The Mathematics of Endurance Training

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Abstract:

Cycling like any sport, is heavily influenced by research around training and training philosophy. Data has allowed for more insights on the distribution of training intensity and the evaluation of fitness. Researchers such as Stephen Seiler or Matveyev have revealed that there are specific amounts of intensity required to optimize performance. Additionally, there are different recommended ratios of intensity relative to the time until the target race. Looking at cyclist data collected, it should be possible to develop models that can evaluate fitness and training efficacy using existing knowledge of best practices.

Introduction:

Just how common is that middle-aged person that receives the sudden impulse to run 26.2 miles uninterrupted without any prior training? A lot of the mentality around athletics and fitness is just that some people are born with it, others aren't. However, study after study demonstrate that fitness is important for everyone, and—more importantly—attainable for everyone. Years of research consistently show that improvement is based on three things: training, diet, and rest. For most, this is obvious, but the real issue is finding the specific prescriptions in these three categories to maximize the desired physical results. For a cyclist, a sport that is so largely determined by the fitness of a rider, it's important that the training, diet, and rest is prescribed perfectly to get the rider to the finish line first.

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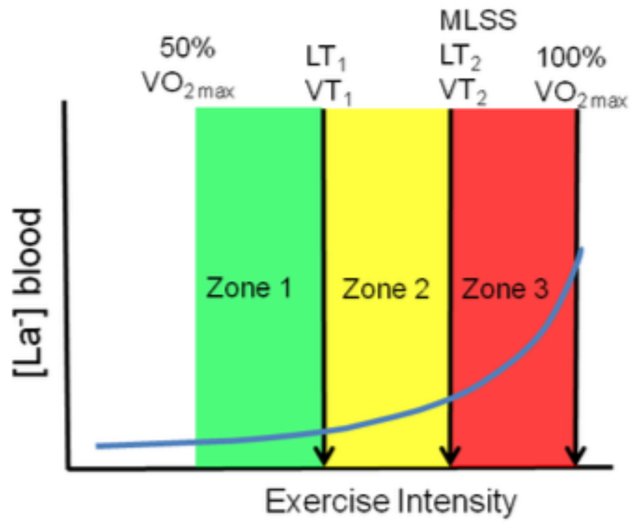
Training Background

Typically, when people first start to work towards an endurance goal, the first thing they'll do is go as hard as they can. "No pain, no gain" as Professor in Sports Science Stephen Seiler says. The issue when performing a maximal effort, however, is that if done day after day, the quality of the workout drops dramatically. It's not possible to repetitively push the limit without rest in between. Hard exercise essentially destroys the athlete's muscle fibers and depletes their glycogen—the stored carbohydrate used for energy. When exercise is done with a damaged body, it's unrealistic to expect peak results. As a result, too much intensity can lead to fatigue, burnout, plateauing, or even injury.

Stephen Seiler's work is the backbone of a lot of top cyclists' training philosophy. While he was studying in Norway, he recounts the story of watching a professional runner run down flat roads, but the moment the road began to pitch up, she would start walking. What he realized was that a lot of top athletes across every endurance discipline from running to cycling to cross-country skiing were spending most of their time exercising at low intensity so that when they performed high intensity exercise, they were relatively fresh. This idea was the basis behind the training philosophy he coined called "polarized training." According to his observations, it seemed that world class athletes spent about 80% of their training load at low-moderate intensity and the other 20% at high intensity without much work done at the "sort of hard" middle intensity. This was based off a three-zone model seen in the figure below **(1)**.



Alternative models exist for roughly approximating the different intensity zones, another such model is the five-zone model seen in the figure below.

Intensity zone	VO_2 (%max)	Heart rate (%max)	Lactate ($mmol.L^{-1}$)	Duration within zone
1	45-65	55-75	0.8-1.5	1-6 h
2	66-80	75-85	1.5-2.5	1-3 h
3	81-87	85-90	2.5-4	50-90 min
4	88-93	90-95	4-6	30-60 min
5	94-100	95-100	6-10	15-30 min

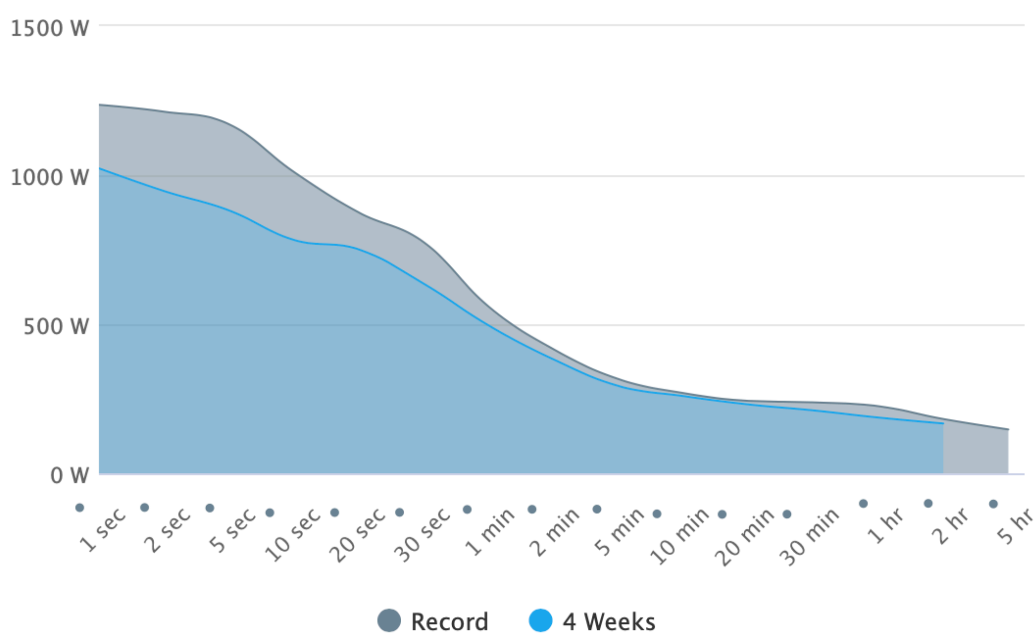
The heart rate scale is slightly simplified compared to the actual scale used by the Norwegian Olympic Federation, which is based primarily on decades of testing of cross-country skiers, biathletes, and rowers.

As with most things, the more specific one tries to be with intensity, the more complicated it becomes. The general idea is that with unlimited time and resources, the optimal training distribution is roughly 80% of the training volume spent at low intensity and the rest of the 20% spent at different degrees of high intensity. Intuitively, with a lot of time to train, the best way to utilize the time is by using all of it, and the only way to sustainably do that is if most of it is low to moderate intensity. When cyclists don’t have a lot of time (like most people that work or have other obligations), the zone distribution can vary with other approaches including sweetspot training or pyramidal training **(2).** Within the context of cycling, how is this intensity measured?

As seen in the figure above, there are multiple ways of approximating intensity. For example, researchers will evaluate how long someone could spend within the zone, how much lactate was present in their blood, or what percent of their V02 max they were working at (VO2 max being the maximum amount of oxygen the body can utilize during aerobic exercise). However, for everyday amateur cyclists, the best measures of intensity are rate of perceived exertion (RPE), heart rate, and power. Each of these measures has their flaws. For example, rate of perceived exertion could also be based on psychological factors affecting the cyclist that day, nor is it a very consistently measurable metric. Heart rate is measurable, but the issue is that it varies person to person and is easily affected by factors like stress, weather, or fatigue. Power is considered the “holy grail” of measuring intensity because it can’t be cheated. It is purely how much energy is being put into the pedals. The issue with this metric is that if an athlete is especially fatigued, their expected power output can vary greatly. For this reason, an important statistic to look at is the ratio between heart rate and power. When someone can produce more power at a lower heart rate, it essentially means their body is working less hard to expend even more energy. This is an indicator of improvement.

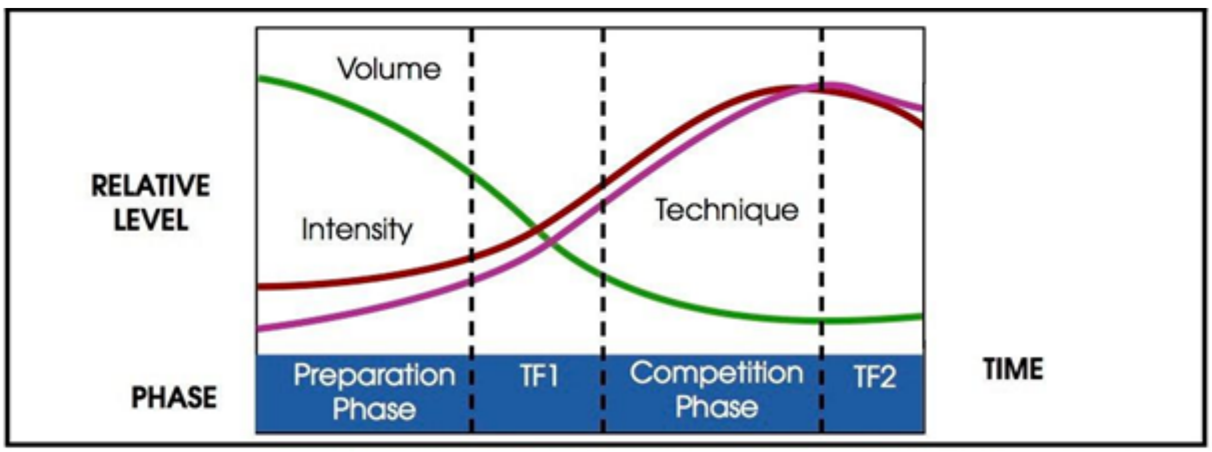
With respect to power, there are a lot of key statistics for summarizing a cyclist’s specialty and their general fitness. The main statistic most people train for is Functional Threshold Power (FTP). This is the maximum amount of power a cyclist can produce for around an hour at peak fitness. This statistic is a good indicator for race success because the higher it is, the easier it would be for a cyclist to stay involved without getting tired.

Another key statistic is peak power. Just like in running, some people are better at short, hard efforts but worse at longer ones. For these sprinters, it's important to have enough fitness to stay in the mix so they have energy for a maximal effort to the finish line in the last 20 seconds. Using all-time power records over efforts of different length, one can construct what's called a power curve. This curve indicates the maximum amount of power a cyclist can maintain for different spans of time ranging from 1 second to five hours. The figure below is an example.



Finally, within the process of training, another major consideration is what is called "periodization." Essentially, within the year of both off-season and racing, it's important to tailor one's training to specific parts of the year. This periodization surprisingly applies to almost every discipline of endurance or strength related exercise.

In the offseason, the focus is maximizing low-intensity volume, which, on a high-level, helps the body prepare for harder work and increases overall bodily efficiency. This stage is called the base phase. As the racing/competition season develops, the high-volume exercises decrease and is then replaced by more intensity. This is the build phase. Finally, for the peak event, the intensity takes precedence to prepare an athlete for their one identified talent. This is called the specialty phase. The phases listed are cycling specific, however, it applies to a variety of other sports, as displayed by the Matveyev Periodization Model below **(3)**.



Applications

Given the generality of training, it should be theoretically possible to make a variety of sport-specific predictions with enough data. An issue is figuring out what to predict. For fitness, one could estimate a statistic like FTP, but this could be dependent on a variety of factors like a rider's ratio between weight and power, their height, how much they sleep, how stressed they are, what they've been eating, how old they are, whether they've been strength training, how flexible they are, or just the genetic potential encoded in their DNA. The main problem with predicting fitness, though, is that this isn't the only factor that determines race outcomes.

Among world class cyclists, most of them have very high FTPs and great fitness, but the differentiating factor could be their risk tolerance, their race strategy, technical ability, or what their specialty is. If they're a sprinter, then FTP won't be a good indicator of their race winning ability, peak power would be more important. However, FTP would still be important in the sense that it could get the sprinter to the end of the race with enough left in the tank to outdo the other sprinters in the race. For a climber, their raw FTP may not be that high, but if they're going up a hill because they weigh so much less they will have the clear advantage. Thus, predicting something like watts per kilogram might be more important.

So, what should a model try to predict? Theoretically race results are the most important, but to do this requires more information than could reasonably be available. A race could depend on weather conditions, the people racing in it, how technical the course is, or how hilly the course is. These factors, for now, are too complicated to model race success without gathering extensive amounts of data on everyone involved.

Conclusion:

While it may be hard to predict race outcomes, it's undeniable that fitness is one of the most important components in predicting race performance. One commonly asked question is "is my training plan working?". While there are a lot of useful guidelines for training, if the perfect training plan was already developed, people wouldn't be studying it anymore. Using an unsupervised model, it might be worth feeding training data into the model and seeing on an activity-to-activity basis whether then training prescriptions are leading to measurable increases in fitness.

Upon further research, it seems possible to download all a cyclist's training data, upload it into R, clean it, and then build models off it. Using this model, it would also be possible to incorporate data points such as hours of sleep, heart rate variability (an indicator of stress), or weight.

References:

1. Seiler, Stephen, director. *How "Normal People" Can Train like the Worlds Best Edurance Athletes | TED Talk*. *Stephen Seiler: How "Normal People" Can Train like the Worlds Best Edurance Athletes | TED Talk*, TED, 2 Dec. 2019, https://www.ted.com/talks/stephen_seiler_how_normal_people_can_train_like_the_worlds_best_edurance_athletes. Accessed 10 Apr. 2023.

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3. Adams, Andre. "Periodization Training Simplified: A Strategic Guide: NASM Blog." *NASM*, https://blog.nasm.org/periodization-training-simplified.