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CS 3305 Project 2

Part 1

$$a_n = a_{n-1} + \frac{1}{4}a_{n-2};\ a_0 = 0,\ a_1 = 3$$

$$r^n = r^{n-1} + \frac{1}{4}r^{n-2}$$

$$r^n - r^{n-1} - \frac{1}{4}r^{n-2} = 0$$

$$r^2 - r - \frac{1}{4} = 0$$

$$\frac{-(-1) \pm \sqrt{1 - 4(1)(-\frac{1}{4})}}{2(1)} = \frac{1 \pm \sqrt{2}}{2}$$

by quadratic formula, roots are $\frac{1+\sqrt{2}}{2}$ & $\frac{1-\sqrt{2}}{2}$

general form:

$$a_n = \alpha_1\left(\frac{1+\sqrt{2}}{2}\right)^n + \alpha_2\left(\frac{1-\sqrt{2}}{2}\right)^n$$

initial conditions $a_0 = 0,\ a_1 = 3$

$$a_0 = \alpha_1\left(\frac{1+\sqrt{2}}{2}\right)^0 + \alpha_2\left(\frac{1-\sqrt{2}}{2}\right)^0 = \alpha_1 + \alpha_2 = 0 \quad \therefore \alpha_1 = -\alpha_2$$

$$a_1 = \alpha_1\left(\frac{1+\sqrt{2}}{2}\right)^1 + \alpha_2\left(\frac{1-\sqrt{2}}{2}\right)^1 = 3$$

$$-\alpha_2\left(\frac{1+\sqrt{2}}{2}\right) + \alpha_2\left(\frac{1-\sqrt{2}}{2}\right) = 3$$

$$\alpha_2\left(\frac{1-\sqrt{2}}{2} - \frac{1+\sqrt{2}}{2}\right) = 3$$

$$\alpha_2\left(\frac{-2\sqrt{2}}{2}\right) = 3$$

$$-\sqrt{2}\,\alpha_2 = 3$$

$$\alpha_2 = -\frac{3}{\sqrt{2}} \quad \therefore \alpha_1 = \frac{3}{\sqrt{2}}$$

$$\boxed{a_n = \frac{3}{\sqrt{2}}\left(\frac{1+\sqrt{2}}{2}\right)^n - \frac{3}{\sqrt{2}}\left(\frac{1-\sqrt{2}}{2}\right)^n}$$

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Part 2

$a_n = 2^n + 3^n + 4^n$ can be written as $a_n = c_1 a_{n-1} + c_2 a_{n-2} + c_3 a_{n-3}$

using sequence $a_n = 2^n + 3^n + 4^n$

$a_0 = 3$

$a_1 = 9$

$a_2 = 29$

$a_3 = 99$

$a_4 = 353$

$a_5 = 1299$

system of equations:

$a_3 = 99 = c_1(29) + c_2(9) + c_3(3)$

$a_4 = 353 = c_1(99) + c_2(29) + c_3(9)$

$a_5 = 1299 = c_1(353) + c_2(99) + c_3(29)$

$c_1 = 9, \quad c_2 = -26, \quad c_3 = 24$

we need initial conditions:

$a_0 = 3, \ a_1 = 9, \ a_2 = 29$