CS 3305 Hw 3 | Drew Pulliam

Let Set S be recursively defined

Basis Step: $1 \in S$, $7 \in S$

Recursive step: if $x \in S$ and $y \in S$, then $2x + 2y \in S$

(a) prove by structural induction for every $a \in S$, $a \equiv 1 \pmod 3$.
there is some integer k, $a = 3k+1$

basis: $1 \equiv 1 \pmod 3$, $7 \equiv 1 \pmod 3$

Inductive step: assume $x \equiv 1 \pmod 3$ and $y \equiv 1 \pmod 3$

$\therefore x = 3j+1$, $y = 3k+1$ for some ints j, k (def mod 3)

$2x + 2y = 2(3j+1) + 2(3k+1)$

$= 6j + 6k + 4$

$6j + 6k \equiv 0 \pmod 3$

$4 \equiv 1 \pmod 3$

$\therefore 2x + 2y \equiv 1 \pmod 3$

by structural induction, every $a \in S$, $a \equiv 1 \pmod 3$

(b) show that $46 \in S$

$1 \in S$, $7 \in S$ (given)

$2(7) + 2(1) \in S$

$16 \in S$ (rule 1)

$2(16) + 2(7) \in S$

$46 \in S$ (rule 1)

(c) is it true that for any positive integer n, if $n \equiv 1 \pmod 3$, then $n \in S$?

no

$13 \equiv 1 \pmod 3$ $13 \notin S$

every element in S can be written as $2x + 2y$, where x and y are also in S

since all elements of S are > 0, x and y must be less than 13.

$1 \in S$, $7 \in S$ (basis)

$2(1) + 2(1) \in S$

$4 \in S$

$2(4) + 2(1) \in S$

$10 \in S$

all elements of $S < 13$ are:

$1, 4, 7, 10$

$2(7) = 14 > 13$ $\therefore$ 7 can't be used

$2(10) = 20 > 13$ $\therefore$ 10 can't be used

$2(4) + 2(4) = 16 \neq 13$

$2(4) + 2(1) = 10 \neq 13$

$2(1) + 2(1) = 4 \neq 13$

$\therefore$ there is no way to write 13 as $2x + 2y = 13$

$\therefore$ 13 is not in S