CS 3305 HW2
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Consider the divisibility relation on $\mathbb{Z}^+$ (non-negative integers)

Greatest element?
assume a general x
$xk = 0$ $\therefore x \mathrel{R} 0$
$\therefore$ everything relates to 0
$\therefore$ 0 is the greatest element

maximal elements?
because 0 is greatest it is maximal and there are no other maximal elements because everything relates to 0

least element?
1 relates to everything
because $1k = b$ for any b
$\therefore$ 1 is the least element

minimal elements?
because 1 is the least element, it is also the only minimal element

least upper bound of $\{8, 40, 44\}$
least upper bound is multiple of all 3
every multiple of 40 is also a multiple of 8, $\therefore$ count in increments of 40 until finding a multiple of 44
40, 80, 120, 160, 200, 240, 280, 320, 360, 400, (440)

440 is the least upper bound

greatest lower bound of $\{8, 40, 44\}$
greatest lower bound is a factor of all 3 elements
8 only has 4 factors: 1, 2, 4, 8
$8 \not\mathrel{R} 44$, $4 \mathrel{R} 8$, $4 \mathrel{R} 40$, $4 \mathrel{R} 44$
because 4 relates to all 3 elements it is the greatest lower bound