$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$\hat{Y} = X(X'X)^{-1} X'Y = HY$$

$$\hat{\varepsilon} = Y - \hat{Y} = (I - H)Y$$

$$\boxed{Y = X\beta + \varepsilon}$$

$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$X\hat{\beta} = X(X'X)^{-1} X'Y$$

$$\hat{Y} = X\hat{\beta}$$

$$\hat{\varepsilon} = Y - \hat{Y}$$

$$\hat{\varepsilon} = X\beta + \varepsilon - X\hat{\beta}$$

$$X\hat{\beta} + \hat{\varepsilon} = X\beta + \varepsilon$$

$$\sum_{i=1}^{n} \hat{\varepsilon}_i = 0$$

$$\hat{\varepsilon} = 0$$

2 $\sum_{i=1}^{n} y_i = \sum_{i=1}^{n} \hat{y}_i \Rightarrow Y = \hat{Y}$

$$Y = X(X'X)^{-1}X'Y$$

$$Y = HY$$

because $\hat{\varepsilon} = (I - H)Y$

$$\hat{\varepsilon} = IY - HY$$

$$\hat{\varepsilon} = 0$$

$$0 = Y - HY$$

$$HY = Y$$

$$\hat{Y} = Y$$

$$\sum_{i=1}^{n} x_i \hat{\varepsilon}_i = 0$$

$$\sum_{i=1}^{n} x_i \hat{\varepsilon}_i = \sum_{i=1}^{n} \hat{\varepsilon}_i$$

total variability regression variability residual variability

$$\underbrace{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}_{SST}=\underbrace{\sum_{i=1}^{n}(\hat{Y}_i-\bar{Y})^2}_{SSR}+\underbrace{\sum_{i=1}^{n}(Y_i-\hat{Y}_i)^2}_{SSE}$$

$$\sum_{i=1}^{n}(Y_i-\bar{Y})^2-\sum_{i=1}^{n}(Y_i-\hat{Y}_i)^2=\sum_{i=1}^{n}(\hat{Y}_i-\bar{Y})^2$$

$$\sum_{i=1}^{n}Y_i^2-2Y_i\bar{Y}+\bar{Y}^2-\left(\sum_{i=1}^{n}Y_i^2-2Y_i\hat{Y}_i+\hat{Y}_i^2\right)=SSR$$

$$\cancel{\sum_{i=1}^{n}Y_i^2}-2\bar{Y}\sum_{i=1}^{n}Y_i+n\bar{Y}^2\cancel{-\sum_{i=1}^{n}Y_i^2}-2\sum_{i=1}^{n}Y_i\hat{Y}_i+\sum_{i=1}^{n}\hat{Y}_i^2$$

$$-2\bar{Y}\sum_{i=1}^{n}Y_i+\underline{n\bar{Y}^2}\underbrace{-2\sum_{i=1}^{n}Y_i\hat{Y}_i}+\underline{\sum_{i=1}^{n}\hat{Y}_i^2}=SSR$$

$$SSR=\sum_{i=1}^{n}\hat{Y}_i^2-2\bar{Y}\sum_{i=1}^{n}\hat{Y}_i+\sum_{i=1}^{n}\bar{Y}^2$$

$$SSR=\underline{\sum_{i=1}^{n}\hat{Y}_i^2}-2\bar{Y}\sum_{i=1}^{n}\hat{Y}_i+\underline{n\bar{Y}^2}$$

$$\cancel{-2}\left(\bar{Y}\sum_{i=1}^{n} Y_i + \sum_{i=1}^{n} Y_i\hat{Y}_i\right) = \cancel{-2}\,\bar{Y}\sum_{i=1}^{n}\hat{Y}_i$$

$$= \bar{Y}(Y_1 + \dots + Y_n) + (Y_1\hat{Y}_1 + \dots + Y_n\hat{Y}_n)$$

$$\Rightarrow \bar{Y}Y_1 + \dots + \bar{Y}\,Y_n + Y_1\hat{Y}_1 + \dots + Y_n\hat{Y}_n -$$

$$(\bar{Y}\hat{Y}_1 + \dots + \bar{Y}\hat{Y}_n) = 0$$

$$\Rightarrow \bar{Y}Y_i + Y_i\hat{Y}_i - \bar{Y}\hat{Y}_i = 0$$

$$= 0$$

$$S_i = \beta_1 + \beta_2 \cos(\omega_i + \phi) + \varepsilon_i$$

$$\beta_1 = S_i - \beta_2 \cos(\omega_i + \phi) - \varepsilon_i$$

$$\beta_2 = \frac{S_i - \beta_1 - \varepsilon_i}{\cos(\omega_i + \phi)}$$