CÁLCULO DIFERENCIAL

Fórmulas básicas de derivación.

$$\frac{d}{dx}(c)=0$$

$$\frac{d}{dx}(cx)=c$$

$$\frac{d}{dx}\left(cx^{n}\right)=ncx^{n-1}$$

$$\frac{d}{dx}\left(u\pm v\pm w\pm\Lambda\right)=\frac{du}{dx}\pm\frac{dv}{dx}\pm\frac{dw}{dx}\Lambda$$

$$\frac{d}{dx}(cu)=c\frac{du}{dx}$$

$$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$$

$$\frac{d}{dx}(uvw)=uv\frac{dw}{dx}+uw\frac{dv}{dx}+vw\frac{du}{dx}$$

$$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\left(du/dx\right)-u\left(dv/dx\right)}{v^{2}}$$

$$\frac{d}{dx}\left(u^{n}\right)=nu^{n-1}\frac{du}{dx}$$

$$\frac{dF}{dx}=\frac{dF}{du}\frac{du}{dx}$$ (Regla de la cadena)

$$\frac{du}{dx}=\frac{1}{dx/du}$$

$$\frac{dF}{dx}=\frac{dF/du}{dx/du}$$

Derivadas de las Funciones Exponenciales y Logarítmicas

$$\frac{d}{dx}\log_{a}u=\frac{\log_{a}e}{u}\frac{du}{dx}\qquad a>0,\quad a\neq 1$$

$$\frac{d}{dx}\ln u=\frac{d}{dx}\log_{e}u=\frac{1}{u}\frac{du}{dx}$$

$$\frac{d}{dx}a^{u}=a^{u}\ln a\frac{du}{dx}$$

$$\frac{d}{dx}e^{u}=e^{u}\frac{du}{dx}$$

$$\frac{d}{dx}u^{v}=\frac{d}{dx}e^{v\ln u}=e^{v\ln u}\frac{d}{dx}\left[v\ln u\right]=vu^{v-1}\frac{du}{dx}+u^{v}\ln u\frac{dv}{dx}$$

Derivadas de las Funciones Trigonométricas y de las Trigonométricas Inversas

$$\frac{d}{dx}\operatorname{sen}u=\cos u\frac{du}{dx}$$

$$\frac{d}{dx}\cos u=-\operatorname{sen}u\frac{du}{dx}$$

$$\frac{d}{dx}\cot u=-\csc^{2}u\frac{du}{dx}$$

$$\frac{d}{dx}\sec u=\sec u\tan u\frac{du}{dx}$$

$$\frac{d}{dx}\tan u = \sec^2 u \frac{du}{dx} \qquad \frac{d}{dx}\csc u = -\csc u \cot u \frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{sen}^{-1} u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \left[-\tfrac{\pi}{2} < \operatorname{sen}^{-1} u < \tfrac{\pi}{2}\right]$$

$$\frac{d}{dx}\cos^{-1} u = \frac{-1}{\sqrt{1-u^2}}\frac{du}{dx} \qquad \left[0 < \cos^{-1} u < \pi\right]$$

$$\frac{d}{dx}\tan^{-1} u = \frac{1}{1+u^2}\frac{du}{dx} \qquad \left[-\tfrac{\pi}{2} < \tan^{-1} u < \tfrac{\pi}{2}\right]$$

$$\frac{d}{dx}\cot^{-1} u = \frac{-1}{1+u^2}\frac{du}{dx} \qquad \left[0 < \cot^{-1} u < \pi\right]$$

$$\frac{d}{dx}\sec^{-1} u = \frac{1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} +si & 0 < \sec^{-1} u < \frac{\pi}{2} \\ -si & \frac{\pi}{2} < \sec^{-1} u < \pi \end{bmatrix}$$

$$\frac{d}{dx}\csc^{-1} u = \frac{-1}{|u|\sqrt{u^2-1}}\frac{du}{dx} = \frac{\mu 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} -si & 0 < \csc^{-1} u < \frac{\pi}{2} \\ +si & -\frac{\pi}{2} < \csc^{-1} u < 0 \end{bmatrix}$$

Derivadas de las Funciones Hiperbólicas y de las Hiperbólicas Recíprocas

$$\frac{d}{dx}\operatorname{senh} u = \cosh u \frac{du}{dx} \qquad \frac{d}{dx}\coth u = -\operatorname{csch}^2 u \frac{du}{dx}$$

$$\frac{d}{dx}\cosh u = \operatorname{senh} u \frac{du}{dx} \qquad \frac{d}{dx}\operatorname{sech} u = -\operatorname{sech} u \tanh u \frac{du}{dx}$$

$$\frac{d}{dx}\tanh u = \operatorname{sech}^2 u \frac{du}{dx} \qquad \frac{d}{dx}\operatorname{csch} u = -\operatorname{csch} u \coth u \frac{du}{dx}$$

$$\frac{d}{dx}\operatorname{senh}^{-1} u = \frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$$

$$\frac{d}{dx}\cosh^{-1} u = \frac{\pm 1}{\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} + & si & \cosh^{-1} u > 0, u > 1 \\ - & si & \cosh^{-1} u < 0, u < 1 \end{bmatrix}$$

$$\frac{d}{dx}\tanh^{-1} u = \frac{1}{1-u^2}\frac{du}{dx} \qquad \left[-1 < u < 1\right]$$

$$\frac{d}{dx}\coth^{-1} u = \frac{1}{1-u^2}\frac{du}{dx} \qquad \left[u > 1 \quad o \quad u < -1\right]$$

$$\frac{d}{dx}\operatorname{sech}^{-1} u = \frac{\pm 1}{u\sqrt{u^2-1}}\frac{du}{dx} \qquad \begin{bmatrix} - & si & \operatorname{sech}^{-1} u > 0, & 0 < u < 1 \\ + & si & \operatorname{sech}^{-1} u < 0, & 0 < u < 1 \end{bmatrix}$$

$$\frac{d}{dx}\operatorname{csch}^{-1} u = \frac{-1}{|u|\sqrt{1+u^2}}\frac{du}{dx} = \frac{\mu 1}{u\sqrt{1+u^2}}\frac{du}{dx} \qquad \left[- \quad si \quad u > 0, \quad + \quad si \quad u < 0\right]$$

CÁLCULO INTEGRAL

Tablas de Integrales

$$\int u\,dv = uv - \int v\,du$$

$$\int u^n du = \frac{1}{n+1}u^{n+1} + C \qquad n \neq -1$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int e^u\,du = e^u + C$$

$$\int a^u du = \frac{a^u}{\ln a} + C$$

$$\int \operatorname{sen} u\,du = -\cos u + C$$

$$\int \cos u\,du = \operatorname{sen} u + C$$

$$\int \sec^2 u\,du = \tan u + C$$

$$\int \csc^2 u\,du = -\cot u + C$$

$$\int \sec u \tan u\,du = \sec u + C$$

$$\int \csc u \cot u\,du = -\csc u + C$$

$$\int \tan u\,du = \ln|\sec u| + C$$

$$\int \cot u\,du = \ln|\operatorname{sen} u| + C$$

$$\int \sec u\,du = \ln|\sec u + \tan u| + C$$

$$\int \csc u\,du = \ln|\csc u - \cot u| + C$$

$$\int \frac{du}{\sqrt{a^2-u^2}} = \operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a}\ln\left|\frac{u+a}{u-a}\right| + C$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + C$$

$$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int u^2\sqrt{a^2+u^2}\,du = \frac{u}{8}(a^2+2u^2)\sqrt{a^2+u^2} - \frac{a^2}{8}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{u\sqrt{a^2+u^2}} = -\frac{1}{a}\ln\left|\frac{\sqrt{a^2+u^2}+a}{u}\right| + C$$

$$\int \frac{du}{u^2\sqrt{a^2+u^2}} = -\frac{\sqrt{a^2+u^2}}{a^2u} + C$$

$$\int \frac{\sqrt{a^2+u^2}}{u}du = \sqrt{a^2+u^2} - a\ln\left|\frac{a+\sqrt{a^2+u^2}}{u}\right| + C$$

$$\int \frac{\sqrt{a^2+u^2}}{u^2}du = -\frac{\sqrt{a^2+u^2}}{u} + \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{\sqrt{a^2+u^2}} = \ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{u^2du}{\sqrt{a^2+u^2}} = \frac{u}{2}\sqrt{a^2+u^2} - \frac{a^2}{2}\ln\left|u+\sqrt{a^2+u^2}\right| + C$$

$$\int \frac{du}{(a^2+u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2+u^2}} + C$$

$$\int \sqrt{a^2-u^2}\,du =$$

$$\int \sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int u^2\sqrt{a^2-u^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{a^2-u^2} + \frac{a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u}du = \sqrt{a^2-u^2} - a\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{\sqrt{a^2-u^2}}{u^2}du = -\frac{1}{u}\sqrt{a^2-u^2} - \operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{u^2 du}{\sqrt{a^2-u^2}} = -\frac{u}{2}\sqrt{a^2-u^2} + \frac{a^2}{2}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{u\sqrt{a^2-u^2}} = -\frac{1}{a}\ln\left|\frac{a+\sqrt{a^2-u^2}}{u}\right| + C$$

$$\int \frac{du}{u^2\sqrt{a^2-u^2}} = -\frac{1}{a^2 u}\sqrt{a^2-u^2} + C$$

$$\int (a^2-u^2)^{3/2}du = -\frac{u}{8}(2u^2-5a^2)\sqrt{a^2-u^2} + \frac{3a^4}{8}\operatorname{sen}^{-1}\frac{u}{a} + C$$

$$\int \frac{du}{(a^2-u^2)^{3/2}} = \frac{u}{a^2\sqrt{a^2-u^2}} + C$$

$$\int \frac{udu}{a+bu} = \frac{1}{b^2}\left(a+bu-a\ln|a+bu|\right) + C$$

$$\int \frac{u^2du}{a+bu} = \frac{1}{2b^3}\left[(a+bu)^2 - 4a(a+bu) + 2a^2\ln|a+bu|\right] + C$$

$$\int \frac{du}{u(a+bu)} = \frac{1}{a}\ln\left|\frac{u}{a+bu}\right| + C$$

$$\int \frac{du}{u^2(a+bu)} = -\frac{1}{au} + \frac{b}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int u\tan^{-1}u\,du = \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C$$

$$\int \sqrt{u^2-a^2}\,du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int u^2\sqrt{u^2-a^2}\,du = \frac{u}{8}(2u^2-a^2)\sqrt{u^2-a^2} - \frac{a^4}{8}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u}du = \sqrt{u^2-a^2} - a\cos^{-1}\frac{a}{u} + C$$

$$\int \frac{\sqrt{u^2-a^2}}{u^2}du = -\frac{\sqrt{u^2-a^2}}{u} + \ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{du}{\sqrt{u^2-a^2}} = \ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{u^2du}{\sqrt{u^2-a^2}} = \frac{u}{2}\sqrt{u^2-a^2} + \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$$\int \frac{du}{u^2\sqrt{u^2-a^2}} = \frac{\sqrt{u^2-a^2}}{a^2u} + C$$

$$\int \frac{du}{(u^2-a^2)^{3/2}} = -\frac{u}{a^2\sqrt{u^2-a^2}} + C$$

$$\int \frac{u^2du}{\sqrt{a+bu}} = \frac{2}{15b^3}(8a^2+3b^2u^2-4abu)\sqrt{a+bu}$$

$$\int \frac{du}{u\sqrt{a+bu}} = \frac{1}{\sqrt{a}}\ln\left|\frac{\sqrt{a+bu}-\sqrt{a}}{\sqrt{a+bu}+\sqrt{a}}\right| + C,\ \text{si}\ \ a>0$$

$$= \frac{2}{\sqrt{-a}}\tan^{-1}\sqrt{\frac{a+bu}{-a}} + C,\ \text{si}\ \ a<0$$

$$\int \frac{\sqrt{a+bu}}{u}du = 2\sqrt{a+bu} + a\int\frac{du}{u\sqrt{a+bu}}$$

$$\int \frac{\sqrt{a+bu}}{u^2}du = -\frac{\sqrt{a+bu}}{u} + \frac{b}{2}\int\frac{du}{u\sqrt{a+bu}}$$

$$\int \frac{udu}{(a+bu)^2} = \frac{a}{b^2(a+bu)} + \frac{1}{b}\ln|a+bu| + C$$

$$\int u^n\sqrt{a+bu}\,du = \frac{2}{b(2n+3)}\left[u^n(a+bu)^{3/2} - na\int u^{n-1}\sqrt{a+bu}\,du\right]$$

$$\int \frac{du}{u(a+bu)^2} = \frac{1}{a(a+bu)} - \frac{1}{a^2}\ln\left|\frac{a+bu}{u}\right| + C$$

$$\int \frac{u^2du}{(a+bu)^2} = \frac{1}{b^3}\left(a+bu-\frac{a^2}{a+bu} - 2a\ln|a+bu|\right) + C$$

$$\int u\sqrt{a+bu}\,du = \frac{2}{15b^2}(3bu-2a)(a+bu)^{3/2} + C$$

$$\int \frac{udu}{\sqrt{a+bu}} = \frac{2}{3b^2}(bu-2a)\sqrt{a+bu}$$

$$\int \frac{u^n du}{\sqrt{a+bu}} = \frac{2u^n\sqrt{a+bu}}{b(2n+1)} - \frac{2na}{b(2n+1)}\int\frac{u^{n-1}du}{\sqrt{a+bu}}$$

$$\int \frac{du}{u^n\sqrt{a+bu}} = -\frac{\sqrt{a+bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)}\int\frac{du}{u^{n-1}\sqrt{a+bu}}$$

$$\int \csc^3 u\,du = -\tfrac{1}{2}\csc u\cot u + \tfrac{1}{2}\ln|\csc u - \cot u| + C$$

$$\int \operatorname{sen}^n u\,du = -\tfrac{1}{n}\operatorname{sen}^{n-1}u\cos u + \frac{n-1}{n}\int\operatorname{sen}^{n-2}u\,du$$

$$\int \operatorname{sen}^2 u\,du = \tfrac{1}{2}u - \tfrac{1}{4}\operatorname{sen}2u + C$$

$$\int \cos^2 u\,du = \tfrac{1}{2}u + \tfrac{1}{4}\operatorname{sen}2u + C$$

$$\int \tan^2 u\,du = \tan u - u + C$$

$$\int \cot^2 u\,du = -\cot u - u + C$$

$$\int \operatorname{sen}^3 u\,du = -\tfrac{1}{3}\left(2+\operatorname{sen}^2 u\right)\cos u + C$$

$$\int \cos^3 u\,du = \tfrac{1}{3}\left(2+\cos^2 u\right)\operatorname{sen} u + C$$

$$\int \tan^3 u\,du = \tfrac{1}{2}\tan^2 u + \ln\left|\cos u\right| + C$$

$$\int \cot^3 u\,du = -\tfrac{1}{2}\cot^2 u - \ln\left|\operatorname{sen} u\right| + C$$

$$\int \sec^3 u\ du = \tfrac{1}{2}\sec u\ tanu + \tfrac{1}{2}\ln\left|\sec u + tanu\right| + C$$

$$\int \operatorname{sen} au\ \cos bu\ du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$$\int u\operatorname{sen} u\,du = \operatorname{sen} u - u\cos u + C$$

$$\int u\cos u\ du = \cos u + u\operatorname{sen} u + C$$

$$\int u^n \operatorname{sen} u\,du = u^n \cos u + n\int u^{n-1}\cos u\,du$$

$$\int \cos^n u\,du = \tfrac{1}{n}\cos^{n-1} u\operatorname{sen} u + \frac{n-1}{n}\int \cos^{n-2} u\,du$$

$$\int \tan^n u\,du = \frac{1}{n-1}\tan^{n-1} u - \int \tan^{n-2} u\,du$$

$$\int \cot^n u\,du = \frac{-1}{n-1}\cot^{n-1} u - \int \cot^{n-2} u\,du$$

$$\int \sec^n u\,du = \frac{1}{n-1}tanu\sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$$

$$\int \csc^n u\,du = \frac{1}{n-1}\cot u\csc^{n-2} u + \frac{n-2}{n-1}\int \csc^{n-2} u\,du$$

$$\int \operatorname{sen} au\ \operatorname{sen} bu\ du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} - \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$\int \cos au\ \cos bu\,du = \frac{\operatorname{sen}(a-b)u}{2(a-b)} + \frac{\operatorname{sen}(a+b)u}{2(a+b)} + C$$

$$\int u^n \cos u\,du = u^n \operatorname{sen} u - n\int u^{n-1}\operatorname{sen} u\,du$$

$$\begin{aligned}\int \operatorname{sen}^n u\cos^m u\,du &= -\frac{\operatorname{sen}^{n-1} u\cos^{m+1} u}{n+m} + \frac{n-1}{n+m}\int \operatorname{sen}^{n-2} u\cos^m u\,du \\ &= -\frac{\operatorname{sen}^{n+1} u\cos^{m-1} u}{n+m} + \frac{m-1}{n+m}\int \operatorname{sen}^n u\cos^{m-2} u\,du\end{aligned}$$

$$\int u\cos^{-1} u\,du = \frac{2u^2-1}{4}\cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int \operatorname{sen}^{-1} u\,du = u\operatorname{sen}^{-1} u + \sqrt{1-u^2} + C$$

$$\int \cos^{-1} u\,du = u\cos^{-1} u - \sqrt{1-u^2} + C$$

$$\int \tan^{-1} u\,du = u\tan^{-1} u - \tfrac{1}{2}\ln\left(1+u^2\right) + C$$

$$\int u\operatorname{sen}^{-1} u\ du = \frac{2u^2-1}{4}\operatorname{sen}^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$\int ue^{au}\ du = \frac{1}{a^2}(au-1)e^{au} + C$$

$$\int u^n e^{au}\ du = \frac{1}{a}u^n e^{au} - \frac{n}{a}\int u^{n-1}e^{au}du$$

$$\int e^{au}\operatorname{sen} bu\ du = \frac{e^{au}}{a^2+b^2}(a\operatorname{sen} bu - b\cos bu) + C$$

$$\int u^n \operatorname{sen}^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\operatorname{sen}^{-1} u - \int \frac{u^{n+1}du}{\sqrt{1-u^2}}\right],\quad n \neq -1$$

$$\int u^n \cos^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\cos^{-1} u + \int \frac{u^{n+1}du}{\sqrt{1-u^2}}\right],\quad n \neq -1$$

$$\int u^n \tan^{-1} u\,du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1} u - \int \frac{u^{n+1}du}{\sqrt{1+u^2}}\right], n \neq -1$$

$$\int \ln u\,du = u\ln u - u + C$$

$$\int u^n \ln u\ du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$$

$$\int \frac{1}{u\ln u}du = \ln\left|\ln u\right| + C$$

$$\int e^{au}\cos bu\,du=\frac{e^{au}}{a^2+b^2}(a\cos bu+b\,\mathrm{sen}\,bu)+C$$

$$\int \mathrm{senh}\,u\,du=\cosh u+C$$

$$\int \cosh u\,du=\mathrm{senh}\,u+C$$

$$\int \tanh u\,du=\ln\cosh u+C$$

$$\int \coth u\,du=\ln|\mathrm{senh}\,u|+C$$

$$\int \mathrm{sech}\,u\,du=tan^{-1}|\mathrm{senh}\,u|+C$$

$$\int \mathrm{sech}\,u\,du=\ln\left|\tan\tfrac{1}{2}u\right|+C$$

$$\int \mathrm{sech}^2 u\,du=\tanh u+C$$

$$\int \mathrm{csch}^2 u\,du=-\coth u+C$$

$$\int \mathrm{sech}\,u\tanh u\,du=-\mathrm{sech}\,u+C$$

$$\int \mathrm{csch}\,u\coth u\,du=-\mathrm{csch}\,u+C$$

$$\int\sqrt{2au-u^2}\,du=\frac{u-a}{2}\sqrt{2au-u^2}+\frac{a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right)+C$$

$$\int u\sqrt{2au-u^2}\,du=\frac{2u-au-3a^2}{6}\sqrt{2au-u^2}+\frac{a^3}{2}\cos^{-1}\left(\frac{a-u}{a}\right)+C$$

$$\int\frac{\sqrt{2au-u^2}}{u^2}du=\sqrt{2au-u^2}+a\cos^{-1}\left(\frac{a-u}{a}\right)+C$$

$$\int\frac{\sqrt{2au-u^2}}{u^2}du=-\frac{2\sqrt{2au-u^2}}{u}-\cos^{-1}\left(\frac{a-u}{a}\right)+C$$

$$\int\frac{u^2du}{\sqrt{2au-u^2}}=-\frac{(u+3a)}{2}\sqrt{2au-u^2}+\frac{3a^2}{2}\cos^{-1}\left(\frac{a-u}{a}\right)+C$$

$$\int\frac{du}{\sqrt{2au-u^2}}=\cos^{-1}\left(\frac{a-u}{a}\right)+C$$

$$\int\frac{u\,du}{\sqrt{2au-u^2}}=-\sqrt{2au-u^2}+a\cos^{-1}\left(\frac{a-u}{a}\right)+C$$

$$\int\frac{du}{u\sqrt{2au-u^2}}=-\frac{\sqrt{2au-u^2}}{au}+C$$

INTRODUCCIÓN AL CÁLCULO MULTIVARIABLE

PRODUCTO PUNTO ENTRE VECTORES:

$$\mathbf{A}\bullet\mathbf{B}=\|A\|\|B\|\cos\theta \qquad 0\le\theta\le\pi$$

donde θ es el ángulo formado por **A** y **B**

$$\mathbf{A}\bullet\mathbf{B}=A_1B_1+A_2B_2+A_3B_3$$

donde $\mathbf{A}=A_1\hat{\mathbf{i}}+A_2\hat{\mathbf{j}}+A_3\hat{\mathbf{k}}$, $\quad\mathbf{B}=B_1\hat{\mathbf{i}}+B_2\hat{\mathbf{j}}+B_3\hat{\mathbf{k}}$

PRODUCTO CRUZ ENTRE VECTORES

Producto cruz: $\mathbf{AxB}=\begin{vmatrix}\hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}}\\ A_1 & A_2 & A_3\\ B_1 & B_2 & B_3\end{vmatrix}$

$$=(A_2B_3-A_3B_2)\hat{\mathbf{i}}+(A_3B_1-A_1B_3)\hat{\mathbf{j}}+(A_1B_2-A_2B_1)\hat{\mathbf{k}}$$

Magnitud del Producto Cruz $\|\mathbf{AxB}\|=\|\mathbf{A}\|\|\mathbf{B}\|\,\mathrm{sen}\,\theta$

El operador *nabla* se define así:

$$\nabla = \hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}$$

En las fórmulas que vienen a continuación vamos a suponer que *U=U(x,y,z)*, y **A**=**A**(*x,y,z*) tienen derivadas parciales.

Gradiente de *U* = grad *U* $= \nabla U = \left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right)U = \frac{\partial U}{\partial x}\hat{\mathbf{i}} + \frac{\partial U}{\partial y}\hat{\mathbf{j}} + \frac{\partial U}{\partial z}\hat{\mathbf{k}}$

Divergencia de A = div **A** $= \nabla \bullet \mathbf{A} = \left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right) \bullet \left(A_1\,\hat{\mathbf{i}} + A_2\,\hat{\mathbf{j}} + A_3\,\hat{\mathbf{k}}\right)$

$$= \frac{\partial A_1}{\partial x} + \frac{\partial A_2}{\partial y} + \frac{\partial A_3}{\partial z}$$

Rotacional de A = rot **A** $= \nabla \mathbf{x} \mathbf{A} = \left(\hat{\mathbf{i}}\frac{\partial}{\partial x} + \hat{\mathbf{j}}\frac{\partial}{\partial y} + \hat{\mathbf{k}}\frac{\partial}{\partial z}\right) \mathbf{x} \left(A_1\,\hat{\mathbf{i}} + A_2\,\hat{\mathbf{j}} + A_3\,\hat{\mathbf{k}}\right)$

$$= \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_1 & A_2 & A_3 \end{vmatrix}$$

$$= \left(\frac{\partial A_3}{\partial y} - \frac{\partial A_2}{\partial z}\right)\hat{\mathbf{i}} + \left(\frac{\partial A_1}{\partial z} - \frac{\partial A_3}{\partial x}\right)\hat{\mathbf{j}} + \left(\frac{\partial A_2}{\partial x} - \frac{\partial A_1}{\partial y}\right)\hat{\mathbf{k}}$$

Laplaciano de *U* = $\nabla^2 U = \nabla \bullet (\nabla U) = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$

Integrales Múltiples

$$\int_{x=a}^{b}\int_{y=f_1(x)}^{f_2(x)} F(x,y)\,dydx$$

$$= \int_{x=a}^{b}\left\{\int_{y=f_1(x)}^{f_2(x)} F(x,y)\,dy\right\}dx$$

donde $y = f_1(x)$ e $y = f_2(x)$ son las ecuaciones de las curvas HPG y PGQ respectivamente, mientras que a y b son las abscisas de los puntos P y Q. Esta integral también se puede escribir así:

$$\int_{y=c}^{d}\int_{x=g_1(y)}^{g_2(y)} F(x,y)\,dxdy = \int_{y=c}^{d}\left\{\int_{x=g_1(y)}^{g_2(y)} F(x,y)\,dx\right\}dy$$

donde $x = g_1(y)$, $x = g_2(y)$ son las ecuaciones de las curvas HPG y PGQ respectivamente, mientras que c y d son las ordenadas de H y G.

Estas son las llamadas integrales dobles o integrales de área. Los anteriores conceptos se pueden ampliar para considerar integrales triples o de volumen así como integrales múltiples en más de tres dimensiones.