Aufgabe 1

1

1.1

$f : M \to \mathbb{R}$

$$||f||_1 := \sum_{x \in M} |f(x)|$$

$$||f||_\infty := \sup_{x \in M} ||f(x)||_{\mathbb{R}} = \sup\{|f(x)| : x \in M\}$$

f(0) = 2, f(1) = -4, f(2) = 8, f(3) = -4
g(0) = 5, g(1) = 1, g(2) = 7, g(3) = -3

$$||f - g||_1 = \sum_{x \in M} |f(x) - g(x)|$$

$= |2 - 5| + | - 4 - 1| + |8 - 7| + | - 4 - (-3)|$
$= 3 + 5 + 1 + 1$
$= 10$

$$||f - g||_\infty = \sup_{x \in M} ||f(x) - g(x)||_{\mathbb{R}} = \sup\{|f(x) - g(x)| : x \in M\}$$

—f(0) - g(0)— = —2 - 5— = 3
—f(1) - g(1)— = —-4 - 1— = 5
—f(2) - g(2)— = —8 - 7— = 1
—f(3) - g(3)— = —-4 - (-3)— = 1

$$\to \sup_{x \in M} ||f(x) - g(x)||_{\mathbb{R}} = 5$$

1.2

1.2.1 Positive definite

$||f||_1 = 0 \Rightarrow f = 0$

$$||f||_1 = 0 \Leftrightarrow \sum_{x \in M} |f(x)| = 0$$

$$\Rightarrow \sum_{x \in M} |f(x)| = 0$$

$\Rightarrow |f(0)| + |f(1)| + ... + |f(x)| = 0$
$\Rightarrow f(x) = 0 \ \forall x$
$\Rightarrow f = 0$

1.2.2 Homogeneous

$||\alpha f||_1 = |\alpha| \ ||f||_1, \alpha \in \mathbb{R}$

$$||\alpha f||_1 := \sum_{x \in M} |\alpha| \ |f(x)|$$

$$\Rightarrow \sum_{x \in M} |\alpha| \ |f(x)| = |\alpha| \cdot |f(0)| + |\alpha| \cdot |f(1)| + ... + |\alpha| \cdot |f(x)$$

$\Rightarrow |\alpha| \cdot (|f(0)| + |f(1)| + ... + |f(x)|)$

$$\Rightarrow |\alpha| \cdot \sum_{x \in M} |f(x)|$$

$\Rightarrow |\alpha| \ ||f||_1$
Since α has to be positive for our usage, we can write α instead of $|\alpha|$ and thus gain: $\alpha||f||_1$

1.2.3 Triangle inequality

$||f+g||_1 \leq ||f||_1 + ||g||_1$

$-||f||_1 \leq |f(x)| \leq ||f||_1$ and $-||g||_1 \leq |g(x)| \leq ||g||_1$
$\Rightarrow -(|f(x)| + |g(x)|) \leq f(x) + g(x) \leq |f(x)| + |g(x)|$

$$\sum_{k=1}^{n}(a_k + b_k + c_k + \ldots) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k + \sum_{k=1}^{n} c_k + \ldots$$