrt{2gH_0}}{g}\sin\theta_0 \\ &= 4H_0\cos\theta_0\sin\theta_0. \end{aligned}$$

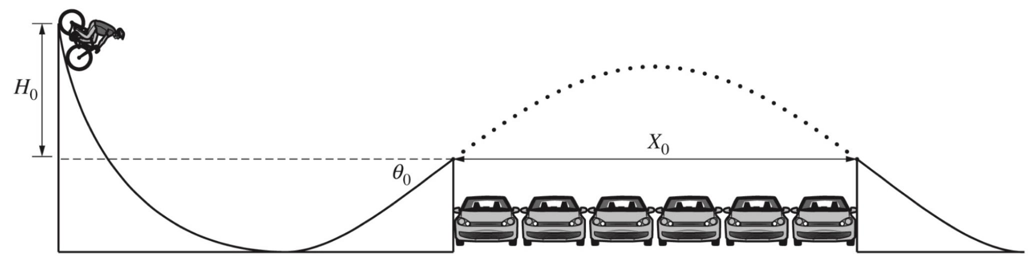
If you remember your double-angle formulae, this is

$$X_0 = 2H_0\sin(2\theta_0).$$

L24-2: Choosing a Model – The Ramp

A stunt cyclist builds a ramp that will allow the cyclist to coast down the ramp and jump over several parked cars, as shown below. To test the ramp, the cyclist starts from rest at the top of the ramp, coasts down to the bottom, jumps over six cars, and lands on a second ramp.

Goal: Derive an expression for X_0.



Note: Figure not drawn to scale.

Problem Credit: College Board ©2021

L24-3: Choosing a Model – Arrow

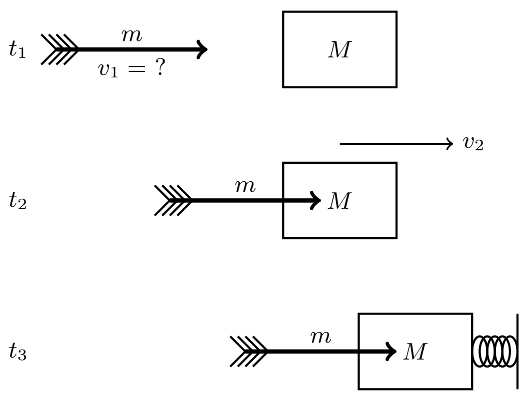
You fire a 0.05 kg arrow at an unknown speed. It embeds in a 0.35 kg block that slides on a frictionless surface until it compresses a spring of spring constant $k = 4000$ N/m a distance of 0.10 m.

What was the speed at which the arrow was fired?

Problem Credit: Etkina College Physics

We begin with a collision between the arrow and the block. The forces between them during the embedding are likely too complicated for the models we understand, and we wouldn't be able to obtain an acceleration that would be usable for kinematics. Collisions suggest using momentum. After the collision, the block has some speed, and it ends at rest after compressing a spring by a known distance. That suggests that we know the final kinetic and potential energies of the system, so energy conservation would allow us to infer the combined velocity of the arrow and block.

Momentum



Our system is the arrow and block. If we assume the collision is very fast, then the impulse from gravity on the arrow is negligible, and therefore momentum is conserved:

$$mv_1 = (m+M)v_2 \implies v_1 = \frac{m+M}{m}v_2.$$

Energy
Our system is the arrow, block and spring.

	K	U_{sp}	E_{total}
t_2	$\frac{1}{2}(m+M)v_2^2$	0	$\frac{1}{2}(m+M)v_2^2$
t_3	0	$\frac{1}{2}kx^2$	$\frac{1}{2}kx^2$

Without friction between the block and the surface, there is no net external work being done on this system, so

$$\frac{1}{2}(m+M)v_2^2 = \frac{1}{2}kx^2 \implies v_2 = \sqrt{\frac{k}{m+M}}x.$$

Putting this together with our answer for the momentum portion, we find that the initial speed of the arrow was

$$\begin{aligned} v_1 &= \frac{m+M}{m}\sqrt{\frac{k}{m+M}}x = \frac{x}{m}\sqrt{k(m+M)} \\ &= \frac{0.1\text{ m}}{0.05\text{ kg}}\sqrt{(4000\text{ N/m})(0.4\text{ kg})} \\ &= 2\frac{\text{m}}{\text{kg}}\sqrt{1600\frac{\text{kg}^2}{\text{s}^2}} \\ &= 80\text{ m/s}. \end{aligned}$$

We are looking for a final speed, but we know our force in terms of position, not time, so kinematics and impulse are not going to help, as we are missing a key piece of information. Instead, we will want to use energy, as force and distance together tell us about the energy being added to or removed from a system, which will tell us how much kinetic energy the particle has at the final position.

Energy

$$K_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(2\text{ kg})(6\text{ m/s})^2 = 36\text{ J};$$
$$W_{\text{net,ext}} = \int_{0\text{ m}}^{4.0\text{ m}} F(x)dx = (-5\text{ N})(3\text{ m}) + \frac{1}{2}(-5\text{ N})(1.0\text{ m}) = -\frac{35}{2}\text{ J}.$$

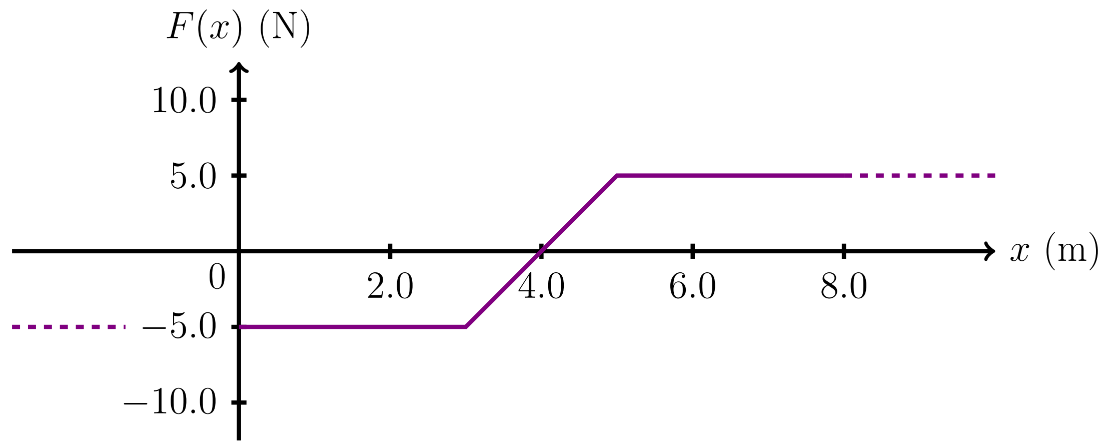
By the work-energy theorem:

$$\begin{aligned}
\Delta E &= W_{\text{net,ext}} \\
\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 &= W_{\text{net,ext}} \\
v_f &= \sqrt{\frac{2\left(W + \frac{1}{2}mv_i^2\right)}{m}} \\
&= \sqrt{\frac{2\left(-\frac{35}{2}\text{ J} + 36\text{ J}\right)}{2\text{ kg}}} \\
&= \sqrt{\frac{37}{2}\frac{\text{m}^2}{\text{s}^2}} \\
&\approx 4.3\text{ m/s}.
\end{aligned}$$

L24-4: Choosing a Model – Force

The plot below shows the net force applied in the x-direction to a 2 kg particle moving parallel to the x-axis. The velocity of the particle at $x = 0$ is +6 m/s in the x-direction.

Find the particle's speed at $x = 4$ m.



Problem Credit: OpenStax University Physics

Main Ideas

- This concludes the course.
- We've tried to introduce you to how a physicist looks at the universe.
- I hope you feel more capable than at the beginning of the term.
- Congratulate yourself for working so hard!