

# Central limit theorem and Convergence

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*July 2, 2014*

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# 1 Exponential Distribution

Simulating a exponential variable with lambda 0.2 and k = 10000

```
n= 40
lambda = 0.2
k=10000
list_of_exponential = array(1:k)

for(i in 1:k){
  list_of_exponential[i] = mean(rexp(n, lambda))
}
```

Let's calculate where the distribution of the mean when it's centered and compare to the theoretical center

```
mean_of_average = mean(list_of_exponential)
mean_of_average
```

```
## [1] 4.994667
```

Meanwhile, the expected value for mean is  $1/\lambda = 5$ .

```
sd_mean = sd(list_of_exponential)
sd_mean
```

```
## [1] 0.7868082
```

The expected Standard Deviation is  $\{[1/(\lambda)^2]/n\}^{(0.5)}$

```
std<-((1/lambda)/sqrt(40))
std
```

```
## [1] 0.7905694
```

1- When we compare the theoretical center we were expecting the average to be 5. it's 5.07.

2- And we were expecting the standard deviation to 0.79 and it's 0.784.

3-For the approximation to a normal, we can check the figure 1 in the appendix. the curve is a normal(5,0.7906) and the histogram is referent to the the mean distribution. We can see that it clearly looks like a normal. To understand how well fitted is the model I have generate 1000 normal variables (Figure 2) as if I wanted to check how they look like, and they are as good as the exponential distribution (even better in my opinion). So we can conclude that when n increases, the exponential converges to a normal.

4- As  $1/\lambda$  is the mean for this distribution we have that at 95% of confidence the mean is in this interval:

```
mean(list_of_exponential) + c(-1,1)*1.96*sd(list_of_exponential)/sqrt(nrow(list_of_exponential))
```

```
## [1] 4.979246 5.010089
```

## 2 Conclusion

From the points above we can conclude that the hypothesis of convergence is consistent for exponential distribution and the mean of n exponential converges to the distribution mean.

### 3 Appendix

```
hist(list_of_exponential, freq= FALSE, col = "blue", main = "Histogram of simulated mean", xlab = "mean of exponential",  
     curve(dnorm(x, mean=5, sd=std),  
           col="red", lwd=2, add=TRUE)
```

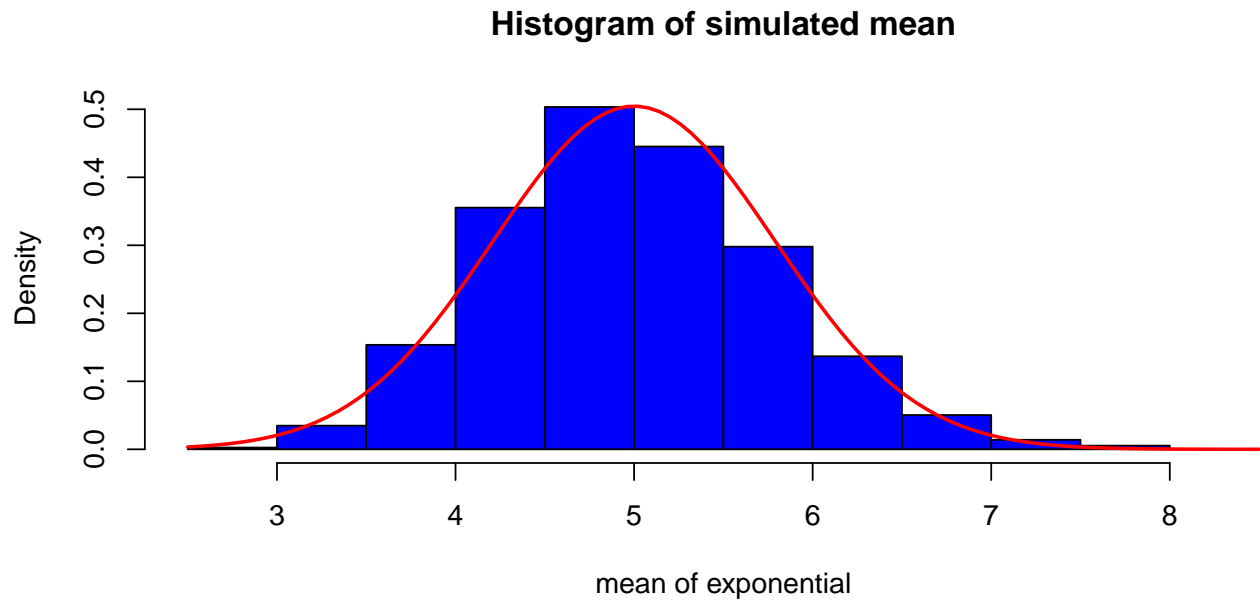


Figure 1: Histogram of Simulated Mean

```
x<-rnorm(1000,5, std)  
hist(x, freq= FALSE, col = "green",main = "Histogram of normal with mean(5,0.7906)")  
curve(dnorm(x, mean=5, sd=std),  
      col="red", lwd=2, add=TRUE)
```

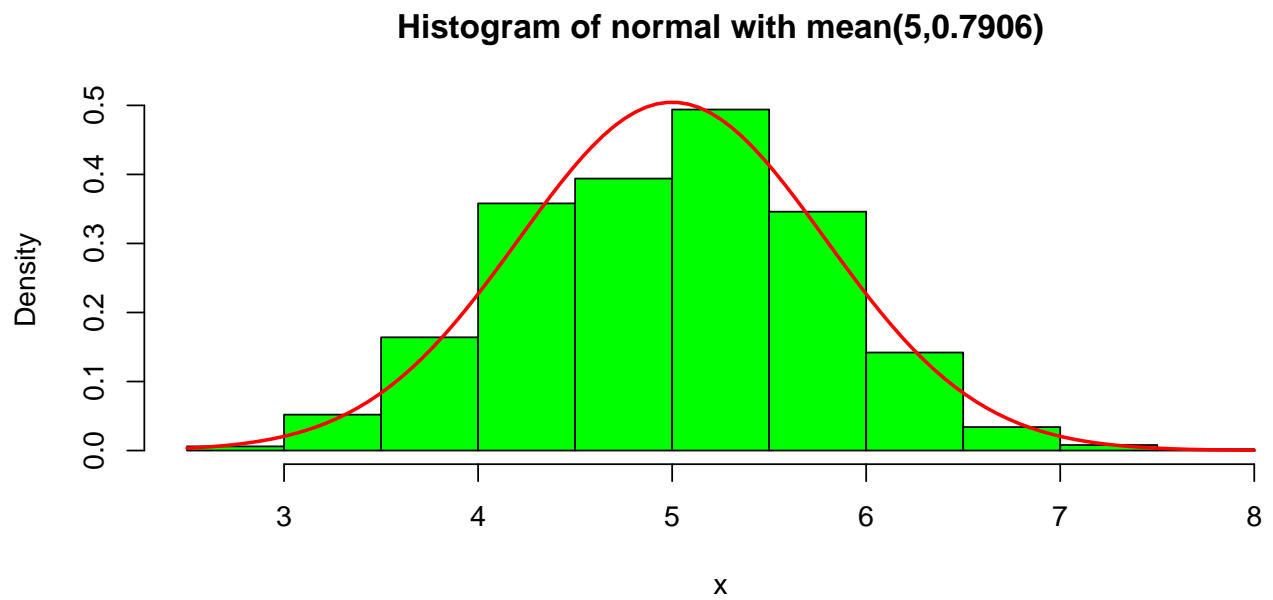


Figure 2: Histogram of a Normal distribution.