

18. The 2d Gaussian is  $G = \frac{1}{2\pi\sigma^2} e^{-r^2/2\sigma^2}$ .  
This can be integrated to yield the quantity enclosed as a function of other parameters.

$$F = \frac{1}{2\pi\sigma^2} \int_0^{2\pi} \int_0^R e^{-r^2/2\sigma^2} dr$$
$$= \sigma^{-2} \int_0^R e^{-r^2/2\sigma^2} dr$$

letting  $s = -r^2/2\sigma^2$ , such that  $ds = -r/\sigma^2$ ,

$$F = - \int_{-R}^0 e^s ds = -[e^s]_{-R}^0 = -[e^{-r^2/2\sigma^2}]_0^R$$

$$F = -[e^{-R^2/2\sigma^2} - e^0]$$

$$F = 1 - e^{-R^2/2\sigma^2}$$

Therefore, the <sup>aperture</sup> radius as a function of sigma is

$$\frac{R}{\sigma} = \sqrt{-2 \ln(1-F)}$$

The FWHM of a Gaussian is  $FWHM = 2\sqrt{2 \ln 2} \sigma$   
 $FWHM = 2.3548 \sigma$

So,  $\sigma = FWHM/2.3548$  and hence

$$\boxed{\frac{R}{FWHM} = \frac{\sqrt{-2 \ln(1-F)}}{2.3548}}$$

$$\frac{R}{FWHM} (50\%) = 0.500$$

$$\frac{R}{FWHM} (95\%) = 1.039$$

$$\frac{R}{FWHM} (98\%) = 1.188$$

Choosing an aperture radius equal to or slightly larger than the FWHM (seeing) gives you 95-98% of the light, so  $R \geq \text{FWHM}$  is a reasonable choice.

$\frac{R}{\text{FWHM}}(99\%) = 1.289$ , so if FWHM can be as large as  $1.2 \text{ FWHM}_0$ , the ideal aperture for ensuring at least 99% of the light is always enclosed would be

$$\frac{R}{1.2 \text{ FWHM}} = 1.289$$

↓

$$\frac{R}{\text{FWHM}} = 1.547, \text{ i.e. an aperture radius of about } 1.5 \times \text{ the seeing,}$$