Topic 1: Exercise 2

Daniel Alonso

November 21th, 2020

Creating function(s)

Function to compute mean vector

```
mean_vector <- function(pis, means) {
    mv <- c()
    for (i in 1:length(pis)) {
        mv <- c(mv, pis[i]*means[,i])
    }
    return(matrix(rowSums(matrix(mv, nrow=2)),nrow=2))
}
```

Testing mean vector

```
pis <- c(0.5, 0.5)
means <- matrix(c(0,0,3,3), nrow=length(pis))
meanvector <- mean_vector(pis,means)
meanvector
```

```
##      [,1]
## [1,]  1.5
## [2,]  1.5
```

Function to compute covariance matrix

```
covariance_matrix <- function(pis, means, sigmas, meanvector) {
    result <- 0
    for (i in 1:length(pis)) {
        result <- result + pis[i]*(sigmas[[i]] + means[,i]%*%t(means[,i]))
    }
    return(result - meanvector%*%t(meanvector))
}
```

Testing covariance matrix

```
pis <- c(0.5, 0.5)
means <- matrix(c(0,0,3,3), nrow=length(pis))
sigmas <- list()
sigmas[[1]] <- matrix(c(1,0.7,0.7,1), nrow=length(pis))
sigmas[[2]] <- matrix(c(1,0.7,0.7,1), nrow=length(pis))
```

```r
covmatrix <- covariance_matrix(pis,means, sigmas, mean_vector(pis,means))
covmatrix
```

```
##      [,1] [,2]
## [1,] 3.25 2.95
## [2,] 2.95 3.25
```

Function to compute correlation matrix

```r
correlation_matrix <- function(covmatrix) {
    delta <- diag(diag(covmatrix)^(-1/2))
    return(delta%*%covmatrix%*%delta)
}
```

Testing correlation matrix

```r
corrmatrix <- correlation_matrix(covmatrix)
corrmatrix
```

```
##           [,1]      [,2]
## [1,] 1.0000000 0.9076923
## [2,] 0.9076923 1.0000000
```

Putting it all together calling other functions

```r
final_function <- function(pis, means, sigmas) {
    meanvector <- mean_vector(pis,means)
    print('Mean Vector:')
    print(meanvector)
    covmatrix <- covariance_matrix(pis,means, sigmas, meanvector)
    print('Covariance Matrix:')
    print(covmatrix)
    corrmatrix <- correlation_matrix(covmatrix)
    print('Correlation Matrix:')
    print(corrmatrix)
}
```

Putting it all together without calling other functions

```
final_function <- function(pis, means, sigmas) {
    # Mean Vector and covariance matrix
    meanvector <- c()
    covmatrix <- 0

    # Single loop for both correlation matrix and mean vector
    for (i in 1:length(pis)) {
        meanvector <- c(meanvector, pis[i]*means[,i])
        covmatrix <- covmatrix + pis[i]*(sigmas[[i]] + means[,i]%*%t(means[,i]))
    }

    # Calculating mean vector and covariance matrix
    meanvector <- matrix(rowSums(matrix(meanvector, nrow=2)),nrow=2)
    covmatrix <- covmatrix - meanvector%*%t(meanvector)

    # Correlation matrix
    delta <- diag(diag(covmatrix)^(-1/2))
    corrmatrix <- delta%*%covmatrix%*%delta

    # Printing results
    print('Mean Vector:')
    print(meanvector)
    print('Covariance Matrix:')
    print(covmatrix)
    print('Correlation Matrix:')
    print(corrmatrix)
}
```

Exercises

Exercise 1

```r
pis <- c(0.5, 0.5)
means <- matrix(c(0,0,3,3), nrow=length(pis))
sigmas <- list()
sigmas[[1]] <- matrix(c(1,0.7,0.7,1), nrow=length(pis))
sigmas[[2]] <- matrix(c(1,0.7,0.7,1), nrow=length(pis))
```

```r
final_function(pis,means,sigmas)
```

```
## [1] "Mean Vector:"
##      [,1]
## [1,]  1.5
## [2,]  1.5
## [1] "Covariance Matrix:"
##      [,1] [,2]
## [1,] 3.25 2.95
## [2,] 2.95 3.25
## [1] "Correlation Matrix:"
##           [,1]      [,2]
## [1,] 1.0000000 0.9076923
## [2,] 0.9076923 1.0000000
```

Correlation analysis for ex.1

We can see that the two variables are highly correlated, it's also a positive correlation at 0.9076923

Exercise 2

```r
pis <- c(0.5, 0.5)
means <- matrix(c(0,0,0,0), nrow=length(pis))
sigmas <- list()
sigmas[[1]] <- matrix(c(1,0.7,0.7,1), nrow=length(pis))
sigmas[[2]] <- matrix(c(1,-0.7,-0.7,1), nrow=length(pis))
```

```r
final_function(pis,means,sigmas)
```

```
## [1] "Mean Vector:"
##      [,1]
## [1,]    0
## [2,]    0
## [1] "Covariance Matrix:"
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
## [1] "Correlation Matrix:"
##      [,1] [,2]
## [1,]    1    0
## [2,]    0    1
```

Correlation analysis for ex.2

We can see that the two variables are not correlated at all (correlation 0) and independent. However given that the means are zero, our covariance matrix is just the identity matrix. Therefore our correlation matrix should also be the same matrix, as $I_n^{-1/2} I_n I_n^{-1/2} = I_n$.

Exercise 3

```r
pis <- c(0.5, 0.5)
means <- matrix(c(-3,3,3,-3), nrow=length(pis))
sigmas <- list()
sigmas[[1]] <- matrix(c(1,0.7,0.7,1), nrow=length(pis))
sigmas[[2]] <- matrix(c(1,0.7,0.7,1), nrow=length(pis))
```

```r
final_function(pis,means,sigmas)
```

```
## [1] "Mean Vector:"
##      [,1]
## [1,]    0
## [2,]    0
## [1] "Covariance Matrix:"
##      [,1] [,2]
## [1,] 10.0 -8.3
## [2,] -8.3 10.0
## [1] "Correlation Matrix:"
##       [,1]  [,2]
## [1,]  1.00 -0.83
## [2,] -0.83  1.00
```

Correlation analysis for ex.3

We can see that the two variables are significantly correlated, it's also a negative correlation at -0.83.

Exercise 4

```r
pis <- c(0.5, 0.5)
means <- matrix(c(-3,-3,3,3), nrow=length(pis))
sigmas <- list()
sigmas[[1]] <- matrix(c(1,-0.7,-0.7,1), nrow=length(pis))
sigmas[[2]] <- matrix(c(1,-0.7,-0.7,1), nrow=length(pis))
```

```r
final_function(pis,means,sigmas)
```

```
## [1] "Mean Vector:"
##      [,1]
## [1,]    0
## [2,]    0
## [1] "Covariance Matrix:"
##      [,1] [,2]
## [1,] 10.0  8.3
## [2,]  8.3 10.0
## [1] "Correlation Matrix:"
##      [,1] [,2]
## [1,] 1.00 0.83
## [2,] 0.83 1.00
```

Correlation analysis for ex.4

We can see that the two variables are significantly correlated, it's also a positive correlation at 0.83.