

### Ungrading Prompts

- (1) Assemble all of your work for the term (Get-Ready assignments, weekly homeworks, lab reports, project submissions).
- (2) Read through everything (your work, your reflections, the feedback you received).
- (3) Answer the following questions based on your review of your work:
  - (a) What portions of your work are you most proud of, and why?
  - (b) Through your work, what did you learn the most about during the term?
  - (c) What areas of your work would benefit the most from further development?
- (4) How many class sessions were you able to attend?
  - (a) If you missed any class sessions, were you able to complete the work for class at a distance?
    - (i) If you could not complete all classwork at a distance, please explain why this was the case.
- (5) Describe your level of engagement during class.
- (6) Roughly how much of the readings for class did you complete as assigned?
- (7) Were you able to turn in all of your assignments on time?
  - (a) If not, please explain why you were not able to do so.
- (8) Reflect on your experience with ungrading; what was your experience with this process at the beginning, middle, and end of the class?
- (9) Please suggest a grade for yourself and provide a brief rationale for your suggestion.

## Lecture 12: Force Analysis and Special Cases

### Announcements

- Mid-Term Ungrading due Saturday at 11:59 p.m.
  - Prompts for Ungrading are in the Syllabus, the Mid-Term Ungrading assignment, and in these lecture notes.
  - Review all of your work and feedback so far, discuss your work, your attendance, and your engagement, then propose a grade for yourself.
  - Citing and quoting your work and feedback as evidence is good.
  - Suggested minimum length of 2 pages.
- Homework 3 assessment completed
  - The Angled Block and the Calculate part of The Penny in the Elevator were not evaluated.
  - Compare your work against the solution. If you have questions or want more feedback, take your work into the Wormhole or office hours.

## A Model for Interactions

- Quantities

- Mass  $m$
  - Force  $\vec{F}$

- Laws

- Net force is proportional to acceleration:  
 $\vec{F}_{net} = m\vec{a}$

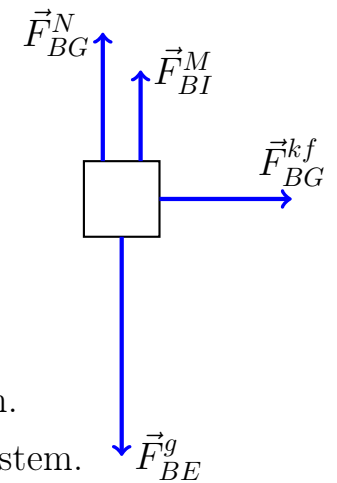
- Forces come in pairs:  $\vec{F}_{AB} = -\vec{F}_{BA}$

- Assumptions

- We can treat multiple objects as a system.

- All forces act as if on the center of the system.

- Diagram



## Solving Problems Using Forces

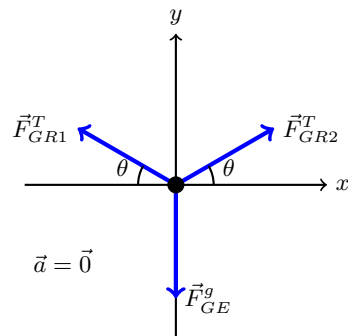
- Identify a system.
- Identify the (external) forces acting on the system.
  - Draw a free-body diagram.
- Identify the acceleration (**not a force**).
  - Static/dynamic equilibrium (acceleration = 0)
  - Dynamics (acceleration not 0)
- Use the laws of motion.
- Reflect on your answer (check units and evaluate special cases).

## Special-Case Analysis

After you solve for a quantity:

- Choose a case that is **special**, not arbitrary.
- Figure out what your quantity **should** be in the case you chose.
- Identify the value of one or more other quantities that corresponds to your **case**.
- Evaluate your answer in the special case.
- Check whether or not your symbolic answer for the case matches what you expected the answer should be.

Let's call the rope on the left rope 1, and the rope on the right rope 2.

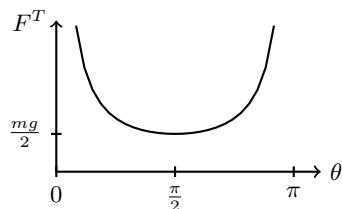


$$\vec{F}^{net} = m\vec{a} = \vec{0}$$

From the  $x$ -component of the net force, we find that the balance of forces and the equal angles require the tensions to be the same. This then can be used to simplify the  $y$ -component of the net force and solve for this one tension.

$$\begin{aligned} 0 &= F_x^{net} & 0 &= F_y^{net} \\ &= -F_{GR1}^T \cos \theta & 0 &= F_{GR1}^T \sin \theta + F_{GR2}^T \sin \theta - F^g \\ &\quad + F_{GR2}^T \cos \theta & 0 &= 2F^T \sin \theta - F^g \\ F_{GR1}^T \cos \theta &= F_{GR2}^T \cos \theta & 2F^T \sin \theta &= F^g = mg \\ F_{GR1}^T &= F_{GR2}^T = F^T & F^T &= \frac{mg}{2 \sin \theta} \end{aligned}$$

### Special Cases



- If the ropes are vertical ( $\theta = \pi/2$ ), then they both hang straight down, and they should each support half of the gymnast's weight (they don't need anything extra to pull in the  $x$ -direction).

$$- F^T = \frac{mg}{2 \sin(\pi/2)} = \frac{mg}{2}$$

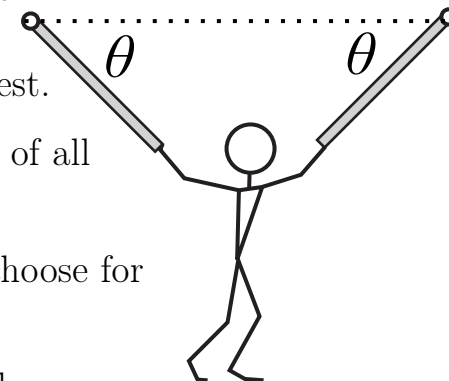
- If the ropes are horizontal with no  $y$ -component, then they need to be extremely taut. In fact, it is impossible for a rope with weight in the middle not to sag a bit (because the ropes need to be tilted to exert an upward force on the suspended mass), so it should require an unrealistic amount of force to make the ropes completely flat.

$$- F^T = \frac{mg}{2 \sin(0)} = \frac{mg}{0} = \infty$$

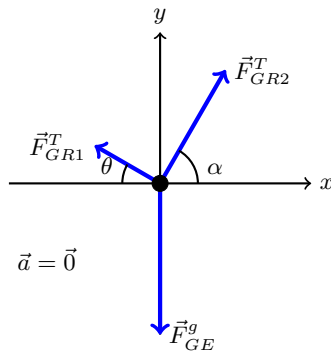
- If the gymnast has no mass, then the ropes don't need to exert any force.  $F^T = \frac{(0 \text{ kg})g}{2 \sin \theta} = 0$

## L12-1: The Gymnast

A gymnast of mass  $m$  is training using two ropes attached to the ceiling, as shown in the figure. The gymnast is suspended at rest.



- Identify and determine the magnitude of all forces acting on the gymnast.
- What special case(s) do you want to choose for this problem? Why?
- Try out at least one special case. (Make sure you know what the answer *should* be in your case!)



### Left vs. Right

Both tensions must have equal  $x$ -components, as the forces balance in the  $x$ -direction. We can relate the magnitudes of the  $y$ -components to the magnitudes of the  $x$ -components by

$$|F_{GR1,y}^T| = |F_{GR1,x}^T| \tan \theta, \quad |F_{GR2,y}^T| = |F_{GR2,x}^T| \tan \alpha,$$

and since the  $x$ -components are equal, this becomes

$$\frac{|F_{GR1,y}^T|}{|F_{GR2,y}^T|} = \frac{\tan \theta}{\tan \alpha}.$$

Since  $\alpha > \theta$ , we know that  $\tan \alpha > \tan \theta$ , and therefore  $|F_{GR2,y}^T| > |F_{GR1,y}^T|$ . This tells us that  $F_{GR2}^T > F_{GR1}^T$ . The left tension is less than the right tension.

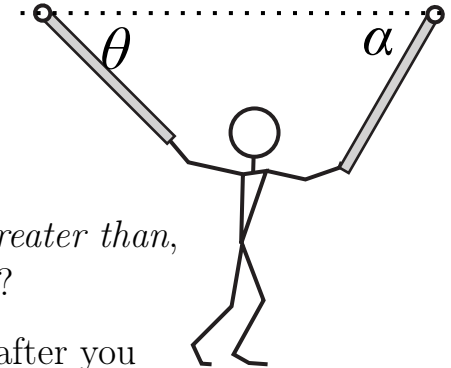
### Special Case

If you set  $\theta = \alpha$ , your equations for both tensions should simplify to what we found before:  $F^T = \frac{mg}{2 \sin \theta}$ .

## L12-2: The Gymnast *Reloaded*

What if the ropes were different angles?

- Don't solve the problem yet!
- You have a problem like this on your homework.
- Do you expect the left tension to be *greater than*, *less than*, or *equal to* the right tension?
- What special case could you evaluate after you solve this problem to check if your answer is right?



## Main Ideas

- When an object is at rest or moving at constant speed, the forces balance and the object is in equilibrium.
- When an object is accelerating, there is a nonzero net force.