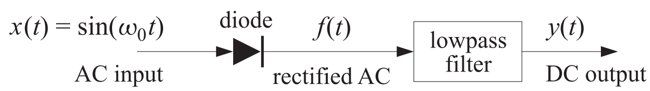
332:347 – Linear Systems Lab – Fall 2016

Lab 5 – S. J. Orfanidis

1. AC/DC half-wave rectifier/converter

A simplified AC to DC converter consists of a diode followed by a lowpass filter, such as a first-order RC filter, as shown below,



The input to the diode is the AC signal, $x(t) = \sin(\omega_0 t)$, and its output, $f(t)$, is half-wave rectified. The lowpass filter smoothes out $f(t)$, effectively producing a DC output. The diode acts as nonlinear device whose input/output relationship can be modeled by the simplified nonlinear rectfication operation,

$$f(t) = x(t)\cdot u\big[x(t)\big] = \begin{cases} x(t)\,, & \text{if } \; x(t) \ge 0 \\ 0\,, & \text{if } \; x(t) < 0 \end{cases}$$

where $u(x)$ is the unit-step function. Thus, the periodic $f(t)$ output of the diode is defined over one period, $T = 2\pi/\omega_0$, by,

$$f(t) = p(t) = \sin(\omega_0 t)\cdot u\big[\sin(\omega_0 t)\big] = \begin{cases} \sin(\omega_0 t)\,, & 0 \le t \le \frac{1}{2}T \\ 0\,, & \frac{1}{2} \le T \le T \end{cases} \tag{1}$$

with Laplace transform,

$$P(s) = \int_0^T p(t)\,e^{-st}\,dt = \int_0^{T/2} \sin(\omega_0 t)\,e^{-st}\,dt = \frac{\omega_0\,(1 + e^{-sT/2})}{s^2 + \omega_0^2} \tag{2}$$

The periodic signal $f(t)$ can be expanded in its Fourier series, with coefficients determined from the Laplace transform $P(s)$ of Eq. (2),

$$f(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t} = c_0 + \sum_{k=1}^{\infty} 2\,\mathrm{Re}\big[c_k e^{jk\omega_0 t}\big]\,, \quad c_k = \frac{1}{T}P(jk\omega_0) \tag{3}$$

The coefficients $c_0, c_{\pm 1}$ are special and can be obtained by taking the proper limits of $P(s)$,

$$P(0) = \frac{T}{\pi}\,, \quad P(\pm j\omega_0) = \frac{\pm T}{4j} \quad \Rightarrow \quad c_0 = \frac{1}{\pi}\,, \quad c_{\pm 1} = \frac{\pm 1}{4j} \tag{4}$$

In this lab, we will use a first-order lowpass filter with transfer function,

$$H(s) = \frac{1}{1 + s\tau} \tag{5}$$

where τ is the time constant of the filter ($\tau = RC$ for an RC filter), that is much longer than the period T, that is, $\tau \gg T$. The long time constant prevents the output from decaying too fast during the off-cycles of the sinusoid. The steady-state output of the filter due to the periodic input $f(t)$ is given by,

$$y_{\text{steady}}(t) = \sum_{k=-\infty}^{\infty} c_k H(jk\omega_0)\,e^{jk\omega_0 t} = c_0 H(0) + \sum_{k=1}^{\infty} 2\,\mathrm{Re}\big[c_k H(jk\omega_0)\,e^{jk\omega_0 t}\big] \tag{6}$$

The average level of the rectified DC output $y(t)$ is equal to the DC-level of the input sinusoid, that is, equal to the Fourier series coefficient c_0 of $f(t)$, since $H(0)=1$. If the input $f(t)$ is taken to be causal, then, as we discussed in the Fourier series notes (set4.pdf), the complete output $y(t)$ of the filter (5) that includes the filter transients is given as follows, for $t \geq 0$,

$$y(t) = Ae^{-t/\tau} + \sum_{k=-\infty}^{\infty} c_k H(jk\omega_0) e^{jk\omega_0 t}, \quad A = -\frac{\tau^{-1}P(-\tau^{-1})}{e^{T/\tau}-1} \tag{7}$$

Lab Procedure

(a) The M-term approximations to the Fourier series expansions of Eqs. (3), (6), and (7) are,

$$\begin{aligned} f_M(t) &= c_0 + \sum_{k=1}^{M} 2\,\mathrm{Re}\big[c_k e^{jk\omega_0 t}\big] \\ y_{M,\mathrm{steady}}(t) &= c_0 H(0) + \sum_{k=1}^{M} 2\,\mathrm{Re}\big[c_k H(jk\omega_0) e^{jk\omega_0 t}\big] \\ y_M(t) &= Ae^{-t/\tau} + c_0 H(0) + \sum_{k=1}^{M} 2\,\mathrm{Re}\big[c_k H(jk\omega_0) e^{jk\omega_0 t}\big] \end{aligned} \tag{8}$$

Compute and plot on the same graph the signals $f_M(t)$ and $y_{M,\mathrm{steady}}(t)$, over three periods, $0 \leq t \leq 3T$, for the following cases,

$$M = 10,\ 30, \quad \text{and} \quad \tau = T,\ 5T,\ 10T \tag{9}$$

You may use **meshgrid** or for-loops to evaluate the sums in Eq. (8). And, it would be useful to separate the $k=1$ term from the others.

The Gibbs ripples are minimal in this problem because $f(t)$ has no discontinuities. But the improvement in using larger M should be evident. Note also the improvement in the DC output as τ gets larger.

(b) For the cases, $M = 30$ and $\tau = 5T,\ 10T$, compute and plot on the same graph the signals $f_M(t)$, $y_{M,\mathrm{steady}}(t)$, and the exact output, $y_M(t)$, from Eq. (8), over 24 periods, $0 \leq t \leq 24T$, which are long enough to observe the transients.

(c) For the same cases and time duration as in part (b), compute the exact output $y_M(t)$ due to the input $f_M(t)$ using the function **lsim**. Do not plot the signals, but rather calculate the error norms of the outputs $y_M(t)$ computed using **lsim** and using the exact formula in Eq. (8).

2. FIR digital filter design using the Fourier series method

A length-L signal $x(n)$ is the sum of a desired signal $s(n)$ and interference $v(n)$:

$$x(n) = s(n) + v(n), \quad 0 \leq n \leq L-1$$

where

$$\begin{aligned} s(n) &= \sin(\Omega_0 n) \\ v(n) &= \sin(\Omega_1 n) + \sin(\Omega_2 n), \quad 0 \leq n \leq L-1 \end{aligned}$$

with

$$\Omega_1 = 0.1\pi, \quad \Omega_0 = 0.2\pi, \quad \Omega_2 = 0.3\pi \qquad [\text{radians/sample}]$$

In order to remove $v(n)$, the signal $x(n)$ is filtered through a bandpass FIR filter that is designed to pass the frequency Ω_0 and reject the interfering frequencies Ω_1, Ω_2. An example of such a filter of order $2M = 150$ can be designed with the Fourier series method using a Hamming window, as discussed in the Fourier series notes (set4.pdf), and has impulse response:

$$h_k = \left[0.54 + 0.46\cos\left(\frac{\pi k}{M}\right)\right] \cdot \left[\frac{\sin(\Omega_b k) - \sin(\Omega_a k)}{\pi k}\right], \quad -M \le k \le M \tag{10}$$

where $[\Omega_a, \Omega_b]$ define the effective passband of the filter. The filter can be made causal by a delay of M samples, that is, we may redefine h_n as follows, for $n = 0, 1, \ldots, 2M$,

$$h_n = \left[0.54 + 0.46\cos\left(\frac{\pi(n-M)}{M}\right)\right] \cdot \left[\frac{\sin(\Omega_b(n-M)) - \sin(\Omega_a(n-M))}{\pi(n-M)}\right] \tag{11}$$

Choose the values $\Omega_a = 0.15\pi$, $\Omega_b = 0.25\pi$ in this lab, so that Ω_0 lies within the passband, and Ω_1, Ω_2 lie in the filter's stopband. To avoid a computational issue at $n = M$, you may use MATLAB's built-in function **sinc**, which is defined as follows:

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$$

Lab Procedure

(a) Let $L = 200$. On the same graph plot $x(n)$ and $s(n)$ versus n over the interval $0 \le n \le L-1$.

(b) Filter $x(n)$ through the filter $h(n)$ using the function **filter**, and plot the filtered output $y(n)$, together with $s(n)$, for $0 \le n \le L-1$. Apart from an overall delay of M samples introduced by the filter, $y(n)$ should resemble $s(n)$ after the M initial transients.

(c) To see what happened to the interference, filter the signal $v(n)$ separately through the filter and plot the output, on the same graph with $v(n)$ itself.

(d) Calculate and plot the magnitude response of the filter over the frequency interval $0 \le \Omega \le 0.4\pi$,

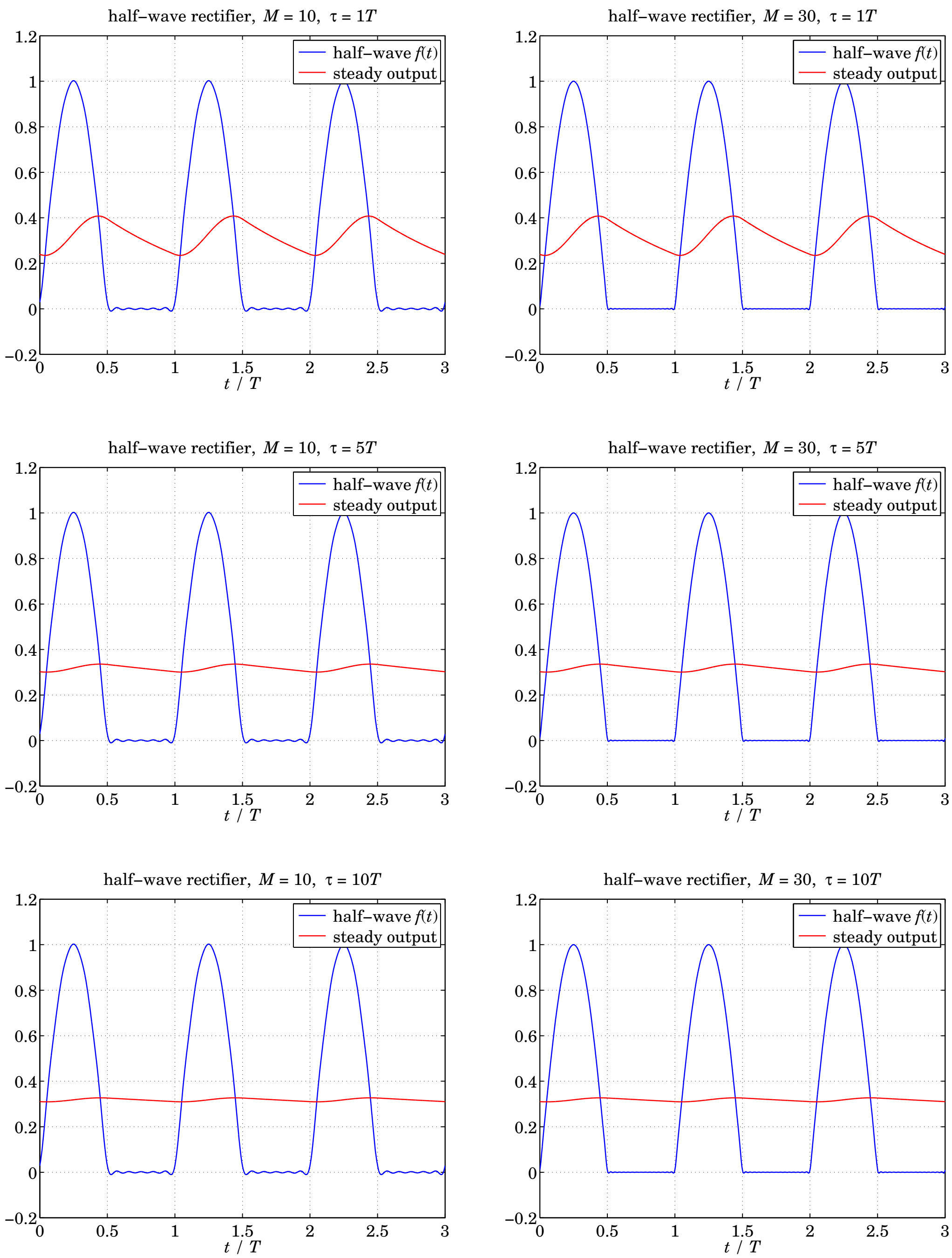
$$\left|H(e^{j\Omega})\right| = \left|\sum_{n=0}^{M} h(n)e^{-j\Omega n}\right|$$

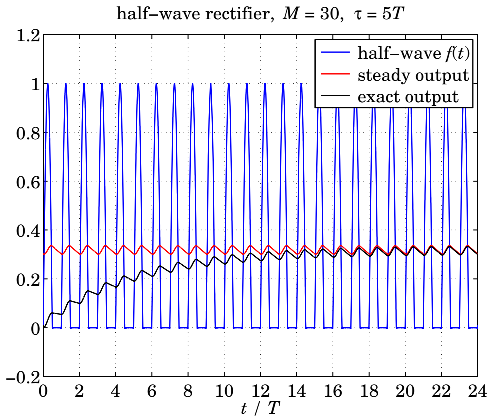
Indicate on that graph the frequencies $\Omega_1, \Omega_0, \Omega_2$. Repeat the plot of $\left|H(e^{j\Omega})\right|$ in dB units. Note that the stopband is down by about 50 dB, which is a property of the Hamming window (other windows provide other amounts of stopband attenuation).

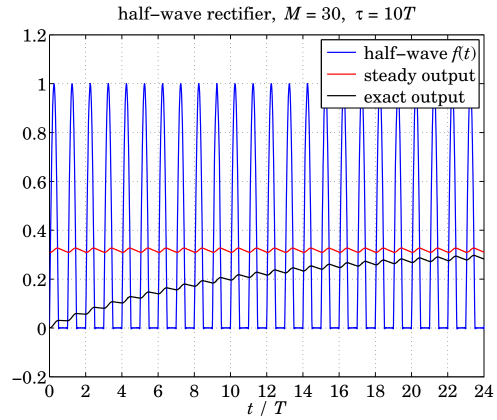
(e) Redesign the filter with $2M = 200$ and repeat parts (a)–(d). Discuss the effect of choosing a longer filter length.

(f) To observe the Gibbs ripples that arise if one uses the Fourier series without windowing, compute and plot the filter response $\left|H(e^{j\Omega})\right|$ in absolute units using rectangular weights, $w_k = 1$, for the two cases of $2M = 150$ and $2M = 200$. Observe the beneficial effect of using a non-rectangular window.

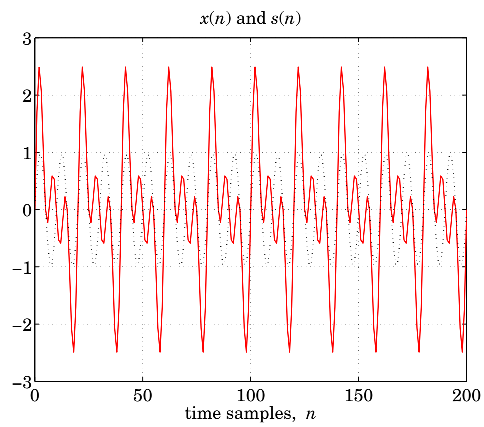
Typical Outputs – Problem 1

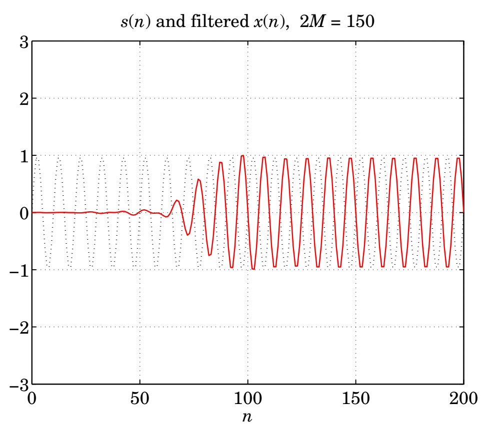


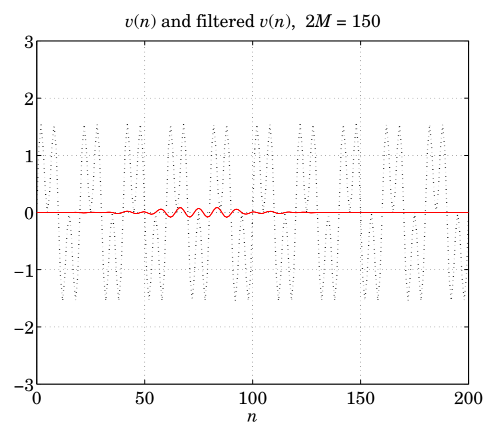


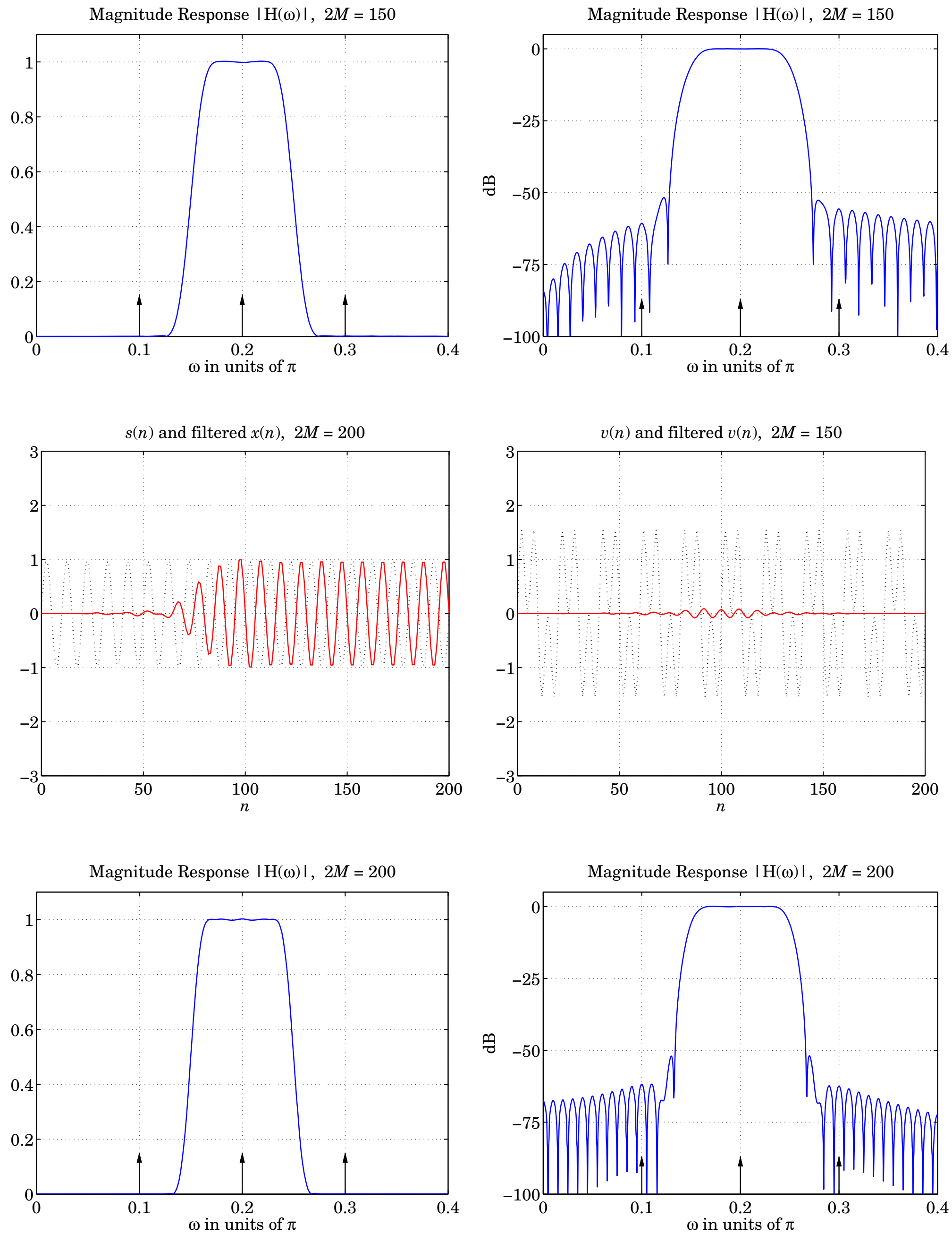


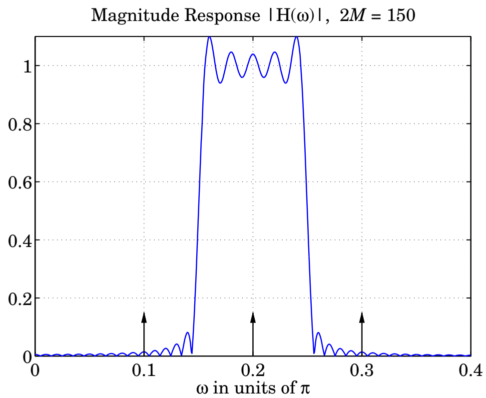
Typical Outputs – Problem 2

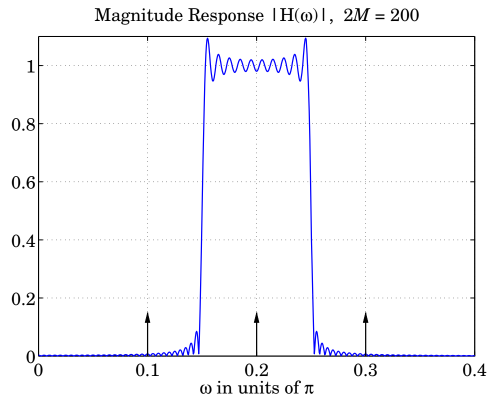







Magnitude Response |H(ω)|, 2M = 150
ω in units of π
Magnitude Response |H(ω)|, 2M = 150
dB
ω in units of π
s(n) and filtered x(n), 2M = 200
n
v(n) and filtered v(n), 2M = 150
n
Magnitude Response |H(ω)|, 2M = 200
ω in units of π
Magnitude Response |H(ω)|, 2M = 200
dB
ω in units of π


Magnitude Response |H(ω)|, 2M = 150
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
ω in units of π


Magnitude Response |H(ω)|, 2M = 200
1
0.8
0.6
0.4
0.2
0
0
0.1
0.2
0.3
0.4
ω in units of π