

Part 1:

Hallway of length  $n$  = <sup>start red</sup> hallways of length  $n-5$  + <sup>start blue</sup> hallways of length  $n-2$

$$a_n = a_{n-5} + a_{n-2}$$

initial conditions  $a_2 = 1, a_5 = 1, a_1 = 0, a_3 = 0$

when  $n \leq 0$   $a_n = 0$  (hallway needs to have positive length)

example use  $n=12$

$$a_{12} = a_7 + a_{10}$$

$$= (a_2 + a_5) + (a_5 + a_8)$$

$$= (a_2 + a_5) + (a_5 + (a_3 + a_6))$$

$$= (a_2 + a_5) + (a_5 + (a_3 + (a_1 + a_4)))$$

$$= (a_2 + a_5) + (a_5 + (a_3 + (a_1 + (a_{-1} + a_2))))$$

$$= 1 + 1 + 1 + 0 + 0 + 0 + 1$$

$$= 4 \checkmark$$

$n=12$

using  $b = \text{blue}$   
 $r = \text{red}$

$rrb$

$brr$

$rbr$

$bbbbb$

4 options total



Part 2:

Part 2 = Part 1 - # hallways with touching reds

$$a_n = a_{n-5} + a_{n-2} - (a_{n-10} + a_{n-10})$$

$$a_n = a_{n-5} + a_{n-2} - 2a_{n-10}$$

Part 3:

using only greens and purples  $a_n = n+1$  because you can have  $0-n$  greens, and the rest purples which results in  $n+1$  options

$\therefore a_n = \text{part 1} + \text{greens/purples}$

$$a_n = a_{n-5} + a_{n-2} + n+1$$

for  $n=12$  you can have  $n+1=13$  green/purple only options

you can start red or blue and get  $a_{n-5}$  or  $a_{n-2}$  options as well

$$\therefore a_n = a_{n-5} + a_{n-2} + n+1$$

covers only green/purple, only red/blue, and combinations of all 4