

Basis:  $(1, 5) \in S$  and  $(3, 7) \in S$

Recursive: if  $(a, b) \in S$  then  $(3a+1, 3b-7) \in S$

if  $(a, b) \in S$  and  $(c, d) \in S$  then  $(2a+d, 2b+c) \in S$

Prove for any  $(x, y) \in S$ ,  $x+4=y$

Basis:  $(1, 5) \in S$ ,  $1+4=5 \checkmark$

$(3, 7) \in S$ ,  $3+4=7 \checkmark$

Inductive step: Assume  $(a, b) \in S$  and  $(c, d) \in S$

$b = a+4$ ,  $d = c+4$  by definition

rule 1:  $(3a+1, 3b-7) \in S$   
 $= (3a+1, 3(a+4)-7) \in S$   
 $= (3a+1, 3a+5) \in S$   
 $(3a+1)+4 = 3a+5 \checkmark$

rule 2:  $(2a+d, 2b+c) \in S$   
 $= (2a+(c+4), 2(a+4)+c) \in S$   
 $= (2a+c+4, 2a+c+8) \in S$   
 $(2a+c+4)+4 = 2a+c+8 \checkmark$

$\therefore$  by SI, if  $(x, y) \in S$ , then  $x+4=y$

Part 2

based on part 1, we can say that  $(1000, 1004)$  MIGHT be in  $S$   
it does follow the rule that if  $(x, y) \in S$ , then  $x+4=y$   
However, part 1 did NOT prove that ALL sets of  $(x, y)$  (where  $x+4=y$ )  $\in S$