Lecture 7: Projectile Motion II

Announcements

- Group Expectations are due at 8pm tonight.
  - Only one submission per group.
  - Fill out the form even if you don't have a group. I will use it to assign you.
- Project Proposal is due on Friday.
  - Not released yet (will be after I form the groups in Canvas).
  - Decide on a topic and a format with your group.
- Make sure you can see your feedback on homework, labs, and Get-Ready assignments!

1a) Understand the Problem

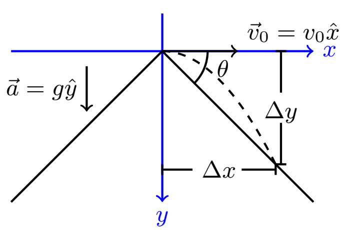
- Known:
  - $v_0 = 25$ m/s, $\theta = 45°$
  - By assumption: $\vec{a} = +g\hat{y}$, $g \approx 9.8$ m/s^2
- Unknown:
  - Δx (horizontal distance traveled), Δy (vertical distance traveled), t_f (time of flight), v_f final speed

1b) Identify Assumptions

- The mountain has a perfectly smooth slope, and the arrow will not travel far enough to reach the bottom.
  - While mountains are not really this smooth, this assumption reflects the picture, and makes it easier to mathematically describe the slope.
  - Mountains are really tall, and reaching the base of the mountain would complicate the problem.
- The arrow is not affected by moving through air.
  - An arrow is fast and aerodynamic, and we don't really know how to handle air resistance or the effects of wind, so we should ignore these effects to make the problem solvable.
- Near Earth's surface ($g \approx 9.8$ m/s^2)
  - So far, all known archery occurs on Earth, and though we are farther up on this mountain, the arrow won't fall so far that the variations in gravity are significant.
- $\vec{a} = +g\hat{y}$
  - Since the object is in free-fall with no wind or air effects, and it isn't traveling far enough for the curvature of the Earth to matter, acceleration is straight down and purely gravitational.

1c) Represent Physically

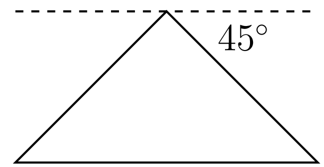
Choosing $\hat{y}$ pointing down saves us a few negative signs in the process of the solution.



L7-1: Archer of the Peak

You (still a long-distance archer) move to the top of the mountain below. The initial speed of your arrow is now 25.0 m/s, and you release it horizontally to the right. Following the steps for solving an A*R*C*S problem, find where the arrow lands.

- *Hint 1:* You will still need to set up separate equations for the x- and y-directions.
- *Hint 2:* You can also relate the horizontal and vertical distances the arrow moves before striking the ground.



2a) Represent Principles
2-D Kinematics: Known Unknown Wanted

$$\Delta x = v_{0x}t_f + \frac{1}{2}a_x {t_f}^2 \qquad \Delta y = v_{0y}t_f + \frac{1}{2}a_y {t_f}^2$$
$$v_{fx} = v_{0x} + a_x t_f \qquad v_{fy} = v_{0y} + a_y t_f$$
$${v_{fx}}^2 = {v_{0x}}^2 + 2a_x\Delta x \qquad {v_{fy}}^2 = {v_{0y}}^2 + 2a_y\Delta y$$

We can avoid the bottom four equations, since we don't really want to know v_f.
Trigonometry: $\tan\theta = \frac{\Delta y}{\Delta x}$

2b) Find Unknown(s) Symbolically
Between $\Delta x = v_{0x}t_f + \frac{1}{2}a_x {t_f}^2$ and $\Delta y = v_{0y}t_f + \frac{1}{2}a_y {t_f}^2$, we have three unknowns (t_f, Δx, Δy). However, we can bring in our one trigonometric equation to relate two of the unknowns, giving us a solvable system of three equations.

$$\Delta x = v_{0x}t_f + \frac{1}{2}a_x {t_f}^2 \qquad \Delta y = v_{0y}t_f + \frac{1}{2}a_y {t_f}^2$$
$$\Delta x = v_0 t_f \qquad \Delta y = \frac{1}{2}g t_f^2$$
$$\tan\theta = \frac{\Delta y}{\Delta x} = \frac{\frac{1}{2}g t_f^2}{v_0 t_f} = \frac{g}{2v_0}t_f \implies t_f = \frac{2v_0}{g}\tan\theta$$
$$\Delta x = \frac{2v_0^2}{g}\tan\theta \qquad \Delta y = \frac{2v_0^2}{g}\tan^2\theta$$

2c) Plug in Numbers
I will actually use $g \approx 10\ \text{m/s}^2$ for simplicity here:

$$\Delta x \approx \frac{2(25\ \text{m/s})^2}{10\ \text{m/s}^2}\tan(45^\circ) = 125\ \text{m}.$$

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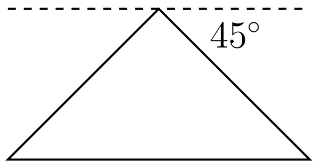

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3a) Units
We want $[\Delta x] = \mathrm{m}$, and we know $[v_0] = \frac{\mathrm{m}}{\mathrm{s}}$, $[g] = \frac{\mathrm{m}}{\mathrm{s}^2}$, and both $[\tan(\theta)] = 1$ and $[2] = 1$ (unitless), so

$$[\Delta x] = \frac{[2][v_0]^2}{[g]}[\tan\theta] = \frac{\mathrm{m}^2/\mathrm{s}^2}{\mathrm{m}/\mathrm{s}^2} = \mathrm{m}$$

3b) Numbers
Δx and Δy are both 125 m, and $\sqrt{\Delta x^2 + \Delta y^2} = 125\sqrt{2}$ m ≈ 176 m, which is comparable to the distance we shot in L6-2: Long-Distance Archer. We achieved it at a lower speed here, since our arrow could fall toward its target (rather than having to keep itself above flat ground for the entire flight).
3c) Symbols

- v_0 increases
  - If it is shot faster, the arrow will have more horizontal speed and have farther to fall (and hence more time in flight), so it should achieve more horizontal distance.
  - Δx increases as v_0 increases, so our equation matches this prediction.
- g increases
  - If g increases (say we do archery on Jupiter), then the arrow will be dragged to the ground more quickly, so it will not travel as far.
  - Δx decreases as g increases, so our equation matches this prediction.
- $\theta = 0^\circ$
  - In this special case, there is no mountain, and an arrow launched horizontally at ground level will hit the ground immediately, having no range.
  - $\Delta x = \frac{2v_0^2}{g}\tan(0^\circ) = 0$

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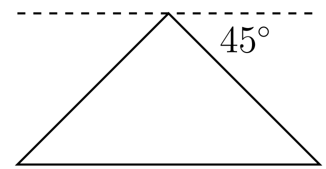

Main Ideas

- We can use the kinematics equations to solve for any quantity of interest when the acceleration is constant.
- Motion in 2 dimensions can be broken down into independent motion in each dimension.