Stochastic Processes: Assignment 1

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Importing libraries

Problem 1

```
#> [1] "n bnq ndrsw awornoc aslnxs rzs dowfr ksfrswc ljicrwq rj nirzjwoms n ljwjcngowif gnllocs aworofz
```

Problem 2

(a)

Let $N(t)$ be the number of cars arriving at a parking lot by time t, according to the proposed scenario, we can model $N(t)$ as a non-homogenous Poisson process. Such process has almost the same process as any other Poisson process, however, its rate is a function of time.

$N(t), t \in [0, \infty)$ is the non-homogenous Poisson process with rate $\lambda(t)$ where:

- $N(0) = 0$
- $N(t)$ has independent increments

We define 8:00 as $t = 0$ with the following integrable function and each unit of t equals to 1 hour:

$$\lambda(t) = \begin{cases} 100 & 0 \leq t \leq \frac{1}{2} \\ 600t - 200 & \frac{1}{2} \leq t \leq \frac{3}{4} \\ 400t - 50 & \frac{3}{4} \leq t \leq 1 \\ -500t + 850 & 1 \leq t \leq 1.5 \end{cases}$$

So,

$E[N(t)] = \{$

lots of latex here

(b)

```
#> $x
#> [1] 0.8
#>
#> $fvec
#> [1] 5.882155e-18
#>
#> $termcd
#> [1] 1
#>
```

```
#> $message
#> [1] "Function criterion near zero"
#>
#> $scalex
#> [1] 1
#>
#> $nfcnt
#> [1] 0
#>
#> $njcnt
#> [1] 0
#>
#> $iter
#> [1] 0
```

```
#> [1] 0.8
```

```
f <- function(l) {
    1 - sum((exp(-l) * l^k)/(factorial(k))) - 0.8
}

tt = c()
aa = seq(0,100,0.1)
for (i in seq(0,100,0.1)) {
    tt = c(tt, f(i))
}
test <- data.frame(val=aa, tt=tt)
```

```
tt = c()
aa = seq(0,200,1)
for (i in aa) {
    tt = c(tt, ppois(149,lambda=i))
}
test <- data.frame(val=aa, tt=tt)
test
#>     val           tt
#> 1     0 1.000000e+00
#> 2     1 1.000000e+00
#> 3     2 1.000000e+00
#> 4     3 1.000000e+00
#> 5     4 1.000000e+00
#> 6     5 1.000000e+00
#> 7     6 1.000000e+00
#> 8     7 1.000000e+00
#> 9     8 1.000000e+00
#> 10    9 1.000000e+00
#> 11   10 1.000000e+00
#> 12   11 1.000000e+00
#> 13   12 1.000000e+00
#> 14   13 1.000000e+00
#> 15   14 1.000000e+00
#> 16   15 1.000000e+00
#> 17   16 1.000000e+00
#> 18   17 1.000000e+00
#> 19   18 1.000000e+00
```

```
#> 20   19 1.000000e+00
#> 21   20 1.000000e+00
#> 22   21 1.000000e+00
#> 23   22 1.000000e+00
#> 24   23 1.000000e+00
#> 25   24 1.000000e+00
#> 26   25 1.000000e+00
#> 27   26 1.000000e+00
#> 28   27 1.000000e+00
#> 29   28 1.000000e+00
#> 30   29 1.000000e+00
#> 31   30 1.000000e+00
#> 32   31 1.000000e+00
#> 33   32 1.000000e+00
#> 34   33 1.000000e+00
#> 35   34 1.000000e+00
#> 36   35 1.000000e+00
#> 37   36 1.000000e+00
#> 38   37 1.000000e+00
#> 39   38 1.000000e+00
#> 40   39 1.000000e+00
#> 41   40 1.000000e+00
#> 42   41 1.000000e+00
#> 43   42 1.000000e+00
#> 44   43 1.000000e+00
#> 45   44 1.000000e+00
#> 46   45 1.000000e+00
#> 47   46 1.000000e+00
#> 48   47 1.000000e+00
#> 49   48 1.000000e+00
#> 50   49 1.000000e+00
#> 51   50 1.000000e+00
#> 52   51 1.000000e+00
#> 53   52 1.000000e+00
#> 54   53 1.000000e+00
#> 55   54 1.000000e+00
#> 56   55 1.000000e+00
#> 57   56 1.000000e+00
#> 58   57 1.000000e+00
#> 59   58 1.000000e+00
#> 60   59 1.000000e+00
#> 61   60 1.000000e+00
#> 62   61 1.000000e+00
#> 63   62 1.000000e+00
#> 64   63 1.000000e+00
#> 65   64 1.000000e+00
#> 66   65 1.000000e+00
#> 67   66 1.000000e+00
#> 68   67 1.000000e+00
#> 69   68 1.000000e+00
#> 70   69 1.000000e+00
#> 71   70 1.000000e+00
#> 72   71 1.000000e+00
```

```
#> 73   72 1.000000e+00
#> 74   73 1.000000e+00
#> 75   74 1.000000e+00
#> 76   75 1.000000e+00
#> 77   76 1.000000e+00
#> 78   77 1.000000e+00
#> 79   78 1.000000e+00
#> 80   79 1.000000e+00
#> 81   80 1.000000e+00
#> 82   81 1.000000e+00
#> 83   82 1.000000e+00
#> 84   83 1.000000e+00
#> 85   84 1.000000e+00
#> 86   85 1.000000e+00
#> 87   86 1.000000e+00
#> 88   87 1.000000e+00
#> 89   88 1.000000e+00
#> 90   89 1.000000e+00
#> 91   90 1.000000e+00
#> 92   91 1.000000e+00
#> 93   92 1.000000e+00
#> 94   93 1.000000e+00
#> 95   94 9.999999e-01
#> 96   95 9.999999e-01
#> 97   96 9.999998e-01
#> 98   97 9.999996e-01
#> 99   98 9.999994e-01
#> 100  99 9.999989e-01
#> 101 100 9.999981e-01
#> 102 101 9.999969e-01
#> 103 102 9.999948e-01
#> 104 103 9.999916e-01
#> 105 104 9.999866e-01
#> 106 105 9.999789e-01
#> 107 106 9.999672e-01
#> 108 107 9.999497e-01
#> 109 108 9.999237e-01
#> 110 109 9.998857e-01
#> 111 110 9.998309e-01
#> 112 111 9.997528e-01
#> 113 112 9.996429e-01
#> 114 113 9.994899e-01
#> 115 114 9.992797e-01
#> 116 115 9.989940e-01
#> 117 116 9.986101e-01
#> 118 117 9.981000e-01
#> 119 118 9.974296e-01
#> 120 119 9.965579e-01
#> 121 120 9.954366e-01
#> 122 121 9.940090e-01
#> 123 122 9.922100e-01
#> 124 123 9.899657e-01
#> 125 124 9.871937e-01
```

```
#> 126 125 9.838030e-01
#> 127 126 9.796951e-01
#> 128 127 9.747652e-01
#> 129 128 9.689035e-01
#> 130 129 9.619977e-01
#> 131 130 9.539345e-01
#> 132 131 9.446032e-01
#> 133 132 9.338985e-01
#> 134 133 9.217234e-01
#> 135 134 9.079929e-01
#> 136 135 8.926372e-01
#> 137 136 8.756045e-01
#> 138 137 8.568643e-01
#> 139 138 8.364097e-01
#> 140 139 8.142587e-01
#> 141 140 7.904564e-01
#> 142 141 7.650744e-01
#> 143 142 7.382114e-01
#> 144 143 7.099916e-01
#> 145 144 6.805631e-01
#> 146 145 6.500952e-01
#> 147 146 6.187753e-01
#> 148 147 5.868054e-01
#> 149 148 5.543975e-01
#> 150 149 5.217697e-01
#> 151 150 4.891418e-01
#> 152 151 4.567302e-01
#> 153 152 4.247447e-01
#> 154 153 3.933837e-01
#> 155 154 3.628314e-01
#> 156 155 3.332543e-01
#> 157 156 3.047996e-01
#> 158 157 2.775927e-01
#> 159 158 2.517365e-01
#> 160 159 2.273111e-01
#> 161 160 2.043738e-01
#> 162 161 1.829596e-01
#> 163 162 1.630826e-01
#> 164 163 1.447376e-01
#> 165 164 1.279016e-01
#> 166 165 1.125365e-01
#> 167 166 9.859069e-02
#> 168 167 8.600170e-02
#> 169 168 7.469845e-02
#> 170 169 6.460334e-02
#> 171 170 5.563443e-02
#> 172 171 4.770729e-02
#> 173 172 4.073672e-02
#> 174 173 3.463825e-02
#> 175 174 2.932942e-02
#> 176 175 2.473080e-02
#> 177 176 2.076681e-02
#> 178 177 1.736636e-02
```

```
#> 179 178 1.446325e-02
#> 180 179 1.199644e-02
#> 181 180 9.910119e-03
#> 182 181 8.153750e-03
#> 183 182 6.681894e-03
#> 184 183 5.454028e-03
#> 185 184 4.434278e-03
#> 186 185 3.591107e-03
#> 187 186 2.896984e-03
#> 188 187 2.328030e-03
#> 189 188 1.863666e-03
#> 190 189 1.486269e-03
#> 191 190 1.180835e-03
#> 192 191 9.346654e-04
#> 193 192 7.370732e-04
#> 194 193 5.791157e-04
#> 195 194 4.533494e-04
#> 196 195 3.536113e-04
#> 197 196 2.748258e-04
#> 198 197 2.128332e-04
#> 199 198 1.642421e-04
#> 200 199 1.263005e-04
#> 201 200 9.678622e-05
write.csv(test, 'test.csv')
```

```
```

```
test <- 0
for (i in 0:143) {
    test <- test + (exp(-140) * 140^i)/(factorial(i))
    print((exp(-140) * 140^i)/(factorial(i)))
}
#> [1] 1.58042e-61
#> [1] 2.212588e-59
#> [1] 1.548812e-57
#> [1] 7.227788e-56
#> [1] 2.529726e-54
#> [1] 7.083232e-53
#> [1] 1.652754e-51
#> [1] 3.305508e-50
#> [1] 5.784639e-49
#> [1] 8.998328e-48
#> [1] 1.259766e-46
#> [1] 1.603338e-45
#> [1] 1.870562e-44
#> [1] 2.014451e-43
#> [1] 2.014451e-42
#> [1] 1.880154e-41
#> [1] 1.645135e-40
#> [1] 1.354817e-39
#> [1] 1.053747e-38
#> [1] 7.764448e-38
#> [1] 5.435114e-37
#> [1] 3.623409e-36
#> [1] 2.305806e-35
```

```
#> [1] 1.403534e-34
#> [1] 8.187281e-34
#> [1] 4.584878e-33
#> [1] 2.46878e-32
#> [1] 1.280108e-31
#> [1] 6.400541e-31
#> [1] 3.089917e-30
#> [1] 1.441961e-29
#> [1] 6.512082e-29
#> [1] 2.849036e-28
#> [1] 1.208682e-27
#> [1] 4.976926e-27
#> [1] 1.99077e-26
#> [1] 7.741884e-26
#> [1] 2.929362e-25
#> [1] 1.079238e-24
#> [1] 3.874189e-24
#> [1] 1.355966e-23
#> [1] 4.630129e-23
#> [1] 1.543376e-22
#> [1] 5.024946e-22
#> [1] 1.598846e-21
#> [1] 4.974189e-21
#> [1] 1.513884e-20
#> [1] 4.50944e-20
#> [1] 1.315253e-19
#> [1] 3.757867e-19
#> [1] 1.052203e-18
#> [1] 2.8884e-18
#> [1] 7.776461e-18
#> [1] 2.05416e-17
#> [1] 5.325599e-17
#> [1] 1.355607e-16
#> [1] 3.389017e-16
#> [1] 8.323902e-16
#> [1] 2.009218e-15
#> [1] 4.767635e-15
#> [1] 1.112448e-14
#> [1] 2.55316e-14
#> [1] 5.7652e-14
#> [1] 1.281156e-13
#> [1] 2.802528e-13
#> [1] 6.036214e-13
#> [1] 1.280409e-12
#> [1] 2.675481e-12
#> [1] 5.508344e-12
#> [1] 1.117635e-11
#> [1] 2.23527e-11
#> [1] 4.407575e-11
#> [1] 8.570284e-11
#> [1] 1.643616e-10
#> [1] 3.109544e-10
#> [1] 5.804482e-10
```

```
#> [1] 1.069247e-09
#> [1] 1.944085e-09
#> [1] 3.489383e-09
#> [1] 6.183717e-09
#> [1] 1.082151e-08
#> [1] 1.870384e-08
#> [1] 3.193338e-08
#> [1] 5.386353e-08
#> [1] 8.977255e-08
#> [1] 1.478607e-07
#> [1] 2.407034e-07
#> [1] 3.873388e-07
#> [1] 6.162209e-07
#> [1] 9.693362e-07
#> [1] 1.507856e-06
#> [1] 2.319779e-06
#> [1] 3.530098e-06
#> [1] 5.314127e-06
#> [1] 7.914657e-06
#> [1] 1.16637e-05
#> [1] 1.700957e-05
#> [1] 2.454989e-05
#> [1] 3.507128e-05
#> [1] 4.959574e-05
#> [1] 6.943404e-05
#> [1] 9.624521e-05
#> [1] 0.0001321013
#> [1] 0.0001795551
#> [1] 0.0002417088
#> [1] 0.0003222784
#> [1] 0.0004256507
#> [1] 0.0005569262
#> [1] 0.0007219414
#> [1] 0.0009272641
#> [1] 0.001180154
#> [1] 0.001488483
#> [1] 0.001860604
#> [1] 0.002305173
#> [1] 0.002830914
#> [1] 0.00344633
#> [1] 0.004159364
#> [1] 0.004977016
#> [1] 0.005904935
#> [1] 0.006946982
#> [1] 0.008104812
#> [1] 0.009377469
#> [1] 0.01076103
#> [1] 0.01224833
#> [1] 0.01382876
#> [1] 0.01548821
#> [1] 0.01720912
#> [1] 0.01897068
#> [1] 0.02074918
```

```
#> [1] 0.02251849
#> [1] 0.02425069
#> [1] 0.02591676
#> [1] 0.02748748
#> [1] 0.02893419
#> [1] 0.03022975
#> [1] 0.03134937
#> [1] 0.03227141
#> [1] 0.03297808
#> [1] 0.03345602
#> [1] 0.03369671
#> [1] 0.03369671
#> [1] 0.03345773
#> [1] 0.03298649
#> [1] 0.03229447
test
#> [1] 0.621196
```

Problem 3

(a)

$$\begin{pmatrix} -\lambda & \lambda & 0 & 0 & \dots \\ \mu & -(\lambda+\mu) & \lambda & 0 & \dots \\ 0 & 2\mu & -(2\mu+\lambda) & \lambda & \dots \\ & \vdots & & \ddots & \end{pmatrix}$$

(b)

We solve the following system:

$$\begin{cases} \sum_{i=1}^{\infty} \pi_i = 1 \\ \lambda\pi_0 = \mu\pi_1 \\ \lambda\pi_1 = 2\mu\pi_2 \\ \vdots \\ \lambda\pi_{n-1} = 2\mu\pi_n \\ \vdots \end{cases}$$

First we have:

$\pi_1 = \frac{\lambda\pi_0}{\mu}$
$\pi_2 = \frac{\lambda^2\pi_0}{2\mu^2}$
$\pi_3 = \frac{\lambda^3\pi_0}{2^2\mu^3}$
$\vdots$
$\pi_n = \frac{\lambda^n\pi_0}{2^{n-1}\mu^n}$
$\vdots$

Then:

$\sum_{i=0}^{\infty} \pi_i = \pi_0 + \frac{\lambda\pi_0}{\mu} + \frac{\lambda^2\pi_0}{2\mu^2} + \dots + \frac{\lambda^n\pi_0}{2^{n-1}\mu^n} + \dots = 1$

And so factoring π_0 we get:

$\pi_0(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2} + \dots) = \pi_0(1 + \sum_{i=1}^{\infty} \frac{1}{2^{i-1}}(\frac{\lambda}{\mu})^i)$

Then multiplying $\frac{2}{2}$ to the summation:

$\pi_0(1 + \frac{2}{2}\sum_{i=1}^{\infty} \frac{1}{2^{i-1}}(\frac{\lambda}{\mu})^i)$

$= \pi_0(2\sum_{i=0}^{\infty}(\frac{\lambda}{2\mu})^i - 1)$

$\pi_0(2(\frac{1}{1-\frac{\lambda}{2\mu}}) - 1) = 1$

$\pi_0 = \frac{1}{2(\frac{1}{1-\frac{\lambda}{2\mu}})-1}$

$\vdots$

$\pi_n = \frac{\lambda^n}{2^{n-1}\mu_n\pi_0}\frac{1}{2(\frac{1}{1-\frac{\lambda}{2\mu}})-1}$

finally:

$\pi_n = \frac{1}{2^{n-1}}(\frac{\lambda}{\mu})^n\pi_0$