$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$\hat{Y} = X(X'X)^{-1} X'Y = HY$$

$$\hat{\varepsilon} = Y - \hat{Y} = (I - H)Y$$

$$\boxed{Y = X\beta + \varepsilon}$$

$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$X\hat{\beta} = X(X'X)^{-1} X'Y$$

$$\hat{Y} = X\hat{\beta}$$

$$\hat{\varepsilon} = Y - \hat{Y}$$

$$\sum_{i=1}^{n} \hat{\varepsilon}_i = 0$$

$$\hat{\varepsilon} = X\beta + \varepsilon - X\hat{\beta}$$

$$\hat{\varepsilon} = 0$$

$$X\hat{\beta} + \hat{\varepsilon} = X\beta + \varepsilon$$

② $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \hat{y}_i \Rightarrow Y = \hat{Y}$

$$Y = X(X'X)^{-1}X'Y$$

$$Y = HY$$

because $\hat{\varepsilon} = (I - H)Y$

$$\hat{\varepsilon} = IY - HY$$

$$\hat{\varepsilon} = 0$$

$$0 = Y - HY$$

$$HY = Y$$

$$\hat{Y} = Y$$

$$\sum_{i=1}^{n} x_i \hat{\varepsilon}_i = 0$$

$$\sum_{i=1}^{n} x_i \hat{\varepsilon}_i = \sum_{i=1}^{n} \hat{\varepsilon}_i$$

$$\overbrace{\underbrace{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}_{SST}}^{\text{total variability}} = \overbrace{\underbrace{\sum_{i=1}^{n}(\hat{Y}_i-\bar{Y})^2}_{SSR}}^{\text{regression variability}} + \overbrace{\underbrace{\sum_{i=1}^{n}(Y_i-\hat{Y}_i)^2}_{SSE}}^{\text{residual variability}}$$

$$\sum_{i=1}^{n}(Y_i-\bar{Y})^2 - \sum_{i=1}^{n}(Y_i-\hat{Y}_i)^2 = \sum_{i=1}^{n}(\hat{Y}_i-\bar{Y})^2$$

$$\sum_{i=1}^{n} Y_i^2 - 2Y_i\bar{Y} + \bar{Y}^2 - \left(\sum_{i=1}^{n} Y_i^2 - 2Y_i\hat{Y}_i + \hat{Y}_i^2\right) = SSR$$

$$\cancel{\sum_{i=1}^{n} Y_i^2} - 2\bar{Y}\sum_{i=1}^{n} Y_i + n\bar{Y}^2 \cancel{- \sum_{i=1}^{n} Y_i^2} - 2\sum_{i=1}^{n} Y_i\hat{Y}_i + \sum_{i=1}^{n}\hat{Y}_i^2$$

$$-2\bar{Y}\sum_{i=1}^{n} Y_i + \underline{n\bar{Y}^2} - \underbrace{2\sum_{i=1}^{n} Y_i\hat{Y}_i} + \underline{\sum_{i=1}^{n}\hat{Y}_i^2} = SSR$$

$$SSR = \sum_{i=1}^{n}\hat{Y}_i^2 - 2\bar{Y}\sum_{i=1}^{n}\hat{Y}_i + \sum_{i=1}^{n}\bar{Y}^2$$

$$SSR = \underline{\sum_{i=1}^{n}\hat{Y}_i^2} - 2\bar{Y}\sum_{i=1}^{n}\hat{Y}_i + \underline{n\bar{Y}^2}$$

$$\cancel{-2}\left(\bar{Y}\sum_{i=1}^{n} Y_i + \sum_{i=1}^{n} Y_i\hat{Y}_i\right) = \cancel{-2}\,\bar{Y}\sum_{i=1}^{n}\hat{Y}_i$$

$$= \bar{Y}(Y_1 + \dots + Y_n) + (Y_1\hat{Y}_1 + \dots + Y_n\hat{Y}_n)$$

$$\Rightarrow \bar{Y}Y_1 + \dots + \bar{Y}\,Y_n + Y_1\hat{Y}_1 + \dots + Y_n\hat{Y}_n$$

$$= \bar{Y}\hat{Y}_1 + \dots + \bar{Y}\hat{Y}_n$$

$$\Rightarrow (\bar{Y} + \hat{Y}_1)Y_1 + \dots + (\bar{Y} + \hat{Y}_1)Y_n$$

$$= \bar{Y}\hat{Y}_1 + \dots + \bar{Y}\hat{Y}_n$$