Exam 2 Review

Directions:

- This review supplements the lectures, class materials, and homework in exam preparation. **Use this review *in addition* to your class notes, lesson notes, exercises, and homework.**
- On both this review and on the exam, you may use a **graphing calculator** and the included probability functions; however, problems may be conceptual or ask for an expression (i.e., non-simplified answer).
- For best results, check your answers *after* fully attempting a problem.

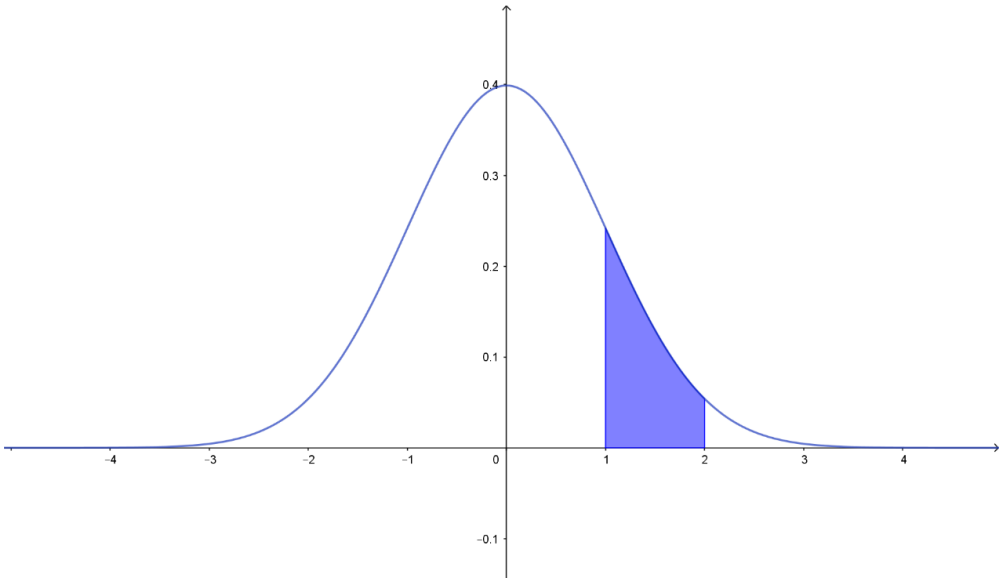
1. [PDF 1] Suppose a probability distribution has the following probability density.

$$f(x) = \begin{cases} kx & 0 \le x \le 2 \\ k(4-x) & 2 < x \le 4 \\ 0 & \text{else} \end{cases}$$

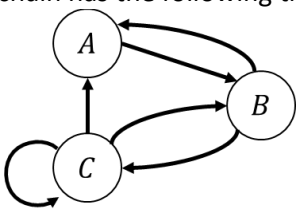
   a. Find the normalizing constant k.
   b. Sketch the probability density.
   c. What are the possible values of the experiment this distribution describes? Which values are more likely?

2. [Normal areas] Use the Normal distribution functions on a calculator to find the area (rounded to four decimal places) under the standard Normal probability density function and…
   a. …to the left of 0.8
   b. …to the left of -0.8
   c. …to the right of 1.36
   d. …to the right of -1.36

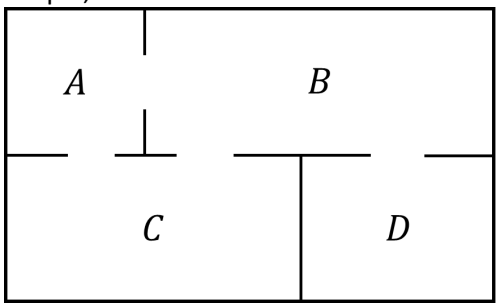
3. [Normal graph 1] The Normal probability curve is graphed below. Find the area of the shaded region.



4. [Downloading time] Suppose we wish to download several files sequentially (one after another), and the average time required for one file is 4 seconds. Assume download times for each of the files are independent and may be modeled with an Exponential distribution.
    a. Find the probability that the next file will require no more than 2 seconds.
    b. Find the probability that the next two files will require no more than 4 seconds.
    c. Compute the probability that the next three files will require at most 9 seconds.
    d. To download 100 such files, It will likely take around _____, give or take _____ or so. (Provide units!)
5. [Normal heights] Suppose for a certain population of people, the average height is 68 inches with an SD of 3 inches, and heights are approximately Normally distributed.
    a. If someone in this population is 60 inches tall, what is his height in standard units? (Round to two decimal places.)
    b. What percentage of people in this population are between 60 and 70 inches tall?
    c. What height (in inches) separates the tallest 10% of this population from the rest?
6. [Gamma CDF] Find a formula for the CDF of the Gamma probability distribution where $n = 4$.
7. [Continuity corrections]
    a. Suppose X is Binomial with $n = 100$ and $p = 0.25$. To use the Normal curve to estimate $P(20 \leq X \leq 30)$, what is the continuity correction?
    b. Suppose we want the chance of getting 37 or more heads during 50 tosses of a coin. If we use the Normal curve to estimate the probability, what is the continuity correction?
8. [Markov diagram] Suppose a Markov chain has the following transition diagram:



All possible transitions are indicated with an arrow. From any given state, each possible transition is equally likely. What proportion of time will the process spend at each state in the long run?
9. [Blue-Red sequence] Suppose a sequence of red and blue dots behaves like a Markov chain. Blue comes after red with a probability of 0.3. Red comes after blue with probability 0.4.
    a. If now there is a red dot, what is the probability of seeing a blue dot 2 entries from now?
    b. If now there is a red dot, what is the probability of seeing a blue dot 3 entries from now?
10. [Markov house] Suppose a board game takes place in a house, and spaces are divided into four rooms. A certain token moves to a different room on each turn. It can only go through a doorway, and does so randomly. (For example, from room A the token will move either to B or C randomly.)



Set up the transition matrix for the location of the token (keep the states in alphabetical order), then find the fraction of time the token will spend in each room after many turns.

11. [PDF 2] Find the normalizing constant k for the following probability density, then find the expected value and standard deviation of the probability distribution.
$$f(x) = kx^2 \quad 0 < x < 1$$

12. [PDF 3] Find the normalizing constant k for the following probability density, then find the expected value and standard deviation of the probability distribution.
$$f(x) = kx^2 \quad 0 < x < 2$$

13. [Family income] In families whose parents make above-average incomes, the children tend also to make above-average incomes, but not as much as their parents. On the other hand, in families whose parents make below-average incomes, the children tend to make more than their parents (though these children still tend to make below-average incomes). Of course there are exceptions, but supposing that this is the average tendency, what is a good explanation?

14. [Height and weight] For a certain population, a health survey yields the following summary statistics:
$$\text{Average height} \approx 68 \text{ inches} \quad \text{SD} \approx 3 \text{ inches}$$
$$\text{Average weight} \approx 170 \text{ pounds} \quad \text{SD} \approx 50 \text{ pounds}$$
$$r \approx 0.5$$
The scatterplot is an oval shaped cloud of points with no remarkable patterns or outliers, and so the regression method is appropriate to use.
    a. Someone from this population is selected randomly. What is your best guess as to his weight?
    b. If someone from this population is 64 inches tall, what is your best guess as to his weight?
    c. Estimate the average weight of people in this population who are 70 inches tall.
    d. Estimate the average height of people in this population who weigh 200 pounds.
    e. Someone from this population loses weight through diet and exercise. If he loses 30 pounds, what does the regression method predict about his height?

15. [Find the CDF] Find the CDF of the probability distribution with the following probability density:
$$f(x) = \begin{cases} 4x^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

16. [CDF] Suppose a probability distribution has the following CDF.
$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{9}x^2 & 0 < x < 3 \\ 1 & x \ge 3 \end{cases}$$
    a. Find the probability of getting a number between 1 and 2.
    b. Find the probability of getting a number less than 1.8.
    c. Find the probability of getting a number more than 2.5.
    d. Find the probability of getting a number more than 3.
    e. Find the PDF.

17. [Uniform distribution] Let $X \sim \text{Uniform}(-10, 10)$.
    a. What is the probability X will be positive?
    b. What is the probability X will be more than 7.75?
    c. What is the probability X will be a whole number?

18. [Exponential time] Suppose the time, in hours, required to complete a process is Exponential with $\lambda = 5$. We begin the process and the timer is at 0:00. (hours:minutes)
    a. What is the probability the process will complete before the timer hits 0:10?
    b. The timer just passed 0:04. Now what is the probability the process will complete before the timer hits 0:10?