Cognitive Modeling Homework 3

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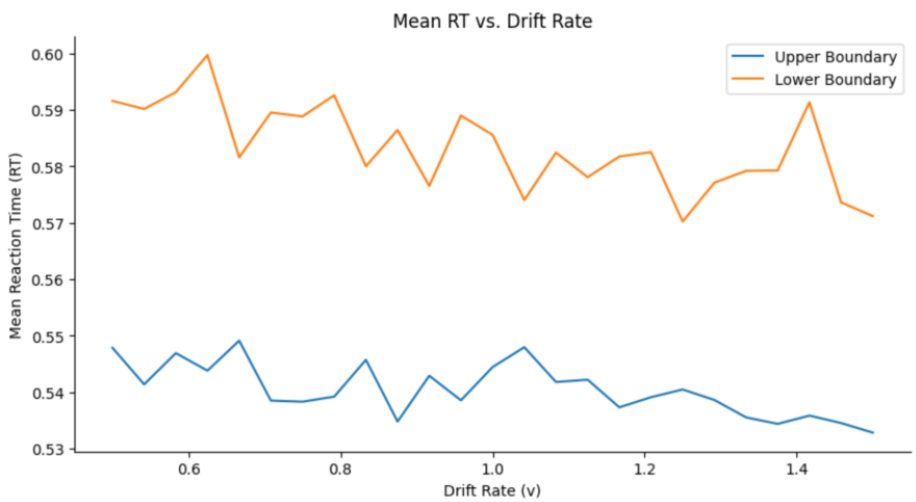
1 Problem 1: True-False Questions (5 points)

Mark all statements which are **FALSE**.

1. The solution of the stochastic integral $\int_0^T \mu \, dW_t$ is $\mu(W_T - W_0)$ and is a random variable itself. **TRUE**
2. The variance of a Wiener process with scale coefficient $\sigma = 1$ at time t is t^2. **FALSE**
3. The standard Drift-Diffusion Model (DDM) assumes that evidence about a dominant alternative accumulates in discrete chunks over time. **FALSE**
4. The first passage time distribution has a closed-form probability density function, but its density can still be evaluated only numerically. **TRUE**
5. The Euler-Maruyama method can only be used to simulate linear stochastic systems. **FALSE**
6. For any Bayesian analysis, the prior will always have a smaller variance than the posterior. **FALSE**
7. In addition to good statistical practices, experimental validation of cognitive models is crucial for ensuring construct validity. **TRUE**
8. Markov chain Monte Carlo (MCMC) methods approximate a complex posterior distribution through a simpler, yet analytically tractable, distribution. **FALSE**
9. For most Bayesian problems, the more data we collect, the less influence does the prior exert on the resulting inferences. **TRUE**
10. The effective sample size (ESS) estimated from MCMC samplers differs from the total number of samples because the samples are not independent (i.e., exhibit non-zero autocorrelation). **TRUE**

2 Problem 2:



The results show a clear difference in mean reaction times (RT) between decisions reaching the upper and lower boundaries. On average, trials that terminate at the lower boundary take longer than those that reach the upper boundary. This asymmetry suggests that the decision-making process is biased towards the upper boundary, leading to faster responses in that direction. As the drift rate increases, the mean RT for both boundaries decreases slightly, which aligns with the expectation that higher drift rates accelerate evidence accumulation, resulting in quicker decisions. However, the lower boundary RTs exhibit more variability compared to the upper boundary, likely due to the starting position bias ($= 0.6$), which places the initial evidence closer to the upper threshold. This means that, on average, decisions favoring the upper boundary require less accumulated evidence and, consequently, less time to reach a decision. The persistent mean difference between the two RT distributions reinforces the idea that the initial bias, combined with drift rate effects, makes reaching the upper boundary both more probable and faster than reaching the lower one.

Varying each parameter in the drift-diffusion model influences the reaction time (RT) distributions in distinct ways. Increasing boundary separation (a) extends the decision threshold, requiring more accumulated evidence before a decision is made. This results in a longer mean RT and greater variability, as decisions take more time and are more dispersed. Adjusting the starting point (β) changes the initial bias in evidence accumulation, shifting the likelihood of reaching one boundary over the other. When β is closer to 1, the lower boundary becomes more likely and takes longer to reach, whereas a lower β favors the upper boundary, leading to faster and more frequent upper-boundary decisions. As non-decision time (τ) increases, the mean RT rises uniformly for both boundaries since τ represents time spent on processes independent of decision-making, such as sensory encoding and motor response. However, τ

does not affect variability in RTs, as it is simply an additive delay. Finally, decreasing the time step (dt) improves the precision of the simulation but does not fundamentally alter the decision process. A smaller dt results in finer time resolution but at the cost of longer computation time, whereas a larger dt may introduce inaccuracies in RT estimates.

3 Problem 3: Prior and Posterior Variance (4 points)

Show that the following identity holds for any given prior and posterior:

$$Var[0] = \mathbb{E}[Var\,[\theta \mid y]] \,+\, Var[\mathbb{E}\,[\theta \mid y]]$$

Clarification of terms:

1. Var[θ] - Prior variance.
2. $Var\,[\mathbb{E}\,[\theta \mid y]]$ – Variance of posterior mean.
3. $\mathbb{E}\,[Var\,[\theta \mid y]]$ – Expected posterior variance.

(a) By the definition of variance,

$$Var[\theta] = \mathbb{E}[\theta^2] - (\mathbb{E}[\theta])^2.$$

(b) Using the law of iterated expectations, we express the expectation of θ in terms of the conditional expectation:

$$\mathbb{E}[\theta] = \mathbb{E}[\mathbb{E}[\theta|y]].$$

(c) Similarly, for the expectation of θ^2,

$$\mathbb{E}[\theta^2] = \mathbb{E}[\mathbb{E}[\theta^2|y]].$$

(d) Expanding the variance using its definition,

$$Var[\theta] = \mathbb{E}[\theta^2] - (\mathbb{E}[\theta])^2.$$

(e) Substituting the conditional expectation,

$$Var[\theta] = \mathbb{E}[\mathbb{E}[\theta^2|y]] - (\mathbb{E}[\mathbb{E}[\theta|y]])^2.$$

(f) From the variance identity,

$$\mathbb{E}[\theta^2|y] = Var[\theta|y] + (\mathbb{E}[\theta|y])^2.$$

(g) Taking expectations on both sides,

$$\mathbb{E}[\mathbb{E}[\theta^2|y]] = \mathbb{E}[Var[\theta|y]] + \mathbb{E}[(\mathbb{E}[\theta|y])^2].$$

(h) Thus,

$$Var[\theta] = \mathbb{E}[Var[\theta|y]] + \mathbb{E}[(\mathbb{E}[\theta|y])^2] - (\mathbb{E}[\mathbb{E}[\theta|y]])^2.$$

(i) Recognizing that the last two terms form the variance of $\mathbb{E}[\theta|y]$, we conclude:

$$Var[\theta] = \mathbb{E}[Var[\theta|y]] + Var[\mathbb{E}[\theta|y]].$$