Problem set # 2, part I

Numerical Methods for Data Science 2020/21

UC3M — *Master on Statistics for Data Science*

Due date: November 8, end of day (through Aula Global).

Note: This is an individual assignment. Evidence of plagiarism will be penalized. Hand in the assignment as a pdf file, with Gurobi–Python code printouts where required.

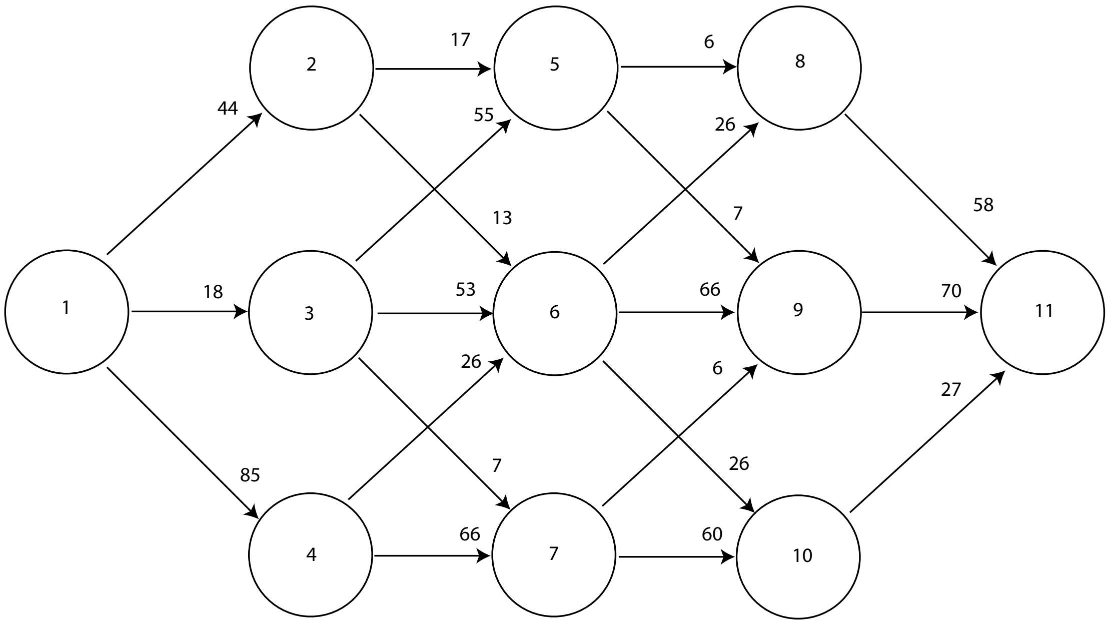

Figure 1: Network for Problem 1.

Problem 1 (33 points). Consider the network shown in Figure 1, where each node represents a city, each arc a direct road between two cities, and the numbers shown on arcs are the corresponding distances, in kms. Consider the problem of finding a shortest route from node 1 to node 11.

(a, 11 points) Formulate the problem as a linear or integer optimization problem. Which model is appropriate? Why?

(b, 11 points) Implement and solve it using Gurobi–Python.

(c, 11 points) Obtain and interpret the optimal dual solution and the reduced costs.

Problem 2 (33 pts). A firm can produce four products, P1, P2, P3 and P4, which can be produced in continuous quantities. The productive process for each product consists of three operations: O1, O2 and O3. The following table shows the required hours of each operation for each product (per unit), and the available hours of productive capacity for each operation.

	P1	P2	P3	P4	Available Hours
O1	3	3	5	6	598
O2	6	3	4	3	287
O3	6	6	2	7	392

The accounting department determines that each unit made and sold of products P1, . . . , P4 contributes to the firm's profits 49 €, 54 €, 47 €, and 51 €, respectively. Furthermore, the manufacture of products P1, ..., P4 requires a process (different for each product) for setting up the production line that costs 993 €, 715 €, 834 €, and 940 €, respectively. It is assumed that all units made are sold.

(a, 17 pts) Formulate the problem as a mixed integer optimization model.

(b, 16 pts) Solve through Gurobi–Python the integer model and its linear relaxation, and compare the solutions obtained.

Problem 3 (34 pts). Consider a binary knapsack model with n objects, where the weight w_j and the value r_j of object $j = 1, \ldots, n$ are equal, being given by $w_j = r_j = 2^{k+n+1} + 2^{k+j} + 1$, where $k = \lfloor \ln(2n) \rfloor$, with ln the natural logarithm and $\lfloor \cdot \rfloor$ the integer part (floor). The knapsack's capacity is $b = \lfloor \sum_j w_j/2 \rfloor$.

(a, 17 pts) Implement in Gurobi–Python this knapsack problem, and its linear relaxation, and solve them for the case $n = 24$ objects. Discuss the results, indicating the recorded solution time and the number of nodes explored by Branch & Bound.

(b, 17 pts) Identify a valid inequality of the type seen in class. Add it to the formulation, and resolve the integer model. Compare the results obtained with those of part (a), indicating the recorded solution time and the number of nodes explored by Branch & Bound as reported by Gurobi.