CS 3305 HW 4
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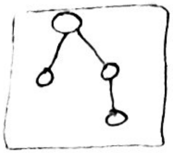
ⓐ Draw B_2



ⓑ The number of vertices of $B_k = 2^k$
the definition of binary trees is that B_k consists of 2 copies of B_{k-1}
∴ number of vertices of B_k = double vertices B_{k-1}
B_0 has 1 vertex $- 2^0 = 1$
∴ B_1 has 1×2 vertices $- 2^1 = 2$
⋮
B_k has $B_{k-1} \times 2$ vertices $- 2^k$

ⓒ Prove there are $C(k, l)$ vertices at level l of B_k
Base: B_0 contains a single node and $C(0,0) = 1$ ✓
for a tree of height $h+1$ it contains 2 trees of height h one connected to the root of the other
∴ nodes at level l of $h+1$ = nodes at level l in h + nodes at level $l-1$ in h (since the trees are offset one level)
∴ nodes at level l of $h+1 = C(h, l) + C(h, l-1)$
$= C(h+1, l)$ by Pascal's Identity
∴ there are $C(k, l)$ vertices at level l of B_k