Assignment 1

Group 1

November 22th, 2020

Problem 1

a)

Markov chain criteria:

1- The probability of being in a state only depends on the previous state.

2- It's a stochastic process.

X = The chain hits state j at time n

X_n is the scenario at time n

All states have finite expected return times and are communicated with each other, also the MC is irreducible, therefore its stationary distribution is **unique**.

b)

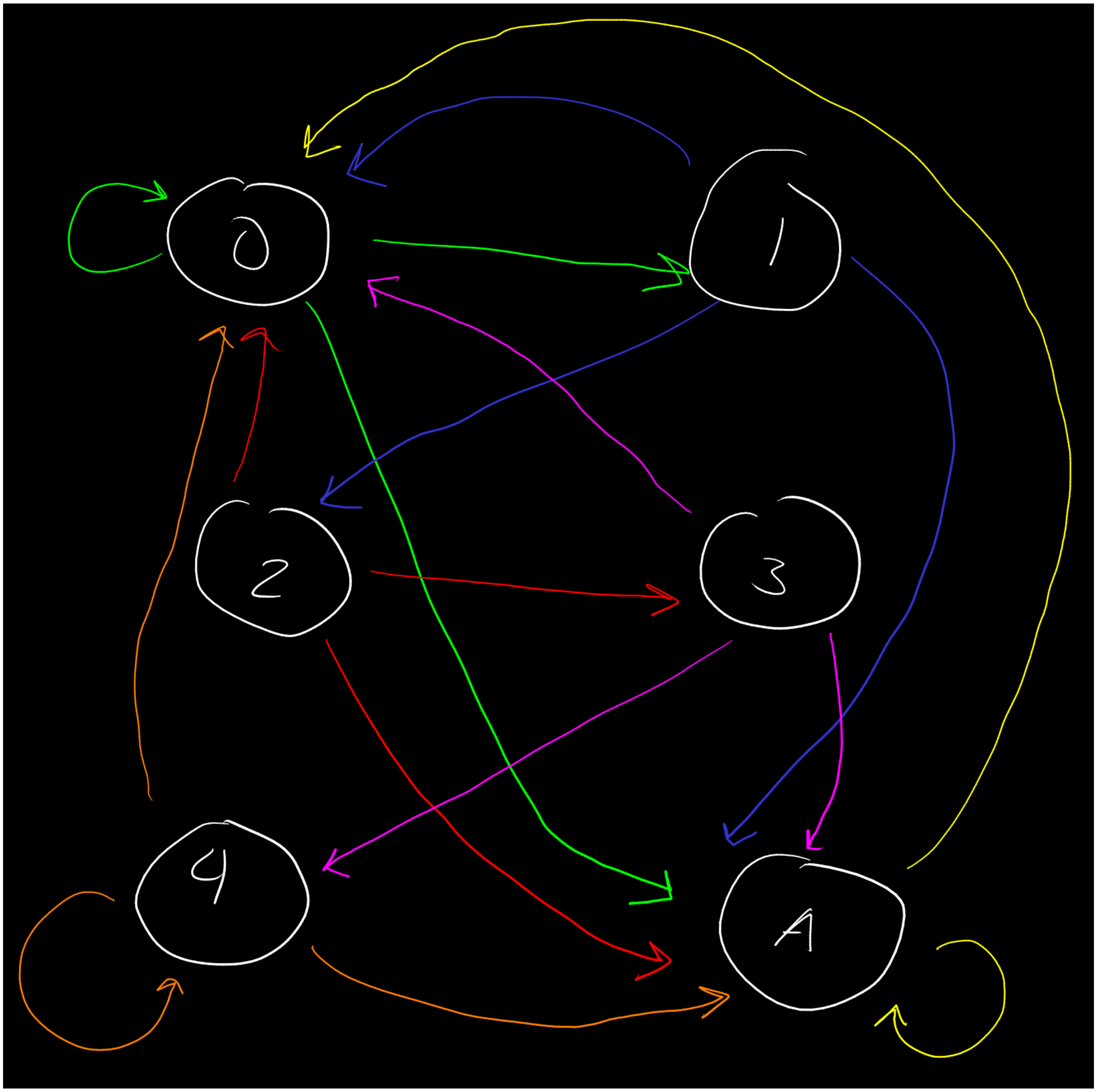

Figure 1: Graph for prob.1

Problem 2

a)

We set up the following system of equations:

$\sum_{i=0} P_{i,0}\pi_i = \pi_1$

$\sum_{i=1} \pi_i = 1$

$(1-p)\pi_1 = \pi_2 \, \ldots \, (1-p)\pi_{n-2} = \pi_{n-1} \, \ldots$

For the first equation, each $P_{i,0} = p$, therefore:

$\sum_{i=0} P_{i,0}\pi_i = \pi_1 \Rightarrow p\sum_{i=1} \pi_i = \pi_1$

$p = \pi_1$

$(1-p)p = \pi_2 \; (1-p)^2 p = \pi_3 \, \ldots \, (1-p)^{n-1} p = \pi_n \, \ldots$

Then, we get:

Because our MC is an irreducible infinite state MC, we have a unique stationary distribution π, $\pi_i = \frac{1}{\mu_i}$ and all states have expected finite return times then we have:

$E[T_i|X_0 = i] = \mu_i = \frac{1}{\pi_i}$

b)

Because it has a unique stationary distribution, it can only have one communication class (it is irreducible), all states are recurring states and there is no transient state.