

# Week 1 exercises

Daniel Alonso

November 18th, 2020

Importing libraries

```
library(ggplot2)
library(foreach)
library(dplyr)
```

## Exercise 1

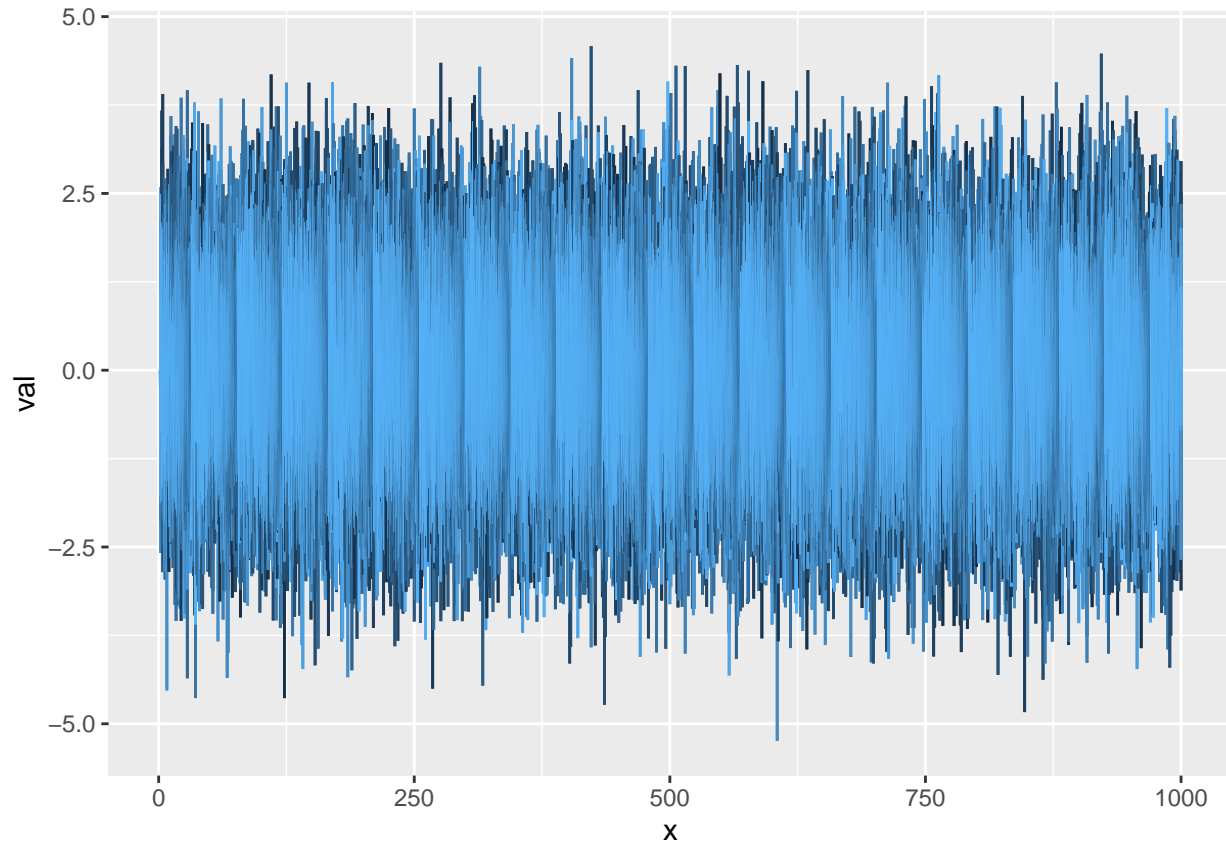
Simulating 100 trajectories of length  $n = 1000$  for  $X$  and  $Y$ .

```
Traj_X <- data.frame()
Traj_Y <- data.frame()
for (k in 1:100) {
  X <- data.frame(x=1:1000, val=rep(0,1000), run=rep(k,1000))
  Y <- data.frame(x=1:1000, val=rep(0,1000), run=rep(k,1000))
  X_r <- rep(0,1000)
  Y_r <- rep(0,1000)
  for (i in 1:1000) {
    if (i > 1) {
      X_r[i] <- 0.5*X_r[i-1]+rnorm(1)
      Y_r[i] <- 2*Y_r[i-1]+rnorm(1)
    }
  }
  X$val <- X_r
  Y$val <- Y_r
  Traj_X <- rbind(Traj_X, X)
  Traj_Y <- rbind(Traj_Y, Y)
}
```

## a - Plotting simulated trajectories

### Trajectories X

```
ggplot(data = Traj_X, aes(x=x, y=val)) +  
  geom_line(aes(colour=run), show.legend=FALSE)
```



### Trajectories Y

The values of Y are too large for ggplot to show.

## b - Use the simulated trajectories to estimate the mean and the covariance functions

```
mean_X = rep(0,100)  
mean_Y = rep(0,100)  
for (i in 1:100) {  
  x = Traj_X %>% filter(run == i) %>% select(val)  
  y = Traj_Y %>% filter(run == i) %>% select(val)  
  mean_X[i] = mean(x$val)  
  mean_Y[i] = mean(y$val)  
}
```

The mean of all trajectories of X is the following:

```
mean(mean_X)
```

```
## [1] -0.006578448
```

The mean of all trajectories of Y is the following:

```
mean(mean_Y)
```

```
## [1] 5.338702e+296
```

The mean of the covariances of all combinations of trajectories is the following:

```
covs = rep(0,100*100)
cnt = 0
for (i in 1:100) {
  for (j in 1:100) {
    x = Traj_X %>% filter(run == i) %>% select(val)
    y = Traj_Y %>% filter(run == j) %>% select(val)
    cnt = cnt + 1
    covs[cnt] = cov(x,y)
  }
}
mean(covs)
```

```
## [1] 1.260156e+295
```

**c - Is the process stationary? Is it weakly stationary?**

**d - If the process is weakly stationary, use the function `acf` to display the autocorrelation function and compare with your own estimate**