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**Research Project – Topic Brief**

## Quantum Optimization for the Vehicle Routing Problem (VRP)

**INTRODUCTION**

The Vehicle Routing Problem (VRP) is a fundamental combinatorial optimization challenge in logistics, aiming to determine the most efficient routes for a fleet of vehicles to service a set of customers. Over the years, VRP and its different versions have become a popular topic in research. However, since the problem can be defined in many versions (many ways with many different assumptions), researchers need to be aware that VRP itself has a broad range of variants [1]. Given its NP-hard nature, classical algorithms often struggle with large-scale instances due to computational limitations.

Quantum computing offers a paradigm for approaching and solving such problems. Algorithms like the Quantum Approximate Optimization Algorithm (QAOA) [2] and Quantum Walk-based Optimization [3] have shown promise in tackling NP-hard tasks. This research proposes to explore these quantum techniques to develop efficient approximation methods for the VRP.

**SCOPE AND BOUNDARIES**

*Vehicle Routing Problem*

VRP has numerous real-world applications, including logistics, supply chain management, and last-mile delivery services. Various VRP variants exist, such as the Capacitated VRP (CVRP), Time-Window VRP (VRPTW), and Stochastic VRP (SVRP), each introducing unique constraints that impact solution complexity. Classical methods struggle to find optimal solutions as problem size increases due to exponential computational growth.

This diagram is one method of finding solution to VRP [4]:
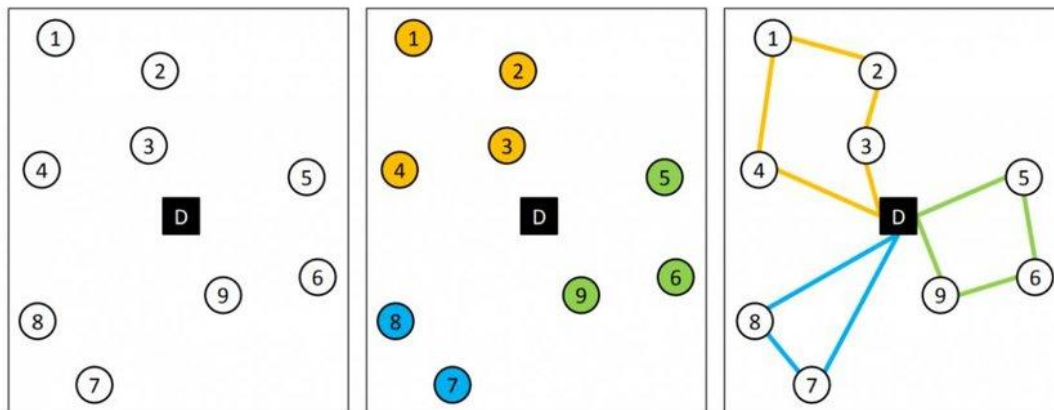


Unassigned Customers (Left Panel)

- Customers (represented by numbered circles) are scattered around a central depot (black square labeled "D").
- At this stage, no routes have been assigned.

Customer Clustering (Middle Panel)

- Customers are grouped into different sets based on their locations and other factors such as demand or distance (this is where heuristic comes into play).

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- Each color represents a different vehicle that will serve a specific group (with certain heuristics) of customers.

Optimized Routes (Right Panel)

- Vehicles are assigned optimized routes that start and end at the depot (D).
- Each route ensures that all customers are served efficiently while minimizing travel distance and cost.

***Quantum Optimization***

Quantum computing offers a novel approach to solving VRP by leveraging quantum superposition and entanglement to explore multiple solutions simultaneously. Quantum algorithms like QAOA and Quantum Walk-based methods have shown potential in improving combinatorial optimization efficiency. However, implementing these algorithms for practical VRP scenarios requires addressing efficiency, scalability, and quantum hardware limitations.


**BACKGROUND AND CHALLENGES**

***Classical Approaches to VRP***

Traditional methods for solving the VRP include:

- Exact Algorithms: Techniques such as Branch-and-Bound for CVRP [5]
- Metaheuristic Approaches: Algorithms like Genetic Algorithms (GA) and Ant Colony Optimization (ACO) offer approximate solutions more feasibly but may not guarantee optimality [6]

***Quantum Programming Approaches to VRP***

Quantum Optimization Techniques offer a better solution than a classical one:

- Quantum Approximate Optimization Algorithm (QAOA): QAOA is designed to find approximate solutions to combinatorial problems by leveraging quantum superposition and entanglement [7].
- Quantum Walk-based Optimization: Quantum walks, the quantum analog of classical random walks, have been applied to search and optimization problems, offering potential speedups over classical methods [8].
- Variational Quantum Eigensolver (VQE): VQE is another quantum algorithm that has been studied for solving VRP. Currently, Noisy-Intermediate Scale Quantum (NISQ) devices can only handle small VRP instances, but as quantum hardware improves, both VQE and QAOA are expected to perform better and potentially surpass classical methods [9].
- QUBO-Based Formulation for VRP: A novel Quadratic Unconstrained Binary Optimization (QUBO) formulation has been proposed for the Capacitated Vehicle Routing Problem (CVRP), incorporating a time-table representation to model the time evolution of each vehicle. This formulation successfully integrates time-based and capacity constraints, allowing for dynamic updates as vehicles visit different cities [10].

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**PROPOSED RESEARCH OBJECTIVES**

1. Develop quantum optimization approaches for VRP using QAOA and Quantum Walk-based methods.
2. Compare quantum and classical algorithms by analyzing solution quality, computational efficiency, and scalability.
3. Investigate the effectiveness of quantum approximations in solving VRP and their ability to find near-optimal solutions.
4. Implement a quantum-based VRP solution with the potential to outperform classical methods in specific scenarios.

**RESEARCH QUESTIONS**

All of the discussion leads to research that needs to be done which includes around these four questions:

RQ1. How can spatial datasets be transformed into a VRP formulation suitable for quantum optimization?

RQ2. How can quantum optimization techniques such as QAOA, Quantum Walk, VQE, and/or Quantum Annealing provide a more effective solution for VRP compared to classical methods?

RQ3. In terms of efficiency and scalability, how do quantum algorithms compare to state-of-the-art classical VRP solvers?

RQ4. What are the key limitations of using quantum algorithms for VRP, and how can these challenges be addressed?

**PROPOSED METHODOLOGY**

Quantum Computing Approaches

- Formulation: Represent the VRP in a quantum framework suitable for QAOA, Quantum Walk algorithms, and VQE.
- Theoretical Calculations: Analyze mathematical models and complexity to understand how quantum algorithms can outperform classical methods in VRP domain. This includes identifying cases where quantum optimization provides faster solutions or better approximations.
- Implementation: Develop and simulate quantum circuits using platforms like PennyLane or Qiskit (or other compatible library)

***Benchmarking Strategy***

- Classical Comparison: Utilize established solution using classical methods to obtain baseline solutions.
- Performance Metrics: Compare quantum and classical approaches based on solution quality, computational time, and scalability.

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*Datasets*

- Data Acquisition: Employ real-world datasets from sources like Victoria's open data portal [11].
- Data Processing: Transform raw data into graph representations that will be compatible with quantum algorithms.

**EXPECTED OUTCOMES, CONTRIBUTIONS, AND CHALLENGES**

*Expected Outcomes and Contributions*

- Development of quantum-based approximation methods: Providing new strategies for solving VRP using quantum computing.
- Comparative Analysis: Offering insights into the practical advantages and limitations of quantum versus classical approaches for VRP.

*Challenges and Limitations*

Quantum Hardware Constraints: Real quantum cloud is expensive. Use a simpler graph to simulate in real quantum computer and use a simulator to simulate bigger graph.

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