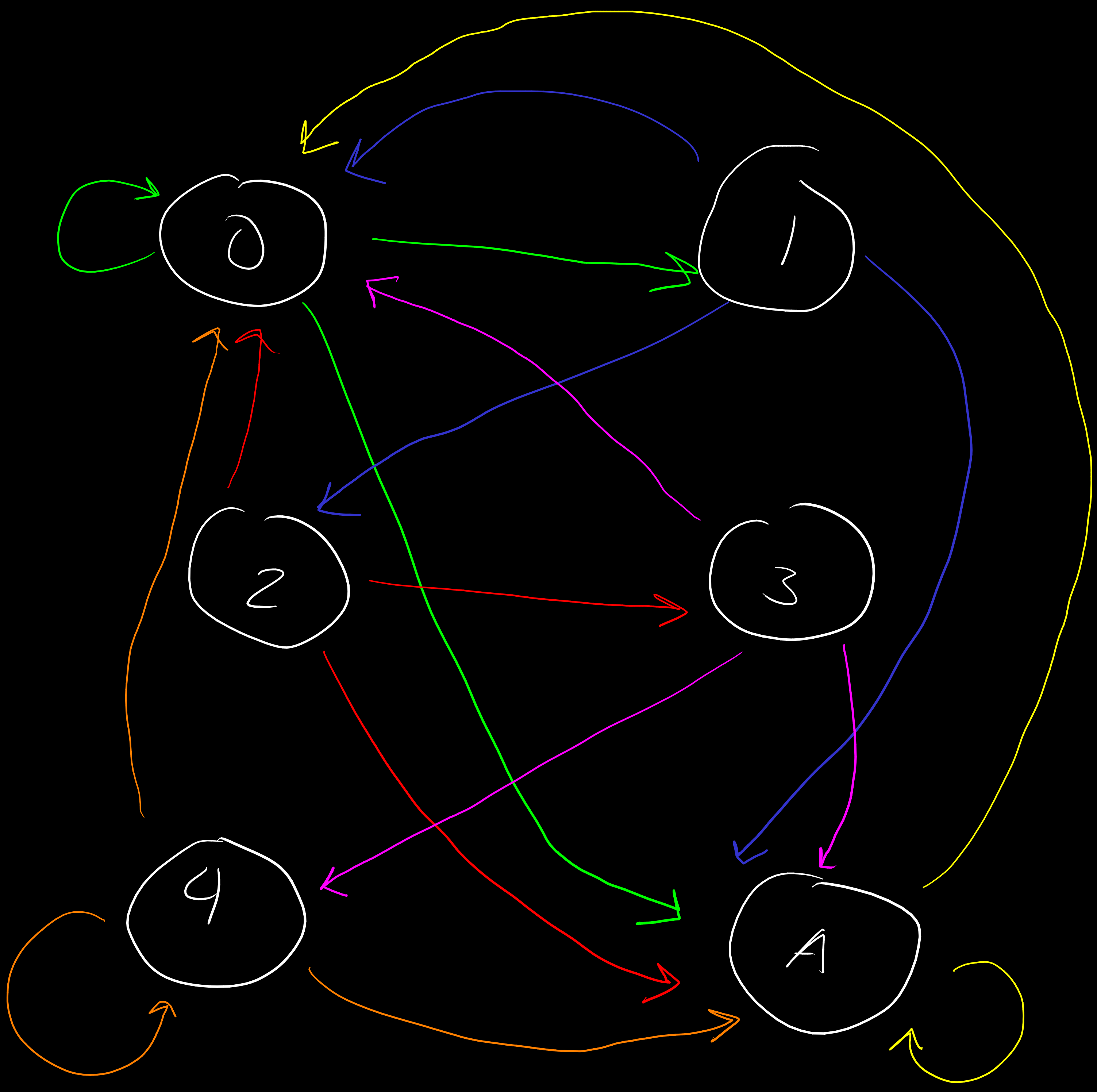
State	the wold
1
2
3



$P_{i0} = p$ $\qquad\qquad\qquad\qquad$ $\mathbb{N} \cup \{0\}$

$P_{i,i+1} = 1 - p$

$P_{ij} = 0 \quad \text{if } j \neq 0, i+1 \;, \forall i \geq 0$

$$
\begin{array}{cc|c}
 & & j \\
 & & \begin{array}{ccccc} 0 & 1 & 2 & 3 & \cdots \end{array} \\
i & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ \cdot \end{array} &
\begin{bmatrix}
p & 1-p & 0 & 0 & \cdots \\
p & 0 & 1-p & 0 & \cdots \\
p & 0 & 0 & 1-p & \cdots \\
p & 0 & 0 & \cdots & \\
\vdots & & \vdots & & \vdots \\
p & \cdots & \cdots & \cdots & \cdots
\end{bmatrix}
\end{array}
$$

$p \in (0,1)$

$$\begin{cases} \sum_{i=0}^{n} p_{i0}\pi_i = \pi_1 \\ \sum_{i=1}^{n} \pi_i = 1 \\ (1-p)\pi_1 = \pi_2 \\ \vdots \\ (1-p)\pi_{n-2} = \pi_{n-1} \\ \vdots \end{cases}$$

$\Rightarrow \quad p\sum_{i=1}^{n} \pi_i = \pi_1$

$p = \pi_1$

$(1-p)p = \pi_2$

$(1-p)^2 p = \pi_3$

$(1-p)^{n-2} p = \pi_{n-1}$

$$p + (1-p)p + (1-p)^2 p + \dots + (1-p)^{n-1} p = \sum_{i=1}^{n} \pi_i$$

$$p \sum_{i=0} (1-p)^k = \sum_{i=1}^{n} \pi_i = 1$$

$$p \cdot \frac{1}{1-(1-p)} = 1$$

$$\begin{bmatrix} p & (1-p)p & \cdots & (1-p)^n p \end{bmatrix} \begin{bmatrix} p & 1-p & \cdots & 0 \\ \vdots & \ddots & & \\ \vdots & & \ddots & \\ & & & \ddots \end{bmatrix} = \pi$$

$$\pi \times P = \pi$$

$$\pi_1 = p^2 + (1-p)p^2 + \dots + (1-p)^n p^2$$

$$= p^2 \left(\sum_{i=1}^{n} (1-p)^i \right) = \frac{p^2}{1-(1-p)} = p$$

$$\pi_2 = p(1-p)$$

$$\pi_3 = p(1-p)^2$$

$$\vdots$$

$$\pi_n = p(1-p)^{n-1}$$