Fluid Simulation

CIS 563 Spring 2013, Hw3 Recitation

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Overview

- Math & Physics
- MAC Grid
- 2x2x1 Example
- Data Structure
- Starter Kit Workflow
- Extras
- Tips

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- if $\vec{f}$ is vector:
  - Divergence: $\operatorname{div}(\vec{f}) = \nabla \cdot \vec{f}$
  - Curl: $curl(\vec{f}) = \nabla \times \vec{f}$

Math: Calculus

- Vector differential operator ∇ in 3D Euclidian space
- $\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix}$

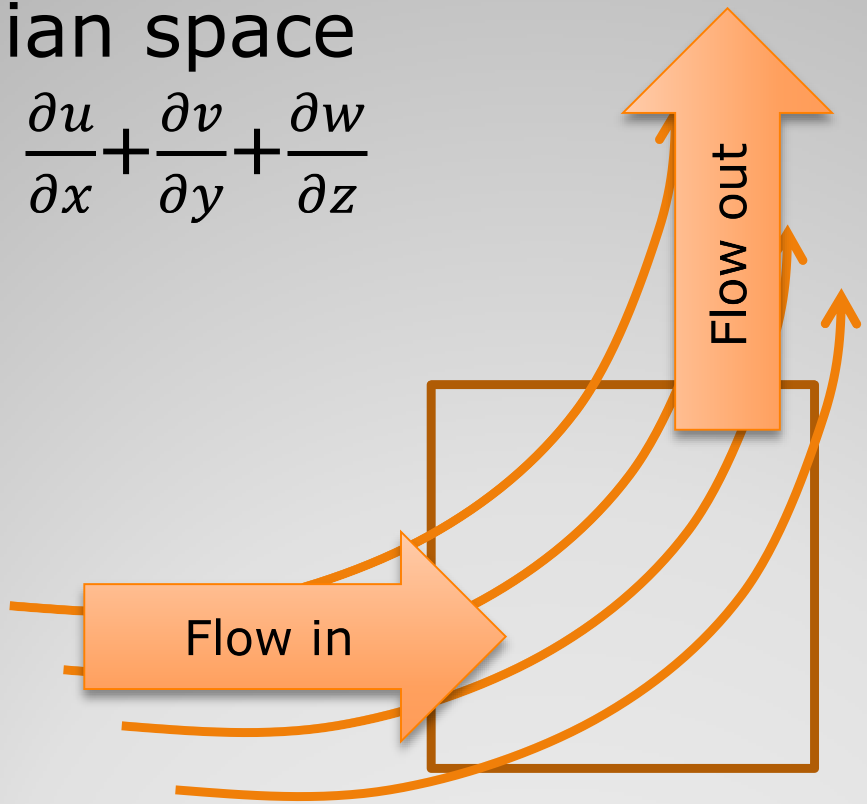
Math: Calculus

- Divergence of vector $\vec{f} = (u, v, w)^T$ in 3D Euclidian space
- $\nabla \cdot \vec{f} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \cdot \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

Math: Calculus

Math: Calculus

- Divergence of vector $\vec{f} = (u, v, w)^T$ in 3D Euclidian space
- $\nabla \cdot \vec{f} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$



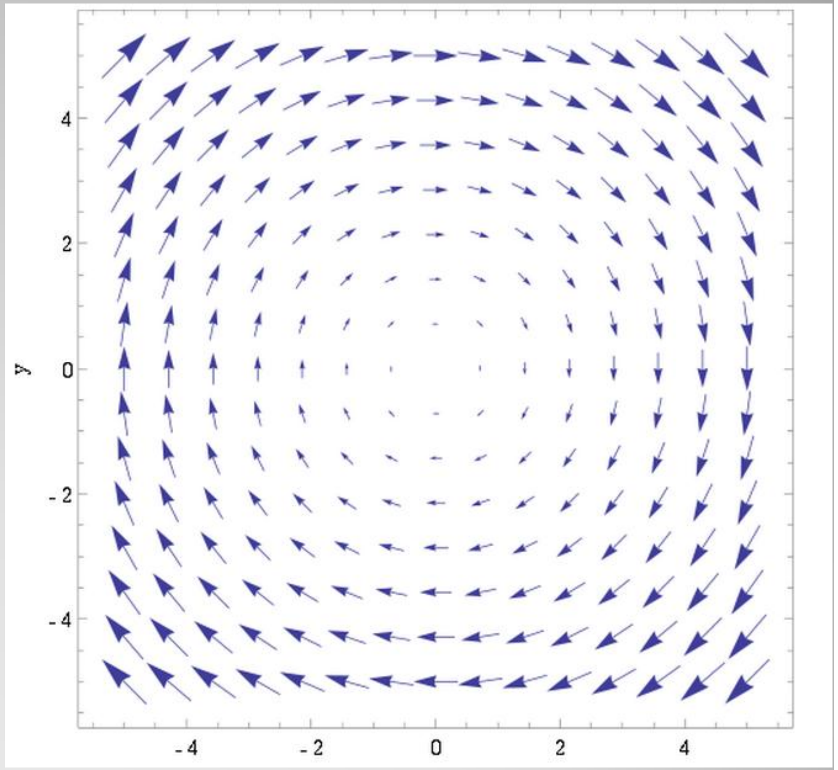
- Divergence of vector $\vec{f} = (u, v, w)^T$ in 3D Euclidian space
- $\nabla \cdot \vec{f} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$
  - $\nabla \cdot \vec{f} > 0$: source
  - $\nabla \cdot \vec{f} < 0$: sink
  - $\nabla \cdot \vec{f} = 0$: "divergence free"

Math: Calculus

- Curl of vector $\vec{f} = (u, v, w)^T$ in 3D Euclidian space
- $\nabla \times \vec{f} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} u \\ v \\ w \end{pmatrix} = \begin{pmatrix} \frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \\ \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \end{pmatrix}$

Math: Calculus

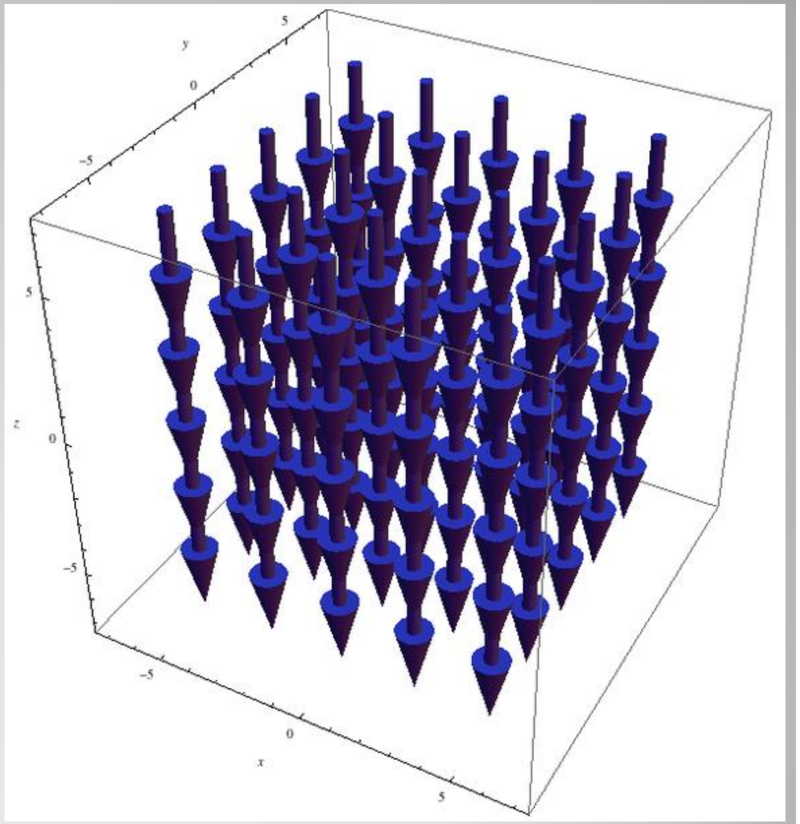
- Example:
- $\vec{f}(x,y,z) = y \cdot \vec{x} - x \cdot \vec{y}$
- $\begin{cases} u = y \\ v = -x \\ w = 0 \end{cases}$



[Figure from http://en.wikipedia.org/wiki/Curl_(mathematics)]

Math: Calculus

- Example:
- $\vec{f}(x,y,z) = y \cdot \vec{x} - x \cdot \vec{y}$
- $\nabla \times \vec{f} = \begin{pmatrix} \frac{\partial 0}{\partial y} - \frac{\partial y}{\partial z} \\ \frac{\partial(-x)}{\partial z} - \frac{\partial 0}{\partial x} \\ \frac{\partial(-x)}{\partial x} - \frac{\partial y}{\partial y} \end{pmatrix}$
- $\nabla \times \vec{f} = (0,0,-2)^T$



[Figure from http://en.wikipedia.org/wiki/Curl_(mathematics)]

Math: Calculus

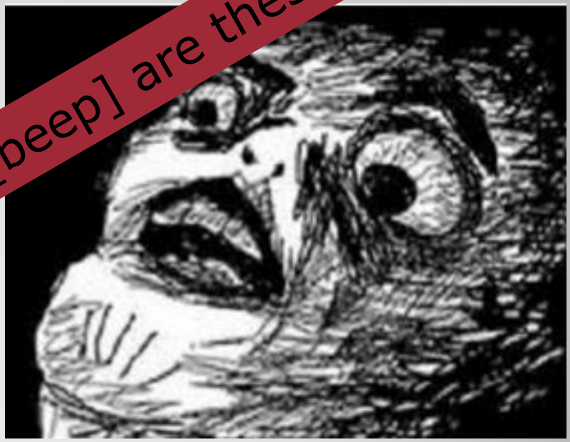
- Summary:
  - Divergence: $\mathrm{div}(\vec{f}) = \nabla \cdot \vec{f}$
    - Measures the magnitude of a vector field’s source or sink at a given point.
  - Curl: $curl(\vec{f}) = \nabla \times \vec{f}$
    - Measure the rotation of a 3d vector field

Math: Calculus

- Fluid equation:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \vec{g} + \nu \nabla \cdot \nabla \vec{u}$$

$$\nabla \cdot \vec{u} = 0$$



Physics: Navier-Stokes Eq

Physics: Navier-Stokes Eq

- Momentum equation: f = ma
  - $\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \vec{g} + \nu \nabla \cdot \nabla \vec{u}$
  - Rearrange:

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = \vec{g} + \left(-\frac{1}{\rho} \nabla p\right) + \nu \nabla \cdot \nabla \vec{u}$$

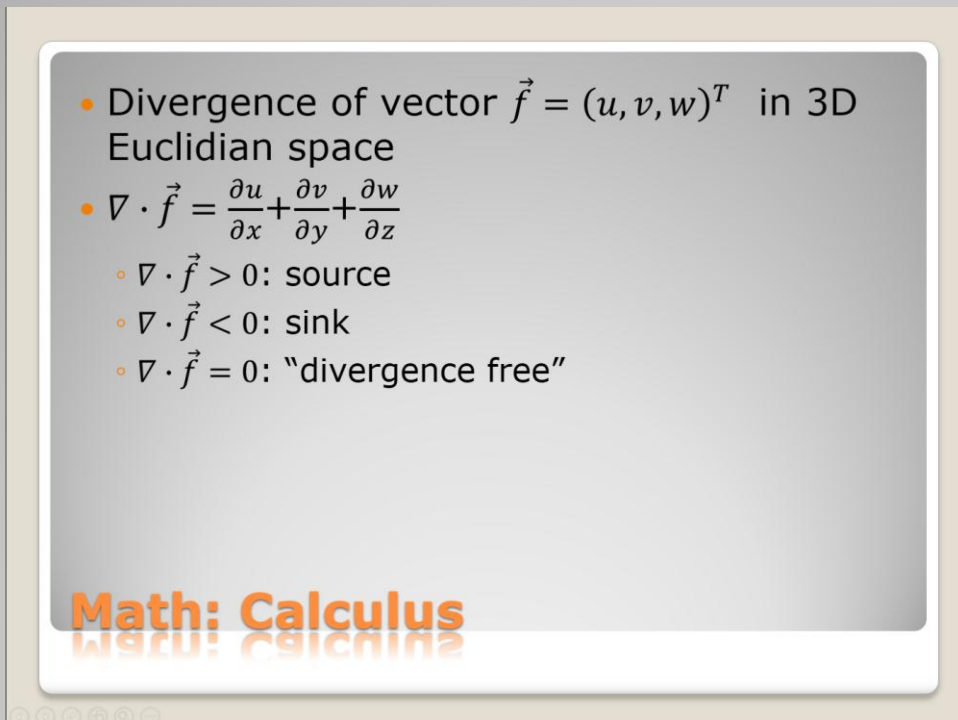
Material Derivative

External Force

Pressure Force

Viscosity Force

- Constraint equation: divergence free
  - $\nabla \cdot \vec{u} = 0$



Physics: Navier-Stokes Eq

Physics: Navier-Stokes Eq

- Fluid equation:

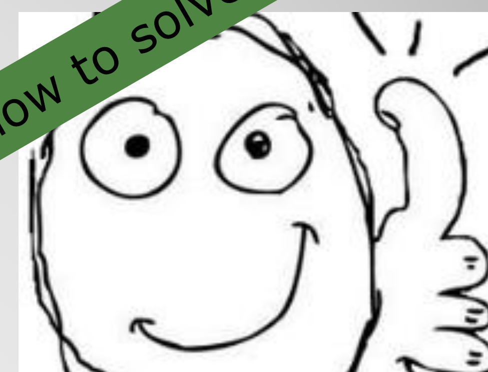
F=ma

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \vec{g} + \nu \nabla \cdot \nabla \vec{u}$$

Div=0

$$\nabla \cdot \vec{u} = 0$$

Great! But how to solve them?



Physics: Navier-Stokes Eq

- Solve PDE by splitting:
  - Toy example:
    - $\frac{\partial q}{\partial t} + 1 + 2 = 3 + 4$
    - Given q^n, how do we find q^{n+1}?
    - Step 1: rearrange $\frac{\partial q}{\partial t} = -1 - 2 + 3 + 4$
    - Step 2: split and solve $\begin{cases} q^{(1)} = q^n + dt * (-1) \\ q^{(2)} = q^{(1)} + dt * (-2) \\ q^{(3)} = q^{(2)} + dt * (3) \\ q^{n+1} = q^{(3)} + dt * (4) \end{cases}$
    - Step 3: get result $q^{n+1} = q^n + 4dt$

Physics: Navier-Stokes Eq

- Solve PDE by splitting:
  - N-S equation
    - $\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \vec{g} + \nu \nabla \cdot \nabla \vec{u}$, $\nabla \cdot \vec{u} = 0$
    - Given $\vec{u}^{n}$, how do we find $\vec{u}^{n+1}$?
    - Step 1, rearrange:

$$\frac{\partial \vec{u}}{\partial t} = -\vec{u} \cdot \nabla \vec{u} + \vec{g} + \nu \nabla \cdot \nabla \vec{u} - \frac{1}{\rho} \nabla p$$

Physics: Navier-Stokes Eq

- Solve ODE by splitting:
  - N-S equation
    - $\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \vec{g} + \nu \nabla \cdot \nabla \vec{u}, \; \nabla \cdot \vec{u} = 0$
    - Given $\vec{u}^{n}$, how do we find $\vec{u}^{n+1}$?
    - Step 2, split:

Advection

Gravity

Viscosity

Projection

$$\begin{cases} solve \frac{\partial \vec{u}}{\partial t} = -\vec{u} \cdot \nabla \vec{u}, using\ \vec{u}^{n}, save\ the\ result\ to\ \vec{u}^{(1)} \\ sovle \frac{\partial \vec{u}}{\partial t} = \vec{g}, using\ \vec{u}^{(1)}, save\ the\ result\ to\ \vec{u}^{(2)} \\ solve \frac{\partial \vec{u}}{\partial t} = \nu \nabla \cdot \nabla \vec{u}, using\ \vec{u}^{(2)}, save\ the\ result\ to\ \vec{u}^{(3)} \\ solve \frac{\partial \vec{u}}{\partial t} = -\frac{1}{\rho} \nabla p, using\ \vec{u}^{(3)}, save\ the\ result\ to\ \vec{u}^{n+1} \\ keep\ \nabla \cdot \vec{u}^{n+1} = 0 \end{cases}$$

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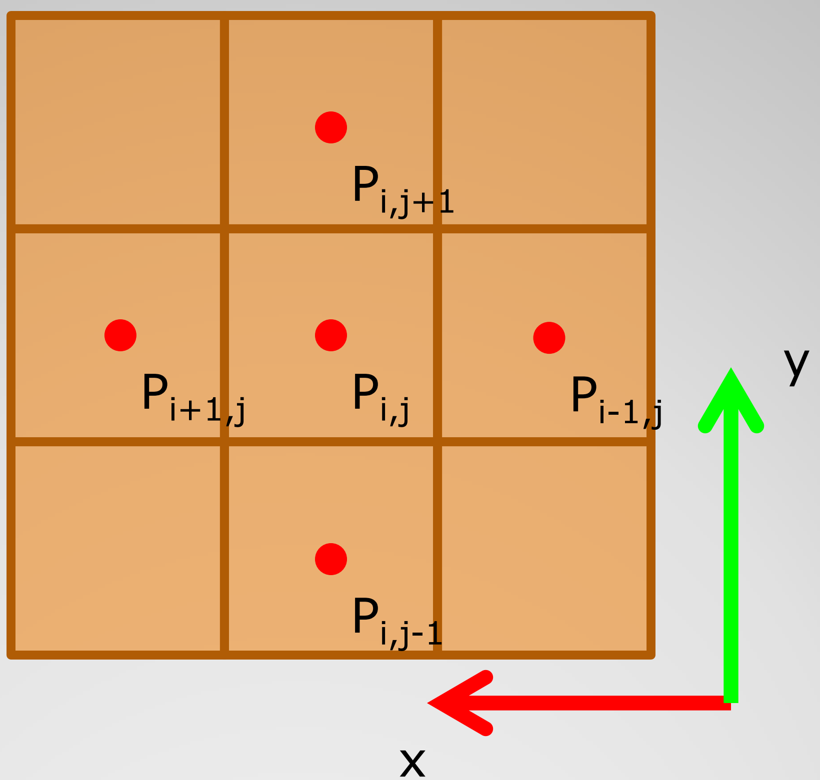
- Marker and Cell Grid / Staggered Grid
  - Detail described in Bridson's notes, page 15
  - If I say any thing about page number in today's class, I'm referring the Bridson's notes.

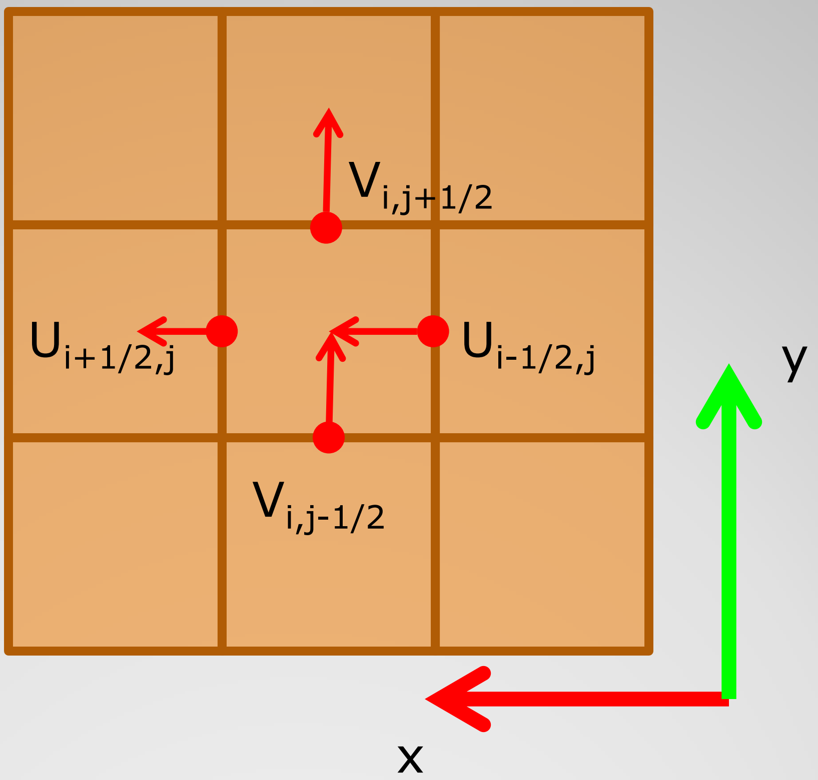
MAC Grid

- 2D case, center quantities:



MAC Grid

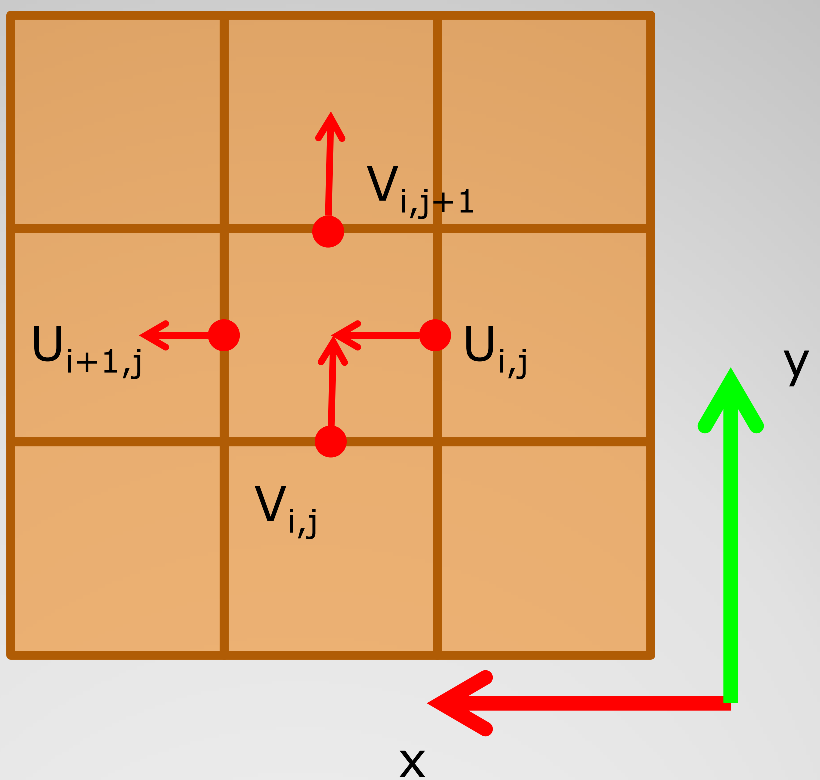
- 2D case, staggered quantities:



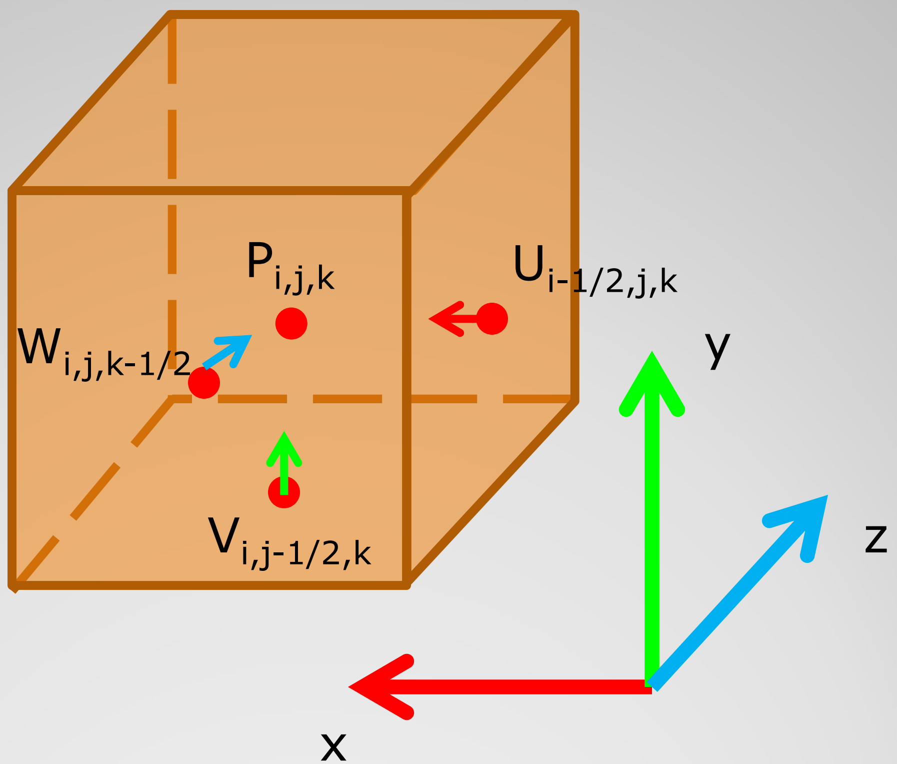
MAC Grid

MAC Grid

- 2D case, staggered quantities: (Programming)

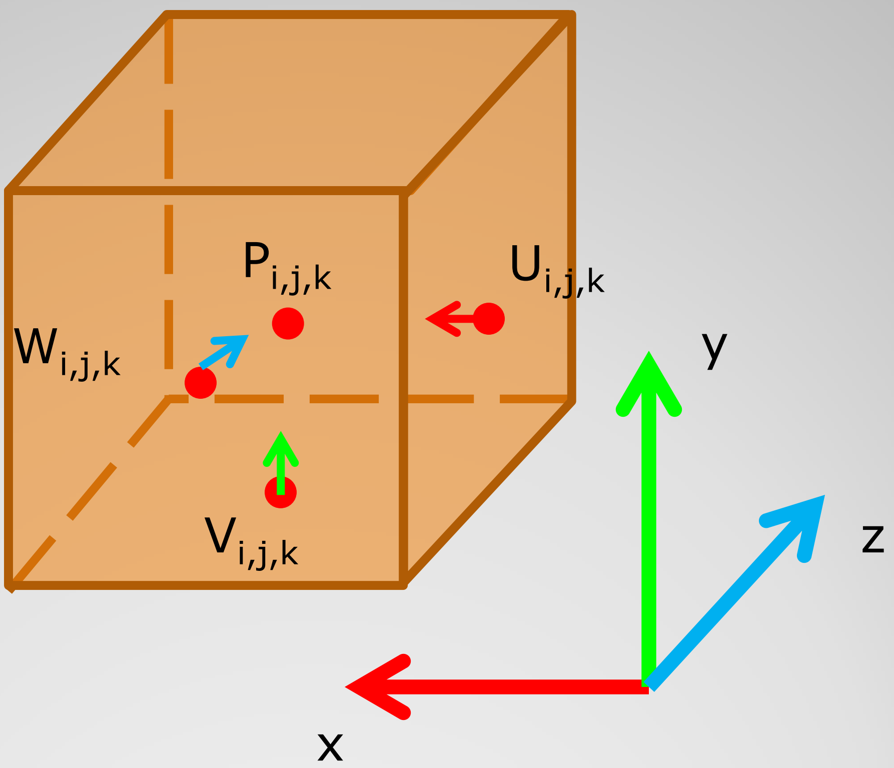


- 3D case:



MAC Grid

- 3D case: (Programming)

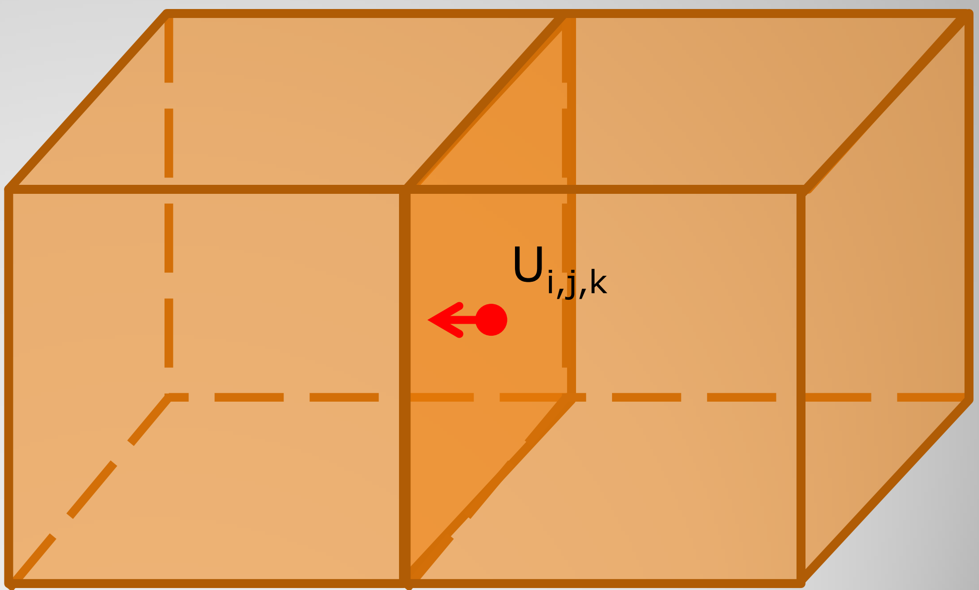


MAC Grid

- All the elements inside are SCALAR.
  - Pressure, temperature, density… etc
  - Also velocity: U, V, W.
- We solve the bad ½ cases by ADDING another ½.
  - $U_{i-1/2, j, k} \rightarrow U_{i, j, k}$
  - $U_{i+1/2, j, k} \rightarrow U_{i+1, j, k}$

MAC Grid

- Simple case of interpolation
  - What's the velocity at the red point?
  - $Vec3(U_{i,j,k},\ \frac{1}{4}*(V_{i,j,k}+V_{i-1,j,k}+V_{i,j+1,k}+V_{i-1,j+1,k}),\ \frac{1}{4}*(W_{i,j,k}+W_{i-1,j,k}+W_{i,j,k+1}+W_{i-1,j,k+1}))$



MAC Grid

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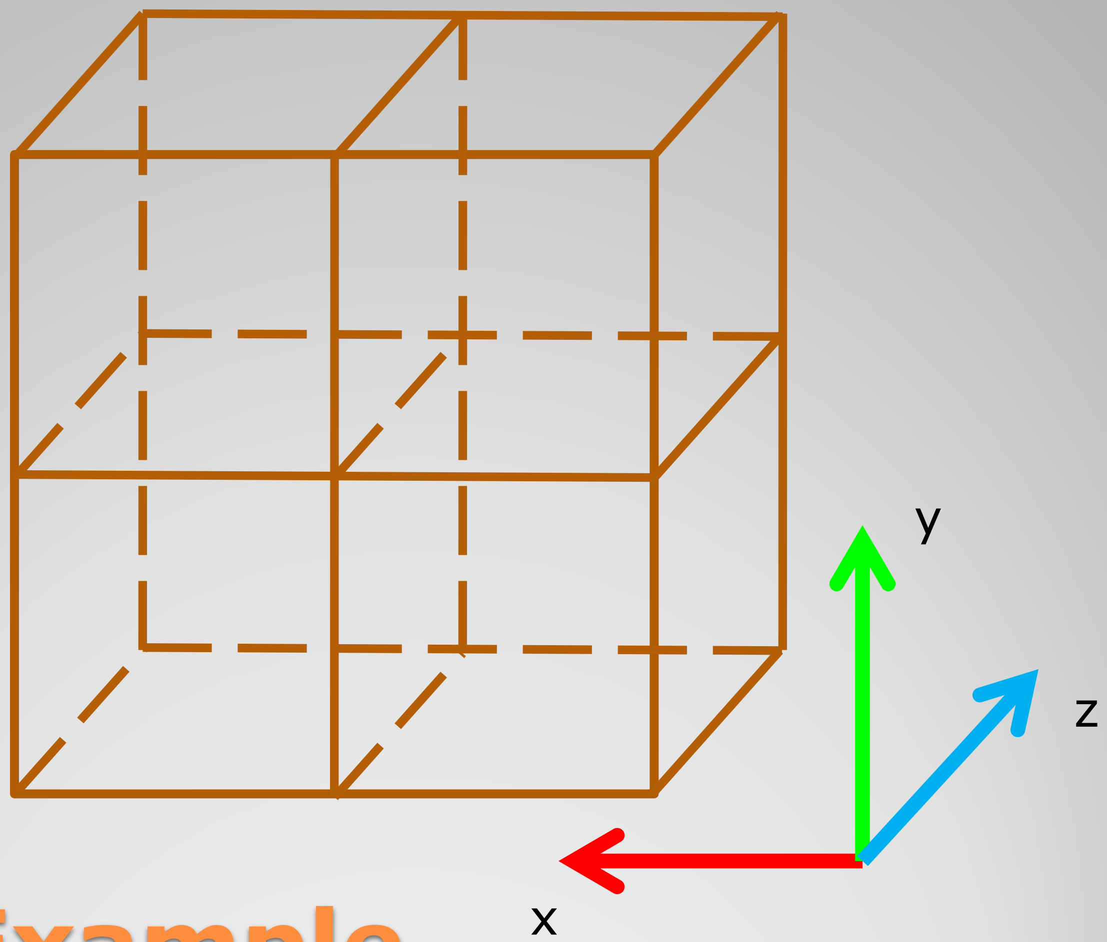
- Example of simple simulation
  - Dropped the external and viscosity force

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = \cancel{\vec{g}} + \cancel{\nu \nabla \cdot \nabla \vec{u}}$$

$$\nabla \cdot \vec{u} = 0$$

2x2x1 Example

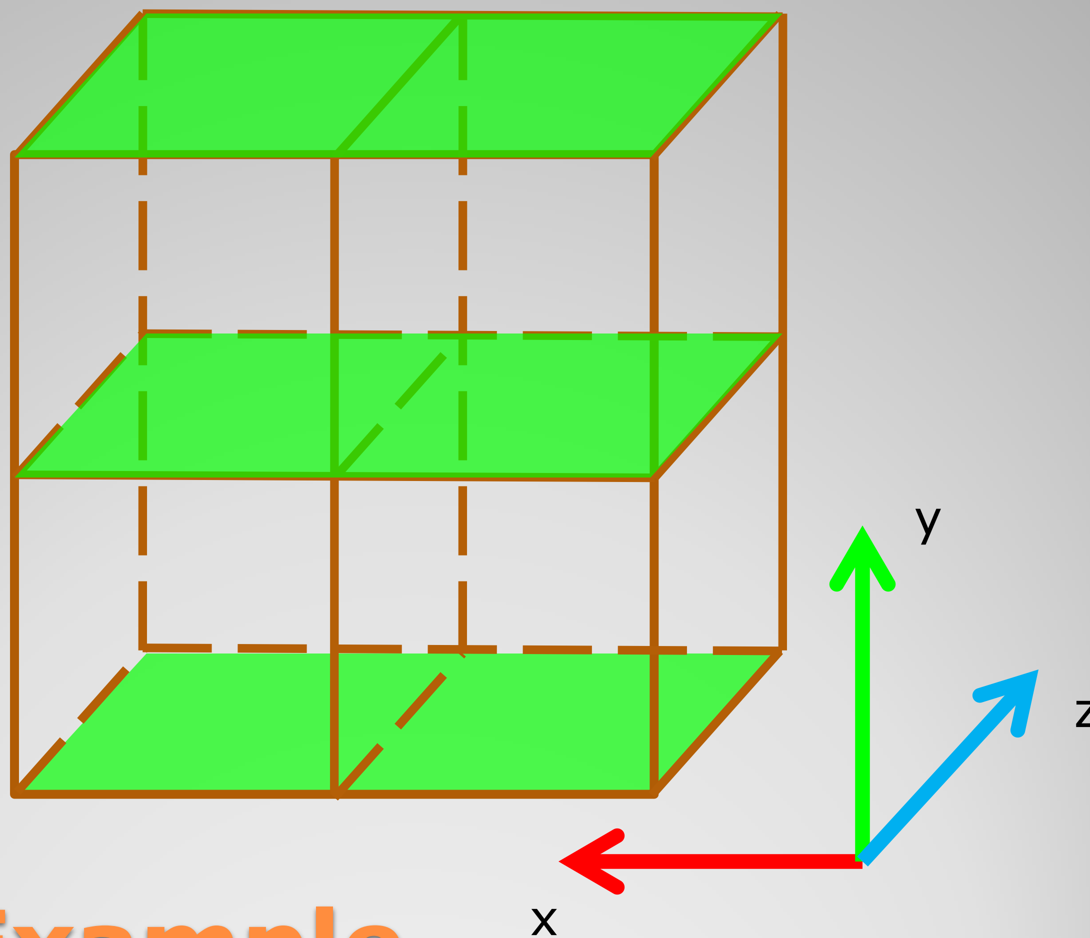
- A good start…



2x2x1 Example

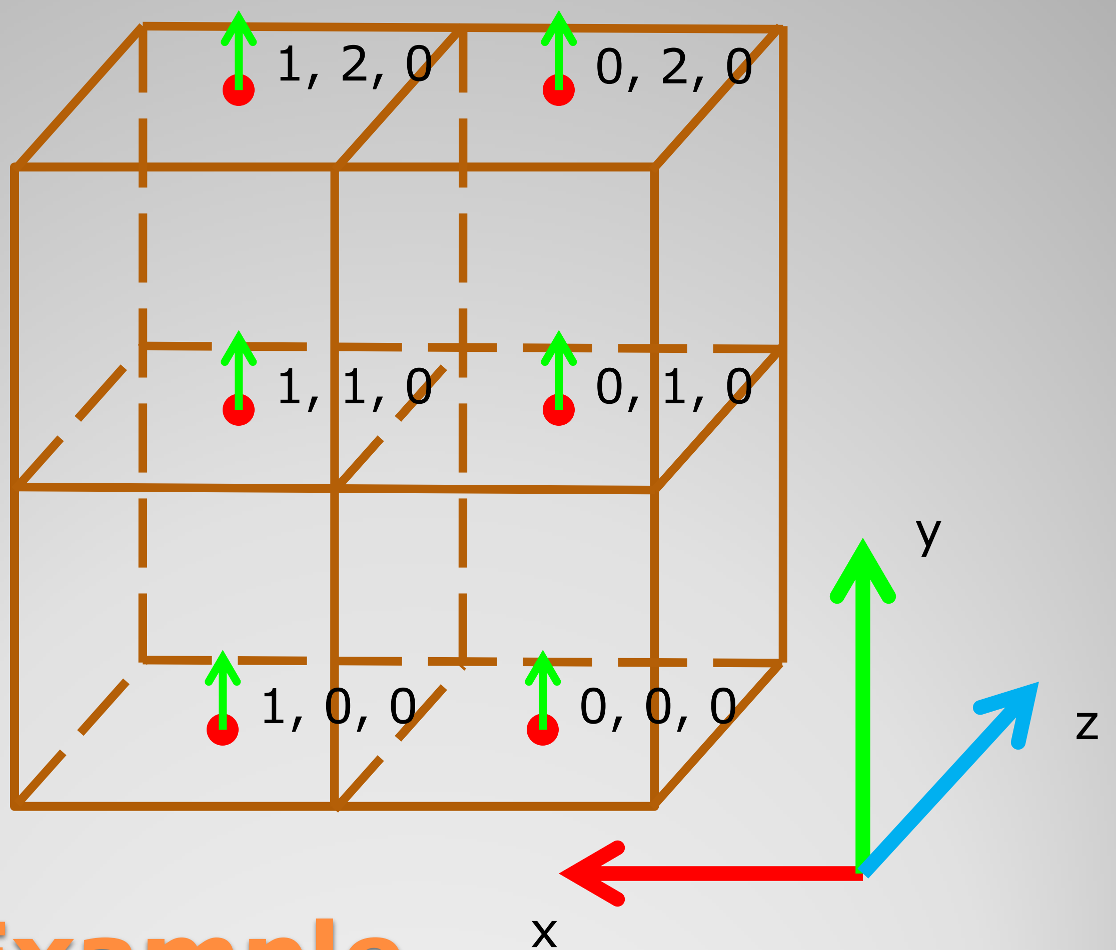
2x2x1 Example

- How many y-velocities (v)?



2x2x1 Example

- What are the indices?



2x2x1 Example

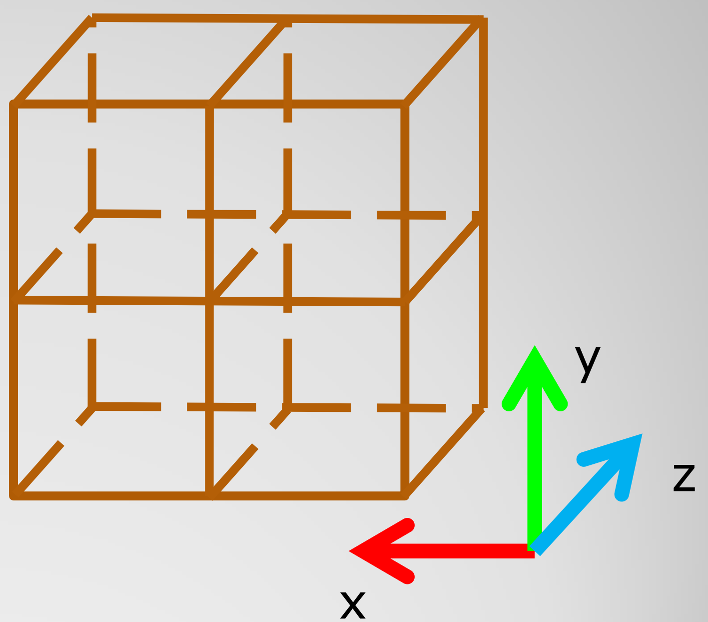
2x2x1 Example

- Ditto for x and z
  - x velocities: u

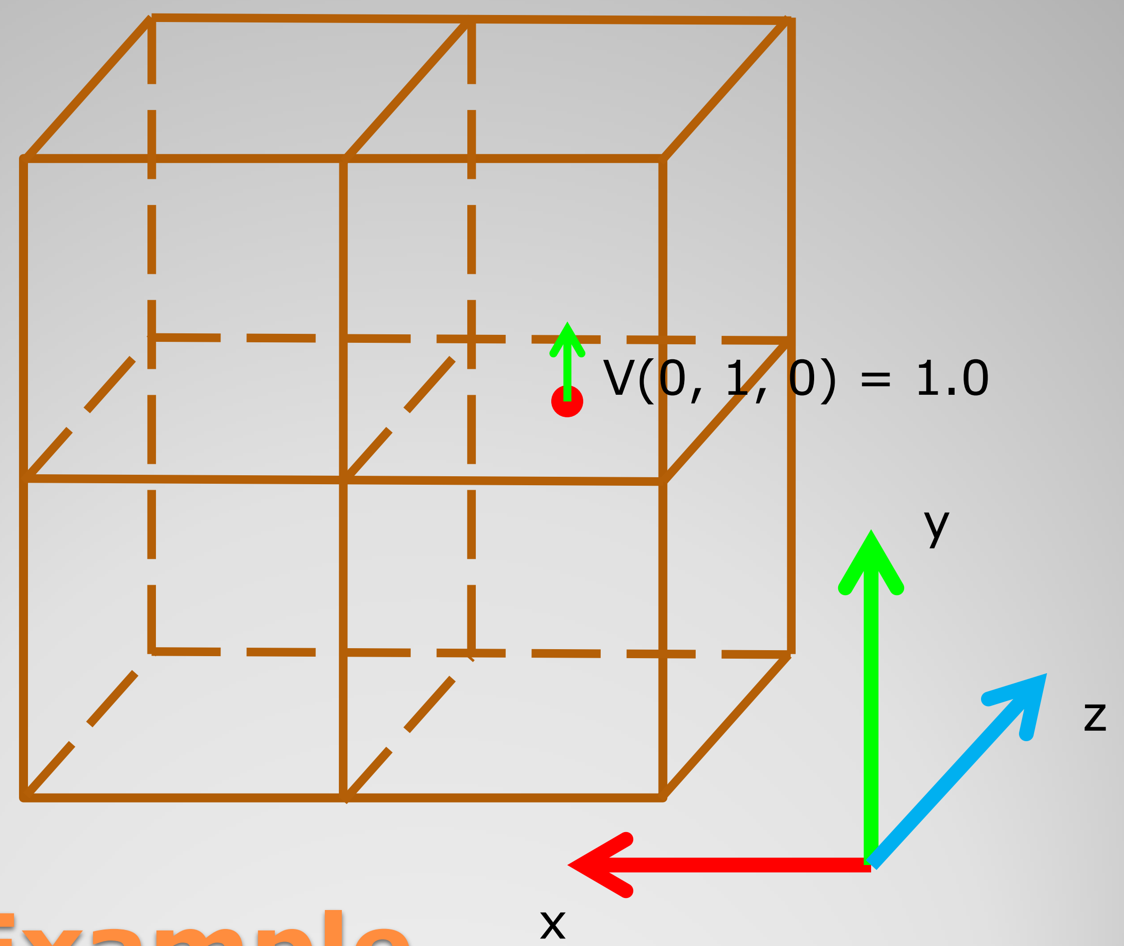
    (2, 1, 0) (1, 1, 0) (0, 1, 0)
    (1, 0, 0) (1, 0, 0) (0, 0, 0)
  - z velocities: w
    - front

      (1, 1, 0) (0, 1, 0)
      (0, 1, 0) (0, 0, 0)
    - back

      (1, 1, 1) (0, 1, 1)
      (0, 1, 1) (0, 0, 1)



- Source



2x2x1 Example

- After setting up the source
  - What is the current u?
  - All zero
  - What is the current v?
  - 1 at v(0, 1, 0), 0 at the rest of them
  - What is the current w?
  - All zero

2x2x1 Example

- N-S Equation

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = 0$$

$$\nabla \cdot \vec{u} = 0$$

2x2x1 Example

- Advection Part Split from N-S Equation

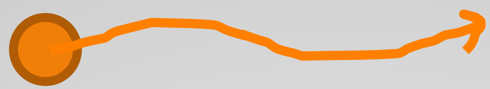
$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = 0$$

2x2x1 Example

2x2x1 Example

- Change a view (Page 5)
  - Lagrangian: treat fluid as particles, q can be any quantity carried by one particle (temperature, density, or velocity).

$$q(t,\vec{x})$$



  - In the advection step, we assume that there's no outside force affecting the particle. (Traveling with the velocity field)

$$\frac{\partial}{\partial t}q(t,\vec{x})=0$$

2x2x1 Example

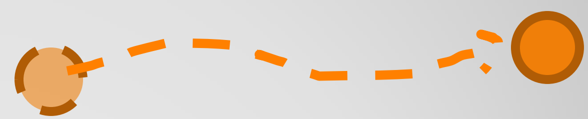
- Change a view (Cont'd)

$$\frac{\partial}{\partial t} q(t, \vec{x})$$

$$= \frac{\partial q}{\partial t} + \nabla q \cdot \frac{d\vec{x}}{dt}$$

$$\boxed{= \frac{\partial q}{\partial t} + \nabla q \cdot \vec{u}}$$

$$\frac{\partial}{\partial t} q(t, \vec{x}) = 0$$



2x2x1 Example

- Advect Velocity

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = 0$$

$$\vec{u} = (u, v, w)$$

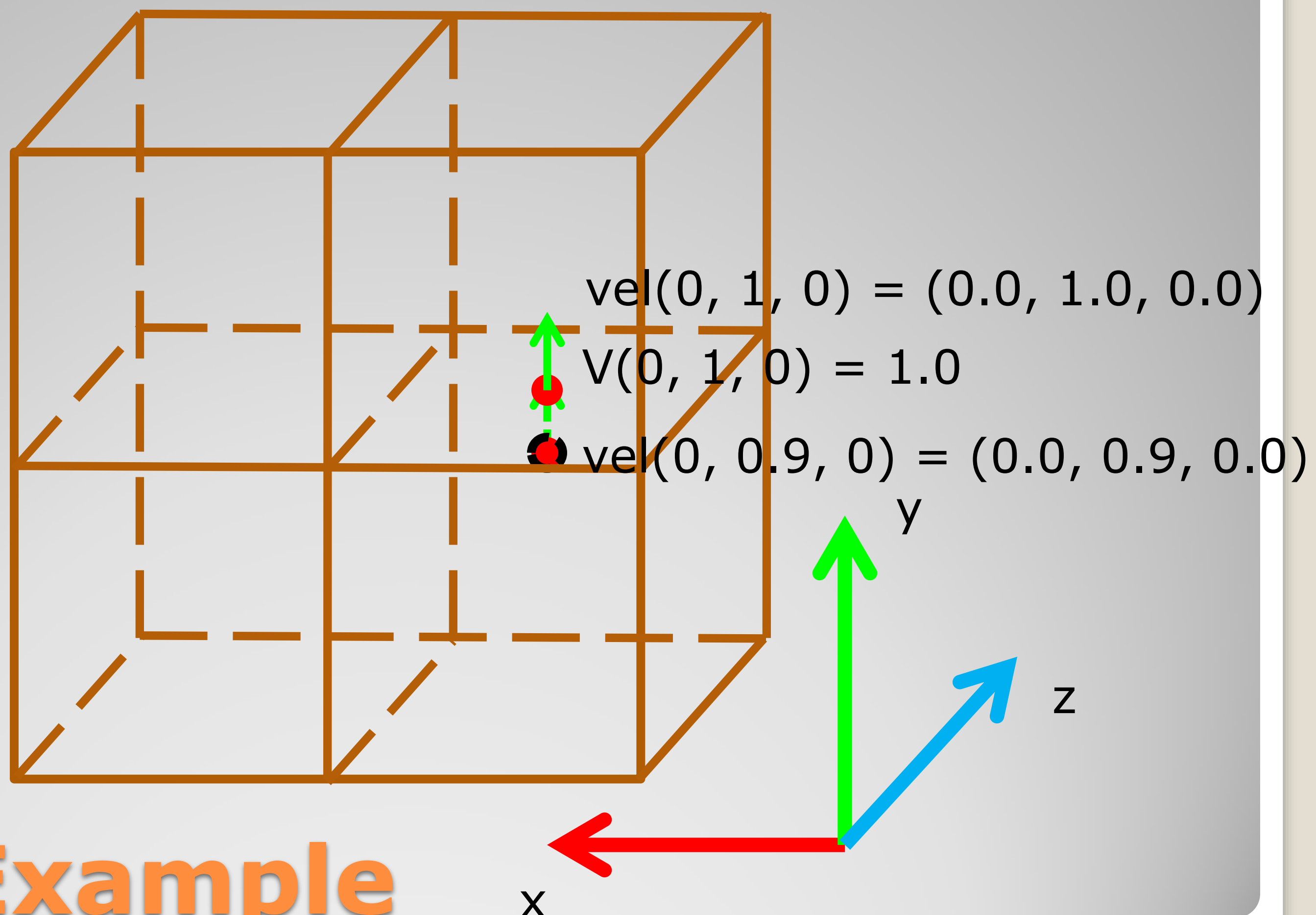
$$\frac{\partial u}{\partial t} + \vec{u} \cdot \nabla u = 0$$

$$\boxed{\frac{\partial v}{\partial t} + \vec{u} \cdot \nabla v = 0}$$

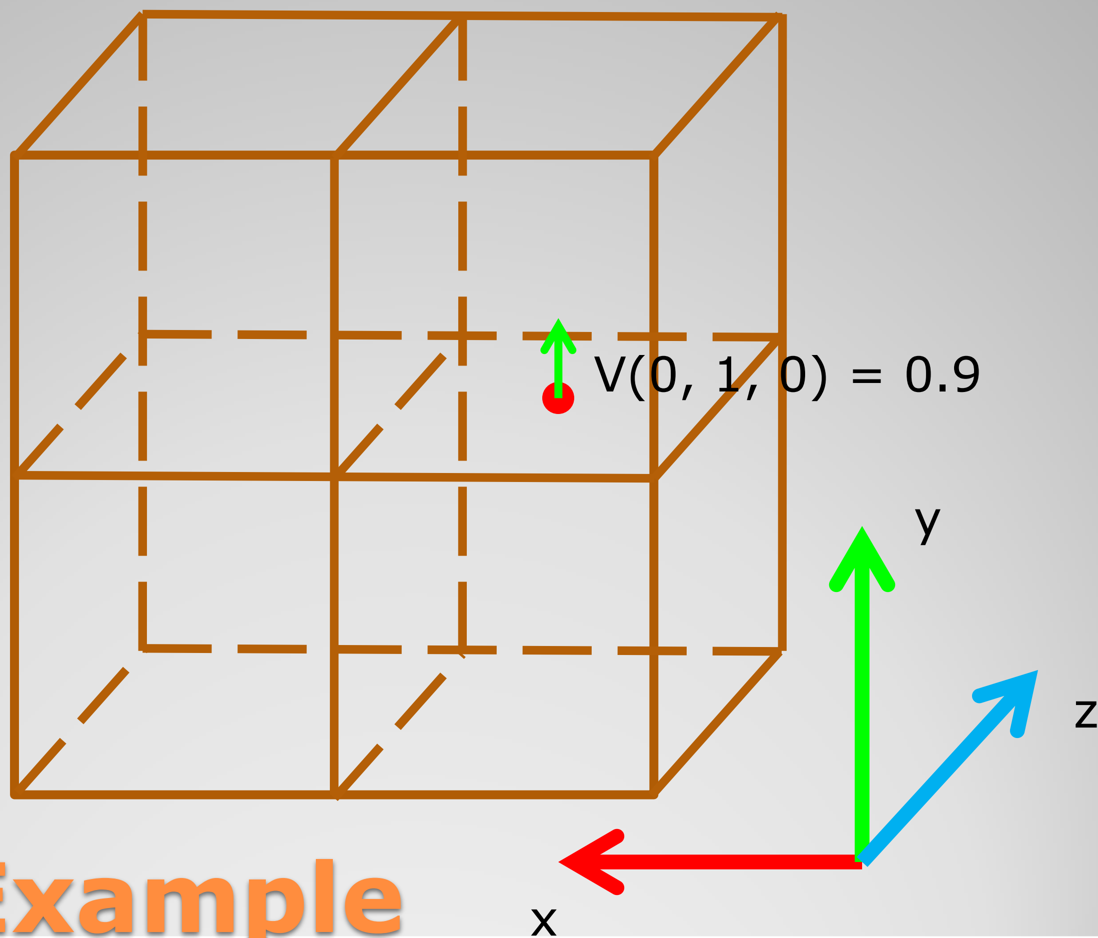
$$\frac{\partial w}{\partial t} + \vec{u} \cdot \nabla w = 0$$

2x2x1 Example

- Advect Velocity (Trace back)
  - Assume dt = 0.1, dx = dy = dz = 1.0



- Advect Velocity (Update)
  - Assume dt = 0.1, dx = dy = dz = 1.0



2x2x1 Example

- N-S Equation

$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} + \frac{1}{\rho} \nabla p = 0$$

$$\nabla \cdot \vec{u} = 0$$

2x2x1 Example

- Projection Part Split from N-S Equation

$$\frac{\partial \vec{u}}{\partial t} + \frac{1}{\rho}\nabla p = 0$$

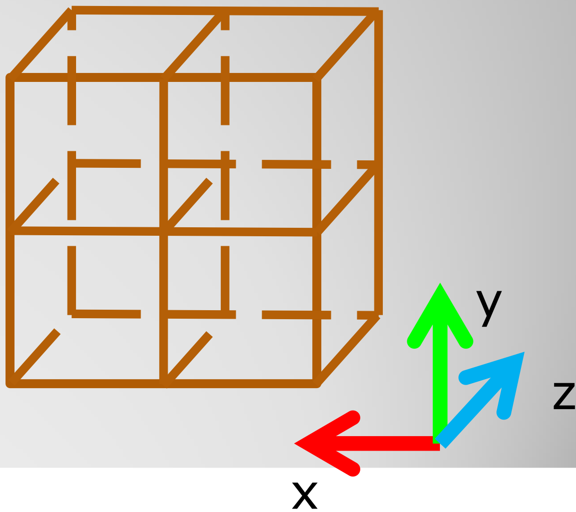
$$\nabla \cdot \vec{u} = 0$$

2x2x1 Example

- Projection

$$\vec{u}^{\,n+1} = \vec{u}^{\,n} - \Delta t \frac{1}{\rho} \nabla p$$

$$\nabla \cdot \vec{u}^{\,n+1} = 0$$



2x2x1 Example

- Projection

$$\nabla \cdot \vec{u}^{\,n+1} = \frac{u^{n+1}_{i+1/2,j,k} - u^{n+1}_{i-1/2,j,k}}{\Delta x} + \frac{v^{n+1}_{i,j+1/2,k} - v^{n+1}_{i,j-1/2,k}}{\Delta x} + \frac{w^{n+1}_{i,j,k+1/2} - w^{n+1}_{i,j,k-1/2}}{\Delta x} = 0$$

$$\frac{1}{\Delta x}\left[\left(u_{i+1/2,j,k} - \frac{\Delta t}{\rho}\frac{p_{i+1,j,k} - p_{i,j,k}}{\Delta x}\right) - \left(u_{i-1/2,j,k} - \frac{\Delta t}{\rho}\frac{p_{i,j,k} - p_{i-1,j,k}}{\Delta x}\right)\right.$$
$$+\left(v_{i,j+1/2,k} - \frac{\Delta t}{\rho}\frac{p_{i,j+1,k} - p_{i,j,k}}{\Delta x}\right) - \left(v_{i,j-1/2,k} - \frac{\Delta t}{\rho}\frac{p_{i,j,k} - p_{i,j-1,k}}{\Delta x}\right)$$
$$\left.+\left(w_{i,j,k+1/2} - \frac{\Delta t}{\rho}\frac{p_{i,j,k+1} - p_{i,j,k}}{\Delta x}\right) - \left(w_{i,j,k-1/2} - \frac{\Delta t}{\rho}\frac{p_{i,j,k} - p_{i,j,k-1}}{\Delta x}\right)\right] = 0$$

2x2x1 Example

- Projection (Page 29-30, general case)

$$\frac{\Delta t}{\rho}\left(\frac{6p_{i,j,k}-p_{i+1,j,k}-p_{i,j+1,k}-p_{i,j,k+1}-p_{i-1,j,k}-p_{i,j-1,k}-p_{i,j,k-1}}{\Delta x^2}\right)$$

$$=-\left(\frac{u_{i+1/2,j,k}-u_{i-1/2,j,k}}{\Delta x}+\frac{v_{i,j+1/2,k}-v_{i,j-1/2,k}}{\Delta x}+\frac{w_{i,j,k+1/2}-w_{i,j,k-1/2}}{\Delta x}\right)$$

2x2x1 Example

- Projection (Solid Wall boundary on (i+1,j,k))

$$\frac{\Delta t}{\rho}\left(\frac{\overset{5}{\cancel{6}}p_{i,j,k}-\overset{0}{\boxed{\cancel{p_{i+1,j,k}}}}-p_{i,j+1,k}-p_{i,j,k+1}-p_{i-1,j,k}-p_{i,j-1,k}-p_{i,j,k-1}}{\Delta x^2}\right)$$

$$=-\left(\frac{\overset{u_{solid}}{\boxed{\cancel{u_{i+1/2,j,k}}}}-u_{i-1/2,j,k}}{\Delta x}+\frac{v_{i,j+1/2,k}-v_{i,j-1/2,k}}{\Delta x}+\frac{w_{i,j,k+1/2}-w_{i,j,k-1/2}}{\Delta x}\right)$$

2x2x1 Example

- Projection (Free Surface boundary on (i+1,j,k))

$$\frac{\Delta t}{\rho}\left(\frac{6p_{i,j,k}-\overset{0}{\boxed{\cancel{p_{i+1,j,k}}}}-p_{i,j+1,k}-p_{i,j,k+1}-p_{i-1,j,k}-p_{i,j-1,k}-p_{i,j,k-1}}{\Delta x^2}\right)$$

$$=-\left(\frac{u_{i+1/2,j,k}-u_{i-1/2,j,k}}{\Delta x}+\frac{v_{i,j+1/2,k}-v_{i,j-1/2,k}}{\Delta x}+\frac{w_{i,j,k+1/2}-w_{i,j,k-1/2}}{\Delta x}\right)$$

2x2x1 Example

- Projection (Matrix Equation)

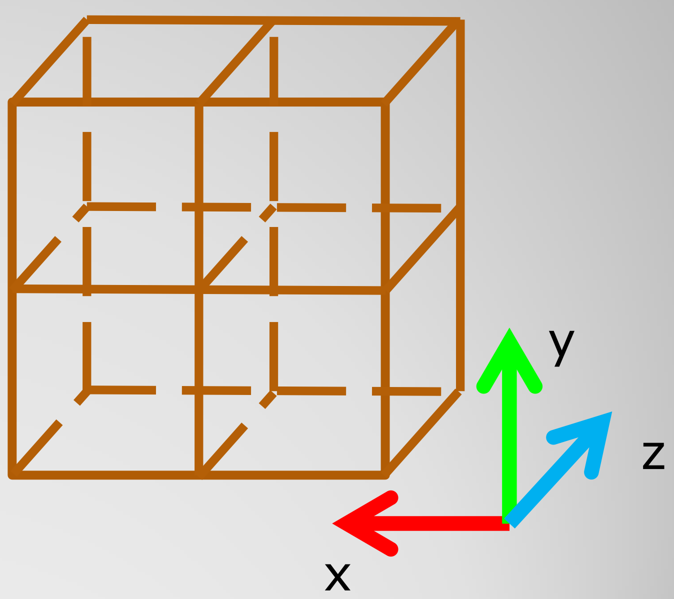
$$\begin{pmatrix} 6 & -1 & \dots & 0 \\ -1 & 5 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 6 \end{pmatrix} \begin{pmatrix} p_{0,0,0} \\ p_{1,0,0} \\ \vdots \\ p_{I,J,K} \end{pmatrix} = -\frac{(\Delta x)^2 * \rho}{dt} \begin{pmatrix} \nabla \cdot \vec{u}_{0,0,0} \\ \nabla \cdot \vec{u}_{1,0,0} \\ \vdots \\ \nabla \cdot \vec{u}_{I,J,K} \end{pmatrix}$$

$$Ap = d$$

2x2x1 Example

- Projection
  - What is dimension of the A matrix?
  - What does the A matrix look like?

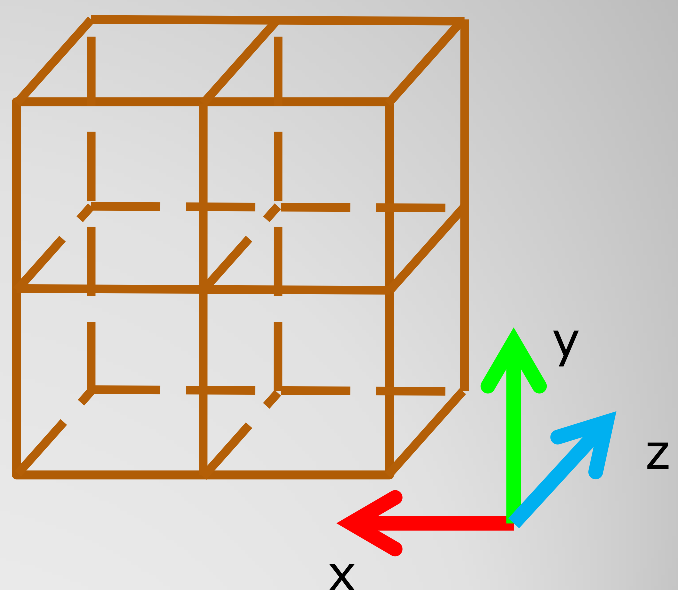
$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix}$$



2x2x1 Example

- Projection
  - What is the divergence column?

$$\begin{pmatrix} 0.9 \\ 0 \\ -0.9 \\ 0 \end{pmatrix}$$



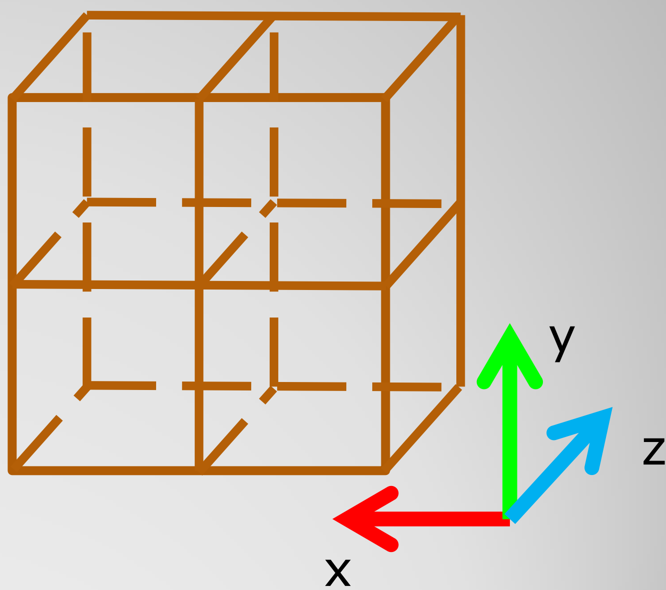
2x2x1 Example

- Projection
  - Multiply by the constant…

$$-\frac{(\Delta x)^2 * \rho}{dt} = -10$$

Here I use dx = 1 and dt = 0.1
But in your framework, they are different

$$\begin{pmatrix} -9 \\ 0 \\ 9 \\ 0 \end{pmatrix}$$

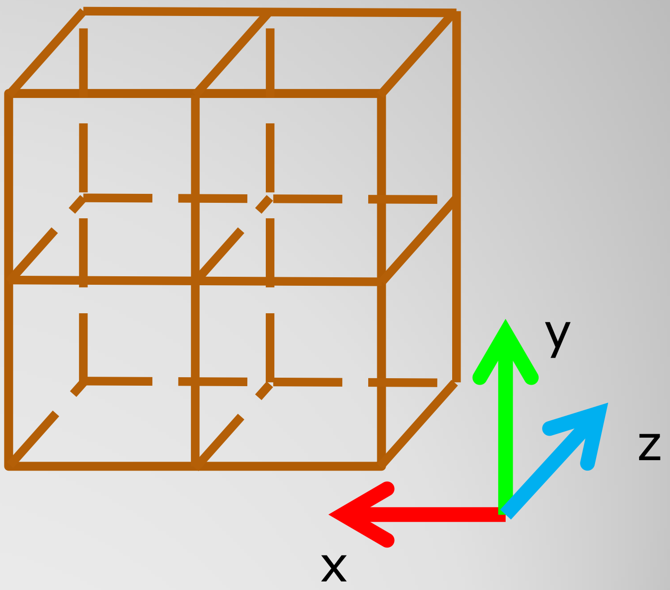


2x2x1 Example

- Projection
  - Solve for p

$$\begin{pmatrix} 2 & -1 & -1 & 0 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ 0 & -1 & -1 & 2 \end{pmatrix} \begin{pmatrix} p_{0,0,0} \\ p_{1,0,0} \\ p_{0,1,0} \\ p_{1,1,0} \end{pmatrix} = \begin{pmatrix} -9 \\ 0 \\ 9 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} p_{0,0,0} \\ p_{1,0,0} \\ p_{0,1,0} \\ p_{1,1,0} \end{pmatrix} = \begin{pmatrix} -3.375 \\ -1.125 \\ 3.375 \\ 1.125 \end{pmatrix}$$



2x2x1 Example

- Projection
  - Update velocity u^{n+1}: $\vec{u}^{n+1} = \vec{u}^{n} - \Delta t \frac{1}{\rho} \nabla p$

$$u_{1,0,0}{}^{n+1} = u_{1,0,0}{}^{n} - \Delta t \frac{1}{\rho} \frac{p_{1,0,0} - p_{0,0,0}}{\Delta x}$$

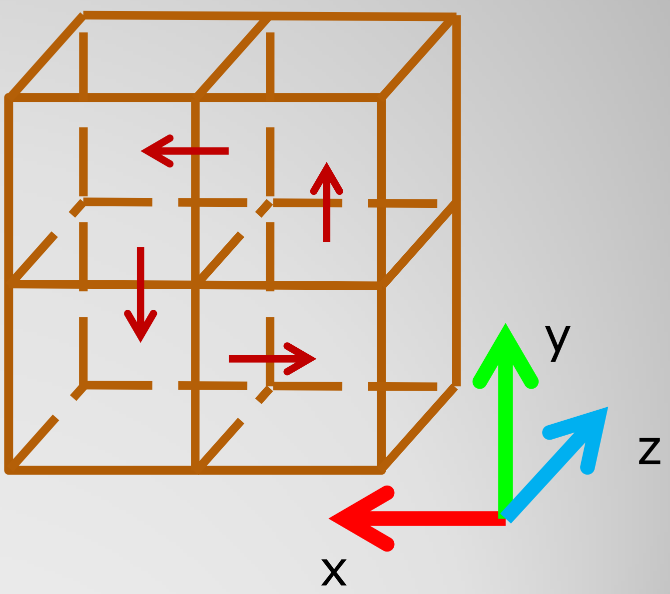
$$= 0 - 0.1 * (1/1) * (-1.125 - (-3.375))$$

$$= -0.225$$

$$u_{1,1,0}{}^{n+1} = 0.225$$

$$v_{0,1,0}{}^{n+1} = 0.225$$

$$v_{1,1,0}{}^{n+1} = -0.225$$



2x2x1 Example

- Math & Physics
- MAC Grid
- 2x2x1 Example
- Data Structure
- Starter Kit Workflow
- Extras
- Tips

Overview

Data Structure

- Classes
  - GridData
  - GridDataMatrix
  - MACGrid
- Useful Constants and Macros

Data Structure

- GridData
  - Key member:
    - std::vector<double> mData;
  - Key methods:
    - virtual double interpolate(const vec3& pt)
    - Indexing with "()" notation.
    - Arithmetic operations defined in MACGrid.h
  - Subclasses:
    - GridDataX
    - GridDataY
    - GridDataZ

Data Structure

- GridDataMatrix
  - Key members:
    - GridData diag;
    - GridData plusI;
    - GridData plusJ;
    - GridData plusK;
  - Key methods:
    - Arithmetic operations defined in MACGrid.h

Data Structure

- MACGrid
  - Key Members:
    - GridDataX mU;
    - GridDataY mV;
    - GridDataZ mW;
    - GridData mP
    - GridData mD;
    - GridData mT;
    - GridDataMatrix AMatrix;
  - Key Methods:
    - Main work flow functions.
    - Drawing functions.

Data Structure

- Useful Constants
  - Mostly defined in constants.cpp
    - const int theDim[3]
  - But dt defined in SmokeSim::step()
- Useful Macros
  - Mostly defined in mac_grid.cpp
  - FOR_EACH_CELL
  - FOR_EACH_FACE
  - Recommended to write your own:
    - FOR_EACH_FACE_X
    - FOR_EACH_FACE_Y … etc.

Overview

- Math & Physics
- MAC Grid
- 2x2x1 Example
- Data Structure
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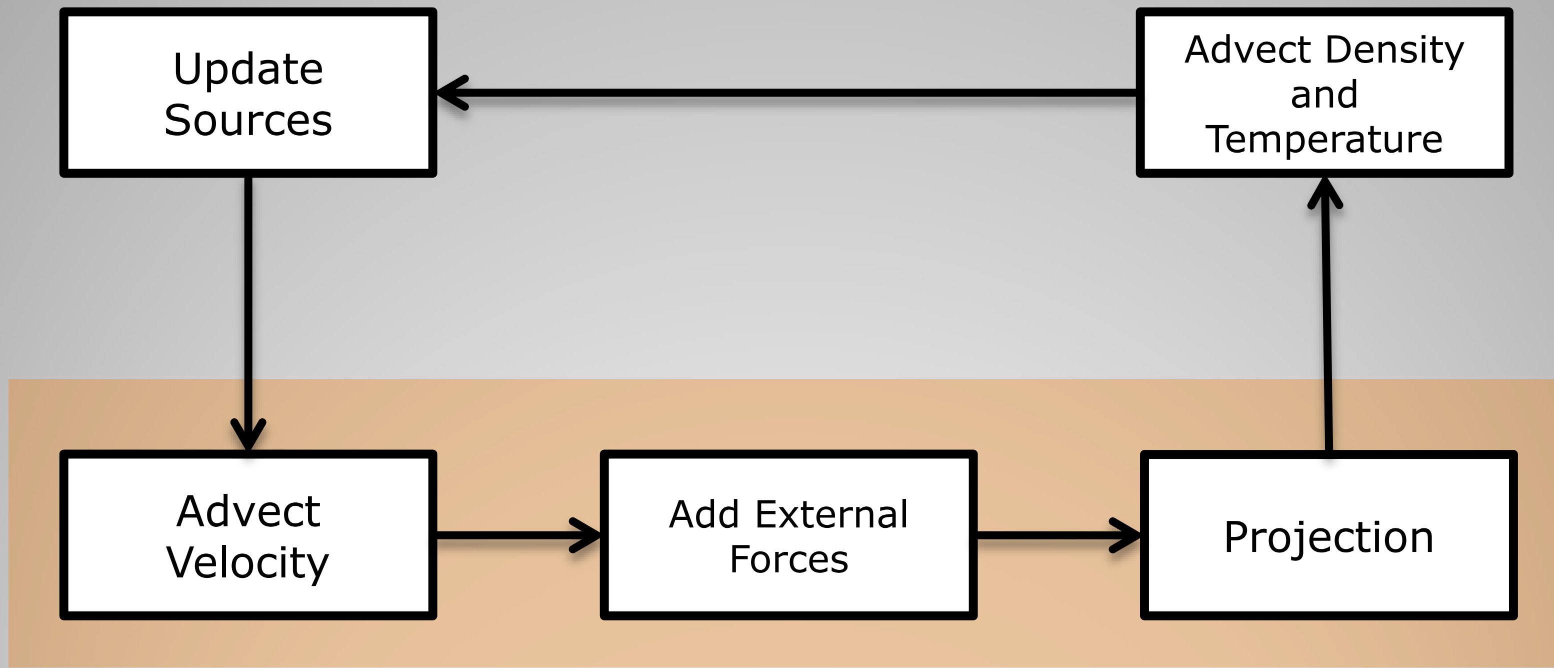
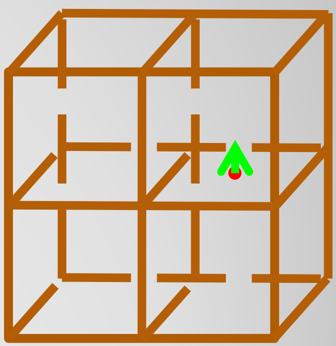
- Tips

- Main simulation loop



Workflow

- Update Sources
  - Set temperature/density: Think it as a filling some cubic spaces will smoke
  - Set velocity: Think it as a fan blowing at some faces.
  - Note: do not set invalid velocity on boundaries.



Workflow

- Advect Velocity
  - Save the current velocity as a copy. (target)
  - Trace back and update the target velocity.
  - Copy it back to the current velocity.

  - Quiz:
    - Why are we using a copy?
      - We are using current velocity field to change velocities
    - How many velocities do I need to advect in the 2x2x1 case?
      - 6+6+8=20

Workflow

- External Forces
  - Get a copy (target)
  - Calculate forces and update the copy (Explicit Euler integration)
  - Copy it back

Workflow

- External Forces
  - Kind of forces:
    - Buoyancy (Typo in our code…)
    - Vorticity Confinement Forces
    - Gravity
    - Viscosity
    - Surface Tension

Smoke Sim

Water Sim

Workflow

- External Forces
  - Buoyancy+Gravity: Page 45
    - Only acts on vertical velocities.

$$f_{\text{buoy}} = (0, -\alpha s + \beta(T - T_{\text{amb}}), 0)$$

Workflow

- External Forces
  - Vorticity Confinement Force: Page 46,47
    - Goal: enhance the vortices
    - vorticity measurement: curl

$$\vec{\omega} = \nabla \times \vec{u}$$

    - Find the local maximum (vortex center):

$$\vec{N} = \frac{\nabla|\vec{\omega}|}{||\nabla|\vec{\omega}|||}$$

    - Add force:

$$f_{\text{conf}} = \epsilon \Delta x (\vec{N} \times \vec{\omega})$$

Workflow

Workflow

- Projection
  - Build A Matrix.
    - setUpAMatrix() defined in MACGrid.
    - Need to modify setUpAMatrix() if you are putting other solids into the scene.
  - Build d vector.
  - Solve for p from Ap = d. (call conjugateGradient ())
  - Update velocity.
  - Sanity check. (Divergence Free).

$$\begin{pmatrix} 6 & -1 & \dots & 0 \\ -1 & 5 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 6 \end{pmatrix} \begin{pmatrix} p_{0,0,0} \\ p_{1,0,0} \\ \vdots \\ p_{I,J,K} \end{pmatrix} = -\frac{(dx)^2}{dt * \rho} \begin{pmatrix} \nabla \cdot \vec{u}_{0,0,0} \\ \nabla \cdot \vec{u}_{1,0,0} \\ \vdots \\ \nabla \cdot \vec{u}_{I,J,K} \end{pmatrix}$$

- Advect Other Quantities:
  - Temperature
    - For buoyancy
  - Density
    - For gravity / For display

Workflow

Overview

- Math & Physics
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- Tips

- Sharper interpolation
  - Hermite Cubic Interpolation: Described in page 47.

Extras (make it shaper)

- Add Objects:
  - Interactive objects. (Cell Sized, like Tetris)
  - Arbitrary objects. (Deal with half fill cases, curved boundaries. (Page 38))

Extras (make it interesting)

- Fluid Viscosity:
  - Adding viscosity to external force.
  - Document it and compare the difference.

Extras (make it different)

Extras (make it faster)

- Preconditioner for faster conjugate gradient solver.
  - Described in page 32 with conjugate gradient solver.
  - Document which preconditioner you use.
  - Compare the speed before/after you apply your preconditioner.

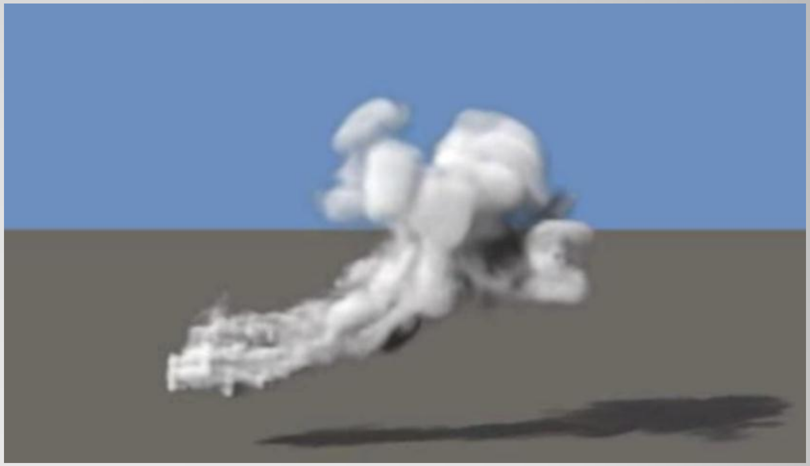
- Port parts of your solve to the GPU using CUDA. (cg_solver is probably the best part to accelerate!)

  OR

- Make a totally different solver.
  - Sparse Matrix Solvers on the GPU: Conjugate Gradients and Multigrid
  - A parallel multigrid Poisson solver for fluids simulation on large grids

Extras (make it faster)

- Port the data out.
  - Inside SmokeSim::grabScreen() Function, you can uncomment lines of code to port your data out.
- Use volumetric renderer you made in 560.
- Use Maya.
- Etc.



[Peter Kutz, 2011]

Extras (render it out)

- Math & Physics
- MAC Grid
- 2x2x1 Example
- Data Structure
- Starter Kit Workflow
- Extras
- Tips

Overview

- How to start this assignment
  - Find the place that you need to fill by searching “TODO”
  - Start with a 2x2x1 sample, because you can verify everything by hand!
  - Start with
    - void MACGrid::updateSources()
    - void MACGrid::advectVelocity(double dt)
    - void MACGrid::project(double dt)

Tips for you

- Save some time for simulation and rendering
  - For instance, under a 50x50x20 setting, simulating one frame may cost you 2 minutes, rendering it using naive volumetric renderer may cost you 10 minutes…
- Typical Length: less than 600 frames is fine.
  - If your smoke fills the entire space, you can hardly see anything, even if it is flowing.
  - Rendering costs more time than simulation.

Tips for you

Tips for you

- If you still have late days, use it…
  - Late days can not be used on your final project
- Correction of due date:
  - 03/11/2013 11:59 PM

Acknowledgement

- Thanks for Aline Normoyle and Peter Kutz setting up the starter code.
- The 2x2x1 example is taken from Aline's recitation in 2011.

Good Luck ☺