$$\hat{\sigma}^2 = \frac{(Y - X\hat{\beta})'(Y - X\hat{\beta})}{n}$$

$$\ell(\sigma^2 \mid X) = \sum_{i=1}^{n} \log\left(\frac{1}{\sqrt{2\pi\sigma^2}}\right) - \frac{(Y_i - (\beta_0 + \dots + \beta_k X_{ik}))^2}{2\sigma^2}$$

$$\propto -\frac{n}{2}\log(\sigma^2) - \frac{(Y - X\beta)'(Y - X\beta)}{2\sigma^2}$$

$$\frac{\partial \ell}{\partial \sigma}\left(-\frac{n}{2}\log(\sigma^2) - \frac{(Y - X\beta)'(Y - X\beta)}{2\sigma^2}\right) = 0$$

$$-\frac{n}{\cancel{2}}\left(\frac{1}{\sigma^{\cancel{2}}}\right)(\cancel{2}\cancel{\sigma}) - (Y - X\beta)'(Y - X\beta) \cdot (-\cancel{2})(2\sigma^{-3}) = 0$$

$$-\frac{n}{\cancel{\sigma}} + \frac{(Y - X\beta)'(Y - X\beta)}{\sigma^{\cancel{3}}} = 0$$

$$-n + \frac{(Y - X\beta)'(Y - X\beta)}{\sigma^2} = 0$$

$$-\sigma^2 n + (Y - X\beta)'(Y - X\beta) = 0$$

$$\hat{\sigma}^2 = \frac{(Y - X\beta)'(Y - X\beta)}{n}$$