we define 8:00 as $t=0$, 50%

$$\lambda(t) = \begin{cases} \frac{100}{} & \overbrace{0 \le t \le 1/2}^{1\text{ min}} \\ 600t - 200 & 1/2 < t \le 3/4 \quad 15 \\ 400t - 50 & 3/4 < t \le 1 \quad 15 \\ -500t + 850 & 1 < t \le 1.5 \quad 30 \end{cases}$$

$\underline{\underline{P}}$ pois($\overset{150}{X}$, lambda = ?) = 0.8 ✓

$$E[N(t)] = \begin{cases} \int_0^t 100dt = \\ 100t & 0 \le t \le 1/2 \\ \int_{1/2}^t 600t - 200dt + 50 = \\ 300(t^2 - 1/4) - 200(t - 1/2) + 50 & 1/2 < t \le 3/4 \\ \int_{3/4}^t 400t - 50dt + 93.75 = \\ 25(8t^2 - 2t - 3) + 93.75 & 3/4 < t \le 1 \\ \int_1^t -500t + 850dt + 168.75 = \\ -50(5t^2 - 17t + 12) + 168.75 & 1 < t \le 1.5 \end{cases}$$

$$25(8t^2 - 2t - 3) + 93.75 - 93.06 = 0$$

$$t_1 = -0.497261$$

$$\boxed{t_2 = 0.747261}$$

$$E[W(t)] = \begin{cases} 100t & 0 \le t \le 1/2 \\ 300(t^2 - 1/4) - 200(t - 1/2) + 50 & 1/2 < t \le 3/4 \\ 25(8t^2 - 2t - 3) + 93.75 & 3/4 < t < 0.94468 \\ 150 & t \ge 0.94468 \end{cases}$$

$$\frac{e^{-\lambda} \lambda^k}{k!}$$

$$\frac{e^{-\lambda} \lambda^1}{1!} + \dots + \frac{e^{-\lambda} \lambda^{149}}{149!} = 0.8.$$

$$\sum_{0}^{149} p(x = k) = 0.8 \qquad \underline{p(x \le 150)}$$

$$\sum_{k=0}^{149} \frac{e^{-\lambda} \lambda^k}{k!} = 1$$

λ?

1.609438

$K = 1,149$

$$0.2 \quad \sum_{K=1}^{149} \frac{e^{-\lambda} \cdot \lambda^{K}}{K!} - 0.8 = 0$$

✓

$1,6043\ldots$

☆ 3 a

$$Q = \begin{bmatrix} -\lambda & \lambda & 0 & 0 & \cdots \\ \mu & -(\lambda+\mu) & \lambda & 0 & \cdots \\ 0 & 2\mu & -(2\mu+\lambda) & \lambda & \cdots \\ & & \ddots & \ddots & \ddots \end{bmatrix}$$

3b

$$\pi Q = 0$$

$$\begin{cases} \sum_{i=1}^{\infty} \pi_i = 1 \\ \lambda \pi_0 = \mu \pi_1 \\ \lambda \pi_1 = 2\mu \pi_2 \\ \vdots \\ \lambda \pi_{n-1} = 2\mu \pi_n \\ \vdots \end{cases}$$

$$\pi_1 = \frac{\lambda \pi_0}{\mu}, \quad \pi_2 = \frac{\lambda^2 \pi_0}{2\mu^2}$$

$$\pi_3 = \frac{\lambda^3 \pi_0}{2^2 \mu^3}, \ldots, \pi_n = \frac{\lambda^n \pi_0}{2^{n-1} \mu^n}$$

$$\sum \pi_i = \pi_0 + \frac{\lambda \pi_0}{\mu} + \frac{\lambda^2 \pi_0}{2\mu^2} + \cdots$$

$$\cdots + \frac{\lambda^n \pi_0}{2^{n-1}\mu^n} + \cdots$$

$$= 1$$

$$\pi_0 \left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2} + \cdots \right)$$

$$= \pi_0 \left(1 + \sum_{i=1}^{\infty} \frac{1}{2^{i-1}} \left(\frac{\lambda}{\mu} \right)^i \right)$$

$$\pi_0 \left(1 + \frac{2}{2} \sum_{i=1}^{\infty} \frac{1}{2^{i-1}} \left(\frac{\lambda}{\mu} \right)^i \right) =$$

$$\pi_0 \left(2 \sum_{i=0}^{\infty} \left(\frac{\lambda}{2\mu} \right)^i - 1 \right) =$$

$$\pi_0 \left(2 \left(\frac{1}{1 - \frac{\lambda}{2\mu}} \right) - 1 \right) = 1$$

$$\pi_0 = \frac{1}{2\left(\frac{1}{1-\frac{\lambda}{2\mu}}\right) - 1}$$

$$\pi_1 = \frac{\lambda}{\mu}\pi_0 = \frac{\lambda}{\mu}\left(\frac{1}{2\left(\frac{1}{1-\frac{\lambda}{2\mu}}\right) - 1}\right)$$

$$\pi_2 = \frac{\lambda^2}{2\mu^2}\left(\frac{1}{2\left(\frac{1}{1-\frac{\lambda}{2\mu}}\right) - 1}\right)$$

$$\vdots$$

$$\pi_n = \frac{\lambda^n}{2^{n-1}\mu^n}\overbrace{\left(\frac{1}{2\left(\frac{1}{1-\frac{\lambda}{2\mu}}\right) - 1}\right)}^{\text{el coso } = \pi_0}$$

$$\pi_n = \frac{1}{2^{n-1}} \cdot \left(\frac{\lambda}{\mu}\right)^n \pi_0$$

the infinite sum converges when

$$\left|\frac{\lambda}{2\mu}\right| < 1$$

in which case the stat. dist. p exists

3 b

$$L = \sum_{n=0}^{\infty} \pi_n \cdot n$$

Si $n = \infty$

$$\sum_{i=0}^{n-1} i a^i = \frac{a - \cancelto{0}{na^n} + \cancelto{0}{(n-1)a^{n+1}}}{(1-a)^2}$$

$$L = \pi_0 \left[1 + \sum_{n=0}^{\infty} \left(\frac{\lambda}{\mu}\right)^n \cdot \frac{n}{2^{n-1}} \right]$$

$$L = \pi_0 \left[1 + 2 \sum_{n=0}^{\infty} \left(\frac{\lambda}{2\mu}\right)^n \cdot n \right]$$

$$L = \pi_0 \left[1 + 2 \left(\frac{\frac{\lambda}{2\mu}}{\left(1 - \frac{\lambda}{2\mu}\right)^2} \right) \right]$$

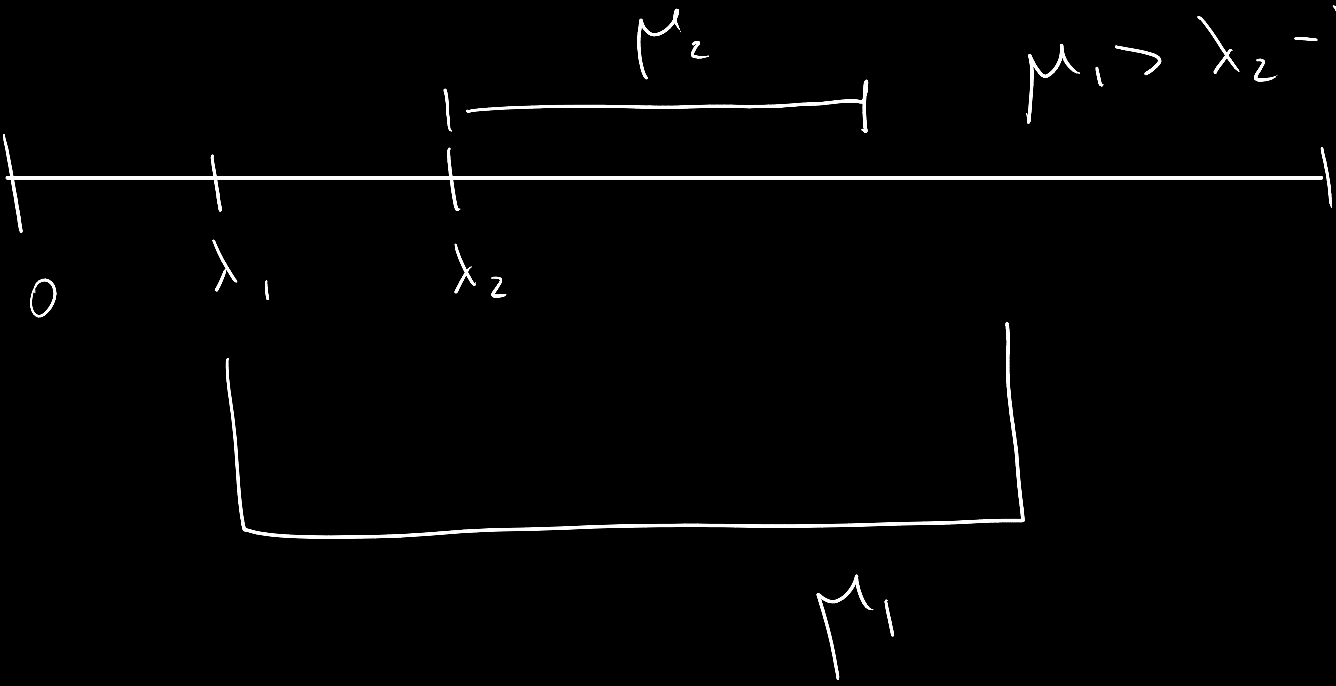
3c

μ_i = tiempo de atención de i

λ_i = tiempo de espera en la cola de i

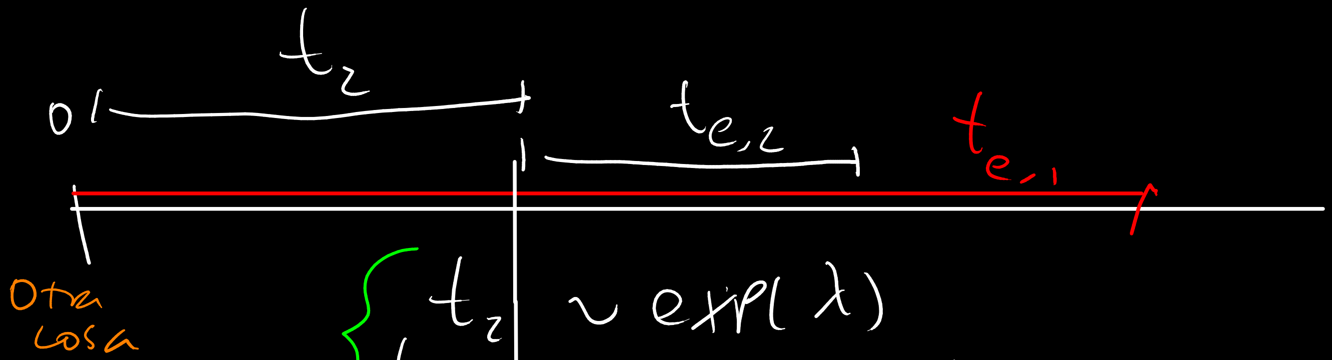
$\lambda_2 > \lambda_1$

$\mu_1 > \lambda_2 - \lambda_1 + \mu_2$



$$P(\mu_1 > \lambda_2 - \lambda_1 + \mu_2 \mid \lambda_2 > \lambda_1)$$

$$= \frac{P(\mu_1 > \lambda_2 - \lambda_1 + \mu_2) \cap P(\lambda_2 > \lambda_1)}{P(\lambda_2 > \lambda_1)}$$



$$\begin{cases} t_2 \sim \exp(\lambda) \\ t_{e,1}, t_{e,2} \sim \exp(\mu) \end{cases}$$

$$P(t_{e,1} > t_2 + t_{e,2})$$

$$1 - \mathrm{pexp}(\mu + \lambda,\ \mathrm{rate} = \underbrace{\mu}_{\text{incid/tiempo}})$$

$$Q = \begin{bmatrix} -\lambda & \lambda & 0 & 0 & \cdots \\ \mu & -(\lambda+\mu) & \lambda & 0 & \cdots \\ 0 & 2\mu & -(2\mu+\lambda) & \lambda & \cdots \\ & & \vdots & \ddots & \ddots \end{bmatrix}$$

$$\pi Q = 0$$

$$\begin{cases} \sum_{i=1}^{n} \pi_i = 1 \\ \lambda \pi_0 = \mu \pi_1 \\ \lambda \pi_1 = 2\mu \pi_2 \\ \vdots \\ \lambda \pi_{n-1} = 2\mu \pi_n \end{cases}$$