Problem Set #1.1

Thursday, August 17, 2023 1:54 PM

1) $z = a + ib = re^{i\theta}$ $\qquad |z| =$

Maclaunin Series is Tayl

$$\boxed{f(x) = f(0) + x f'(0) + \frac{x^2}{2}}$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!}$$

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!}$$

$$e^{ix} = 1 + ix - \frac{x^2}{2} - \frac{ix^3}{3!} + x^4 + ix^5$$

$(a^2+b^2)^{1/2}$

...on Series about $x=0$

$$\boxed{\ldots f''(0) + \frac{x^3}{3!} f'''(0) + \ldots}$$

$$- \frac{x^6}{\ldots} + \ldots$$

$3!$ $4!$ $5!$

$\mathbb{R}: 1 - \frac{x^2}{2} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots$

$I: ix - \frac{ix^3}{3!} + \frac{ix^5}{5!} + \dots = i\left[x - \right.$

$$e^{ix} = \cos x + i \sin x$$ ✓✓

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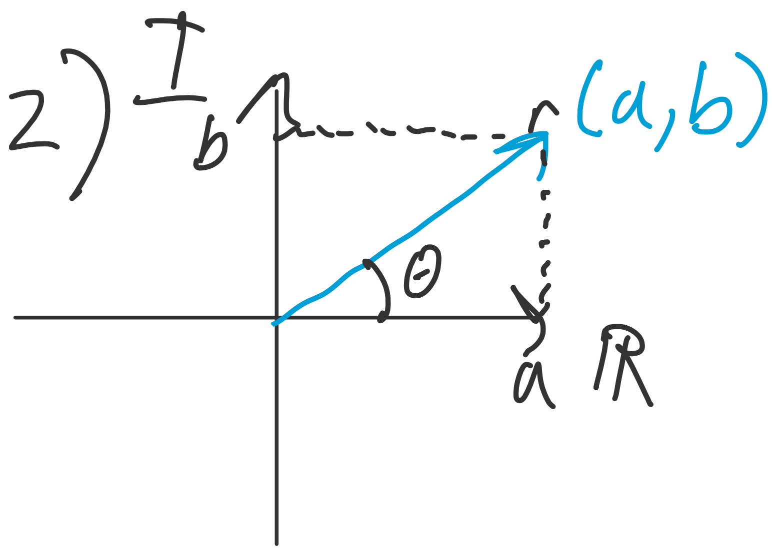
2)



SOH CAH T

$\sin(\theta) = \frac{b}{r} \Rightarrow b =$

$\cos(\theta) = \frac{a}{r} \Rightarrow a =$

$6!$

$- \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots]$

TOA

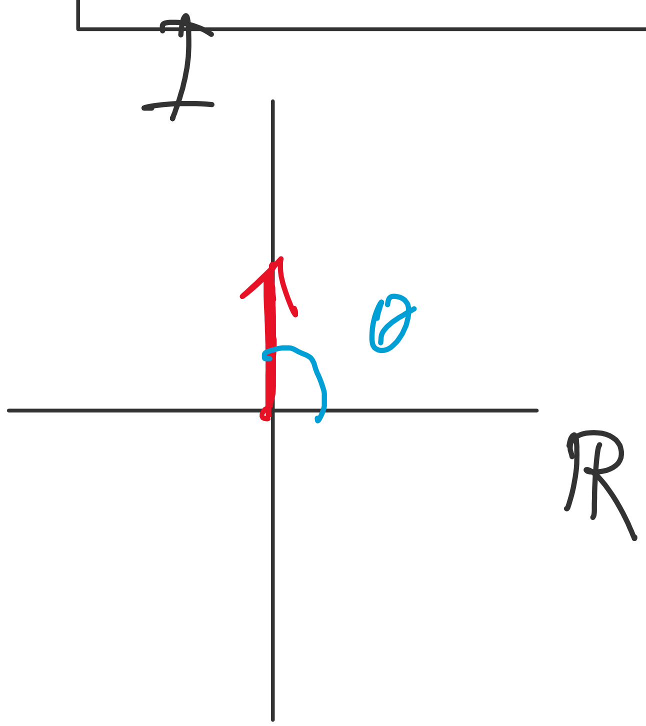
$r \sin\theta$

$= r\cos\theta$

3) $z = re^{i\theta}$ $w = e^{i\phi}$ $z \cdot w$

using $\boxed{e^x e^y = e^{(x+y)}}$ z

- modulus does not change
- argument is $\theta + \phi$



$z_c = a + ib = R$

no real par

$= re^{i\theta} e^{i\phi}$

$\cdot w = re^{i(\theta+\phi)}$

$+ I$

$\Rightarrow i \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

ntion

$$\boxed{\theta = \frac{\pi}{2}}$$

4) a. only # possible i

b. $z^* = re^{-i\theta} = r[\cos\theta$

c. start with $z + z^*$

$$z + z^* = (a+ib) + (a-ib)$$

$$\boxed{a = \frac{1}{2}(z+z^*)}$$

$$z - z^* = (a+ib) - (a-ib)$$

$$\boxed{b = \frac{z-z^*}{2i}}$$

s zero | $z^{*\cancel{*}} = z$

$\theta - i\sin\theta]$

$= 2a \cancel{+ib} - \cancel{ib}$

$= \cancel{a} - \cancel{a} + 2ib$

d. $zz^* = (a+ib)(a-ib)$

$= a^2 - (-1)b^2 = \boxed{a^2}$

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5) Euler's is: $e^{i\theta} = \cos\theta$

$e^{i(\alpha+\beta)} = e^{i\alpha} e^{i\beta}$ see probl

$= [\cos\alpha + i\sin\alpha] \cdot [\cos\beta$

$\cos\alpha\cos\beta + i\cos\alpha\sin\beta + i\sin\alpha\ \mathrm{co}$

$= a^2 - \cancel{iab} + \cancel{iab} - i^2 b^2$

$\boxed{+b^2 \in \mathbb{R}}$

$+ i \sin\theta$

lem above

$+ i \sin\beta]$

$s\beta + i^2 \sin\alpha \sin\beta$

group $\mathbb{R}$ and $+$

$$= [\cos\alpha\cos\beta - \sin\alpha\sin\beta] + i\ [$$

So from Euler's we have

$$e^{i(\alpha+\beta)} = \cos(\alpha+\beta) + i\sin(\alpha+$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha$$

$$\sin(\alpha+\beta) = \cos\alpha\sin\beta + \sin\alpha\ c$$

these are trig iden

$\Big[\cos\alpha \sin\beta + \sin\alpha \cos\beta\Big]$

$+\beta) =$

$\sin\beta$

$\cos\beta$

titios !!!

6) $(2+i)(3+i) = 6+2i+3i$

$$i + i^2 = 6 - 1 + 5i = 5(1+i)$$