Biostatistics Task 2

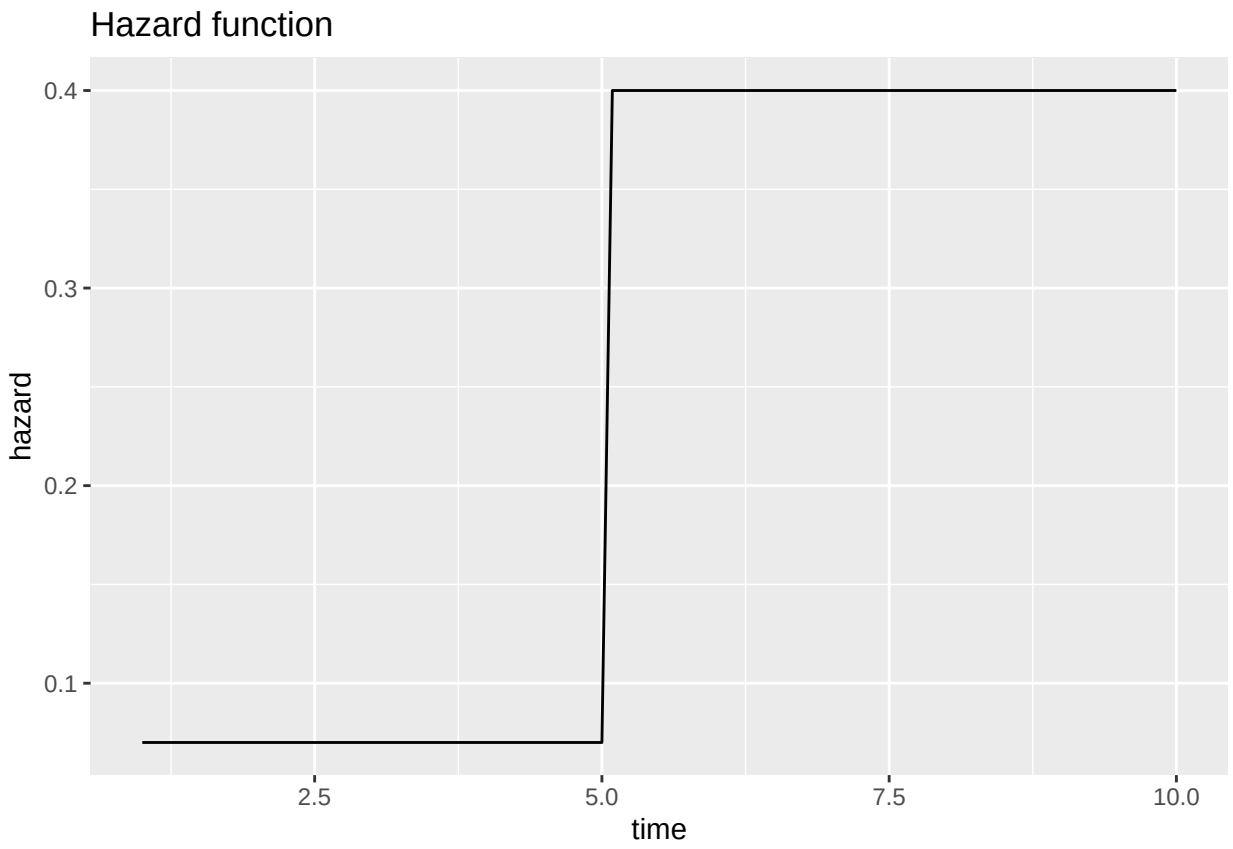
Danyu Zhang & Daniel Alonso

May 21st, 2021

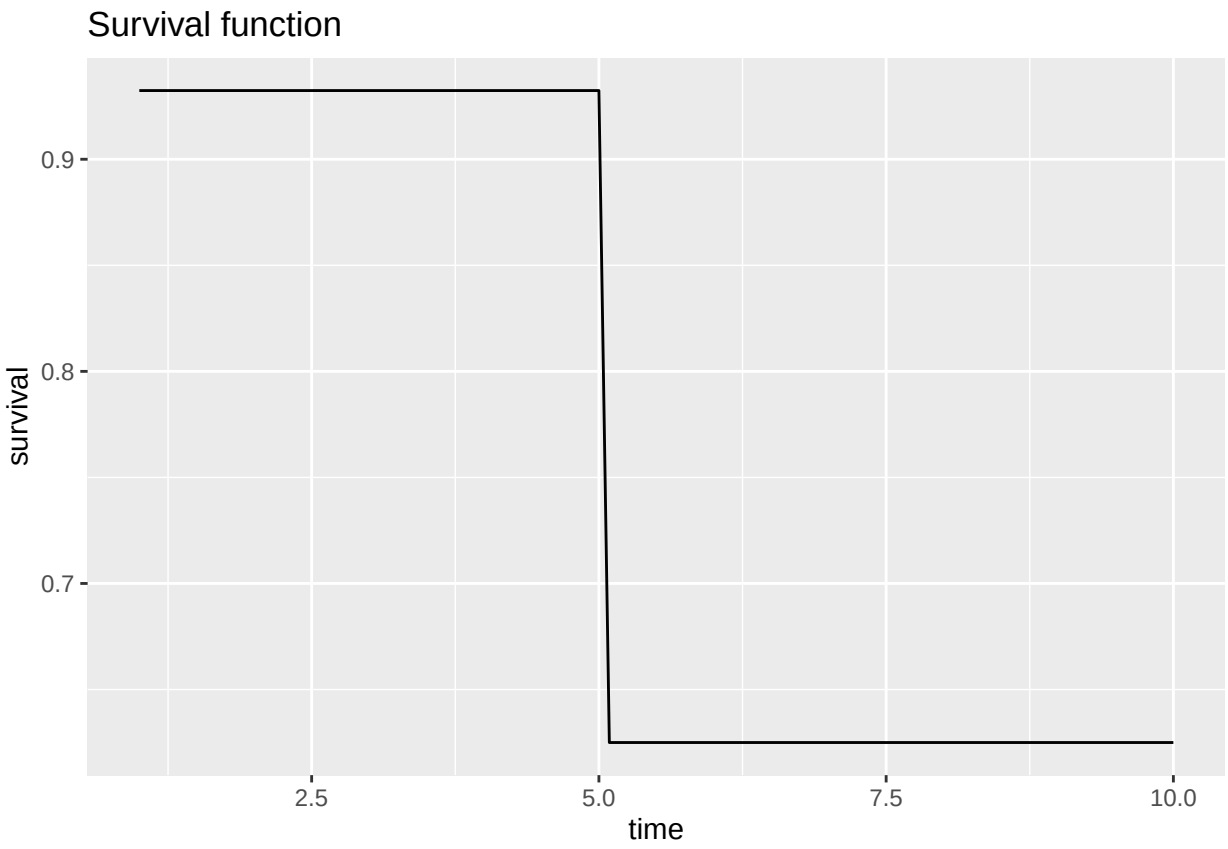
```
library(survival)
library(ggplot2)
library(coin)
```

Exercise 1

Hazard plot



Survival plot

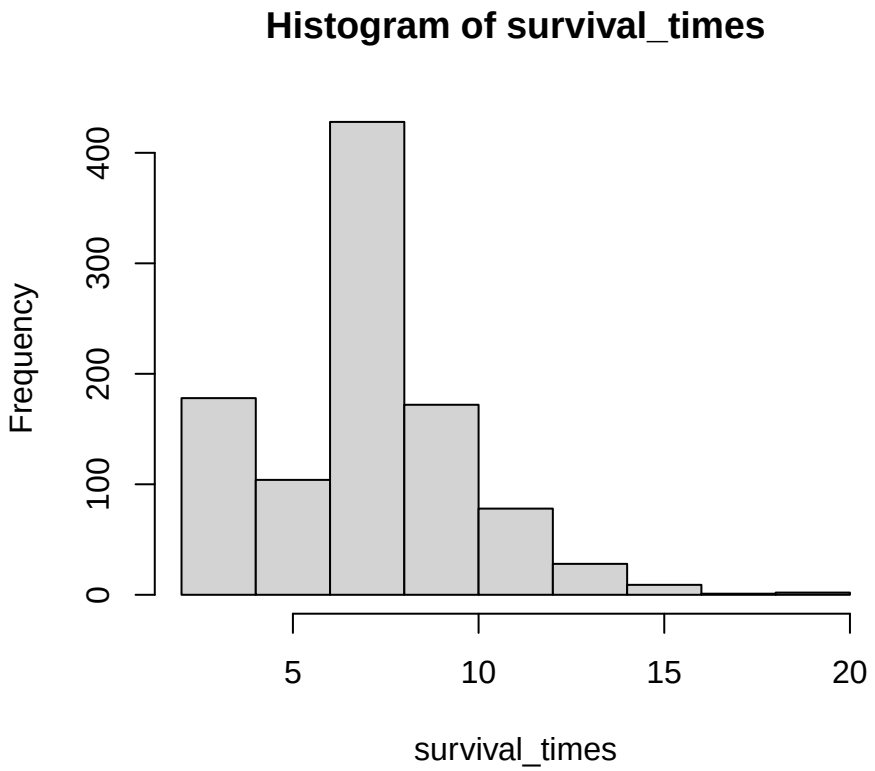


Survival times simulation

```
# number of trials
trials <- 1000
time_length <- 1000
survival_prob <- c(1, rep(0.9323938, 5), rep(0.6250023, time_length-6))

# t is in seconds
survival_times <- rep(0,trials)
for (i in 1:trials) {
    time = 1
    while (time < time_length) {
        if (runif(1,0,1) > survival_prob[time]) {
            break
        } else {
            time = time + 1
        }
    }
    survival_times[i] <- time
}
```

Histogram for survival times



Median survival time

```
#> [1] 7
```

Exercise 2

Given the following density function:

$$f(y) = (\lambda_0 + \lambda_1 y)e^{-\lambda_0 y - \frac{1}{2}\lambda_1 y^2}$$

We obtain the survival function as follows:

$$\begin{aligned} S(t) = P(T > t) &= \int_t^\infty (\lambda_0 + \lambda_1 y)e^{-\lambda_0 y - \frac{1}{2}\lambda_1 y^2} dy \\ &= \lim_{b\to\infty} [-e^{\frac{\lambda_1 b^2}{2} - \lambda_0 b}] + e^{\frac{-\lambda_1 t^2}{2} - \lambda_0 t} \\ &= 0 + e^{\frac{-\lambda_1 t^2}{2} - \lambda_0 t} \\ S(t) &= e^{\frac{-\lambda_1 t^2}{2} - \lambda_0 t},\ \lambda_1 \in \mathbb{R}, \lambda_0 > 0 \end{aligned}$$

We obtain the hazard function as follows:

$$h(t) = \frac{f(t)}{S(t)} = \frac{(\lambda_0 + \lambda_1 t)e^{-\lambda_0 t - \frac{1}{2}\lambda_1 t^2}}{e^{-\lambda_0 t - \frac{1}{2}\lambda_1 t^2}} = \lambda_0 + \lambda_1 t$$

$$h(t) = \lambda_0 + \lambda_1 t$$

And the cumulative hazard function:

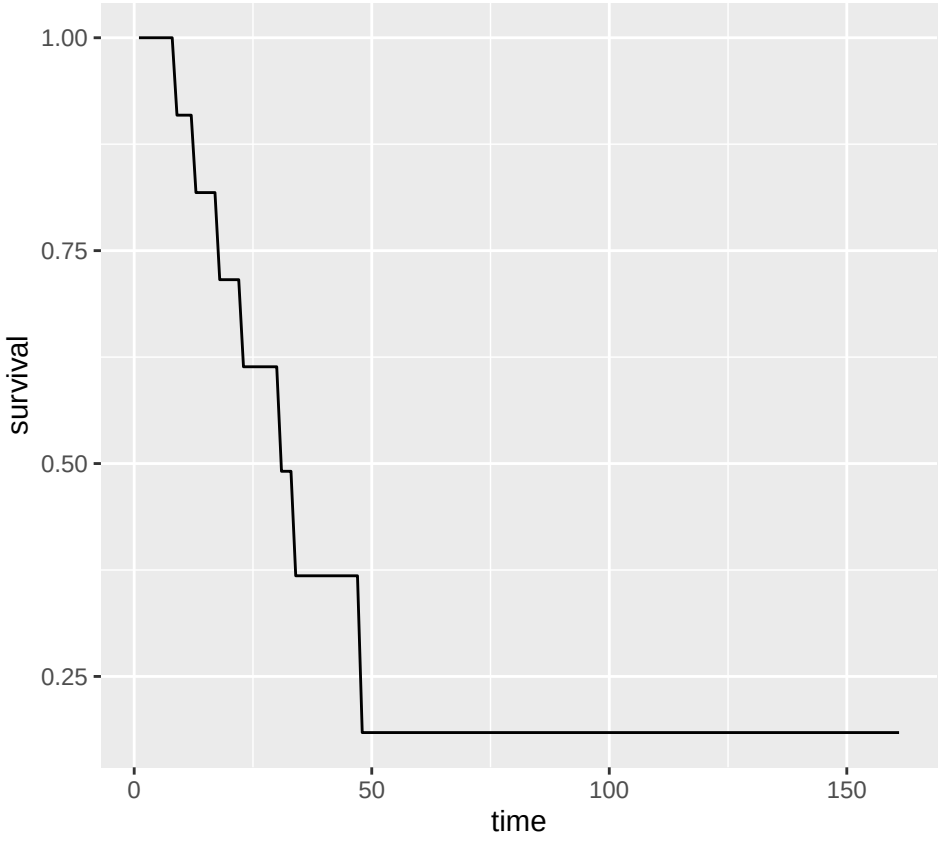
$$H(t) = -log(S(t)) = \frac{\lambda_1 t^2}{2} + \lambda_0 t$$

Exercise 3

KM estimator implementation of the survival function

Utilizing the function to obtain the survival function for the leukaemia dataset

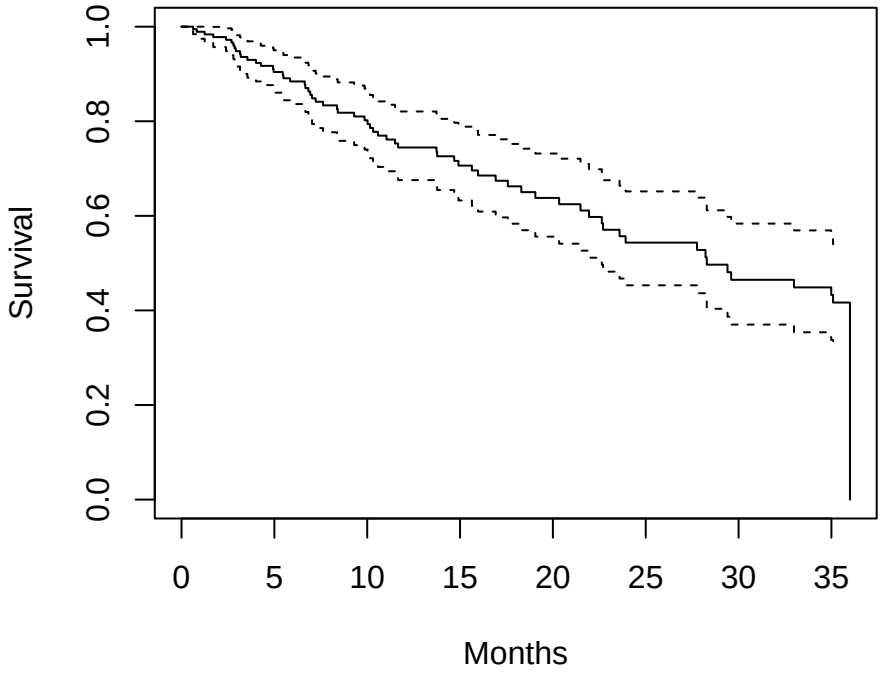
The survival probabilities are as follows, and these change over time at the times displayed on the *time* column of this table:



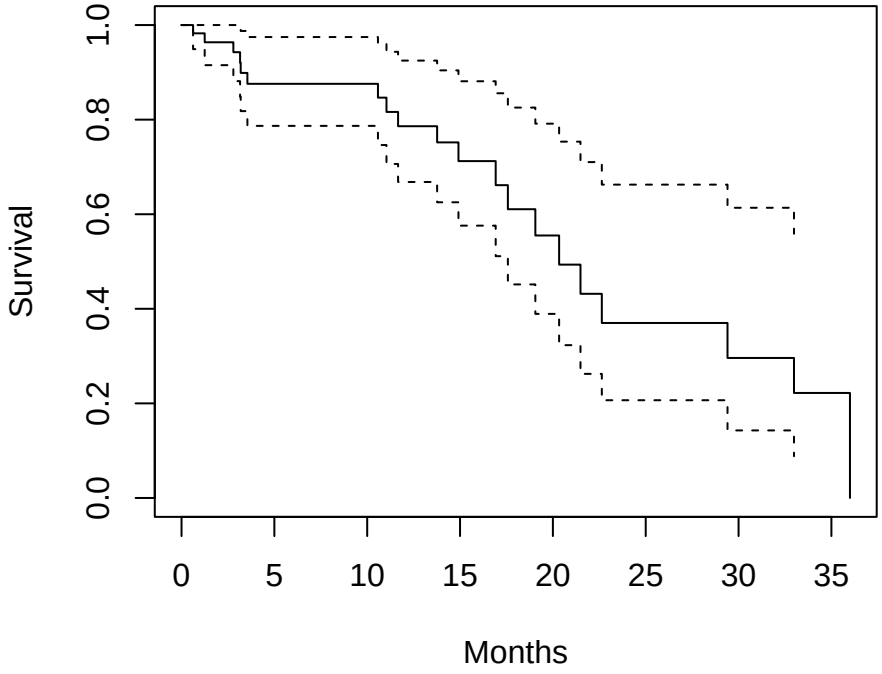
Exercise 4

KM estimate of the survival function

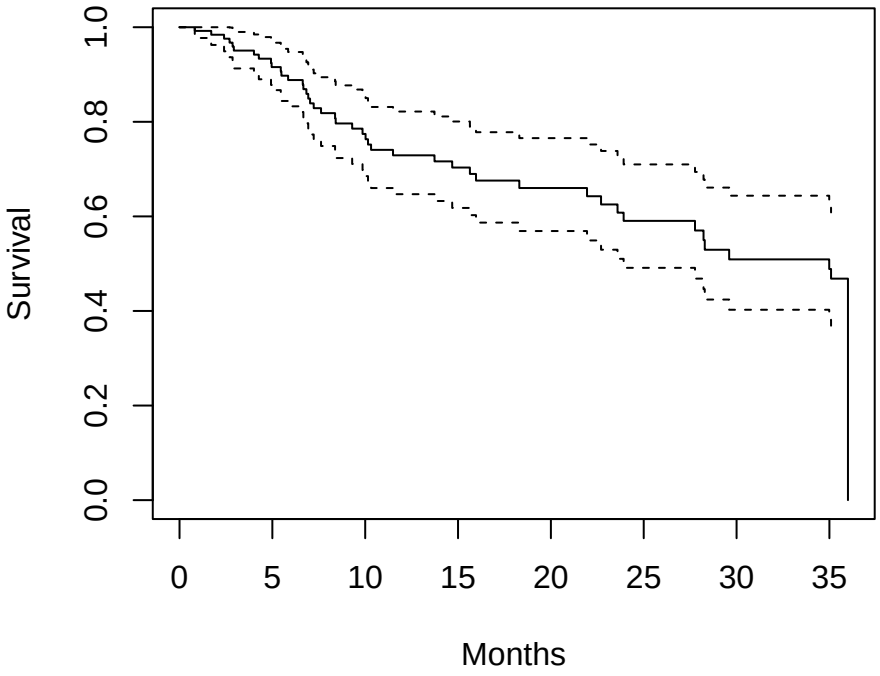
We can see the estimate as follows:



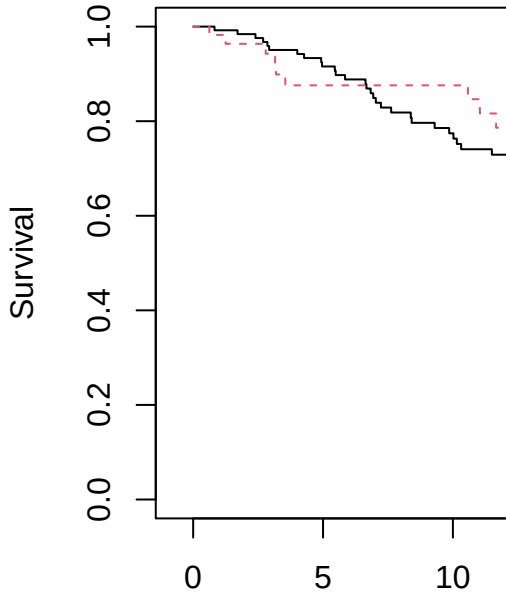
Survival function: with crimes against persons



Survival function: without crimes against persons



Comparing both curves



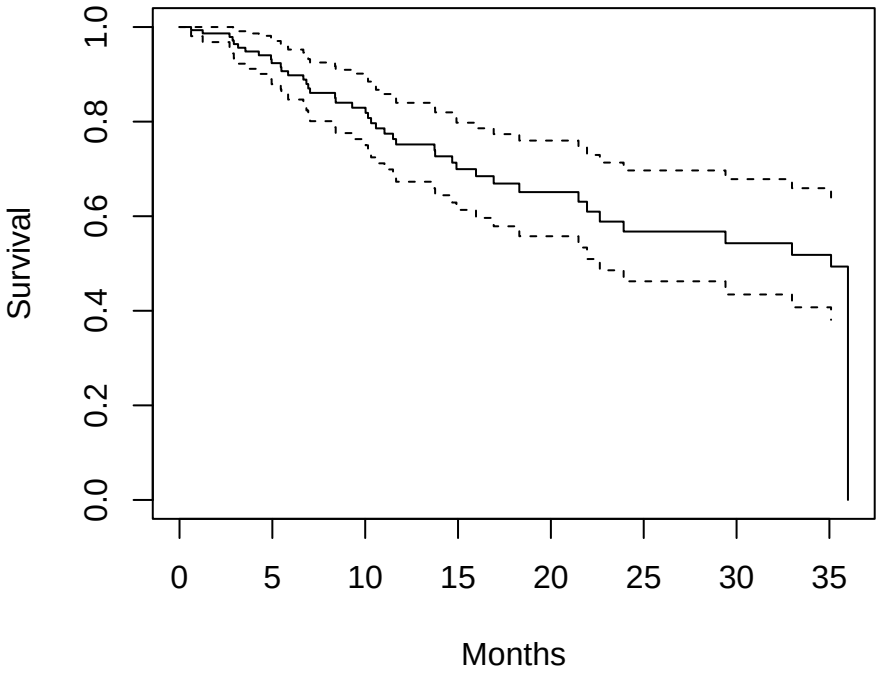
We can see that the curve for nonpersonal crimes decays faster after the 15th month.

Low-rank test

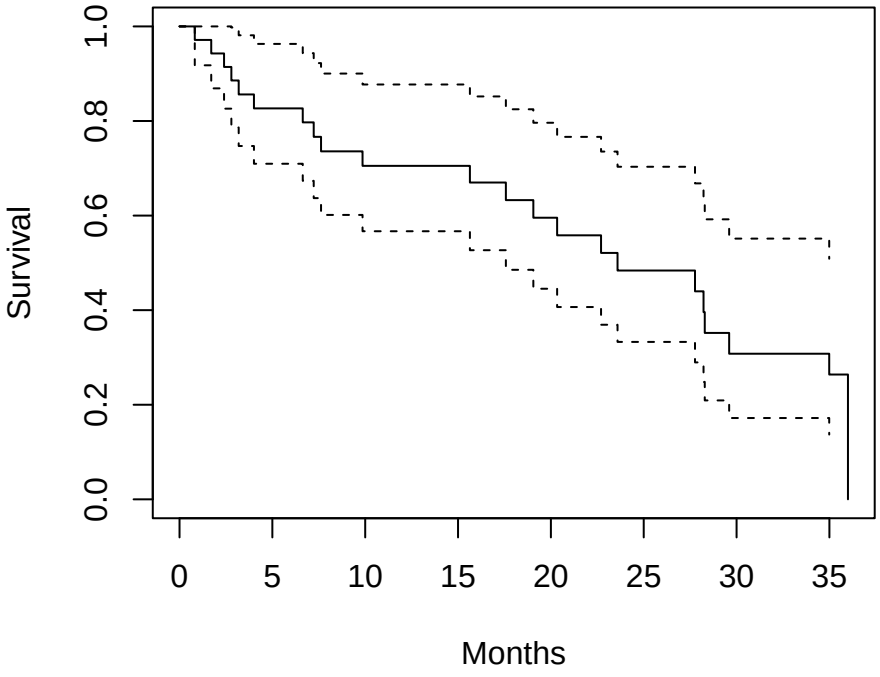
```
#> Call:
#> survdiff(formula = Surv(months, censor) ~ personal, data = henning)
#>
#>                N Observed Expected (O-E)^2/E (O-E)^2/V
#> personal=0 133        66     69.7     0.201      1.24
```

```
#> personal=1  61       22     18.3     0.769      1.24
#>
#>  Chisq= 1.2  on 1 degrees of freedom, p= 0.3
```

Survival function: with crimes against property

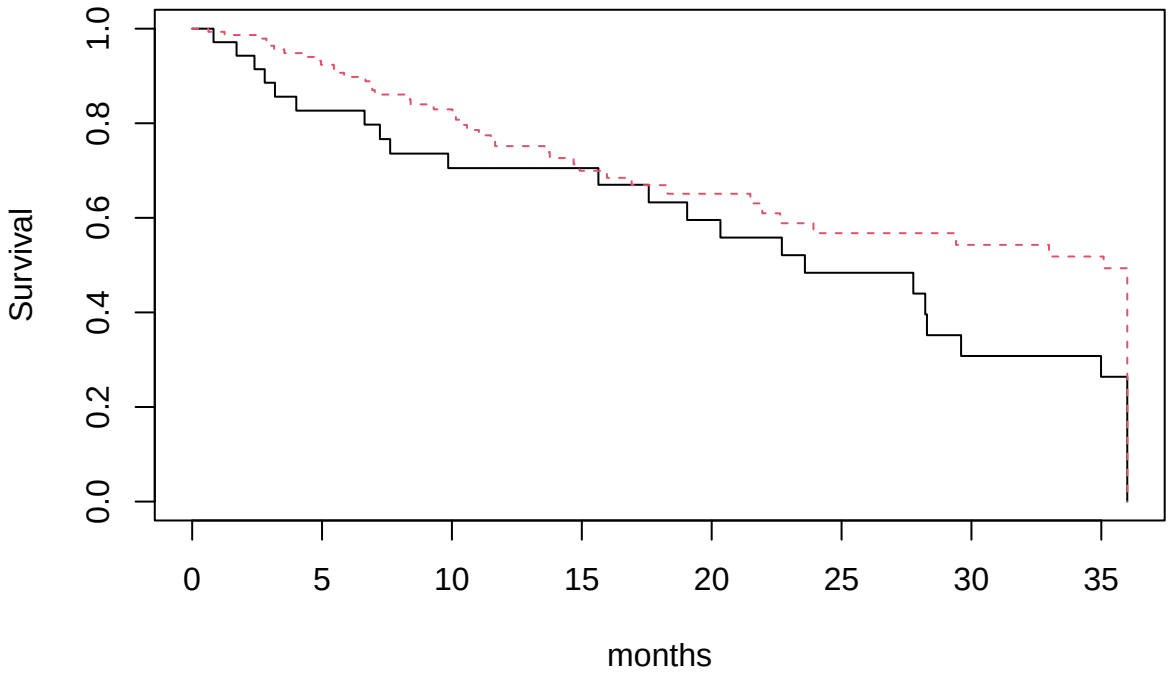


Survival function: with crimes against property



Comparing both curves

We can see that the curve for nonpersonal crimes decays faster after the 15th month.



Low-rank test

```
#> Call:
#> survdiff(formula = Surv(months, censor) ~ property, data = henning)
#>
#>                N Observed Expected (O-E)^2/E (O-E)^2/V
#> property=0  36       27     21.2     1.593      2.98
#> property=1 158       61     66.8     0.505      2.98
#>
#>  Chisq= 3  on 1 degrees of freedom, p= 0.08
```

Fitting a Cox regression

Converting personal and property to leveled factors with labels yes/no.

```
#>      id      months censor personal property        cage
#> 1     1  0.06570842      0      yes      yes  -1.6751978
#> 2     2  0.13141684      0       no      yes -10.4828637
#> 3     3  0.22997947      0      yes      yes  -4.4267378
#> 4     4  0.29568789      0       no      yes -11.3288596
#> 5     5  0.29568789      0      yes      yes  -7.1645886
#> 6     6  0.32854209      0      yes       no  -2.8689007
#> 7     7  0.49281314      0       no      yes  -6.4281067
#> 8     8  0.62422998      1      yes      yes  22.4507434
#> 9     9  0.68993840      0      yes      yes -12.4869705
#> 10   10  0.72279261      0      yes      yes  -9.4096262
#> 11   11  0.78850103      0       no      yes  -8.5170869
#> 12   12  0.82135524      1       no       no  -4.3528158
#> 13   13  0.88706366      0      yes      yes  -5.6341300
#> 14   14  1.05133470      0      yes      yes  -4.7662313
#> 15   15  1.08418891      0       no      yes  -4.5691060
#> 16   16  1.21560575      0       no      yes  -8.5964845
#> 17   17  1.24845996      1      yes      yes  19.7621739
#> 18   18  1.28131417      0       no      yes   8.1947543
#> 19   19  1.37987680      0       no      yes  -3.2412484
#> 20   20  1.51129363      0       no      yes  -7.8819055
#> 21   21  1.60985626      0       no      yes  -5.5081889
#> 22   22  1.70841889      1       no       no   0.2577249
```

```
#> 23   23  2.06981520      0       no      yes   1.9989980
#> 24   24  2.13552361      0       no      yes  -8.4404271
#> 25   25  2.23408624      0      yes      yes  -0.9058617
#> 26   26  2.36550308      0       no      yes  -7.3014811
#> 27   27  2.39835729      0      yes      yes  -0.4212621
#> 28   28  2.39835729      1       no       no  -6.0530212
#> 29   29  2.52977413      0      yes      yes  -6.0968268
#> 30   30  2.52977413      0      yes      yes  -8.1118850
#> 31   31  2.56262834      0      yes      yes  -4.7279014
#> 32   32  2.69404517      1       no      yes   0.3480740
#> 33   33  2.79260780      1      yes       no  27.0612841
#> 34   34  2.79260780      0       no       no -10.8469979
#> 35   35  2.85831622      1       no      yes  -5.5848487
#> 36   36  2.92402464      1       no      yes  -9.0318028
#> 37   37  2.98973306      0       no      yes  -2.9400848
#> 38   38  3.05544148      0      yes      yes  -3.7587022
#> 39   39  3.12114990      0      yes      yes  -2.5814264
#> 40   40  3.15400411      1      yes      yes   8.2933170
#> 41   41  3.18685832      1      yes       no   2.4972868
#> 42   42  3.18685832      0      yes      yes  -2.5677371
#> 43   43  3.21971253      0      yes      yes   1.4350007
#> 44   44  3.31827515      0       no      yes  -5.9818370
#> 45   45  3.54825462      1      yes      yes  -1.2426173
#> 46   46  3.81108830      0      yes      yes   0.3015305
#> 47   47  4.00821355      1       no       no  -1.8120903
#> 48   48  4.04106776      0      yes      yes   0.4630637
#> 49   49  4.27104723      1       no      yes   1.9853087
#> 50   50  4.30390144      0       no      yes  -3.8846433
#> 51   51  4.66529774      0       no      yes   3.6636113
#> 52   52  4.79671458      0       no      yes  -9.2152388
#> 53   53  4.89527721      0       no      yes   0.4767530
#> 54   54  4.92813142      1       no      yes  -3.4328980
#> 55   55  4.96098563      1       no      yes  10.4178892
#> 56   56  5.19096509      0       no      yes  10.8942752
#> 57   57  5.42094456      0       no      yes  -6.0694483
#> 58   58  5.45379877      1       no      yes  -2.7922409
#> 59   59  5.45379877      0       no      yes  -5.7135277
#> 60   60  5.48665298      1       no      yes  -8.2077097
#> 61   61  5.65092402      0      yes      yes  -5.6615085
#> 62   62  5.84804928      0       no      yes  -7.1372101
#> 63   63  5.84804928      0      yes      yes -10.5841642
#> 64   64  5.84804928      1       no      yes  -3.7231101
#> 65   65  6.04517454      0       no      yes   0.5780534
#> 66   66  6.24229979      0       no      yes  -3.3726652
#> 67   67  6.30800821      0       no      yes  -5.6012758
#> 68   68  6.40657084      0       no      yes  -6.2036030
#> 69   69  6.63655031      1       no       no  17.4076222
#> 70   70  6.66940452      0       no      yes  -4.0543901
#> 71   71  6.66940452      1       no      yes  -9.0564435
#> 72   72  6.80082136      0       no       no  -1.2261902
#> 73   73  6.80082136      0       no      yes  -8.0954579
#> 74   74  6.83367556      1       no      yes  10.7601205
#> 75   75  6.93223819      1       no      yes   0.7505380
#> 76   76  7.03080082      0      yes      yes   1.6841452
#> 77   77  7.03080082      1       no      yes   3.3405449
#> 78   78  7.09650924      0       no      yes  -2.2857385
#> 79   79  7.19507187      0       no      yes -11.7285858
#> 80   80  7.22792608      1       no       no  25.0955072
#> 81   81  7.35934292      0      yes      yes  -1.0975112
#> 82   82  7.42505133      0       no      yes  -0.9387159
#> 83   83  7.49075975      0       no      yes  -8.0434387
#> 84   84  7.62217659      1       no       no  16.3617632
#> 85   85  7.68788501      0       no      yes  -4.2734182
#> 86   86  7.78644764      0       no      yes   4.7039946
#> 87   87  7.98357290      0       no      yes  -6.9127063
#> 88   88  8.37782341      1       no      yes  -0.1036714
#> 89   89  8.41067762      1       no      yes  -4.4732813
#> 90   90  9.06776181      0      yes      yes   9.7252129
```

```
#> 91   91  9.10061602      0      yes      yes  -5.1988117
#> 92   92  9.29774127      1       no      yes  13.8210377
#> 93   93  9.36344969      0       no      yes  -0.4595920
#> 94   94  9.49486653      0       no      yes  -6.3843011
#> 95   95  9.85626283      1       no       no   4.8107708
#> 96   96 10.02053388      1       no      yes  -5.4999753
#> 97   97 10.15195072      1       no      yes   6.2015990
#> 98   98 10.31622177      1       no      yes  19.6033786
#> 99   99 10.57905544      1      yes      yes   3.6636113
#> 100 100 10.94045175      0      yes      yes  11.0202163
#> 101 101 11.00616016      0       no      yes  -1.9435071
#> 102 102 11.03901437      1      yes      yes   4.1098810
#> 103 103 11.23613963      0       no      yes   9.3884573
#> 104 104 11.49897331      1       no      yes  -4.8866967
#> 105 105 11.66324435      1      yes      yes   3.5979029
#> 106 106 11.69609856      0       no       no  16.2029679
#> 107 107 11.76180698      0      yes      yes  -4.5608925
#> 108 108 12.02464066      0      yes       no   0.8025572
#> 109 109 12.12320329      0       no      yes  -0.2460396
#> 110 110 12.25462012      0       no      yes   5.1666914
#> 111 111 12.64887064      0       no      yes -11.2138699
#> 112 112 13.14168378      0      yes      yes  -6.2884763
#> 113 113 13.17453799      0       no      yes  -9.0126379
#> 114 114 13.63449692      0       no       no  -6.3952525
#> 115 115 13.73305955      1       no      yes   2.7081014
#> 116 116 13.76591376      1      yes      yes   9.3282246
#> 117 117 13.96303901      0      yes      yes  -0.8155126
#> 118 118 14.02874743      0       no      yes  -4.4267378
#> 119 119 14.09445585      0      yes      yes   3.1680603
#> 120 120 14.39014374      0      yes      yes   6.8668967
#> 121 121 14.68583162      1       no      yes   6.7491691
#> 122 122 14.88295688      0       no      yes   1.1256236
#> 123 123 14.91581109      0       no      yes  -8.7908720
#> 124 124 14.91581109      1      yes      yes  -0.3582915
#> 125 125 15.40862423      0      yes      yes  -5.0865598
#> 126 126 15.63860370      1       no       no  15.9894155
#> 127 127 15.70431211      0       no      yes  -4.7853963
#> 128 128 15.73716632      0       no      yes  -8.0927200
#> 129 129 15.77002053      0      yes       no -11.2467241
#> 130 130 15.96714579      1       no      yes  22.3905107
#> 131 131 16.06570842      0      yes      yes   9.7005723
#> 132 132 16.16427105      0      yes      yes  -1.7628090
#> 133 133 16.91991786      1      yes      yes  15.6992033
#> 134 134 16.98562628      0       no      yes   1.0133717
#> 135 135 17.18275154      0       no      yes  -6.6252320
#> 136 136 17.34702259      0       no      yes  -7.4712279
#> 137 137 17.34702259      0       no      yes   2.2235018
#> 138 138 17.57700205      1      yes       no  15.6061164
#> 139 139 17.67556468      0      yes      yes  14.7272663
#> 140 140 18.23408624      0       no      yes   4.2632006
#> 141 141 18.29979466      1       no      yes   4.5534128
#> 142 142 19.05544148      1      yes       no  -1.8887501
#> 143 143 19.08829569      0      yes      yes  -2.4910773
#> 144 144 19.41683778      0       no      yes  -0.1556906
#> 145 145 19.48254620      0       no      yes  11.6006407
#> 146 146 20.07392197      0       no      yes  -5.2864229
#> 147 147 20.33675565      1      yes       no   1.7142615
#> 148 148 21.48665298      1      yes      yes   6.9189159
#> 149 149 21.51950719      0       no      yes  -8.2460396
#> 150 150 21.94661191      1       no      yes   1.8319891
#> 151 151 22.63655031      1      yes      yes   4.1317838
#> 152 152 22.70225873      1       no       no  -4.3747186
#> 153 153 23.58932238      1       no       no   7.8059795
#> 154 154 23.91786448      1       no      yes  -0.7936098
#> 155 155 24.83778234      0       no       no  -2.0913511
#> 156 156 26.44763860      0       no      yes  -7.9394004
#> 157 157 26.51334702      0       no       no   4.4219959
#> 158 158 26.87474333      0       no      yes  -7.9613032
```

```
#> 159 159 27.23613963      0       no      yes   7.3761369
#> 160 160 27.76180698      1       no       no   9.4569035
#> 161 161 28.22176591      1       no       no  15.4829131
#> 162 162 28.28747433      1       no       no   4.3727146
#> 163 163 28.97741273      0      yes      yes  -2.2556221
#> 164 164 29.40451745      1      yes      yes   1.2871568
#> 165 165 29.60164271      1       no       no  32.8682656
#> 166 166 32.98562628      1      yes      yes  -5.7929253
#> 167 167 34.98973306      1       no       no   7.5623108
#> 168 168 35.08829569      1       no      yes   4.9777797
#> 169 169 36.00000000      1       no      yes  -9.6177029
#> 170 170 36.00000000      1       no      yes  -3.9558275
#> 171 171 36.00000000      1       no       no  -3.7313237
#> 172 172 36.00000000      1       no      yes -10.3185927
#> 173 173 36.00000000      1       no      yes   1.4651171
#> 174 174 36.00000000      1       no      yes   3.5376701
#> 175 175 36.00000000      1       no      yes  10.6451308
#> 176 176 36.00000000      1      yes      yes  -5.7272169
#> 177 177 36.00000000      1       no      yes  -6.4089418
#> 178 178 36.00000000      1       no       no  -6.9428227
#> 179 179 36.00000000      1       no      yes   0.8737413
#> 180 180 36.00000000      1      yes      yes  -1.8832744
#> 181 181 36.00000000      1       no      yes  -8.5581546
#> 182 182 36.00000000      1       no      yes  -2.4856016
#> 183 183 36.00000000      1       no      yes  -3.7367994
#> 184 184 36.00000000      1       no      yes  -3.8955947
#> 185 185 36.00000000      1       no      yes  -9.2617823
#> 186 186 36.00000000      1       no      yes  -6.8990171
#> 187 187 36.00000000      1      yes       no  -7.2001806
#> 188 188 36.00000000      1       no      yes  -5.1002491
#> 189 189 36.00000000      1       no       no   0.8463628
#> 190 190 36.00000000      1       no       no  15.4035155
#> 191 191 36.00000000      1       no      yes  -2.3404955
#> 192 192 36.00000000      1       no      yes   1.8648433
#> 193 193 36.00000000      1       no      yes  10.3631322
#> 194 194 36.00000000      1       no       no   8.3070062
```

Running the cox regression fit.

```
#> Call:
#> coxph(formula = Surv(months, censor) ~ cage + personal + property,
#>     data = henning)
#>
#>   n= 194, number of events= 88
#>
#>                 coef exp(coef) se(coef)      z Pr(>|z|)
#> cage         0.04384   1.04481  0.01236  3.548 0.000389 ***
#> personalyes  0.34006   1.40504  0.25678  1.324 0.185382
#> propertyyes -0.06842   0.93387  0.25255 -0.271 0.786457
#> ---
#> Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
#>
#>             exp(coef) exp(-coef) lower .95 upper .95
#> cage           1.0448     0.9571    1.0198     1.070
#> personalyes    1.4050     0.7117    0.8494     2.324
#> propertyyes    0.9339     1.0708    0.5693     1.532
#>
#> Concordance= 0.603  (se = 0.044 )
#> Likelihood ratio test= 14.74  on 3 df,   p=0.002
#> Wald test            = 16.16  on 3 df,   p=0.001
#> Score (logrank) test = 16.43  on 3 df,   p=9e-04
```

According to the p-value on our Wald test, we can see that

The dummy variables personal and property are not significant, given that they have a large p-val. The variable cage, which is centered age (in years) at time of release is significant, and positive, which means that it increases the probability of survival as it increases.

Exercise 5

Given a hazard function $h(t) = c$, where $c > 0$:

We obtain the cumulative hazard function $H(t)$:

$$\begin{aligned} H(t) &= \int_0^t h(u)du \\ &= c\int_0^t du \\ &= ct \end{aligned}$$

With this, we derive the survival function $S(t)$:

$$\begin{aligned} H(t) &= ct \\ H(t) &= -log(S(t)) \\ ct &= -log(S(t)) \\ S(t) &= e^{-ct} \end{aligned}$$

And then we obtain the density function $f(t)$:

$$\begin{aligned} h(t) &= \frac{f(t)}{S(t)} \\ c &= \frac{f(t)}{e^{-ct}} \\ f(t) &= ce^{-ct} \end{aligned}$$

Calculating median failure time with $c = 5$

We note the functions with $c = 5$ are:

$$\begin{aligned} h(t) &= 5 \\ H(t) &= 5t \\ S(t) &= e^{-5t} \\ f(t) &= 5e^{-5t} \end{aligned}$$

Exercise 6