Beyond the Two-Level System

December 17, 2022

Beyond the Two-Level System

1 Report

1.1 Introduction

Most of the algorithms and function used by quantum computer use only two qubit states: the ground state |0>, and the first excited state |1>. However, higher excited states of qubits exist, and can be accessed probabilistically, or by actively driving the qubit to a higher state. Imperfections in realistic systems can sometimes excite qubits into these higher energy states, which can cause problems for algorithms working with only the two main qubit states.

1.2 Problem and Method

The goal of the experiment was to gain a clearer understanding of the higher level excited states of qubits. To do this, Qiskit Pulse was used to perform tests on the IBM quantum systems to determine how to access, classify, and drive to the higher energy states of qubits.

The first step was to calibrate the qubit to determine the base properties at the beginning of the experiment. Then a frequency sweep was used to determine the frequency needed to drive the qubit from the ground state to the first excited state.

Next, another frequency sweep was used to determine the frequency needed to drive the qubit from the first excited state to the second excited state.

Once this was done, an IQ discriminator was built to classify the three qubit states (|0>, |1>, and |2>). This revealed that the qubit had a probability of being in each of the different states, and each was represented by a probability distribution. An attempt to classify the probabilities into each of the different states revealed that not every probability could be

classified in a particular state, and that some of the probabilities were classified into qubit states higher than the second excited state.

Finally, the probabilities of leakage into qubit states higher than the second excited state were studied. This was done by driving a ground state qubit to the first excited state with increasing power and determining the probability that the qubit would jump to a higher energy state using the IQ discriminator and classifications created.

$$\alpha \equiv \omega_1 - \omega_0 \equiv \omega^{1\to 2} - \omega^{0\to 1}$$

1.3 Results

1.4 Conclusion

2 Code

2.1 Setup

[3]:
```
## general qiskit imports

import warnings
warnings.filterwarnings('ignore')
from qiskit.tools.jupyter import *
from qiskit import IBMQ
import numpy as np
from qiskit import pulse
from qiskit.circuit import Parameter
from qiskit.circuit import QuantumCircuit, Gate
from qiskit import schedule
from qiskit.tools.monitor import job_monitor
import matplotlib.pyplot as plt
from scipy.optimize import curve_fit
from qutip import destroy
```

[4]:
```
## save IBMQ account token

IBMQ.
 ↪save_account('109f33eab1cac8f7fb13d4132013ccb6409a348c9b3fde61e13fdf41730a7949e314860eb3f9d
 ↪overwrite=True)
```

[5]:
```
## load IBMQ account and set the appropriate backend

IBMQ.load_account()
provider = IBMQ.get_provider(hub='ibm-q', group='open', project='main')
backend = provider.get_backend('ibmq_manila')
```

[6]:
```
## verify that the backend supports Pulse features by checking the backend␣
 ↪configuration

backend_config = backend.configuration()
```

[7]:
```
##  find the sampling time for the backend pulses within the backend␣
 ↪configuration

dt = backend_config.dt
print(f"Sampling time: {dt*1e9} ns")
```

```
Sampling time: 0.2222222222222222 ns
```

```
[8]: ## use granurality to determine the length of the pulse

acquire_alignment = backend.configuration().
  ↪timing_constraints['acquire_alignment']
granularity = backend.configuration().timing_constraints['granularity']
pulse_alignment = backend.configuration().timing_constraints['pulse_alignment']
```

```
[9]: ## find least common multiple

lcm = np.lcm(acquire_alignment, pulse_alignment)
print(f"Least common multiple of acquire_alignment and pulse_alignment: {lcm}")
```

```
Least common multiple of acquire_alignment and pulse_alignment: 16
```

```
[10]: ## access estimates for qubit frequencies and default programs to enact basic␣
  ↪quantum operators

backend_defaults = backend.defaults()
```

2.2 Task 1

2.2.1 Find the frequency of the $|0\rangle \rightarrow |1\rangle$ transition

```
[11]: ## define frequency range for sweep in search of the qubit, restricting to a␣
  ↪window of 40 MHz around the estimated qubit frequency

# unit conversion factors -> all backend properties returned in SI (Hz, sec,␣
  ↪etc.)
GHz = 1.0e9 # Gigahertz
MHz = 1.0e6 # Megahertz
us = 1.0e-6 # Microseconds
ns = 1.0e-9 # Nanoseconds

# We will find the qubit frequency for the following qubit.
qubit = 0

# The sweep will be centered around the estimated qubit frequency.
center_frequency_Hz = backend_defaults.qubit_freq_est[qubit]
print(f"Qubit {qubit} has an estimated frequency of {center_frequency_Hz / GHz}␣
  ↪GHz.")

# scale factor to remove factors of 10 from the data
scale_factor = 1e-7

# We will sweep 40 MHz around the estimated frequency
frequency_span_Hz = 40 * MHz
# in steps of 1 MHz.
frequency_step_Hz = 1 * MHz
```

```
# We will sweep 20 MHz above and 20 MHz below the estimated frequency
frequency_min = center_frequency_Hz - frequency_span_Hz / 2
frequency_max = center_frequency_Hz + frequency_span_Hz / 2
# Construct an np array of the frequencies for our experiment
frequencies_GHz = np.arange(frequency_min / GHz,
                            frequency_max / GHz,
                            frequency_step_Hz / GHz)

print(f"The sweep will go from {frequency_min / GHz} GHz to {frequency_max /␣
 ↪GHz} GHz \
in steps of {frequency_step_Hz / MHz} MHz.")
```

```
Qubit 0 has an estimated frequency of 4.962283256340941 GHz.
The sweep will go from 4.94228325634094 GHz to 4.98228325634094 GHz in steps of
1.0 MHz.
```

[12]:
```
## define helper functions for pulse flow

# drive pulse of a frequency
def get_closest_multiple_of(value, base_number):
    return int(value + base_number/2) - (int(value + base_number/2) %␣
 ↪base_number)

# determine the length of the pulse
def get_closest_multiple_of_16(num):
    return get_closest_multiple_of(num, granularity)

# adjust the length of the delay
def get_dt_from(sec):
    return get_closest_multiple_of(sec/dt, lcm)
```

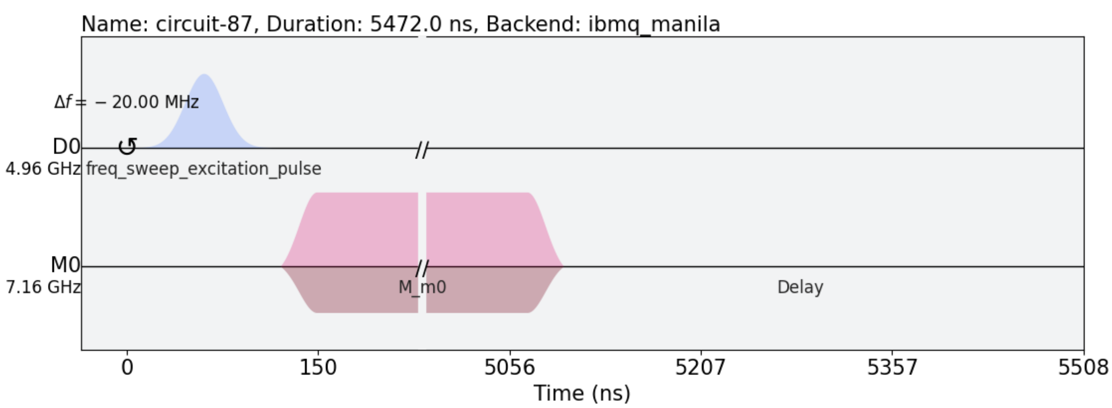
[13]:
```
## create a pulse schedule

# Drive pulse parameters
drive_sigma_sec = 0.015 * us # actual width of the gaussian pulse
drive_duration_sec = drive_sigma_sec * 8 # truncating parameter, as gaussians␣
 ↪don't have a natural finite length
drive_amp = 0.05

# Create the base schedule, start with drive pulse acting on the drive channel
freq = Parameter('freq')
with pulse.build(backend=backend, default_alignment='sequential',␣
 ↪name='Frequency sweep') as sweep_sched:
    drive_duration = get_closest_multiple_of_16(pulse.
 ↪seconds_to_samples(drive_duration_sec))
    drive_sigma = pulse.seconds_to_samples(drive_sigma_sec)
```

```
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(freq, drive_chan)
    # Drive pulse samples
    pulse.play(pulse.Gaussian(duration=drive_duration,
                              sigma=drive_sigma,
                              amp=drive_amp,
                              name='freq_sweep_excitation_pulse'), drive_chan)
```

[14]:
```
## create sweep

sweep_gate = Gate("sweep", 1, [freq])

qc_sweep = QuantumCircuit(1, 1)

qc_sweep.append(sweep_gate, [0])
qc_sweep.measure(0, 0)
qc_sweep.add_calibration(sweep_gate, (0,), sweep_sched, [freq])

# Create the frequency settings for the sweep (MUST BE IN HZ)
frequencies_Hz = frequencies_GHz*GHz
exp_sweep_circs = [qc_sweep.assign_parameters({freq: f}, inplace=False) for f␣
 ↪in frequencies_Hz]
```

[15]:
```
## draw sweep schedule

sweep_schedule = schedule(exp_sweep_circs[0], backend)
sweep_schedule.draw(backend=backend)
```

[15]:



[16]:
```
## run assembled program on backend

num_shots_per_frequency = 1024

job = backend.run(exp_sweep_circs,
```

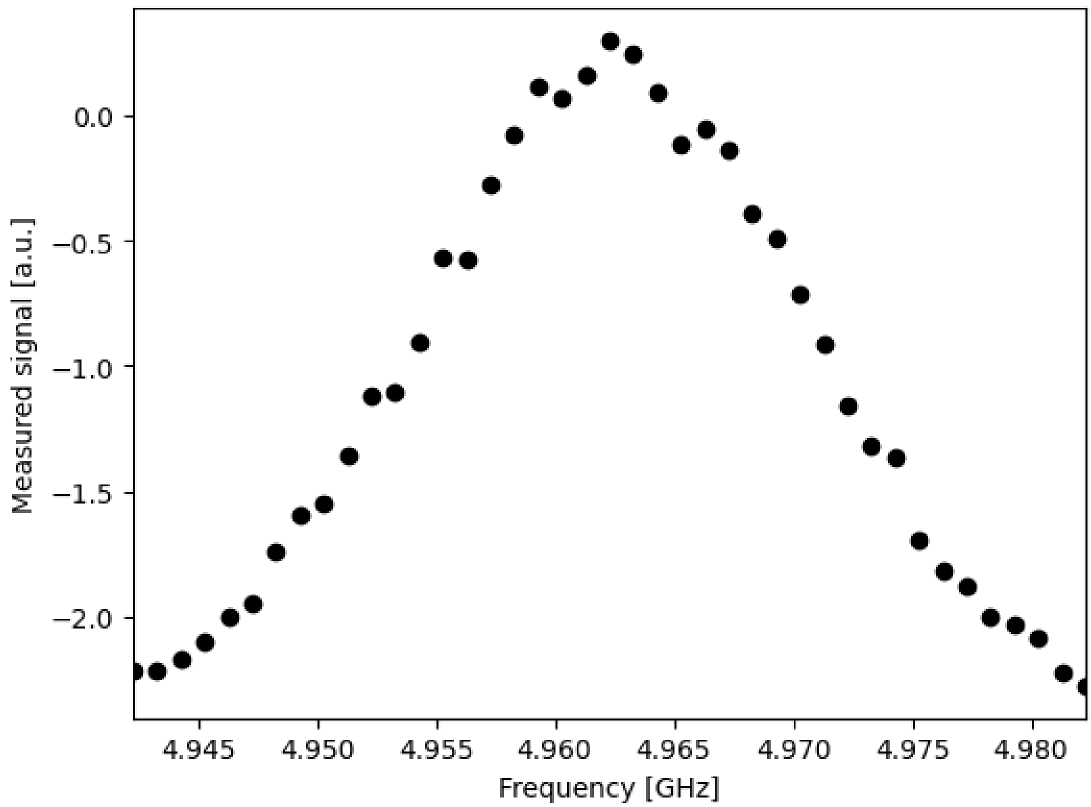
```
                    meas_level=1,
                    meas_return='avg',
                    shots=num_shots_per_frequency)
```

[17]:
```
## monitor job status

job_monitor(job)
```

```
Job Status: job has successfully run
```

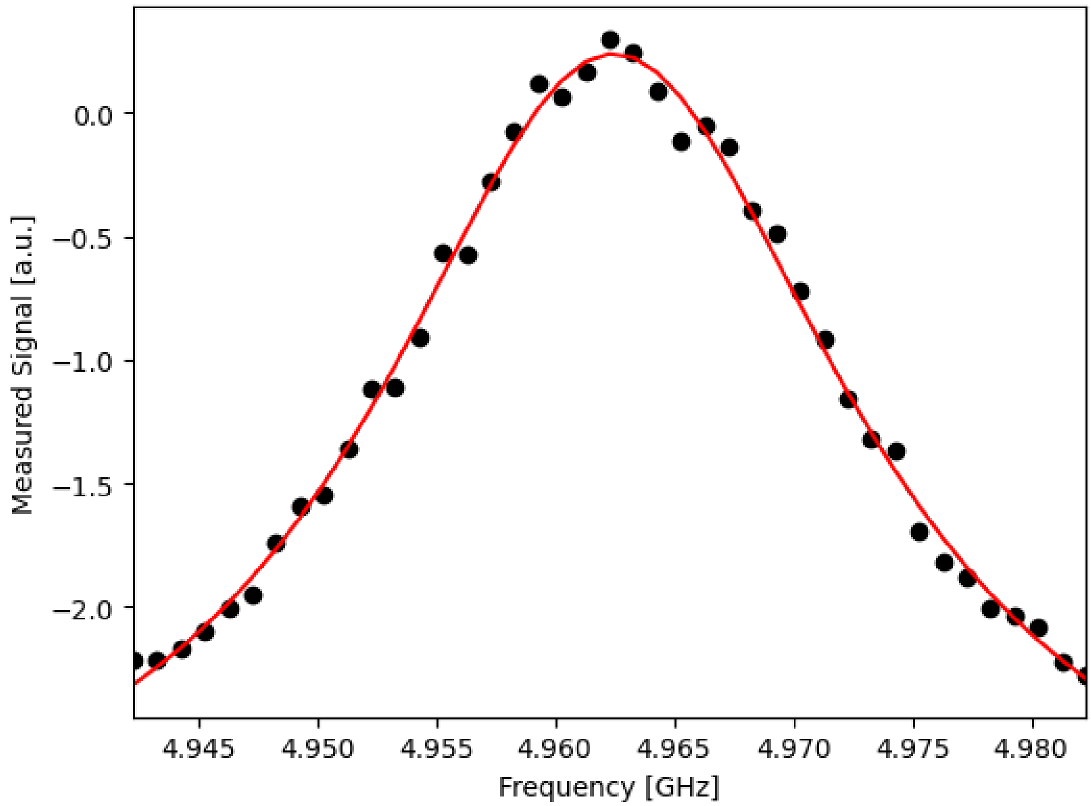
[18]:
```
## retrieve job results

frequency_sweep_results = job.result(timeout=120)
```

[19]:
```
## plot job results

sweep_values = []
for i in range(len(frequency_sweep_results.results)):
    # Get the results from the ith experiment
    res = frequency_sweep_results.get_memory(i)*scale_factor
    # Get the results for `qubit` from this experiment
    sweep_values.append(res[qubit])

plt.scatter(frequencies_GHz, np.real(sweep_values), color='black') # plot real␣
 ↪part of sweep values
plt.xlim([min(frequencies_GHz), max(frequencies_GHz)])
plt.xlabel("Frequency [GHz]")
plt.ylabel("Measured signal [a.u.]")
plt.show()
```



[20]:
```
## fit values to a curve

def fit_function(x_values, y_values, function, init_params):
    fitparams, conv = curve_fit(function, x_values, y_values, init_params)
    y_fit = function(x_values, *fitparams)

    return fitparams, y_fit

fit_params, y_fit = fit_function(frequencies_GHz,
                                 np.real(sweep_values),
                                 lambda x, A, q_freq, B, C: (A / np.pi) * (B /␣
 ↪((x - q_freq)**2 + B**2)) + C,
                                 [1, 4.975, 1, -2] # initial parameters for␣
 ↪curve_fit
                                )

plt.scatter(frequencies_GHz, np.real(sweep_values), color='black')
plt.plot(frequencies_GHz, y_fit, color='red')
plt.xlim([min(frequencies_GHz), max(frequencies_GHz)])

plt.xlabel("Frequency [GHz]")
```

```
plt.ylabel("Measured Signal [a.u.]")
plt.show()
```



[21]:
```
## derive frequency using peak

A, rough_qubit_frequency, B, C = fit_params
rough_qubit_frequency = rough_qubit_frequency*GHz # make sure qubit freq is in␣
↪Hz
print(f"Updated qubit frequency estimate from "
      f"{round(backend_defaults.qubit_freq_est[qubit] / GHz, 5)} GHz to␣
↪{round(rough_qubit_frequency/GHz, 5)} GHz.")
```

```
Updated qubit frequency estimate from 4.96228 GHz to 4.96247 GHz.
```

2.2.2 Calibrate the π pulse for the $|0\rangle \to |1\rangle$ transition using a Rabi experiment

[22]:
```
## Set Rabi experiment parameters

# Rabi experiment parameters
num_rabi_points = 75
```

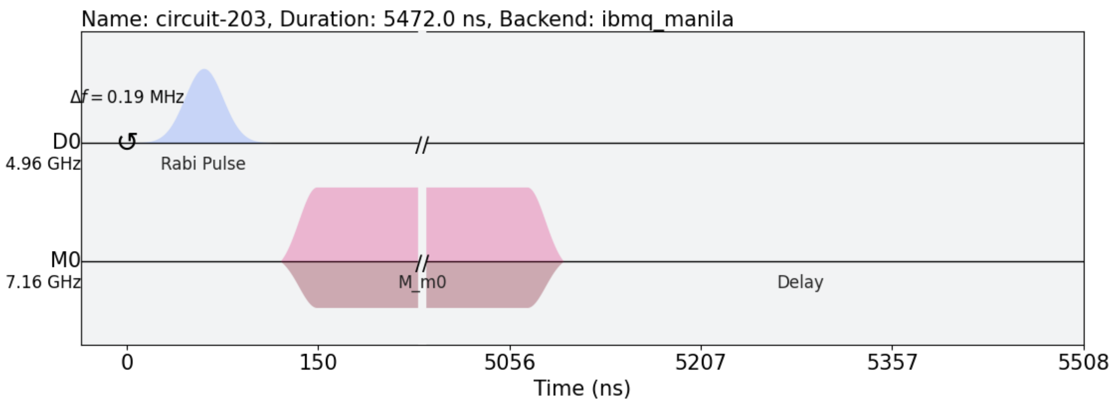
```
# Drive amplitude values to iterate over: 50 amplitudes evenly spaced from 0 to␣
↪0.75
drive_amp_min = 0
drive_amp_max = 1.0
drive_amps = np.linspace(drive_amp_min, drive_amp_max, num_rabi_points)
```

[23]:
```
## Build Rabi experiments

# A drive pulse at the qubit frequency, followed by a measurement, vary the␣
↪drive amplitude each time
drive_amp = Parameter('drive_amp')
with pulse.build(backend=backend, default_alignment='sequential', name='Rabi␣
↪Experiment') as rabi_sched:
    drive_duration = get_closest_multiple_of_16(pulse.
↪seconds_to_samples(drive_duration_sec))
    drive_sigma = pulse.seconds_to_samples(drive_sigma_sec)
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(rough_qubit_frequency, drive_chan)
    pulse.play(pulse.Gaussian(duration=drive_duration,
                              amp=drive_amp,
                              sigma=drive_sigma,
                              name='Rabi Pulse'), drive_chan)
```

[24]:
```
## create Rabi circuit

rabi_gate = Gate("rabi", 1, [drive_amp])

qc_rabi = QuantumCircuit(1, 1)

qc_rabi.append(rabi_gate, [0])
qc_rabi.measure(0, 0)
qc_rabi.add_calibration(rabi_gate, (0,), rabi_sched, [drive_amp])

exp_rabi_circs = [qc_rabi.assign_parameters({drive_amp: a}, inplace=False) for␣
↪a in drive_amps]
```

[25]:
```
## draw Rabi schedule

rabi_schedule = schedule(exp_rabi_circs[-1], backend)
rabi_schedule.draw(backend=backend)
```

[25]:



[26]:
```python
## run assembled program on backend

num_shots_per_point = 1024

job = backend.run(exp_rabi_circs,
                  meas_level=1,
                  meas_return='avg',
                  shots=num_shots_per_point)
```

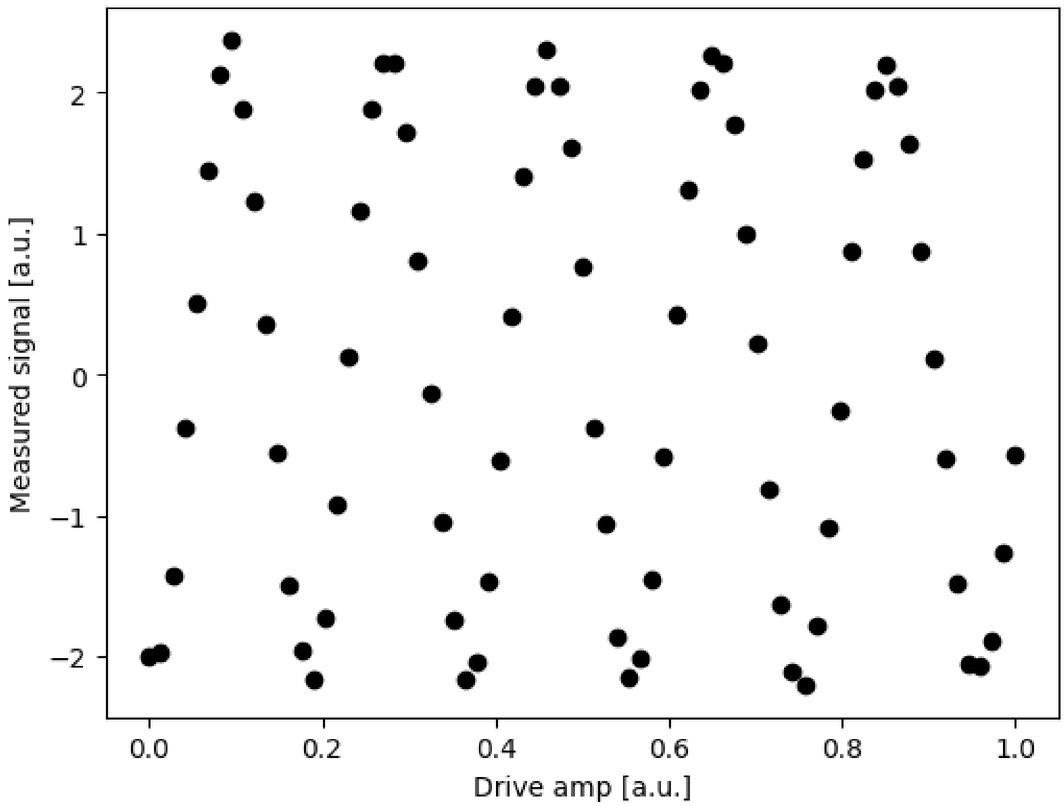
[27]:
```python
## monitor job status

job_monitor(job)
```

```
Job Status: job has successfully run
```

[28]:
```python
## retrieve job results

rabi_results = job.result(timeout=120)
```

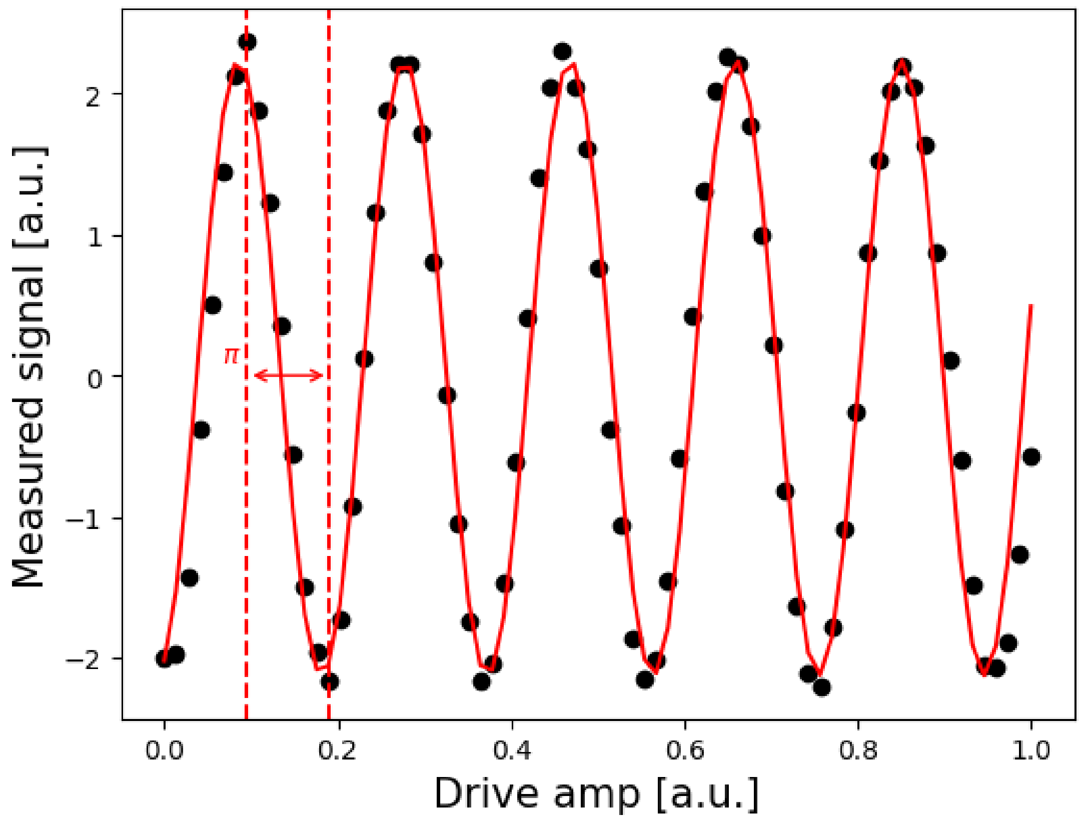
[29]:
```python
## plot job results

# center data around 0
def baseline_remove(values):
    return np.array(values) - np.mean(values)

rabi_values = []
for i in range(num_rabi_points):
    # Get the results for `qubit` from the ith experiment
    rabi_values.append(rabi_results.get_memory(i)[qubit] * scale_factor)

rabi_values = np.real(baseline_remove(rabi_values))

plt.xlabel("Drive amp [a.u.]")
```

```
plt.ylabel("Measured signal [a.u.]")
plt.scatter(drive_amps, rabi_values, color='black') # plot real part of Rabi␣
 ↪values
plt.show()
```



[30]:
```
## fit values to a curve

fit_params, y_fit = fit_function(drive_amps,
                                 rabi_values,
                                 lambda x, A, B, drive_period, phi: (A*np.
 ↪cos(2*np.pi*x/drive_period - phi) + B),
                                 [-2, 0, 0.2, 0])

plt.scatter(drive_amps, rabi_values, color='black')
plt.plot(drive_amps, y_fit, color='red')

drive_period = fit_params[2] # get period of rabi oscillation

pi_amp = drive_period/2
```

```python
plt.axvline(drive_period/2, color='red', linestyle='--')
plt.axvline(drive_period, color='red', linestyle='--')
plt.annotate("", xy=(drive_period, 0), xytext=(drive_period/2,0),␣
 ↪arrowprops=dict(arrowstyle="<->", color='red'))
plt.annotate("$\pi$", xy=(drive_period/2-0.03, 0.1), color='red')

plt.xlabel("Drive amp [a.u.]", fontsize=15)
plt.ylabel("Measured signal [a.u.]", fontsize=15)
plt.show()
```



[31]:
```python
## print pi amplitude

print(f"Pi Amplitude = {round(pi_amp, 5)}")
```

```
Pi Amplitude = 0.09556
```

2.3 Task 2

2.3.1 Find the frequency of the $|1 \rightarrow |2$ transition

[32]:
```
## define smaller range sweep
backend_properties = backend.properties()

default_anharmonicity = backend_properties.qubits[qubit][3].value # Default anharmonicity in GHz

num_freqs = 75
drive_power = 0.15
sweep_freqs = default_anharmonicity*GHz + np.linspace(-30*MHz, 30*MHz, num_freqs)

# there are pulse parameters of the single qubit drive in IBM devices
x12_duration = 160
x12_sigma = 40
```

[33]:
```
# helper function

def get_job_data(job, average):
    job_results = job.result(timeout = 120) # timeout parameter set to 120 s
    result_data = []
    for i in range(len(job_results.results)):
        if average: # get avg data
            result_data.append(np.real(job_results.get_memory(i)[qubit] * scale_factor))
        else: # get single data
            result_data.append(job_results.get_memory(i)[:, qubit] * scale_factor)
    return result_data
```

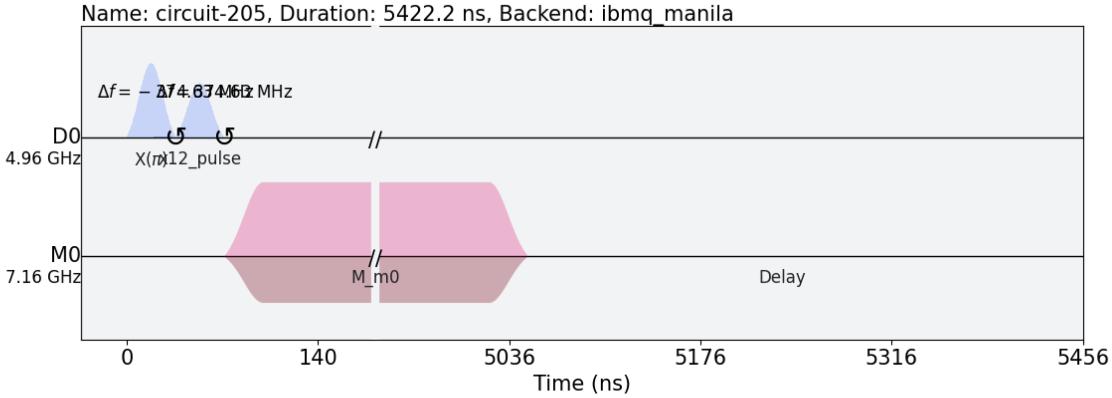
[34]:
```
## create a pulse schedule

freq = Parameter('freq')
with pulse.build(backend=backend, default_alignment='sequential', name='Frequency sweep') as freq12_sweep_sched:
    drive_chan = pulse.drive_channel(qubit)
    with pulse.frequency_offset(freq, drive_chan):
        pulse.play(pulse.Gaussian(duration=x12_duration,
                                  amp=drive_power,
                                  sigma=x12_sigma,
                                  name='x12_pulse'), drive_chan)
```

[35]:
```
## excite qubit to the 1 state
spect_gate = Gate("spect", 1, [freq])
qc_spect = QuantumCircuit(1, 1)
```

```
qc_spect.x(0)
qc_spect.append(spect_gate, [0])
qc_spect.measure(0, 0)
qc_spect.add_calibration(spect_gate, (0,), freq12_sweep_sched, [freq])

exp_spect_circs = [qc_spect.assign_parameters({freq: f}) for f in sweep_freqs]
```

[36]:
```
## draw sweep schedule

freq12_sweep_sched = schedule(exp_spect_circs[0], backend)
freq12_sweep_sched.draw(backend=backend)
```

[36]:



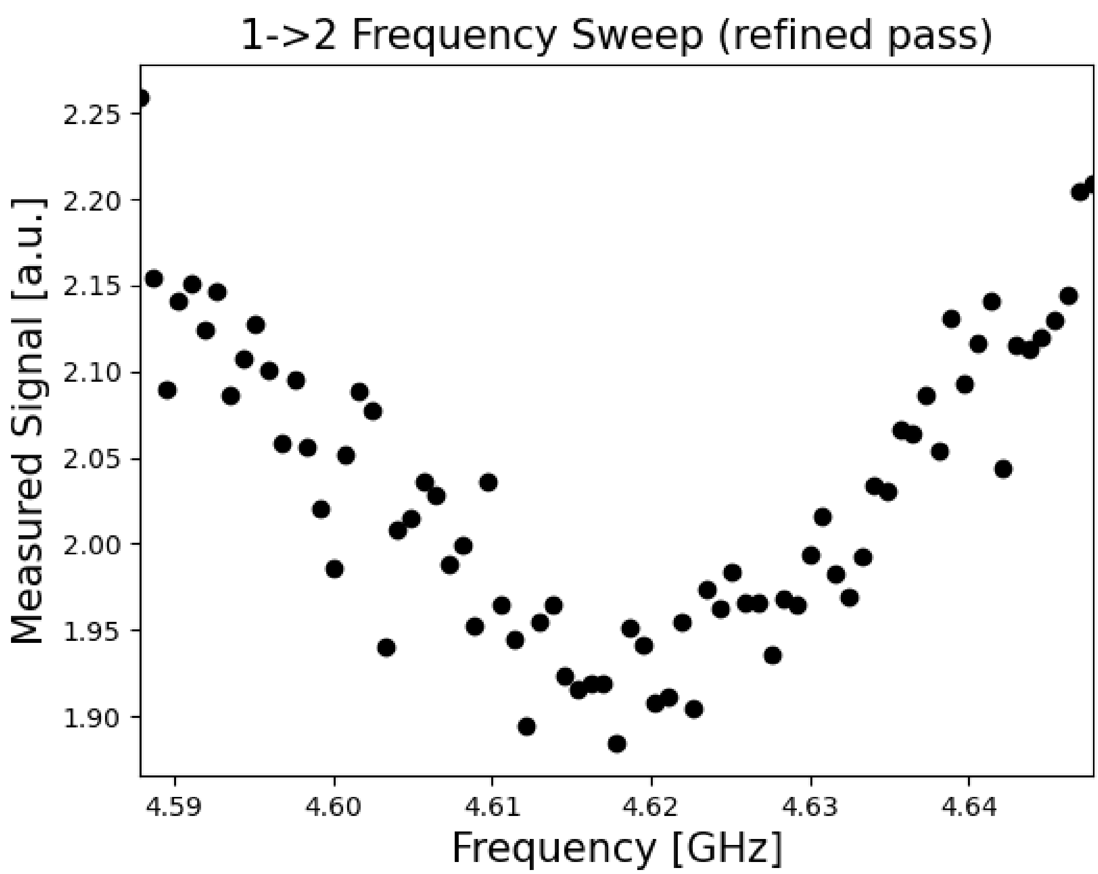
[37]:
```
## run assembled program on backend

num_shots_per_frequency = 1024

excited_freq_sweep_job = backend.run(exp_spect_circs,
                                     meas_level=1,
                                     meas_return='avg',
                                     shots=num_shots_per_frequency)
```

[38]:
```
## monitor job status

job_monitor(excited_freq_sweep_job)
```

```
Job Status: job has successfully run
```

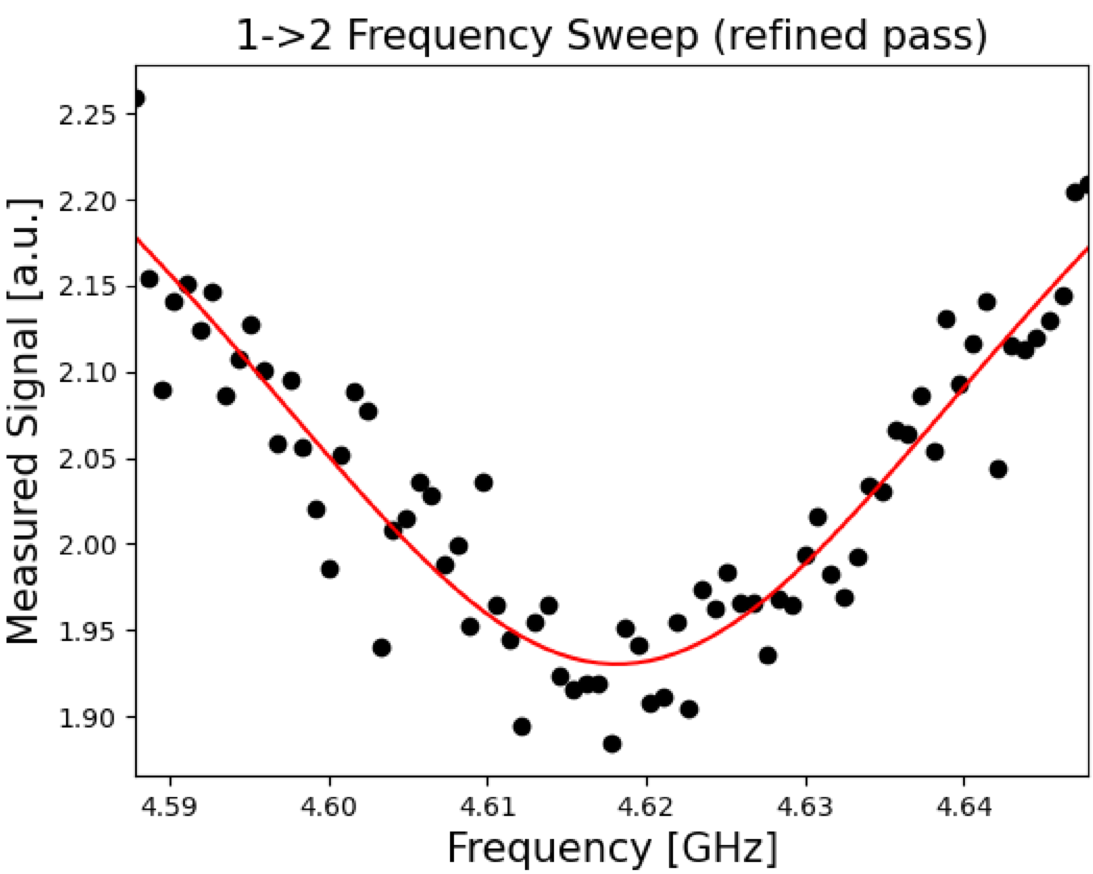
[39]:
```
## retrieve job results

excited_freq_sweep_data = get_job_data(excited_freq_sweep_job, average=True)
excited_sweep_freqs = rough_qubit_frequency + default_anharmonicity*GHz + np.
  ↪linspace(-30*MHz, 30*MHz, num_freqs)
frequency_sweep_results = job.result(timeout=120)
```

[40]:
```
## plot job results

sweep_values = []
for i in range(len(frequency_sweep_results.results)):
    # Get the results from the ith experiment
    res = frequency_sweep_results.get_memory(i)*scale_factor
    # Get the results for `qubit` from this experiment
    sweep_values.append(res[qubit])

plt.scatter(excited_sweep_freqs/GHz, excited_freq_sweep_data, color='black')
plt.xlim([min(excited_sweep_freqs/GHz), max(excited_sweep_freqs/GHz)])
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Measured Signal [a.u.]", fontsize=15)
plt.title("1->2 Frequency Sweep (refined pass)", fontsize=15)
plt.show()
```



[41]:
```
## fit values to a curve

(excited_sweep_fit_params,
```

```
excited_sweep_y_fit) = fit_function(excited_sweep_freqs,
                                  excited_freq_sweep_data,
                                  lambda x, A, q_freq, B, C: (A / np.pi) *␣
↪(B / ((x - q_freq)**2 + B**2)) + C,
                                  [-20, 4.625*GHz, 0.06*GHz, 3*GHz] #␣
↪initial parameters for curve_fit
                                  )

# Note: only plotting the real part of the signal
plt.scatter(excited_sweep_freqs/GHz, excited_freq_sweep_data, color='black')
plt.plot(excited_sweep_freqs/GHz, excited_sweep_y_fit, color='red')
plt.xlim([min(excited_sweep_freqs/GHz), max(excited_sweep_freqs/GHz)])
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Measured Signal [a.u.]", fontsize=15)
plt.title("1->2 Frequency Sweep (refined pass)", fontsize=15)
plt.show()
```



[42]:
```
## derive frequency using peak
```

```
_, qubit_12_freq, _, _ = excited_sweep_fit_params
print(f"1->2 frequency: {round(qubit_12_freq/GHz, 5)} GHz.")
```

```
1->2 frequency: 4.61817 GHz.
```

2.3.2 Calculate the anharmonicity of the qubit

[43]:
```
## Calculate anharmonicity

anharmonicity_01_12 = qubit_12_freq - rough_qubit_frequency
print(f"Updated anharmonicity estimate from "
      f"{round(default_anharmonicity, 5)} GHz to {round(anharmonicity_01_12/
 ↪GHz, 5)} GHz.")
```

```
Updated anharmonicity estimate from -0.34463 GHz to -0.34431 GHz.
```

2.3.3 Calibrate the pulse for the $|1 \rightarrow |2$ transition using a Rabi experiment

[44]:
```
## Set Rabi experiment parameters

num_rabi_points = 75

# Drive amplitude values to iterate over: 75 amplitudes evenly spaced from 0 to␣
 ↪1.0
drive_amp_min = 0
drive_amp_max = 1.0
drive_amps = np.linspace(drive_amp_min, drive_amp_max, num_rabi_points)
```

[45]:
```
## Build Rabi experiments

# A drive pulse at the qubit frequency, followed by a measurement, vary the␣
 ↪drive amplitude each time
amp = Parameter('amp')
with pulse.build(backend=backend, default_alignment='sequential', name='Amp␣
 ↪sweep') as rabi_sched:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_12_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x12_duration,
                              amp=amp,
                              sigma=x12_sigma,
                              name='x12_pulse'), drive_chan)
```

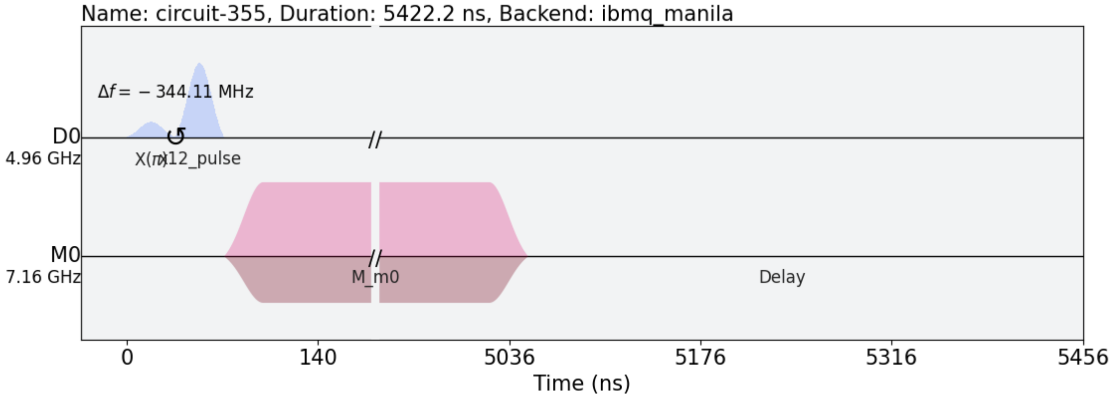
[46]:
```
## create Rabi circuit

rabi_gate = Gate("rabi", 1, [amp])

qc_rabi = QuantumCircuit(1, 1)
```

```
qc_rabi.x(0)
qc_rabi.append(rabi_gate, [0])
qc_rabi.measure(0, 0)
qc_rabi.add_calibration(rabi_gate, (0,), rabi_sched, [amp])

exp_rabi_circs = [qc_rabi.assign_parameters({amp: a}) for a in drive_amps]
```

[47]:

```
## draw Rabi schedule

rabi_schedule = schedule(exp_rabi_circs[-1], backend)
rabi_schedule.draw(backend=backend)
```

[47]:



[48]:

```
## run assembled program on backend

num_shots_per_frequency = 1024

rabi_12_job = backend.run(exp_rabi_circs,
                          meas_level=1,
                          meas_return='avg',
                          shots=num_shots_per_frequency)
```

[49]:

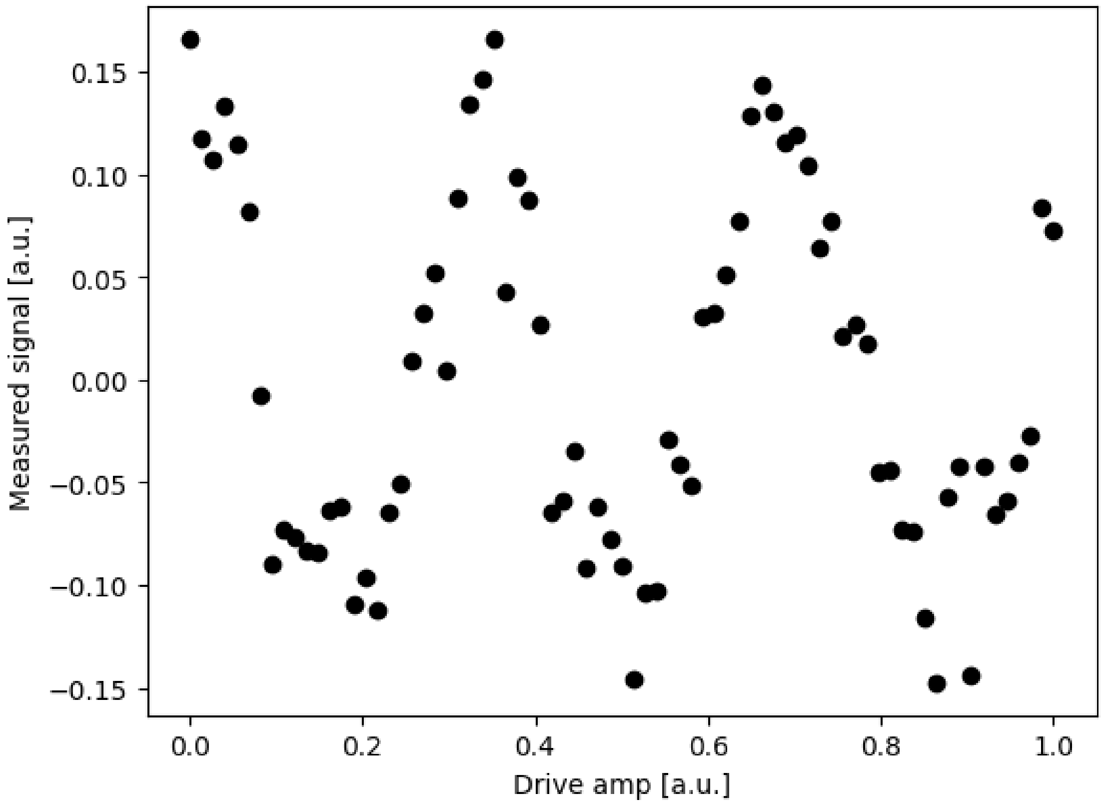
```
## monitor job status

job_monitor(rabi_12_job)
```

```
Job Status: job has successfully run
```

[50]:

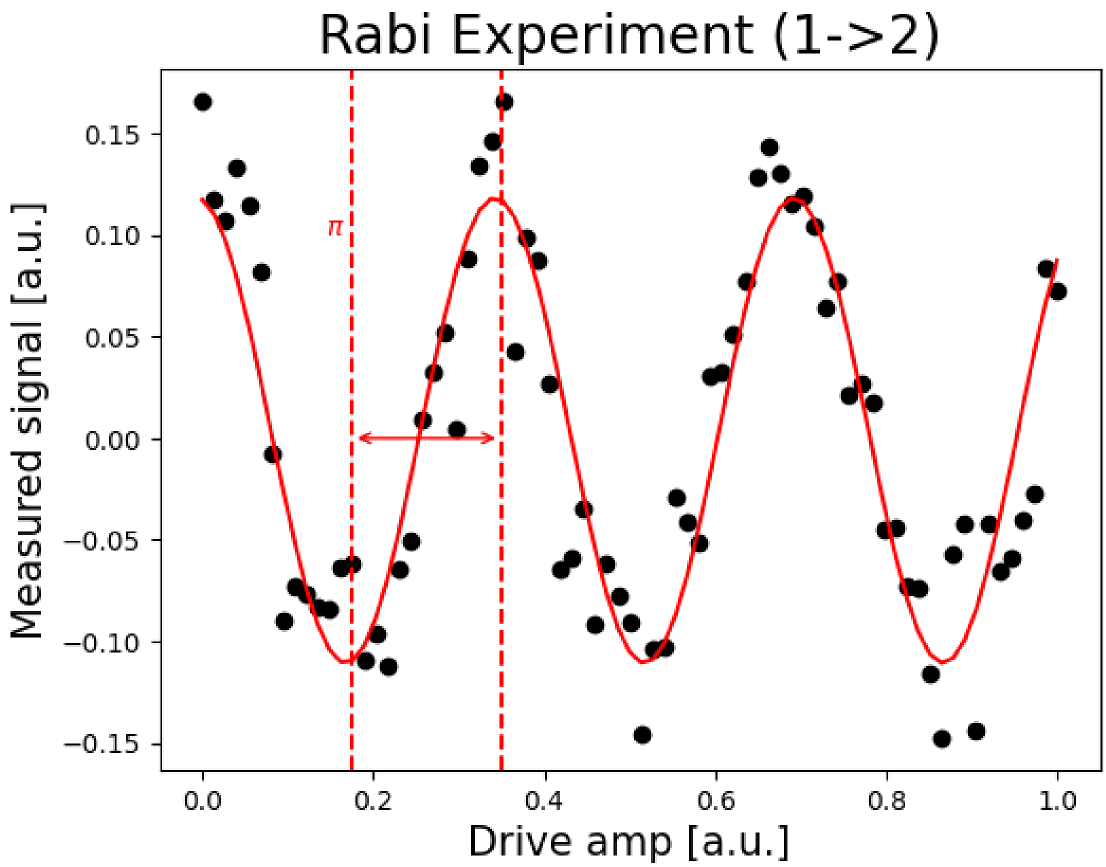
```
## retrieve job results

rabi_12_data = np.real(baseline_remove(get_job_data(rabi_12_job, average=True)))

plt.xlabel("Drive amp [a.u.]")
plt.ylabel("Measured signal [a.u.]")
```

```
plt.scatter(drive_amps, rabi_12_data, color='black') # plot real part of Rabi␣
↪values
plt.show()
```



[51]:
```
## fit values to a curve

(rabi_12_fit_params,
 rabi_12_y_fit) = fit_function(drive_amps,
                               rabi_12_data,
                               lambda x, A, B, drive_12_period, phi: (A*np.
↪cos(2*np.pi*x/drive_12_period - phi) + B),
                               [0.2, 0, 0.3, 0])

plt.scatter(drive_amps, rabi_12_data, color='black')
plt.plot(drive_amps, rabi_12_y_fit, color='red')

drive_12_period = rabi_12_fit_params[2]

pi_amp_12 = drive_12_period/2

plt.axvline(pi_amp_12, color='red', linestyle='--')
```

```
plt.axvline(pi_amp_12+drive_12_period/2, color='red', linestyle='--')
plt.annotate("", xy=(pi_amp_12+drive_12_period/2, 0), xytext=(pi_amp_12,0),
  ↪arrowprops=dict(arrowstyle="<->", color='red'))
plt.annotate("$\pi$", xy=(pi_amp_12-0.03, 0.1), color='red')

plt.xlabel("Drive amp [a.u.]", fontsize=15)
plt.ylabel("Measured signal [a.u.]", fontsize=15)
plt.title('Rabi Experiment (1->2)', fontsize=20)
plt.show()
```



[52]:
```
## print pi amplitude

print(f"Pi Amplitude (1->2) = {round(pi_amp_12, 5)}")
```

```
Pi Amplitude (1->2) = 0.17481
```

2.4 Task 3

2.4.1 Build an IQ discriminator for distinguishing the states into |0 , |1 , and |2

[53]:
```
## Schedule pulse from 1->2

with pulse.build(backend=backend, default_alignment='sequential', name='x12␣
↪schedule') as x12_sched:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_12_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x12_duration,
                              amp=pi_amp_12,
                              sigma=x12_sigma,
                              name='x12_pulse'), drive_chan)
```

[54]:
```
## Create 3 circuits

# 0 state
qc_ground = QuantumCircuit(1, 1)
qc_ground.measure(0, 0)

# 1 state
qc_one = QuantumCircuit(1, 1)
qc_one.x(0)
qc_one.measure(0, 0)

# 2 state
x12_gate = Gate("one_two_pulse", 1, [])
qc_x12 = QuantumCircuit(1, 1)
qc_x12.x(0)
qc_x12.append(x12_gate, [0])
qc_x12.measure(0, 0)
qc_x12.add_calibration(x12_gate, (0,), x12_sched, [])
```

[55]:
```
## run assembled program on backend

num_shots_per_frequency = 1024

IQ_012_job = backend.run([qc_ground, qc_one, qc_x12],
                          meas_level=1,
                          meas_return='single',
                          shots=num_shots_per_frequency)
```

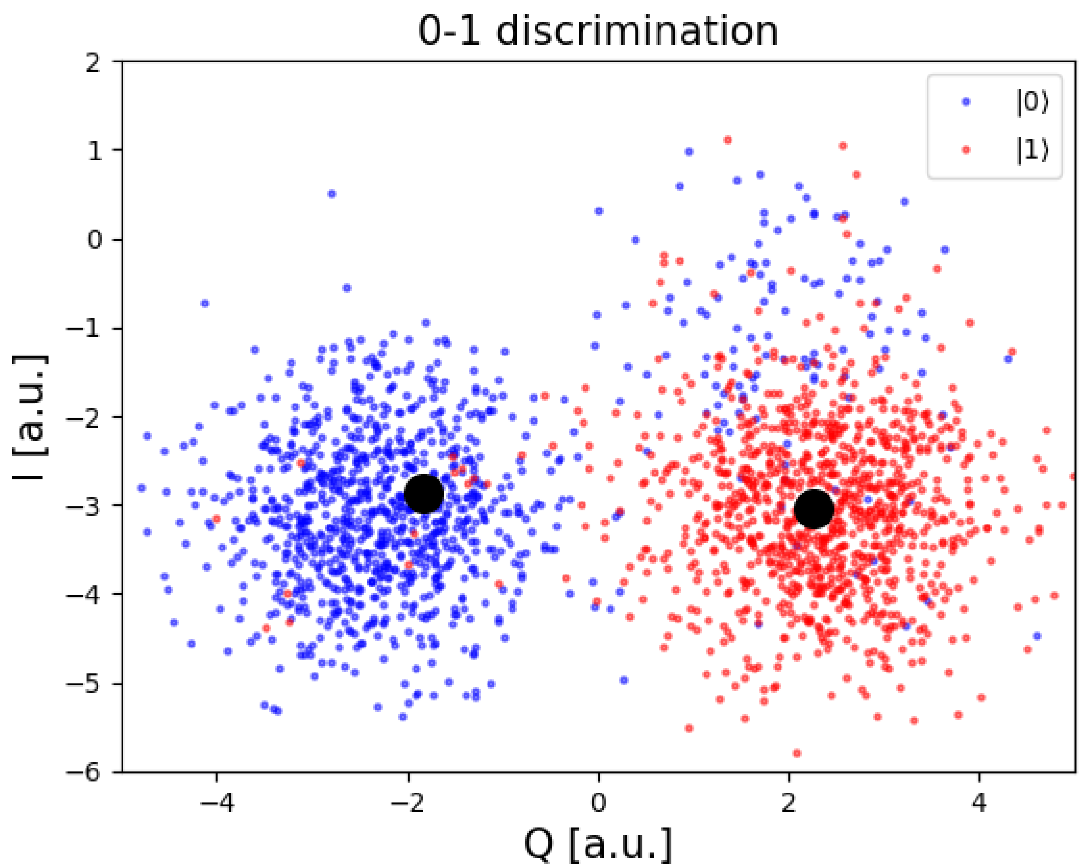
[56]:
```
## monitor job status

job_monitor(IQ_012_job)
```

```
Job Status: job has successfully run
```

[57]:
```python
## retrieve job results

IQ_012_data = get_job_data(IQ_012_job, average=False)
zero_data = IQ_012_data[0]
one_data = IQ_012_data[1]
two_data = IQ_012_data[2]
```

[58]:
```python
## plot the distributions of 0, 1

plt.scatter(np.real(zero_data), np.imag(zero_data),
                s=5, cmap='viridis', c='blue', alpha=0.5, label=r'$|0\rangle$')
# one data plotted in red
plt.scatter(np.real(one_data), np.imag(one_data),
                s=5, cmap='viridis', c='red', alpha=0.5, label=r'$|1\rangle$')

# Plot a large dot for the average result of the 0, 1 and 2 states.
mean_zero = np.mean(zero_data) # takes mean of both real and imaginary parts
mean_one = np.mean(one_data)
plt.scatter(np.real(mean_zero), np.imag(mean_zero),
            s=200, cmap='viridis', c='black',alpha=1.0)
plt.scatter(np.real(mean_one), np.imag(mean_one),
            s=200, cmap='viridis', c='black',alpha=1.0)

plt.xlim(-5, 5)
plt.ylim(-6, 2)
plt.legend()
plt.ylabel('I [a.u.]', fontsize=15)
plt.xlabel('Q [a.u.]', fontsize=15)
plt.title("0-1 discrimination", fontsize=15)
```

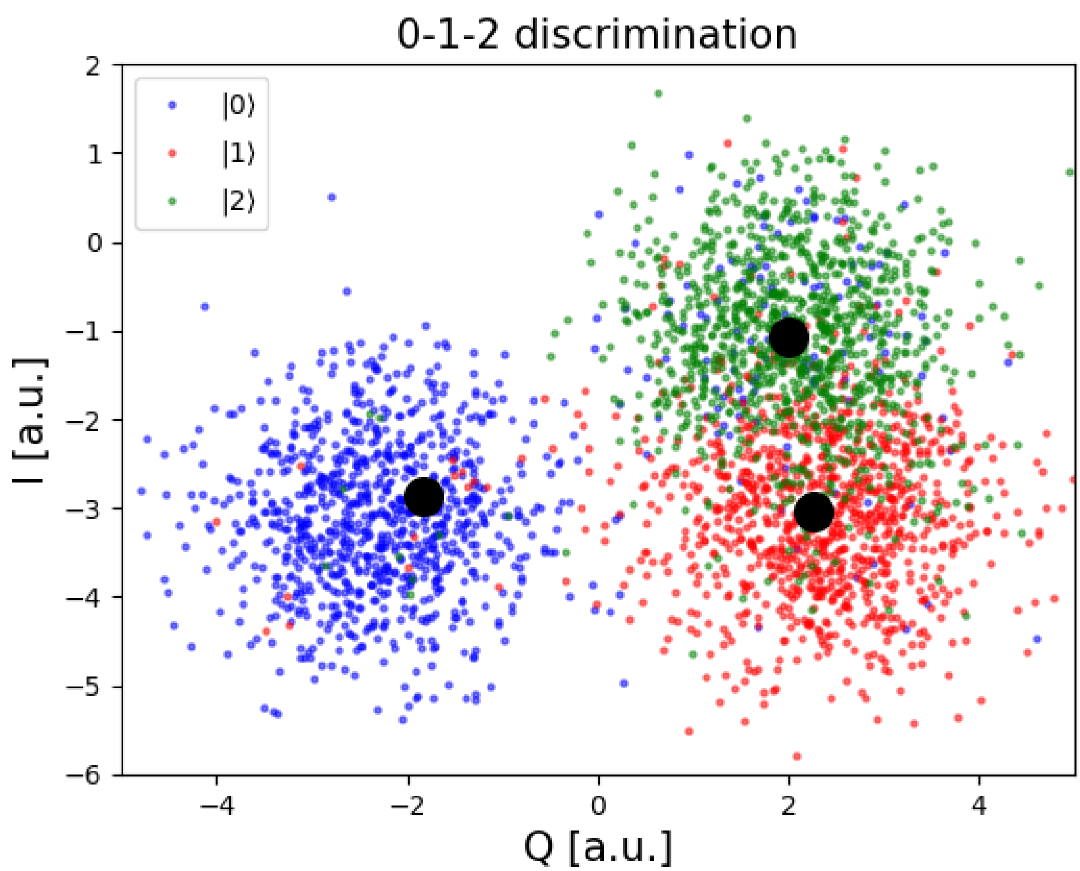
[58]:
```
Text(0.5, 1.0, '0-1 discrimination')
```



[59]:
```
## plot the distributions for 0, 1, 2

# one data plotted in blue
plt.scatter(np.real(zero_data), np.imag(zero_data),
                s=5, cmap='viridis', c='blue', alpha=0.5, label=r'$|0\rangle$')
# one data plotted in red
plt.scatter(np.real(one_data), np.imag(one_data),
                s=5, cmap='viridis', c='red', alpha=0.5, label=r'$|1\rangle$')
# two data plotted in green
plt.scatter(np.real(two_data), np.imag(two_data),
                s=5, cmap='viridis', c='green', alpha=0.5, label=r'$|2\rangle$')

# Plot a large dot for the average result of the 0, 1 and 2 states.
mean_zero = np.mean(zero_data) # takes mean of both real and imaginary parts
mean_one = np.mean(one_data)
mean_two = np.mean(two_data)
plt.scatter(np.real(mean_zero), np.imag(mean_zero),
            s=200, cmap='viridis', c='black',alpha=1.0)
plt.scatter(np.real(mean_one), np.imag(mean_one),
```

```
            s=200, cmap='viridis', c='black',alpha=1.0)
plt.scatter(np.real(mean_two), np.imag(mean_two),
            s=200, cmap='viridis', c='black',alpha=1.0)

plt.xlim(-5, 5)
plt.ylim(-6, 2)
plt.legend()
plt.ylabel('I [a.u.]', fontsize=15)
plt.xlabel('Q [a.u.]', fontsize=15)
plt.title("0-1-2 discrimination", fontsize=15)
```

[59]: Text(0.5, 1.0, '0-1-2 discrimination')

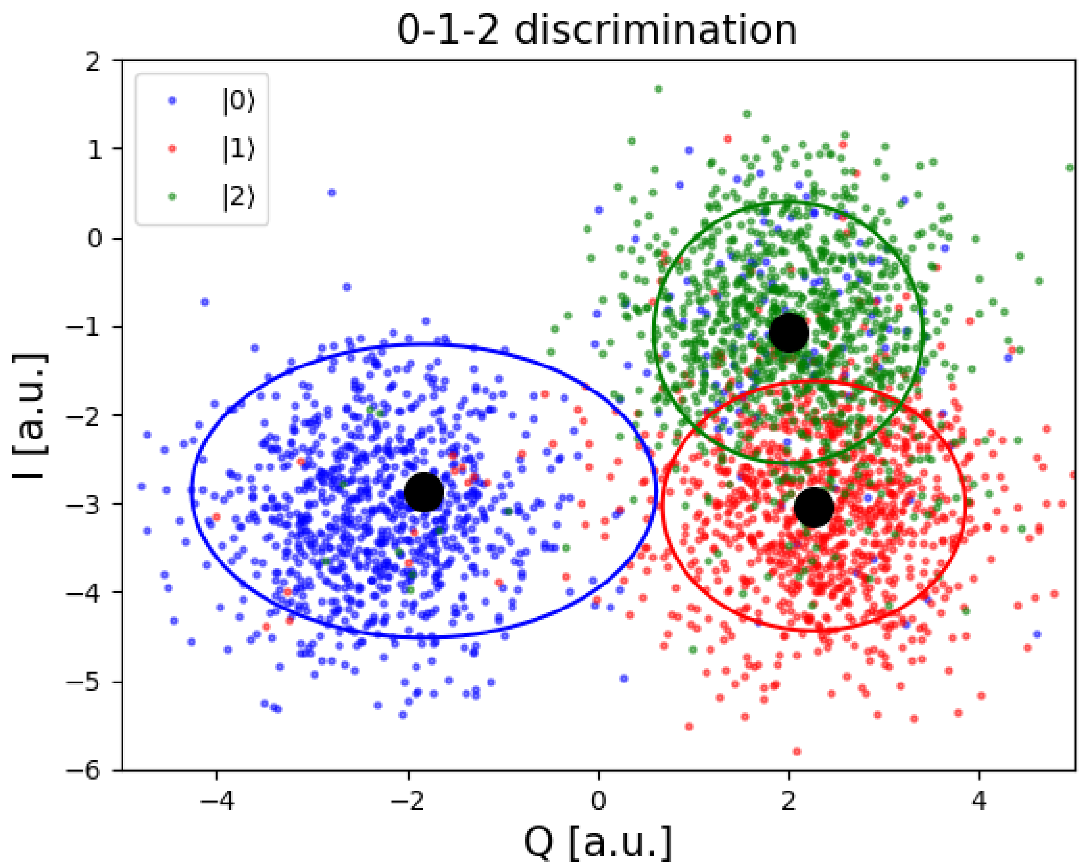


2.4.2 Create a classifier to define the qubit state

[60]:
```
## draw classifier for 0, 1, 2

# helper function
def draw_ellipse(c, q_center, i_center, q_radius, i_radius):
```

```
    t = np.linspace(0,360,360)
    q = q_center + q_radius*np.cos(np.radians(t))
    i = i_center + i_radius*np.sin(np.radians(t))
    plt.plot(q, i, c=c)

# zero data plotted in blue
plt.scatter(np.real(zero_data), np.imag(zero_data),
                s=5, cmap='viridis', c='blue', alpha=0.5, label=r'$|0\rangle$')
# one data plotted in red
plt.scatter(np.real(one_data), np.imag(one_data),
                s=5, cmap='viridis', c='red', alpha=0.5, label=r'$|1\rangle$')
# two data plotted in green
plt.scatter(np.real(two_data), np.imag(two_data),
                s=5, cmap='viridis', c='green', alpha=0.5, label=r'$|2\rangle$')

# Plot a large dot for the average result of the 0, 1 and 2 states.
mean_zero = np.mean(zero_data) # takes mean of both real and imaginary parts
mean_one = np.mean(one_data)
mean_two = np.mean(two_data)

q_center_zero, i_center_zero = np.real(mean_zero), np.imag(mean_zero)
plt.scatter(q_center_zero, i_center_zero,
            s=200, cmap='viridis', c='black',alpha=1.0)
q_center_one, i_center_one = np.real(mean_one), np.imag(mean_one)
plt.scatter(q_center_one, i_center_one,
            s=200, cmap='viridis', c='black',alpha=1.0)
q_center_two, i_center_two = np.real(mean_two), np.imag(mean_two)
plt.scatter(q_center_two, i_center_two,
            s=200, cmap='viridis', c='black',alpha=1.0)

std_devs = 1.5
q_axis_zero, i_axis_zero = std_devs*np.std(np.real(zero_data)), std_devs*np.
 ↪std(np.imag(zero_data))
draw_ellipse('blue', q_center_zero, i_center_zero, q_axis_zero, i_axis_zero)
q_axis_one, i_axis_one = std_devs*np.std(np.real(one_data)), std_devs*np.std(np.
 ↪imag(one_data))
draw_ellipse('red', q_center_one, i_center_one, q_axis_one, i_axis_one)
q_axis_two, i_axis_two = std_devs*np.std(np.real(two_data)), std_devs*np.std(np.
 ↪imag(two_data))
draw_ellipse('green', q_center_two, i_center_two, q_axis_two, i_axis_two)

plt.xlim(-5, 5)
plt.ylim(-6, 2)
plt.legend()
plt.ylabel('I [a.u.]', fontsize=15)
plt.xlabel('Q [a.u.]', fontsize=15)
plt.title("0-1-2 discrimination", fontsize=15)
```

[60]: Text(0.5, 1.0, '0-1-2 discrimination')



[61]:
```
## classify points

def classify_inside(point, ellipses):
    classified = -1
    distance_to_classified = np.Infinity
    ct = 0
    for ellipse in ellipses:
        q_center, i_center, q_axis, i_axis = ellipse
        squared_sum = ((np.real(point) - q_center) / q_axis) ** 2 + ((np.
 ↪imag(point) - i_center) / i_axis) ** 2
        if squared_sum <= 1 and squared_sum < distance_to_classified:
            classified = ct
            distance_to_classified = squared_sum
        ct += 1
    return classified

def classify_closest(point, ellipses):
```

```
    classified = -1
    distance_to_classified = np.Infinity
    ct = 0
    for ellipse in ellipses:
        q_center, i_center, q_axis, i_axis = ellipse
        squared_sum = ((np.real(point) - q_center) / q_axis) ** 2 + ((np.
↪imag(point) - i_center) / i_axis) ** 2
        if squared_sum < distance_to_classified:
            classified = ct
            distance_to_classified = squared_sum
        ct += 1
    return classified
```

2.5 Task 4

2.5.1 Estimate the occupation probability for |0 , |1 , and |2 near the calibrated π pulse of the |0 $\rightarrow$ |1 transition

[62]:

```
## Set Rabi experiment parameters

# Rabi experiment parameters
num_rabi_points = 75
gap = 0.05

# Drive amplitude values to iterate over: 50 amplitudes evenly spaced from 0 to␣
↪0.75
drive_amp_min = pi_amp - gap / 2
drive_amp_max = pi_amp + gap / 2
drive_amps = np.linspace(drive_amp_min, drive_amp_max, num_rabi_points)
```

[63]:

```
## create a pulse schedule

drive_amp = Parameter('drive_amp')
with pulse.build(backend=backend, default_alignment='sequential', name='Rabi␣
↪Experiment') as rabi_sched:
    drive_duration = get_closest_multiple_of_16(pulse.
↪seconds_to_samples(drive_duration_sec))
    drive_sigma = pulse.seconds_to_samples(drive_sigma_sec)
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(rough_qubit_frequency, drive_chan)
    pulse.play(pulse.Gaussian(duration=drive_duration,
                              amp=drive_amp,
                              sigma=drive_sigma,
                              name='Rabi Pulse'), drive_chan)
```

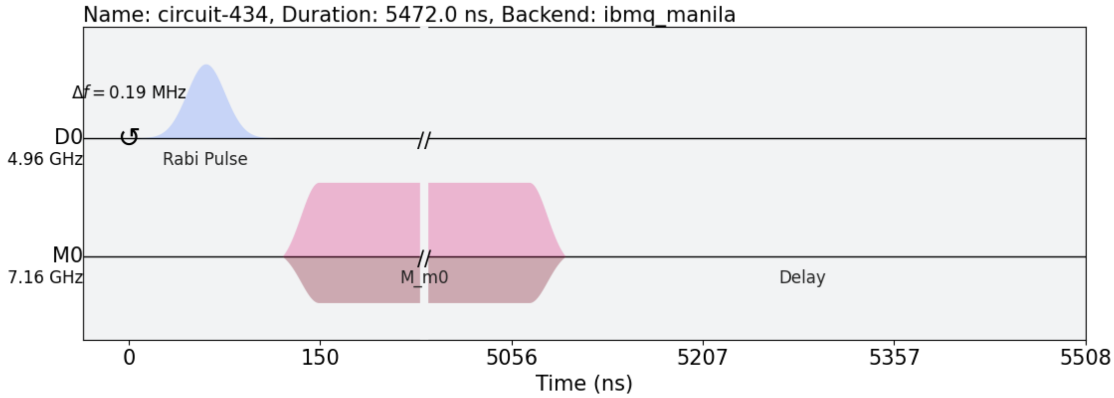
[64]:

```
## create Rabi circuit
```

```
rabi_gate = Gate("rabi", 1, [drive_amp])

qc_rabi = QuantumCircuit(1, 1)

qc_rabi.append(rabi_gate, [0])
qc_rabi.measure(0, 0)
qc_rabi.add_calibration(rabi_gate, (0,), rabi_sched, [drive_amp])

exp_rabi_circs = [qc_rabi.assign_parameters({drive_amp: a}, inplace=False) for␣
 ↪a in drive_amps]
```

[65]:
```
## draw Rabi schedule

rabi_schedule = schedule(exp_rabi_circs[-1], backend)
rabi_schedule.draw(backend=backend)
```

[65]:



[66]:
```
## run assembled program on backend

num_shots_per_point = 1024

job = backend.run(exp_rabi_circs,
                  meas_level=1,
                  meas_return='single',
                  shots=num_shots_per_point)
```

[67]:
```
## monitor job status

job_monitor(job)
```

```
Job Status: job has successfully run
```

[68]:
```
## retrieve job results
```

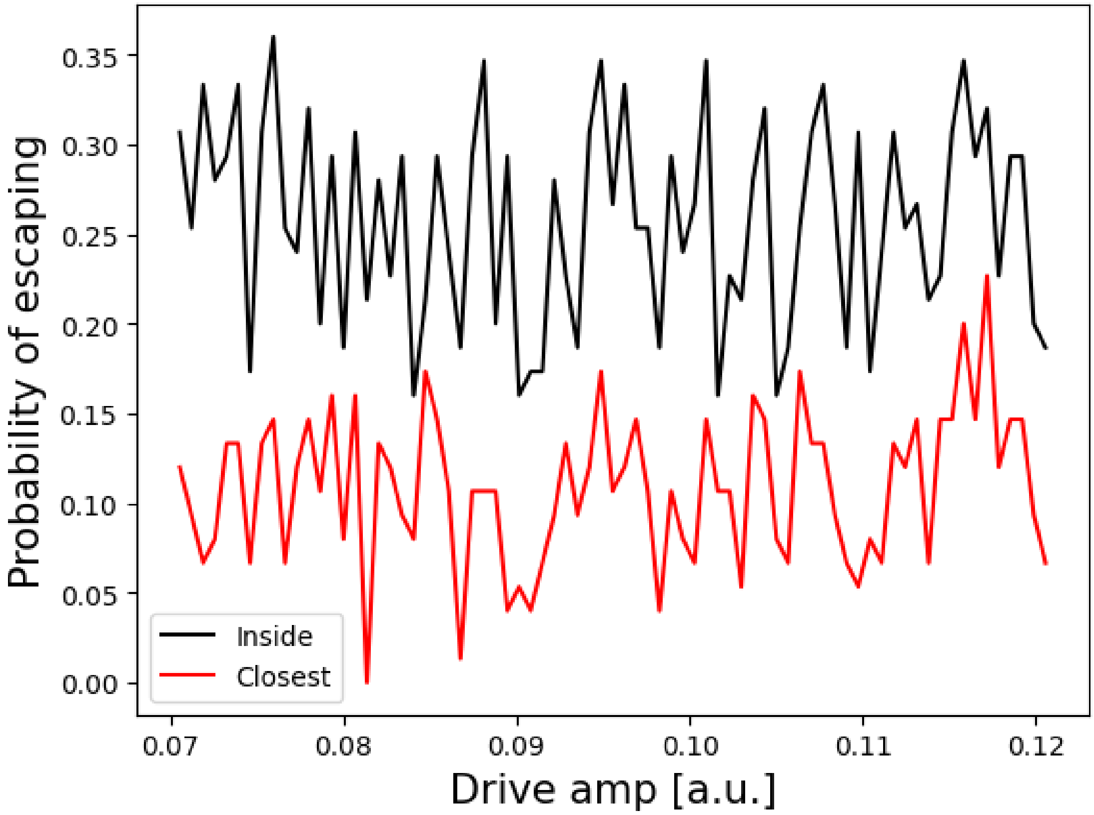
```
rabi_results = job.result(timeout=120)
```

[69]:
```
## classify results

ellipses = [(q_center_zero, i_center_zero, q_axis_zero, i_axis_zero),
            (q_center_one, i_center_one, q_axis_one, i_axis_one),
            (q_center_two, i_center_two, q_axis_two, i_axis_two)]

amp_states = []
amp_classifications_inside = []
amp_classifications_closest = []

for j in range(75):
    states = []
    classifications_inside = []
    classifications_closest = []
    for i in range(num_rabi_points):
        state = rabi_results.get_memory(i)[j]
        states.append(state)
        classifications_inside.append(classify_inside(state * scale_factor,␣
↪ellipses))
        classifications_closest.append(classify_closest(state * scale_factor,␣
↪ellipses))

    amp_states.append(np.array(states))
    amp_classifications_inside.append(np.array(classifications_inside))
    amp_classifications_closest.append(np.array(classifications_closest))
```

[70]:
```
max_probability = 0
probabilities_inside = []
probabilities_closest = []
for j in range(len(amp_classifications_inside)):
    num_escaped_inside = np.count_nonzero(amp_classifications_inside[j] == -1)␣
↪+ np.count_nonzero(amp_classifications_inside[j] == 2)
    probability_inside = num_escaped_inside / num_rabi_points
    probabilities_inside.append(probability_inside)
for j in range(len(amp_classifications_closest)):
    num_escaped_closest = np.count_nonzero(amp_classifications_closest[j] ==␣
↪-1) + np.count_nonzero(amp_classifications_closest[j] == 2)
    probability_closest = num_escaped_closest / num_rabi_points
    probabilities_closest.append(probability_closest)
```

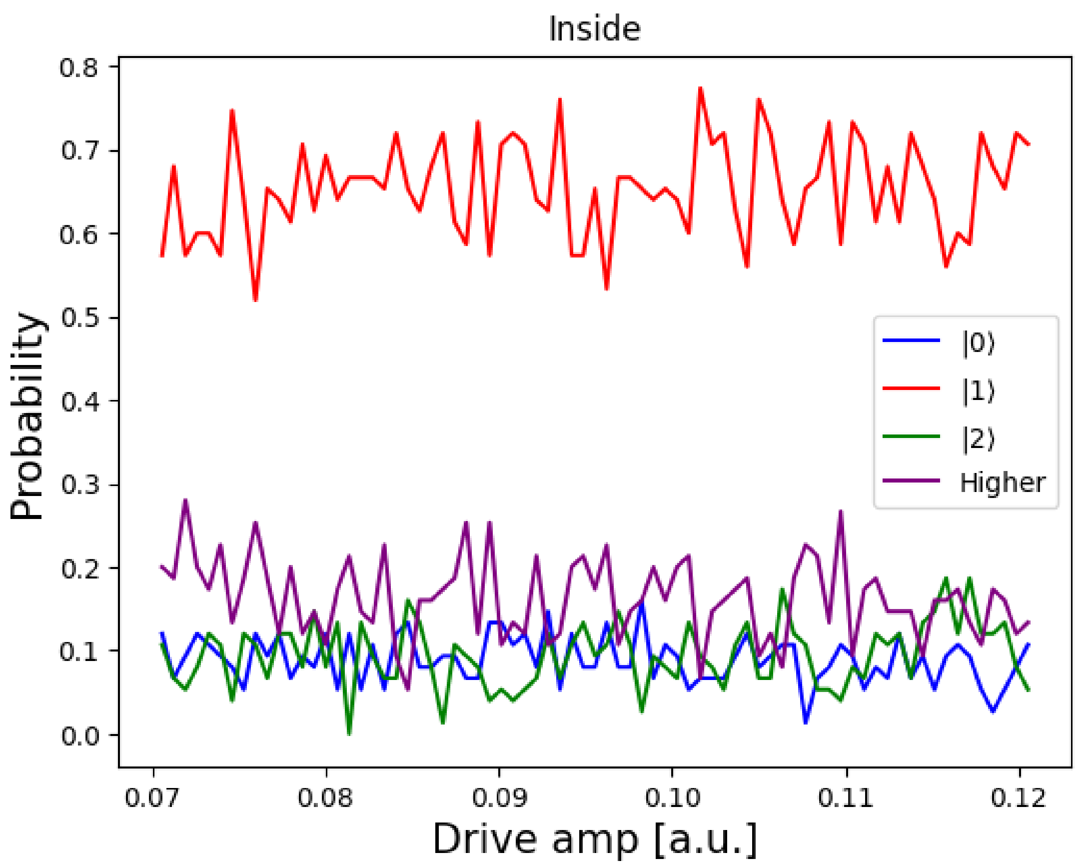
[71]:
```
plt.plot(drive_amps, probabilities_inside, color='black', label='Inside')
plt.plot(drive_amps, probabilities_closest, color='red', label='Closest')

plt.legend()
plt.xlabel("Drive amp [a.u.]", fontsize=15)
```

```
plt.ylabel("Probability of escaping", fontsize=15)
plt.show()
```



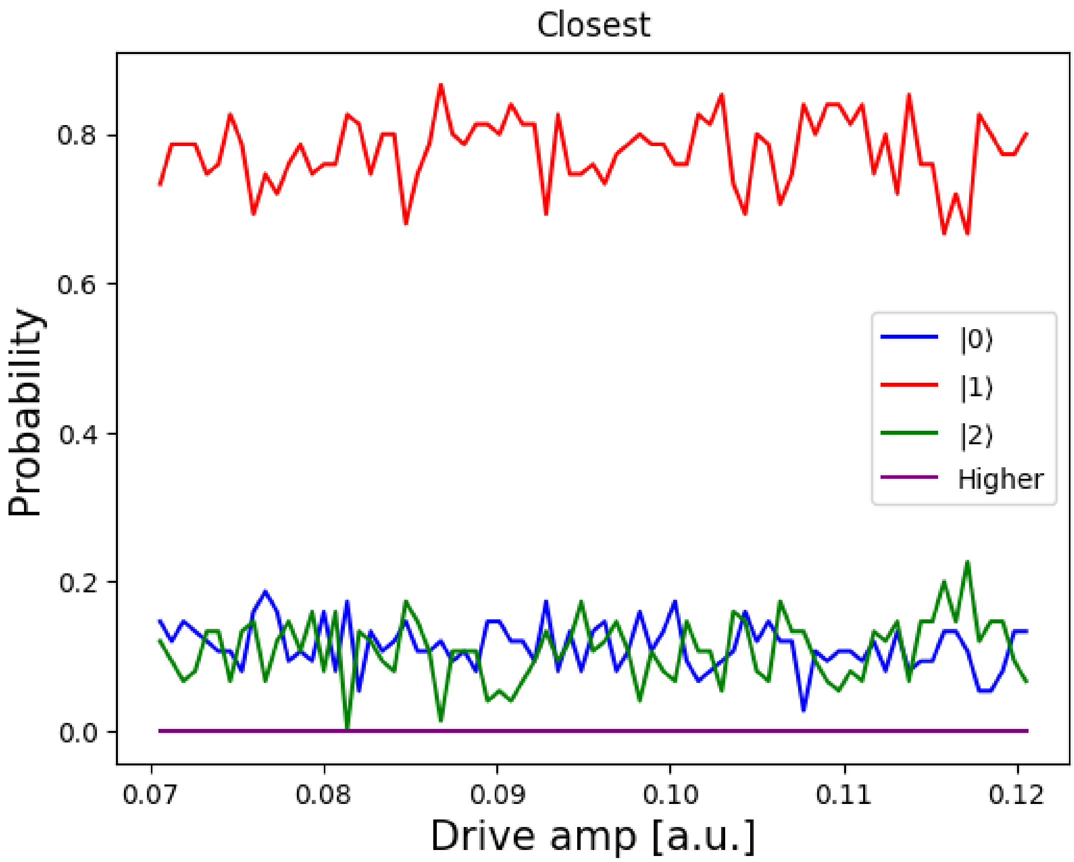
[74]:
```
## plot probabilities

zero_probabilities_inside = []
one_probabilities_inside = []
two_probabilities_inside = []
higher_probabilities_inside = []
for j in range(len(amp_classifications_inside)):
    num_zero = np.count_nonzero(amp_classifications_inside[j] == 0)
    zero_probabilities_inside.append(num_zero / num_rabi_points)
    num_one = np.count_nonzero(amp_classifications_inside[j] == 1)
    one_probabilities_inside.append(num_one / num_rabi_points)
    num_two = np.count_nonzero(amp_classifications_inside[j] == 2)
    two_probabilities_inside.append(num_two / num_rabi_points)
    num_higher = np.count_nonzero(amp_classifications_inside[j] == -1)
    higher_probabilities_inside.append(num_higher / num_rabi_points)
```

```
plt.plot(drive_amps, zero_probabilities_inside, color='blue',␣
 ↪label=r'$|0\rangle$')
plt.plot(drive_amps, one_probabilities_inside, color='red',␣
 ↪label=r'$|1\rangle$')
plt.plot(drive_amps, two_probabilities_inside, color='green',␣
 ↪label=r'$|2\rangle$')
plt.plot(drive_amps, higher_probabilities_inside, color='purple',␣
 ↪label='Higher')

plt.title('Inside')
plt.xlabel("Drive amp [a.u.]", fontsize=15)
plt.ylabel("Probability", fontsize=15)
plt.legend()
plt.show()
```



[77]:
```
## plot probabilities

zero_probabilities_closest = []
one_probabilities_closest = []
```

```python
two_probabilities_closest = []
higher_probabilities_closest = []
for j in range(len(amp_classifications_closest)):
    num_zero = np.count_nonzero(amp_classifications_closest[j] == 0)
    zero_probabilities_closest.append(num_zero / num_rabi_points)
    num_one = np.count_nonzero(amp_classifications_closest[j] == 1)
    one_probabilities_closest.append(num_one / num_rabi_points)
    num_two = np.count_nonzero(amp_classifications_closest[j] == 2)
    two_probabilities_closest.append(num_two / num_rabi_points)
    num_higher = np.count_nonzero(amp_classifications_closest[j] == -1)
    higher_probabilities_closest.append(num_higher / num_rabi_points)

plt.plot(drive_amps, zero_probabilities_closest, color='blue',␣
 ↪label=r'$|0\rangle$')
plt.plot(drive_amps, one_probabilities_closest, color='red',␣
 ↪label=r'$|1\rangle$')
plt.plot(drive_amps, two_probabilities_closest, color='green',␣
 ↪label=r'$|2\rangle$')
plt.plot(drive_amps, higher_probabilities_closest, color='purple',␣
 ↪label='Higher')

plt.title('Closest')
plt.xlabel("Drive amp [a.u.]", fontsize=15)
plt.ylabel("Probability", fontsize=15)
plt.legend()
plt.show()
```



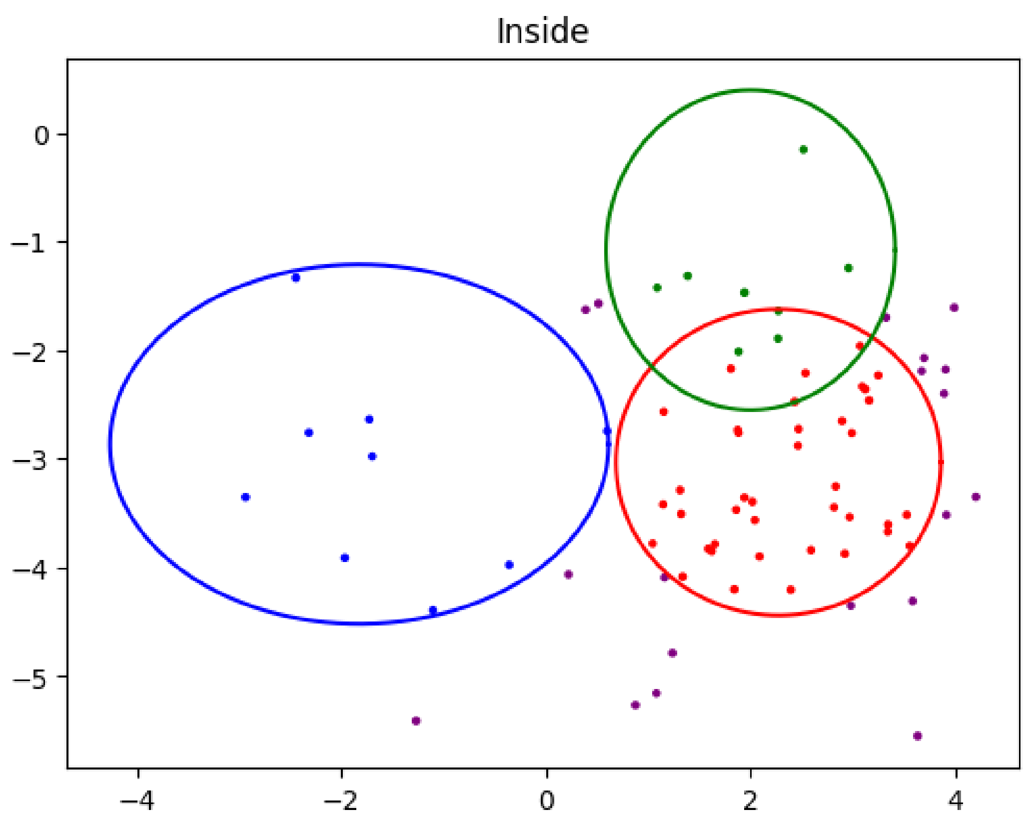
2.5.2 Plot the IQ data for drive power with sizable probability of escaping the manifold

[72]:
```
## find max probability of escape
max_probability_index_inside = np.argmax(probabilities_inside)
max_probability_inside = probabilities_inside[max_probability_index_inside]
max_amps_inside = drive_amps[max_probability_index_inside]
max_states_inside = amp_states[max_probability_index_inside]
max_classifications_inside = ␣
 ↪amp_classifications_inside[max_probability_index_inside]
print(f"Inside")
print(f"Amps = {round(max_amps_inside, 5)}")
print(f"Probability of Escape = {round(max_probability_inside, 5)}")
```

```
Inside
Amps = 0.07596
Probability of Escape = 0.36
```

[73]:
```
## show classifier of max probability of escape
color = np.where(max_classifications_inside == 0, 'blue',
        np.where(max_classifications_inside == 1, 'red',
        np.where(max_classifications_inside == 2, 'green',
        'purple')))
plt.scatter(np.real(max_states_inside) * scale_factor, np.
 ↪imag(max_states_inside) * scale_factor,
                s=5, cmap='viridis', c=color)
draw_ellipse('blue', q_center_zero, i_center_zero, q_axis_zero, i_axis_zero)
draw_ellipse('red', q_center_one, i_center_one, q_axis_one, i_axis_one)
draw_ellipse('green', q_center_two, i_center_two, q_axis_two, i_axis_two)

plt.title('Inside')
```

[73]: Text(0.5, 1.0, 'Inside')



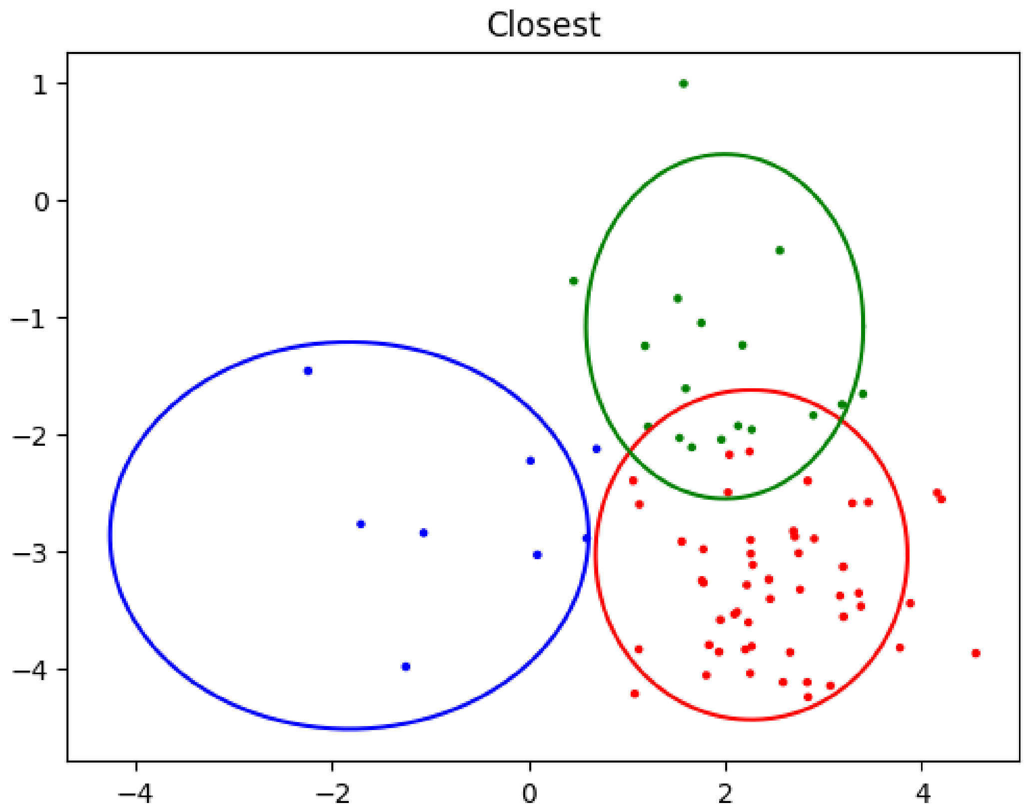
[75]:
```
## find max probability of escape
max_probability_index_closest = np.argmax(probabilities_closest)
max_probability_closest = probabilities_closest[max_probability_index_closest]
max_amps_closest = drive_amps[max_probability_index_closest]
max_states_closest = amp_states[max_probability_index_closest]
```

```
max_classifications_closest =␣
 ↪amp_classifications_closest[max_probability_index_closest]
print(f"Closest")
print(f"Amps = {round(max_amps_closest, 5)}")
print(f"Probability of Escape = {round(max_probability_closest, 5)}")
```

```
Closest
Amps = 0.11718
Probability of Escape = 0.22667
```

[76]:
```
## show classifier of max probability of escape
color = np.where(max_classifications_closest == 0, 'blue',
        np.where(max_classifications_closest == 1, 'red',
        np.where(max_classifications_closest == 2, 'green',
        'purple')))
plt.scatter(np.real(max_states_closest) * scale_factor, np.
 ↪imag(max_states_closest) * scale_factor,
                s=5, cmap='viridis', c=color)
draw_ellipse('blue', q_center_zero, i_center_zero, q_axis_zero, i_axis_zero)
draw_ellipse('red', q_center_one, i_center_one, q_axis_one, i_axis_one)
draw_ellipse('green', q_center_two, i_center_two, q_axis_two, i_axis_two)

plt.title('Closest')
```

[76]:
```
Text(0.5, 1.0, 'Closest')
```



2.6 Bonus

2.6.1 Find the frequency of the $|2\rangle \to |3\rangle$ transition

[78]:
```
## define smaller range sweep
backend_properties = backend.properties()

freq_guess = qubit_12_freq + anharmonicity_01_12
power_guess = 0.275
num_freqs = 100
sweep_freqs = freq_guess + np.linspace(-300*MHz, 300*MHz, num_freqs)

# there are pulse parameters of the single qubit drive in IBM devices
x12_duration = 160
x12_sigma = 40
x23_duration = 160
x23_sigma = 40
```

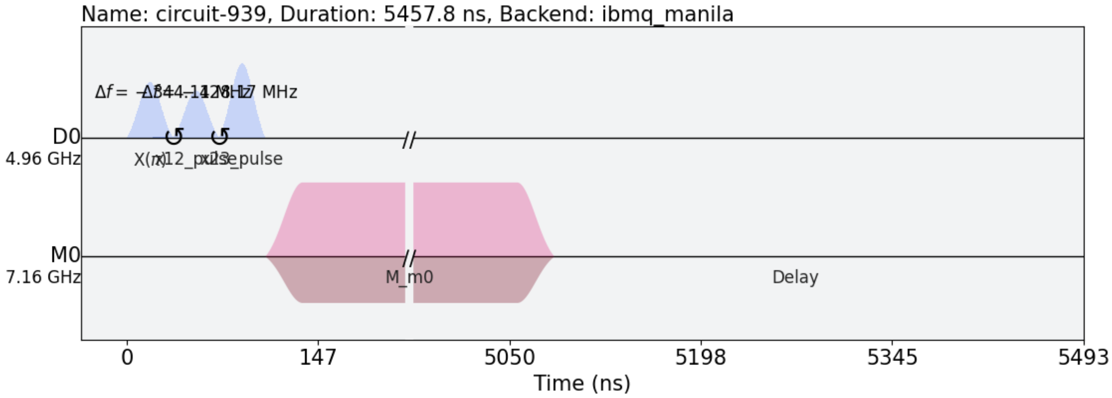
[79]:
```
## create a pulse schedule
with pulse.build(backend=backend, default_alignment='sequential', name='x12␣
↪schedule') as x12_sched:
```

```
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_12_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x12_duration,
                              amp=pi_amp_12,
                              sigma=x12_sigma,
                              name='x12_pulse'), drive_chan)

qubit_23_freq = Parameter('freq')
with pulse.build(backend=backend, default_alignment='sequential',␣
 ↪name='Frequency sweep') as freq23_sweep_sched:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_23_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x23_duration,
                                amp=power_guess,
                                sigma=x23_sigma,
                                name='x23_pulse'), drive_chan)
```

[113]:
```
# prepare state

x12_gate = Gate("one_two_pulse", 1, [])
sweep_23_gate = Gate("two_three_pulse", 1, [qubit_23_freq])
qc_x23 = QuantumCircuit(1, 1)
qc_x23.x(0)
qc_x23.append(x12_gate, [0])
qc_x23.append(sweep_23_gate, [0])
qc_x23.measure(0, 0)
qc_x23.add_calibration(x12_gate, (0,), x12_sched, [])
qc_x23.add_calibration(sweep_23_gate, (0,), freq23_sweep_sched, [qubit_23_freq])

exp_x23_circs = [qc_x23.assign_parameters({qubit_23_freq: f}) for f in␣
 ↪sweep_freqs]
```

[114]:
```
## draw sweep schedule

freq23_sweep_sched = schedule(exp_x23_circs[0], backend)
freq23_sweep_sched.draw(backend=backend)
```

[114]:



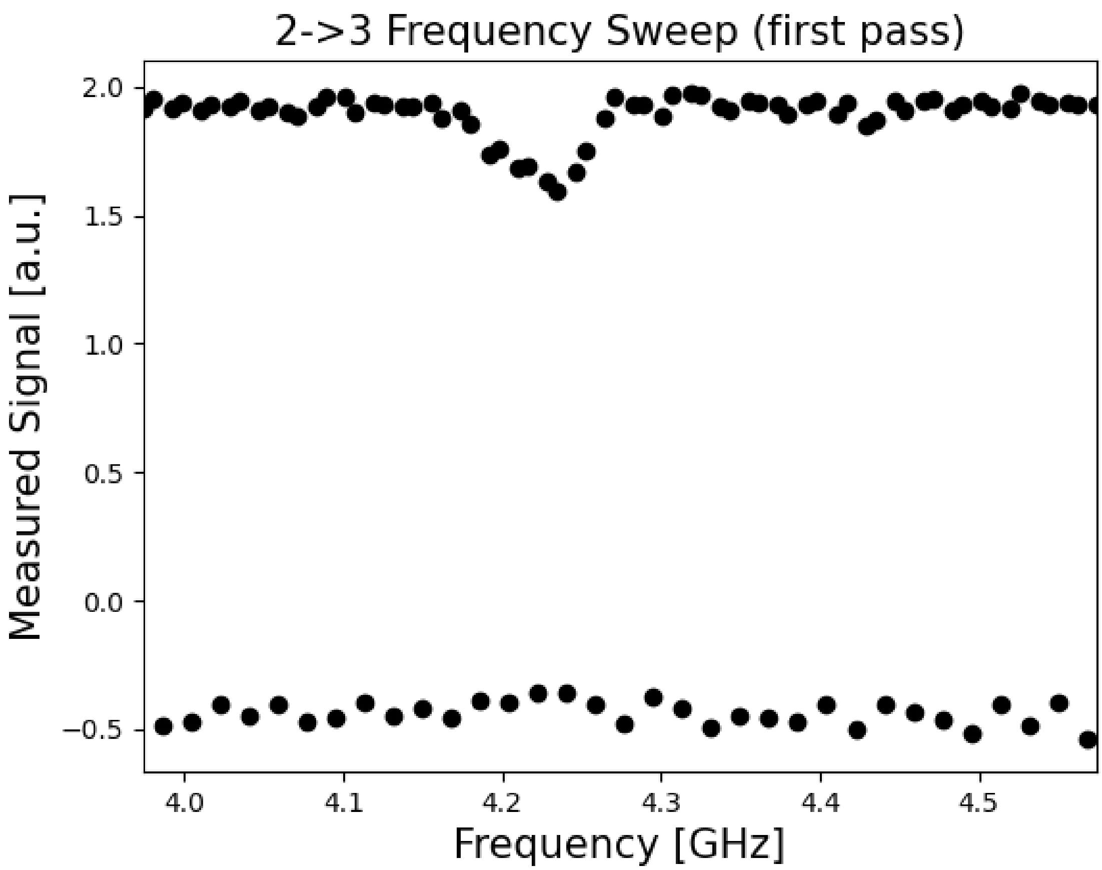
[82]:
```
## run assembled program on backend

num_shots_per_frequency = 1024

excited_freq_sweep_job = backend.run(exp_x23_circs,
                                     meas_level=1,
                                     meas_return='avg',
                                     shots=num_shots_per_frequency)
```

[83]:
```
## monitor job status

job_monitor(excited_freq_sweep_job)
```

Job Status: job has successfully run

[84]:
```
## retrieve job results

excited_freq_sweep_data = get_job_data(excited_freq_sweep_job, average=True)
frequency_sweep_results = job.result(timeout=120)
```

[85]:
```
## plot job results

plt.scatter(sweep_freqs / GHz, excited_freq_sweep_data, color='black')
plt.xlim([min(sweep_freqs / GHz), max(sweep_freqs / GHz)])
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Measured Signal [a.u.]", fontsize=15)
plt.title("2->3 Frequency Sweep (first pass)", fontsize=15)
plt.show()
```



[115]:
```
freq_guess = 4.22

num_freqs = 75
sweep_freqs = freq_guess*GHz + np.linspace(-30*MHz, 30*MHz, num_freqs)
```

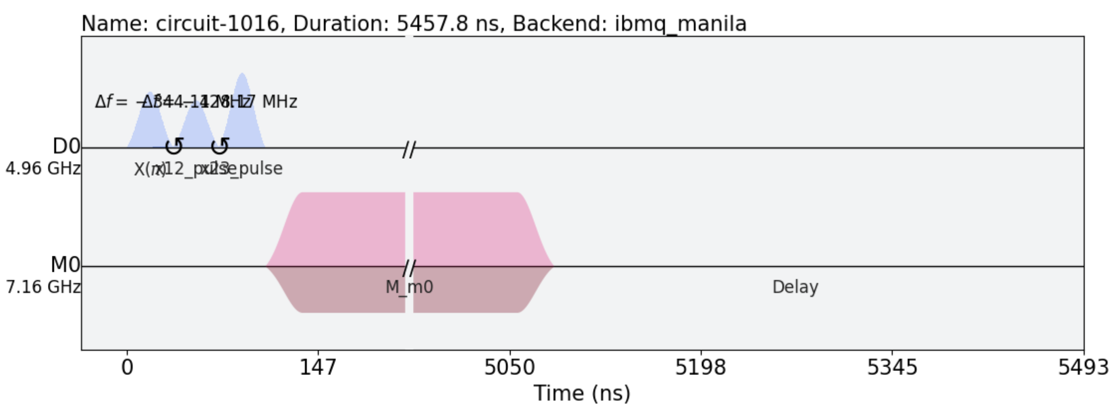
[116]:
```
## create a pulse schedule
with pulse.build(backend=backend, default_alignment='sequential', name='x12␣
↪schedule') as x12_sched:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_12_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x12_duration,
                              amp=pi_amp_12,
                              sigma=x12_sigma,
                              name='x12_pulse'), drive_chan)

qubit_23_freq = Parameter('freq')
with pulse.build(backend=backend, default_alignment='sequential',␣
↪name='Frequency sweep') as freq23_sweep_sched:
    drive_chan = pulse.drive_channel(qubit)
```

```
    pulse.set_frequency(qubit_23_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x23_duration,
                              amp=power_guess,
                              sigma=x23_sigma,
                              name='x23_pulse'), drive_chan)
```

[118]:
```
# prepare state

x12_gate = Gate("one_two_pulse", 1, [])
sweep_23_gate = Gate("two_three_pulse", 1, [qubit_23_freq])
qc_x23 = QuantumCircuit(1, 1)
qc_x23.x(0)
qc_x23.append(x12_gate, [0])
qc_x23.append(sweep_23_gate, [0])
qc_x23.measure(0, 0)
qc_x23.add_calibration(x12_gate, (0,), x12_sched, [])
qc_x23.add_calibration(sweep_23_gate, (0,), freq23_sweep_sched, [qubit_23_freq])

exp_x23_circs = [qc_x23.assign_parameters({qubit_23_freq: f}) for f in␣
 ↪sweep_freqs]
```

[119]:
```
## draw sweep schedule

freq23_sweep_sched = schedule(exp_x23_circs[0], backend)
freq23_sweep_sched.draw(backend=backend)
```

[119]:



[120]:
```
## run assembled program on backend

num_shots_per_frequency = 1024

excited_freq_sweep_job = backend.run(exp_x23_circs,
                                     meas_level=1,
                                     meas_return='avg',
```

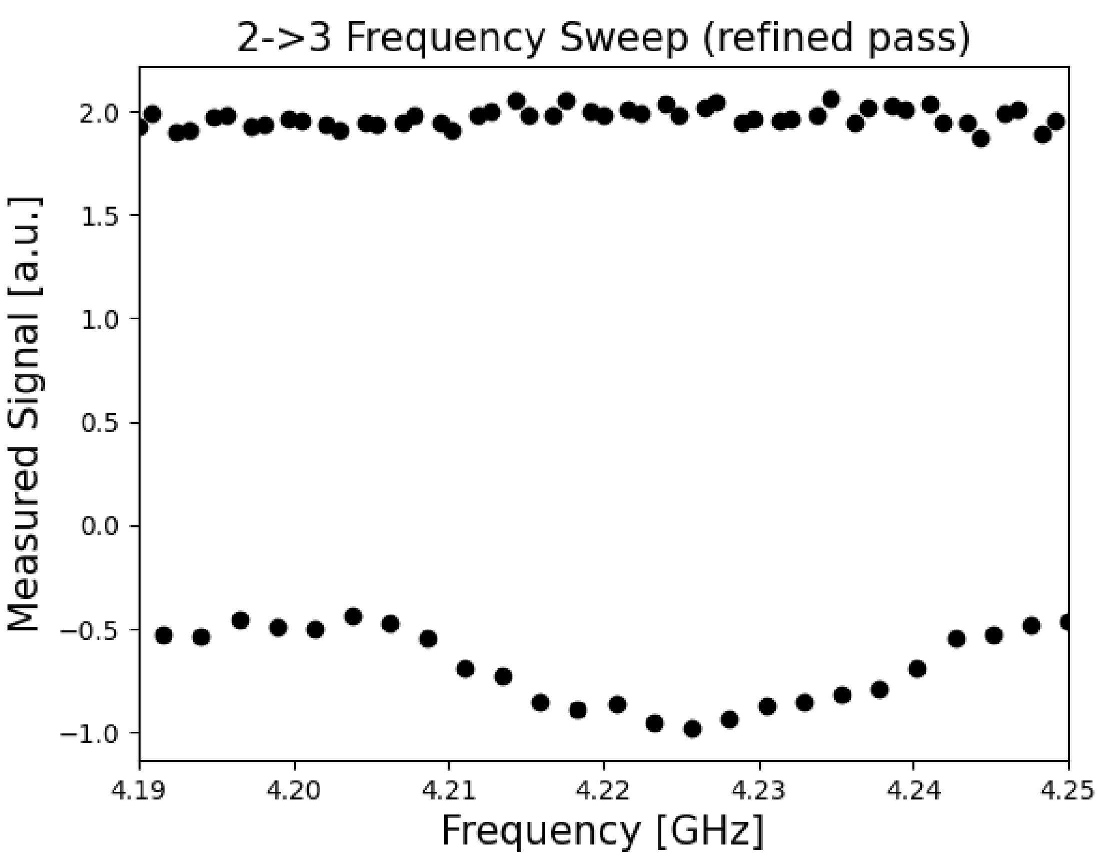
```
                                         shots=num_shots_per_frequency)
```

[121]:
```
## monitor job status

job_monitor(excited_freq_sweep_job)
```

```
Job Status: job has successfully run
```

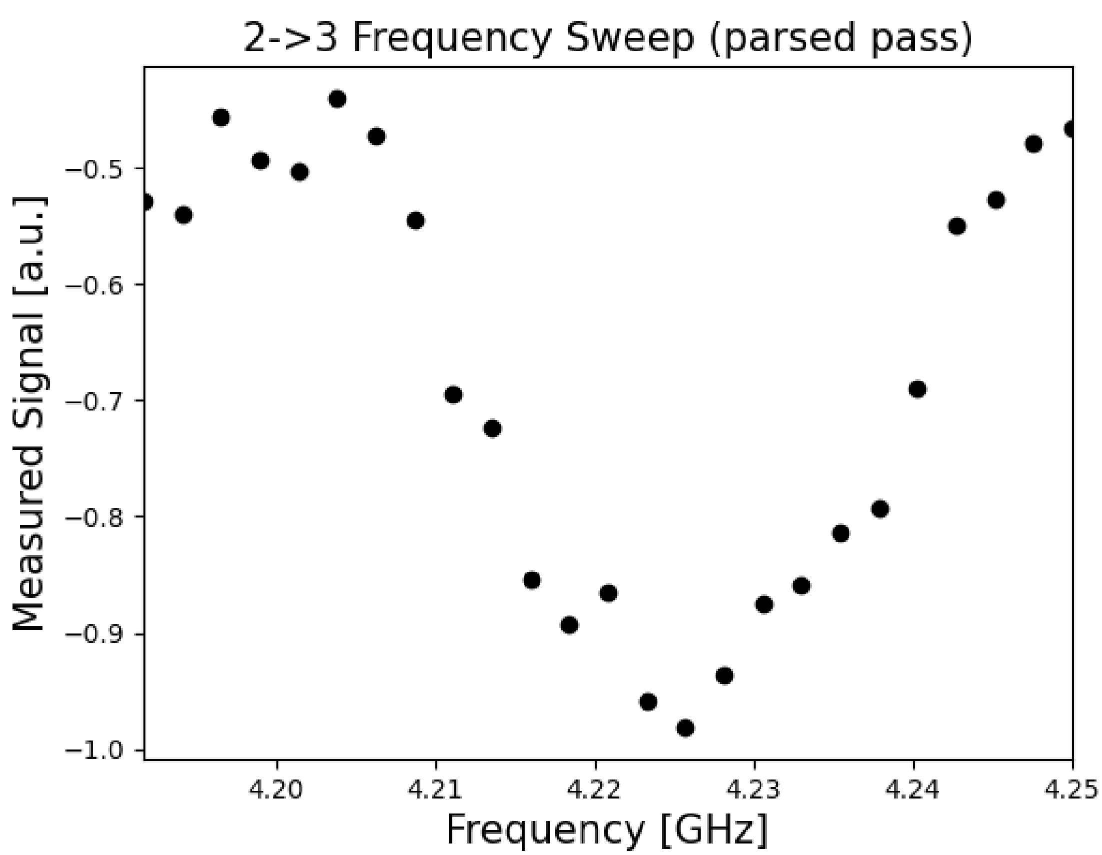
[91]:
```
## retrieve job results

excited_freq_sweep_data = get_job_data(excited_freq_sweep_job, average=True)
frequency_sweep_results = job.result(timeout=120)
```

[123]:
```
## plot job results

plt.scatter(sweep_freqs / GHz, excited_freq_sweep_data, color='black')
plt.xlim([min(sweep_freqs / GHz), max(sweep_freqs / GHz)])
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Measured Signal [a.u.]", fontsize=15)
plt.title("2->3 Frequency Sweep (refined pass)", fontsize=15)
plt.show()
```



[125]:

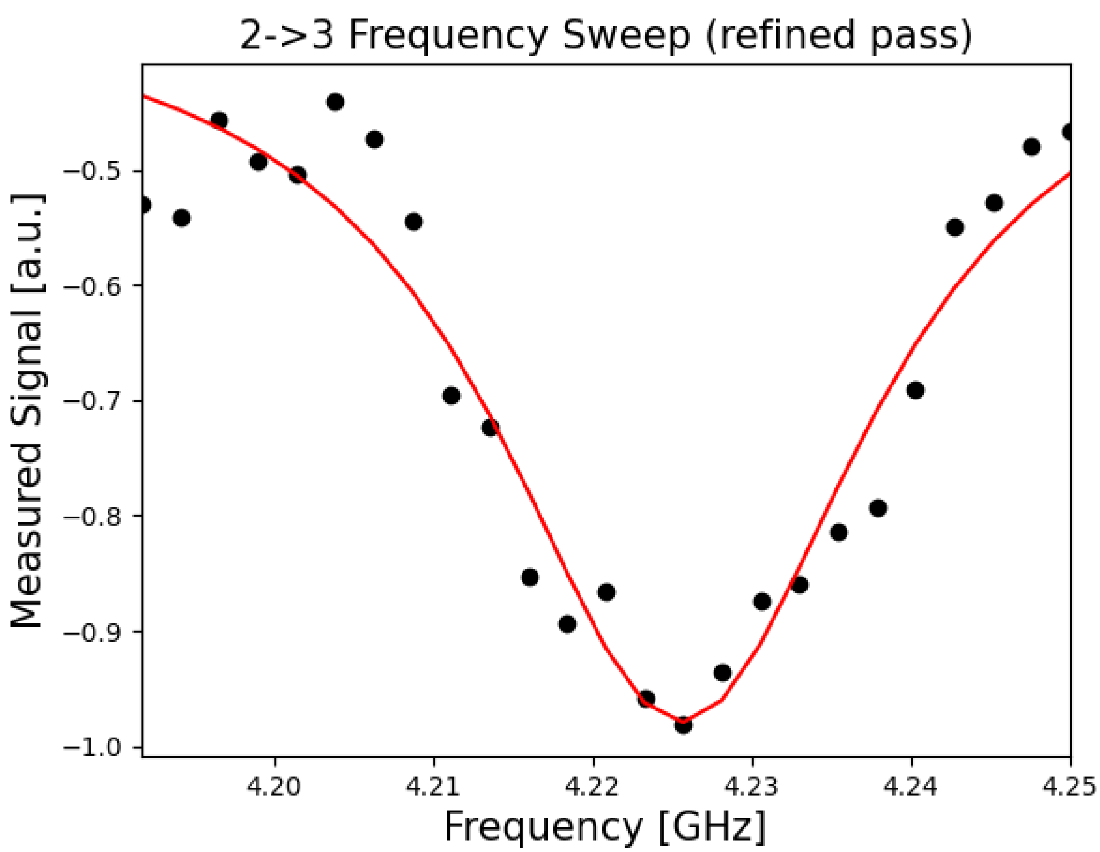
```
## parse job results
num_points = 25
parsed_indices = np.sort(np.argsort(excited_freq_sweep_data)[:num_points])
sweep_freqs_parsed = sweep_freqs[parsed_indices]
excited_freq_sweep_data_parsed = np.
 ↪array(excited_freq_sweep_data)[parsed_indices]

plt.scatter(sweep_freqs_parsed/GHz, excited_freq_sweep_data_parsed,␣
 ↪color='black')
plt.xlim([min(sweep_freqs_parsed/GHz), max(sweep_freqs_parsed/GHz)])
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Measured Signal [a.u.]", fontsize=15)
plt.title("2->3 Frequency Sweep (parsed pass)", fontsize=15)
plt.show()
```



[127]:

```
## fit values to a curve

(excited_sweep_fit_params,
```

```
 excited_sweep_y_fit) = fit_function(sweep_freqs_parsed,
                                     excited_freq_sweep_data_parsed,
                                     lambda x, A, q_freq, B, C: (A / np.pi) *␣
 ↪(B / ((x - q_freq)**2 + B**2)) + C,
                                     [-30, 4.22*GHz, 0.02*GHz, -0.9*GHz] #␣
 ↪initial parameters for curve_fit
                                     )

# Note: only plotting the real part of the signal
plt.scatter(sweep_freqs_parsed/GHz, excited_freq_sweep_data_parsed,␣
 ↪color='black')
plt.plot(sweep_freqs_parsed/GHz, excited_sweep_y_fit, color='red')
plt.xlim([min(sweep_freqs_parsed/GHz), max(sweep_freqs_parsed/GHz)])
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Measured Signal [a.u.]", fontsize=15)
plt.title("2->3 Frequency Sweep (refined pass)", fontsize=15)
plt.show()
```



[128]:
```
## derive frequency using peak

_, qubit_23_freq, _, _ = excited_sweep_fit_params
print(f"2->3 frequency: {round(qubit_23_freq/GHz, 5)} GHz.")
```

```
2->3 frequency: 4.22561 GHz.
```

2.6.2 Calibrate the π pulse for the $|2\rangle \rightarrow |3\rangle$ transition using a Rabi experiment

[129]:
```
## Set Rabi experiment parameters

num_rabi_points = 100

# Drive amplitude values to iterate over: 75 amplitudes evenly spaced from 0 to␣
↪1.0
drive_amp_min = 0
drive_amp_max = 1.0
drive_amps = np.linspace(drive_amp_min, drive_amp_max, num_rabi_points)
```

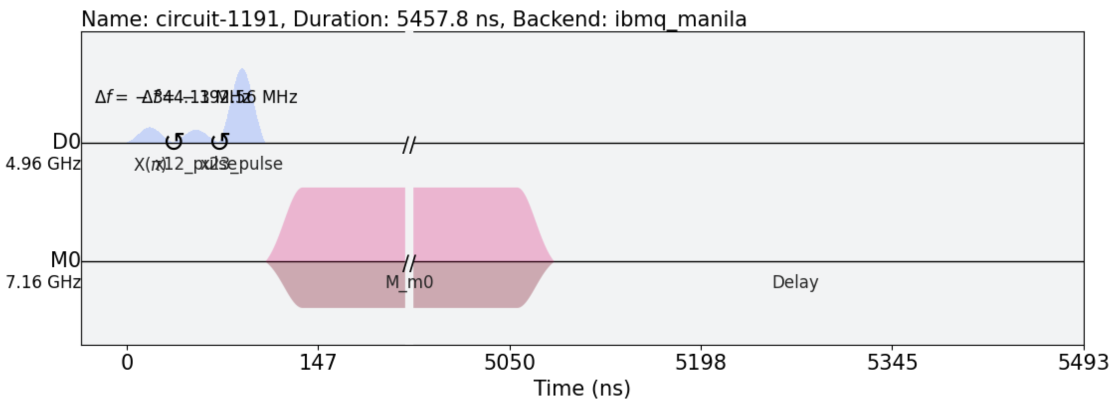
[130]:
```
## Build Rabi experiments

with pulse.build(backend=backend, default_alignment='sequential', name='x12␣
↪schedule') as x12_sched:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_12_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x12_duration,
                              amp=pi_amp_12,
                              sigma=x12_sigma,
                              name='x12_pulse'), drive_chan)

# A drive pulse at the qubit frequency, followed by a measurement, vary the␣
↪drive amplitude each time
amp = Parameter('amp')
with pulse.build(backend=backend, default_alignment='sequential', name='Amp␣
↪sweep') as rabi_sched_23:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_23_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x23_duration,
                              amp=amp,
                              sigma=x23_sigma,
                              name='x23_pulse'), drive_chan)
```

[131]:
```
## create Rabi circuit

x12_gate = Gate("one_two_pulse", 1, [])
rabi_gate = Gate("rabi", 1, [amp])
```

```
qc_rabi = QuantumCircuit(1, 1)

qc_rabi.x(0)
qc_rabi.append(x12_gate, [0])
qc_rabi.append(rabi_gate, [0])
qc_rabi.measure(0, 0)
qc_rabi.add_calibration(x12_gate, (0,), x12_sched, [])
qc_rabi.add_calibration(rabi_gate, (0,), rabi_sched_23, [amp])

exp_rabi_circs = [qc_rabi.assign_parameters({amp: a}) for a in drive_amps]
```

[132]:
```
## draw Rabi schedule

rabi_schedule = schedule(exp_rabi_circs[-1], backend)
rabi_schedule.draw(backend=backend)
```

[132]:



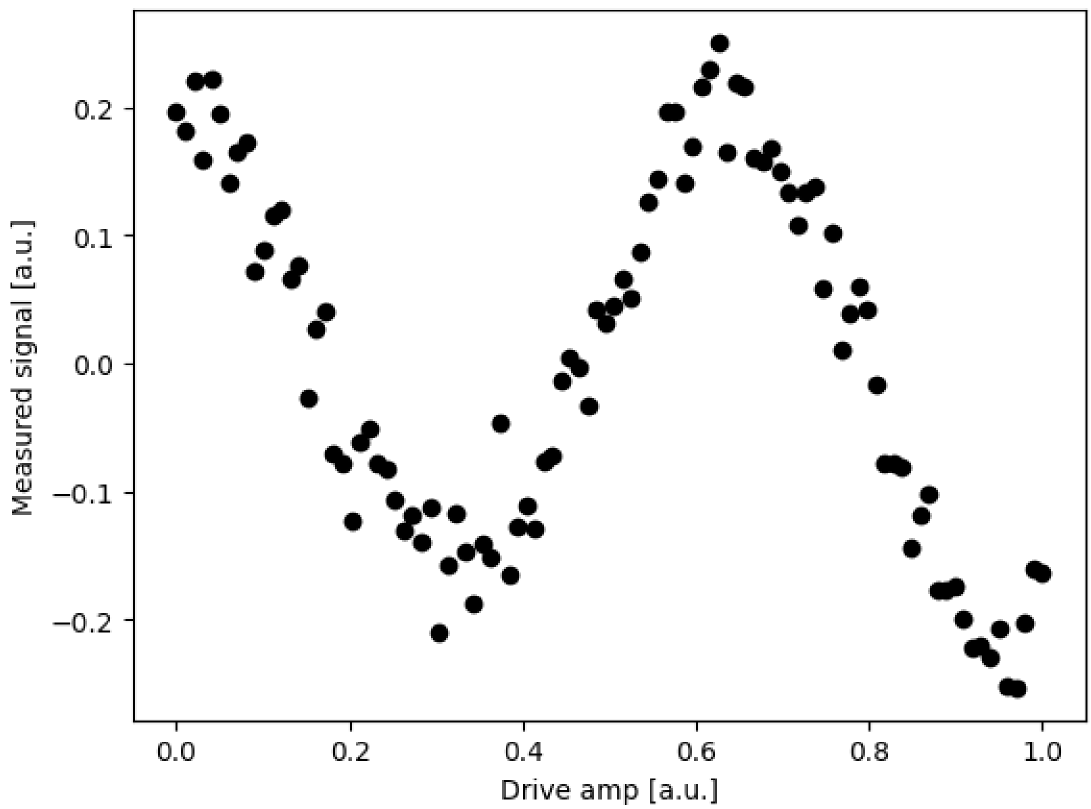
[133]:
```
## run assembled program on backend

num_shots_per_frequency = 1024

rabi_23_job = backend.run(exp_rabi_circs,
                          meas_level=1,
                          meas_return='avg',
                          shots=num_shots_per_frequency)
```

[134]:
```
## monitor job status

job_monitor(rabi_23_job)
```

```
Job Status: job has successfully run
```

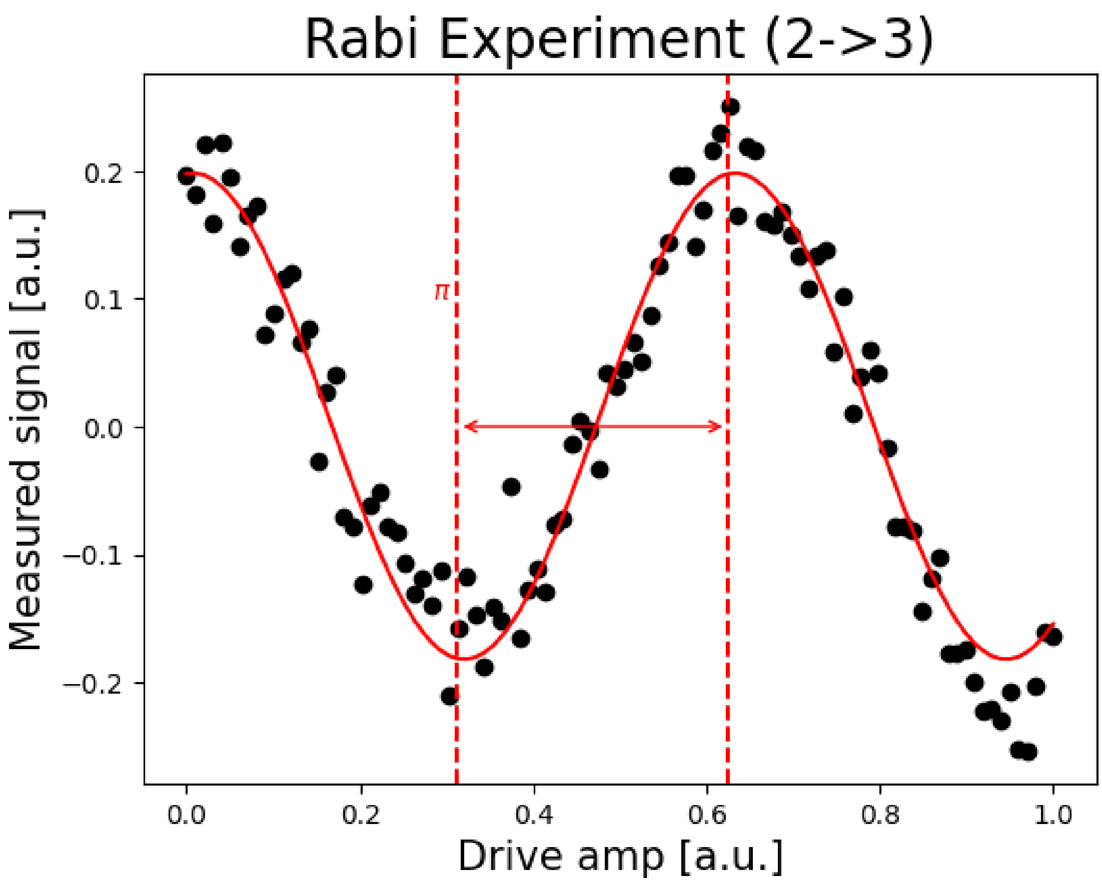
[135]:
```
## retrieve job results
```

```
rabi_23_data = np.real(baseline_remove(get_job_data(rabi_23_job, average=True)))

plt.xlabel("Drive amp [a.u.]")
plt.ylabel("Measured signal [a.u.]")
plt.scatter(drive_amps, rabi_23_data, color='black') # plot real part of Rabi␣
 ↪values
plt.show()
```



[137]:
```
## fit values to a curve

(rabi_23_fit_params,
 rabi_23_y_fit) = fit_function(drive_amps,
                               rabi_23_data,
                               lambda x, A, B, drive_23_period, phi: (A*np.
 ↪cos(2*np.pi*x/drive_23_period - phi) + B),
                               [0.2, -np.pi, 0.3, 0])

plt.scatter(drive_amps, rabi_23_data, color='black')
plt.plot(drive_amps, rabi_23_y_fit, color='red')
```

```
drive_23_period = rabi_23_fit_params[2]

pi_amp_23 = drive_23_period/2

plt.axvline(pi_amp_23, color='red', linestyle='--')
plt.axvline(pi_amp_23+drive_23_period/2, color='red', linestyle='--')
plt.annotate("", xy=(pi_amp_23+drive_23_period/2, 0), xytext=(pi_amp_23,0),␣
 ↪arrowprops=dict(arrowstyle="<->", color='red'))
plt.annotate("$\pi$", xy=(pi_amp_23-0.03, 0.1), color='red')

plt.xlabel("Drive amp [a.u.]", fontsize=15)
plt.ylabel("Measured signal [a.u.]", fontsize=15)
plt.title('Rabi Experiment (2->3)', fontsize=20)
plt.show()
```



[138]:
```
## print pi amplitude

print(f"Pi Amplitude (2->3) = {round(pi_amp_23, 5)}")
```

```
Pi Amplitude (2->3) = 0.31272
```

2.6.3 Build an IQ discriminator for distinguishing the states into |0 , |1 , |2 , and |3

[151]:
```
## Schedule pulse from 1->2 and 2->3

with pulse.build(backend=backend, default_alignment='sequential', name='x12␣
↪schedule') as x12_sched:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_12_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x12_duration,
                              amp=pi_amp_12,
                              sigma=x12_sigma,
                              name='x12_pulse'), drive_chan)

with pulse.build(backend=backend, default_alignment='sequential', name='Amp␣
↪sweep') as x23_sched:
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(qubit_23_freq, drive_chan)
    pulse.play(pulse.Gaussian(duration=x23_duration,
                              amp=pi_amp_23,
                              sigma=x23_sigma,
                              name='x23_pulse'), drive_chan)
```

[152]:
```
## Create 4 circuits

# 0 state
qc_ground = QuantumCircuit(1, 1)
qc_ground.measure(0, 0)

# 1 state
qc_one = QuantumCircuit(1, 1)
qc_one.x(0)
qc_one.measure(0, 0)

# 2 state
x12_gate = Gate("one_two_pulse", 1, [])
qc_two = QuantumCircuit(1, 1)
qc_two.x(0)
qc_two.append(x12_gate, [0])
qc_two.measure(0, 0)
qc_two.add_calibration(x12_gate, (0,), x12_sched, [])

# 3 state
x23_gate = Gate("two_three_pulse", 1, [])
qc_three = QuantumCircuit(1, 1)
qc_three.x(0)
```

```
qc_three.append(x12_gate, [0])
qc_three.append(x23_gate, [0])
qc_three.measure(0, 0)
qc_three.add_calibration(x12_gate, (0,), x12_sched, [])
qc_three.add_calibration(x23_gate, (0,), x23_sched, [])
```

[153]:
```
## run assembled program on backend

num_shots_per_frequency = 1024

IQ_0123_job = backend.run([qc_ground, qc_one, qc_two, qc_three],
                          meas_level=1,
                          meas_return='single',
                          shots=num_shots_per_frequency)
```

[154]:
```
## monitor job status

job_monitor(IQ_0123_job)
```

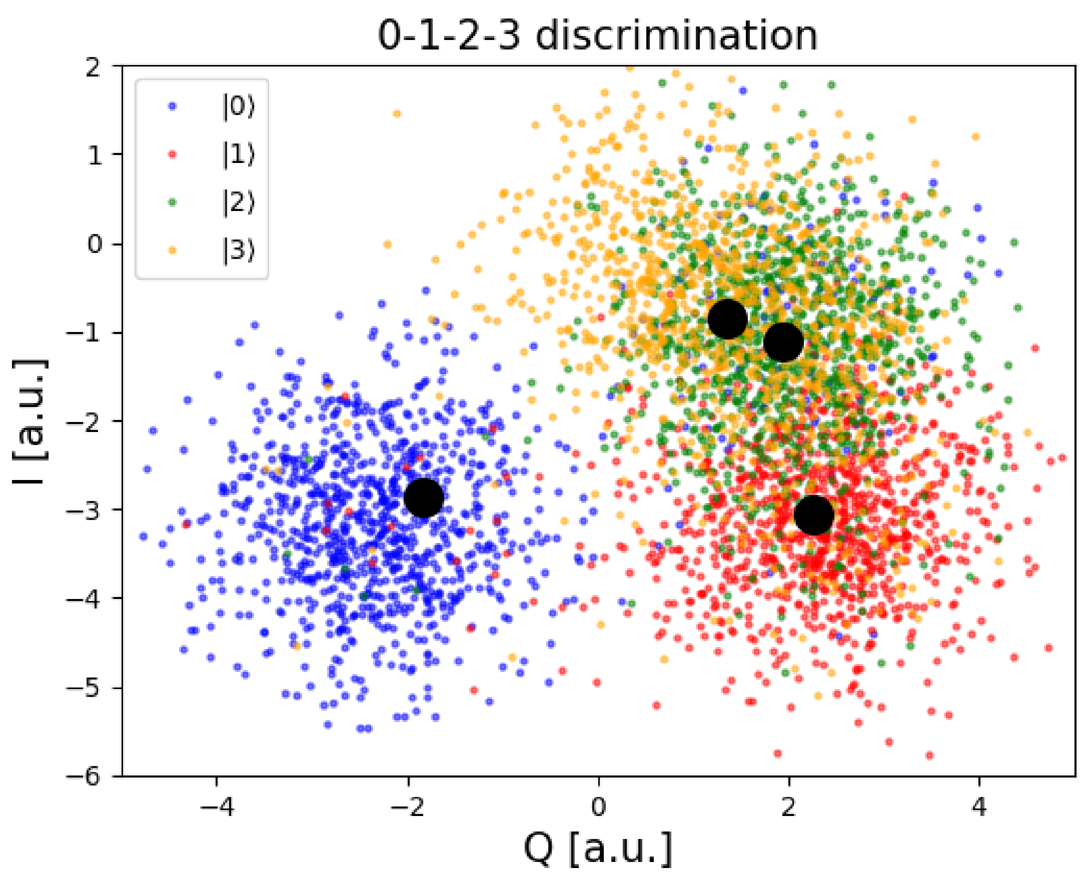
```
Job Status: job has successfully run
```

[155]:
```
## retrieve job results

IQ_0123_data = get_job_data(IQ_0123_job, average=False)
zero_data = IQ_0123_data[0]
one_data = IQ_0123_data[1]
two_data = IQ_0123_data[2]
three_data = IQ_0123_data[3]
```

[156]:
```
## plot the distributions for 0, 1, 2, 3

# one data plotted in blue
plt.scatter(np.real(zero_data), np.imag(zero_data),
                s=5, cmap='viridis', c='blue', alpha=0.5, label=r'$|0\rangle$')
# one data plotted in red
plt.scatter(np.real(one_data), np.imag(one_data),
                s=5, cmap='viridis', c='red', alpha=0.5, label=r'$|1\rangle$')
# two data plotted in green
plt.scatter(np.real(two_data), np.imag(two_data),
                s=5, cmap='viridis', c='green', alpha=0.5, label=r'$|2\rangle$')
# three data plotted in yellow
plt.scatter(np.real(three_data), np.imag(three_data),
                s=5, cmap='viridis', c='orange', alpha=0.5,␣
 ↪label=r'$|3\rangle$')

# Plot a large dot for the average result of the 0, 1 and 2 states.
mean_zero = np.mean(zero_data) # takes mean of both real and imaginary parts
mean_one = np.mean(one_data)
```

```
mean_two = np.mean(two_data)
mean_three = np.mean(three_data)

plt.scatter(np.real(mean_zero), np.imag(mean_zero),
            s=200, cmap='viridis', c='black',alpha=1.0)
plt.scatter(np.real(mean_one), np.imag(mean_one),
            s=200, cmap='viridis', c='black',alpha=1.0)
plt.scatter(np.real(mean_two), np.imag(mean_two),
            s=200, cmap='viridis', c='black',alpha=1.0)
plt.scatter(np.real(mean_three), np.imag(mean_three),
            s=200, cmap='viridis', c='black',alpha=1.0)

plt.xlim(-5, 5)
plt.ylim(-6, 2)
plt.legend()
plt.ylabel('I [a.u.]', fontsize=15)
plt.xlabel('Q [a.u.]', fontsize=15)
plt.title("0-1-2-3 discrimination", fontsize=15)
```

```
[156]: Text(0.5, 1.0, '0-1-2-3 discrimination')
```

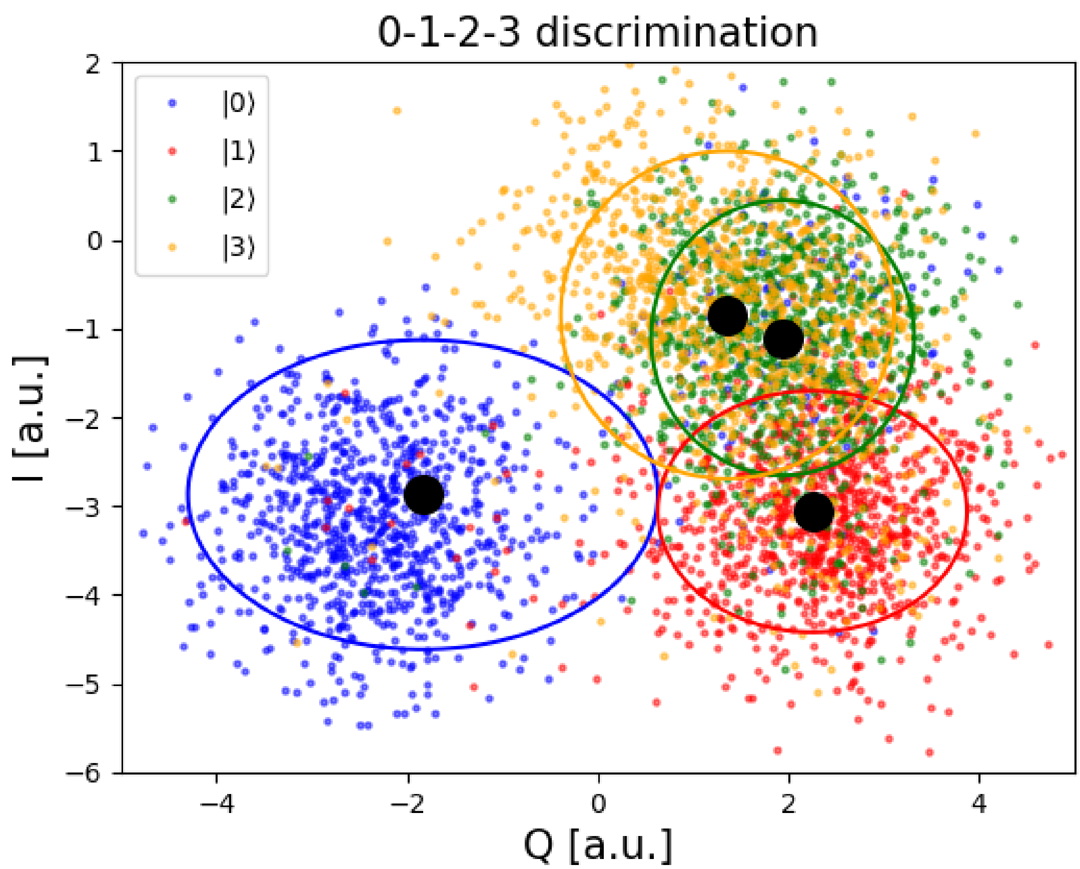


[157]:

```
## draw classifier for 0, 1, 2, 3

# zero data plotted in blue
plt.scatter(np.real(zero_data), np.imag(zero_data),
                s=5, cmap='viridis', c='blue', alpha=0.5, label=r'$|0\rangle$')
# one data plotted in red
plt.scatter(np.real(one_data), np.imag(one_data),
                s=5, cmap='viridis', c='red', alpha=0.5, label=r'$|1\rangle$')
# two data plotted in green
plt.scatter(np.real(two_data), np.imag(two_data),
                s=5, cmap='viridis', c='green', alpha=0.5, label=r'$|2\rangle$')
# three data plotted in yellow
plt.scatter(np.real(three_data), np.imag(three_data),
                s=5, cmap='viridis', c='orange', alpha=0.5,␣
 ↪label=r'$|3\rangle$')

# Plot a large dot for the average result of the 0, 1 and 2 states.
mean_zero = np.mean(zero_data) # takes mean of both real and imaginary parts
mean_one = np.mean(one_data)
mean_two = np.mean(two_data)
mean_three = np.mean(three_data)

q_center_zero, i_center_zero = np.real(mean_zero), np.imag(mean_zero)
plt.scatter(q_center_zero, i_center_zero,
            s=200, cmap='viridis', c='black',alpha=1.0)
q_center_one, i_center_one = np.real(mean_one), np.imag(mean_one)
plt.scatter(q_center_one, i_center_one,
            s=200, cmap='viridis', c='black',alpha=1.0)
q_center_two, i_center_two = np.real(mean_two), np.imag(mean_two)
plt.scatter(q_center_two, i_center_two,
            s=200, cmap='viridis', c='black',alpha=1.0)
q_center_three, i_center_three = np.real(mean_three), np.imag(mean_three)
plt.scatter(q_center_three, i_center_three,
            s=200, cmap='viridis', c='black',alpha=1.0)

std_devs = 1.5
q_axis_zero, i_axis_zero = std_devs*np.std(np.real(zero_data)), std_devs*np.
 ↪std(np.imag(zero_data))
draw_ellipse('blue', q_center_zero, i_center_zero, q_axis_zero, i_axis_zero)
q_axis_one, i_axis_one = std_devs*np.std(np.real(one_data)), std_devs*np.std(np.
 ↪imag(one_data))
draw_ellipse('red', q_center_one, i_center_one, q_axis_one, i_axis_one)
q_axis_two, i_axis_two = std_devs*np.std(np.real(two_data)), std_devs*np.std(np.
 ↪imag(two_data))
draw_ellipse('green', q_center_two, i_center_two, q_axis_two, i_axis_two)
```

```
q_axis_three, i_axis_three = std_devs*np.std(np.real(three_data)), std_devs*np.
 ↪std(np.imag(three_data))
draw_ellipse('orange', q_center_three, i_center_three, q_axis_three,␣
 ↪i_axis_three)

plt.xlim(-5, 5)
plt.ylim(-6, 2)
plt.legend()
plt.ylabel('I [a.u.]', fontsize=15)
plt.xlabel('Q [a.u.]', fontsize=15)
plt.title("0-1-2-3 discrimination", fontsize=15)
```

[157]: Text(0.5, 1.0, '0-1-2-3 discrimination')



2.6.4 Estimate the occupation probability for |0 , |1 , |2 , and |3 near the calibrated π pulse of the |0 → |1 transition

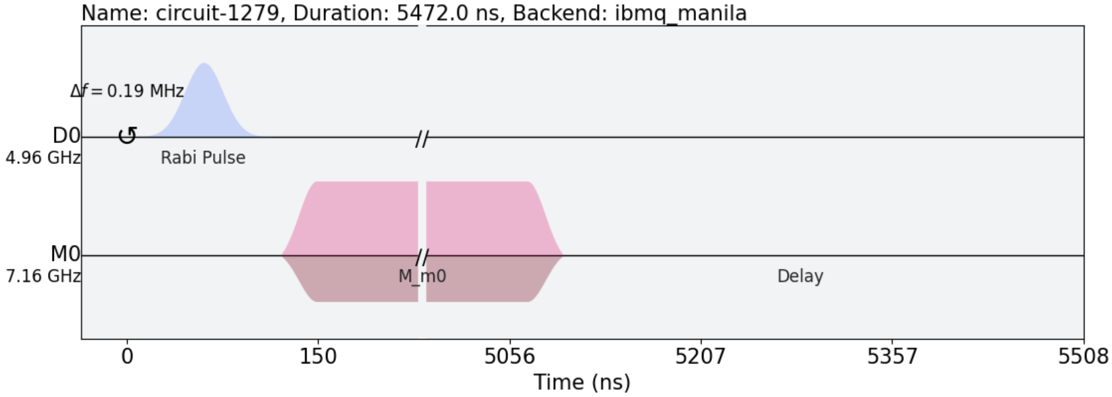
[158]:
```
## Set Rabi experiment parameters

# Rabi experiment parameters
num_rabi_points = 75
gap = 0.05

# Drive amplitude values to iterate over: 50 amplitudes evenly spaced from 0 to␣
↪0.75
drive_amp_min = pi_amp - gap / 2
drive_amp_max = pi_amp + gap / 2
drive_amps = np.linspace(drive_amp_min, drive_amp_max, num_rabi_points)
```

[159]:
```
## create a pulse schedule

drive_amp = Parameter('drive_amp')
with pulse.build(backend=backend, default_alignment='sequential', name='Rabi␣
↪Experiment') as rabi_sched:
    drive_duration = get_closest_multiple_of_16(pulse.
↪seconds_to_samples(drive_duration_sec))
    drive_sigma = pulse.seconds_to_samples(drive_sigma_sec)
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(rough_qubit_frequency, drive_chan)
    pulse.play(pulse.Gaussian(duration=drive_duration,
                              amp=drive_amp,
                              sigma=drive_sigma,
                              name='Rabi Pulse'), drive_chan)
```

[160]:
```
## create Rabi circuit

rabi_gate = Gate("rabi", 1, [drive_amp])

qc_rabi = QuantumCircuit(1, 1)

qc_rabi.append(rabi_gate, [0])
qc_rabi.measure(0, 0)
qc_rabi.add_calibration(rabi_gate, (0,), rabi_sched, [drive_amp])

exp_rabi_circs = [qc_rabi.assign_parameters({drive_amp: a}, inplace=False) for␣
↪a in drive_amps]
```

[161]:
```
## draw Rabi schedule

rabi_schedule = schedule(exp_rabi_circs[-1], backend)
rabi_schedule.draw(backend=backend)
```

[161]:



[162]:
```
## run assembled program on backend

num_shots_per_point = 1024

job = backend.run(exp_rabi_circs,
                  meas_level=1,
                  meas_return='single',
                  shots=num_shots_per_point)
```

[163]:
```
## monitor job status

job_monitor(job)
```

```
Job Status: job has successfully run
```

[164]:
```
## retrieve job results

rabi_results = job.result(timeout=120)
```

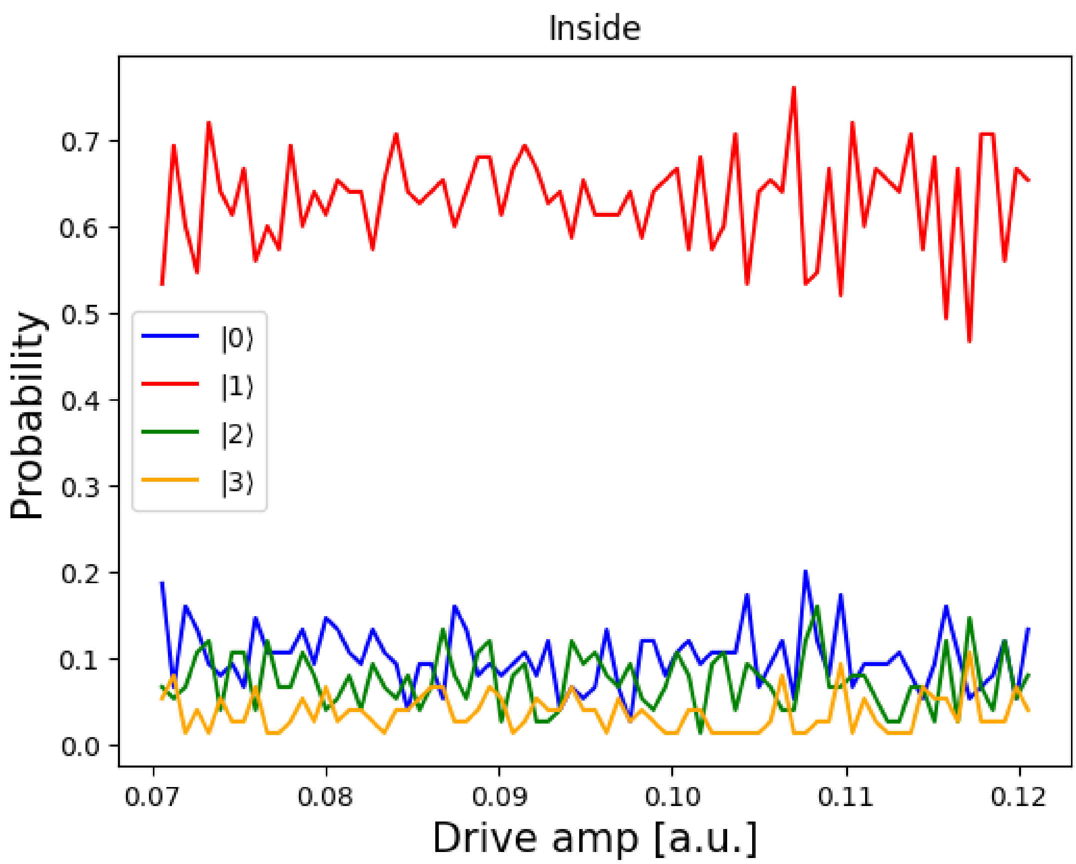
[165]:
```
## classify results

ellipses = [(q_center_zero, i_center_zero, q_axis_zero, i_axis_zero),
            (q_center_one, i_center_one, q_axis_one, i_axis_one),
            (q_center_two, i_center_two, q_axis_two, i_axis_two),
            (q_center_three, i_center_three, q_axis_three, i_axis_three)]

amp_states = []
amp_classifications_inside = []
amp_classifications_closest = []

for j in range(num_rabi_points):
    states = []
    classifications_inside = []
```

```
    classifications_closest = []
    for i in range(num_rabi_points):
        state = rabi_results.get_memory(i)[j]
        states.append(state)
        classifications_inside.append(classify_inside(state * scale_factor,␣
↪ellipses))
        classifications_closest.append(classify_closest(state * scale_factor,␣
↪ellipses))

    amp_states.append(np.array(states))
    amp_classifications_inside.append(np.array(classifications_inside))
    amp_classifications_closest.append(np.array(classifications_closest))
```

[166]:
```
max_probability = 0
probabilities_inside = []
probabilities_closest = []
for j in range(len(amp_classifications_inside)):
    num_escaped_inside = np.count_nonzero(amp_classifications_inside[j] == -1)␣
↪+ np.count_nonzero(amp_classifications_inside[j] == 2)
    probability_inside = num_escaped_inside / num_rabi_points
    probabilities_inside.append(probability_inside)
for j in range(len(amp_classifications_closest)):
    num_escaped_closest = np.count_nonzero(amp_classifications_closest[j] ==␣
↪-1) + np.count_nonzero(amp_classifications_closest[j] == 2)
    probability_closest = num_escaped_closest / num_rabi_points
    probabilities_closest.append(probability_closest)
```

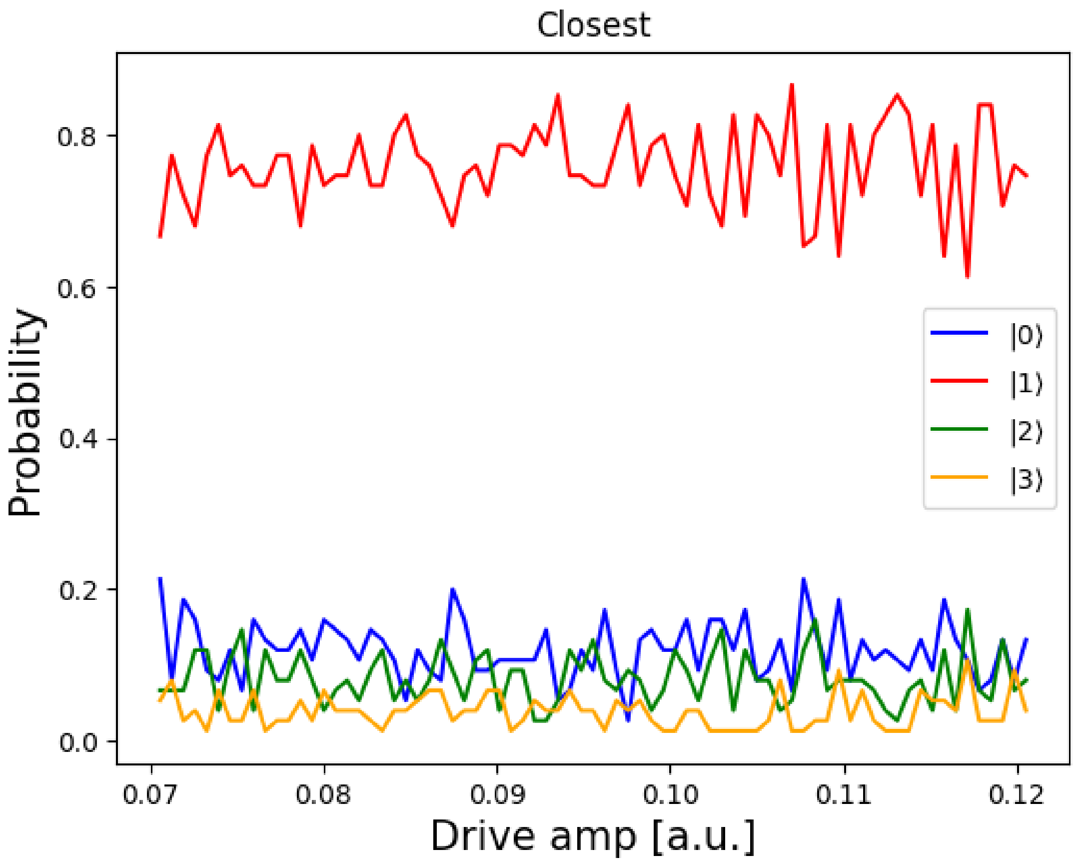
[169]:
```
## plot probabilities

zero_probabilities_inside = []
one_probabilities_inside = []
two_probabilities_inside = []
three_probabilities_inside = []
for j in range(len(amp_classifications_inside)):
    num_zero = np.count_nonzero(amp_classifications_inside[j] == 0)
    zero_probabilities_inside.append(num_zero / num_rabi_points)
    num_one = np.count_nonzero(amp_classifications_inside[j] == 1)
    one_probabilities_inside.append(num_one / num_rabi_points)
    num_two = np.count_nonzero(amp_classifications_inside[j] == 2)
    two_probabilities_inside.append(num_two / num_rabi_points)
    num_three = np.count_nonzero(amp_classifications_inside[j] == 3)
    three_probabilities_inside.append(num_three / num_rabi_points)

plt.plot(drive_amps, zero_probabilities_inside, color='blue',␣
↪label=r'$|0\rangle$')
plt.plot(drive_amps, one_probabilities_inside, color='red',␣
↪label=r'$|1\rangle$')
```

```
plt.plot(drive_amps, two_probabilities_inside, color='green',␣
 ↪label=r'$|2\rangle$')
plt.plot(drive_amps, three_probabilities_inside, color='orange',␣
 ↪label=r'$|3\rangle$')

plt.title('Inside')
plt.xlabel("Drive amp [a.u.]", fontsize=15)
plt.ylabel("Probability", fontsize=15)
plt.legend()
plt.show()
```



[170]:
```
## plot probabilities

zero_probabilities_closest = []
one_probabilities_closest = []
two_probabilities_closest = []
three_probabilities_closest = []
for j in range(len(amp_classifications_closest)):
    num_zero = np.count_nonzero(amp_classifications_closest[j] == 0)
```

```python
    zero_probabilities_closest.append(num_zero / num_rabi_points)
    num_one = np.count_nonzero(amp_classifications_closest[j] == 1)
    one_probabilities_closest.append(num_one / num_rabi_points)
    num_two = np.count_nonzero(amp_classifications_closest[j] == 2)
    two_probabilities_closest.append(num_two / num_rabi_points)
    num_three = np.count_nonzero(amp_classifications_closest[j] == 3)
    three_probabilities_closest.append(num_three / num_rabi_points)

plt.plot(drive_amps, zero_probabilities_closest, color='blue',␣
 ↪label=r'$|0\rangle$')
plt.plot(drive_amps, one_probabilities_closest, color='red',␣
 ↪label=r'$|1\rangle$')
plt.plot(drive_amps, two_probabilities_closest, color='green',␣
 ↪label=r'$|2\rangle$')
plt.plot(drive_amps, three_probabilities_closest, color='orange',␣
 ↪label=r'$|3\rangle$')

plt.title('Closest')
plt.xlabel("Drive amp [a.u.]", fontsize=15)
plt.ylabel("Probability", fontsize=15)
plt.legend()
plt.show()
```



2.6.5 Estimate the occupation probability for |0 , |1 , and |2 near the calibrated frequency of the |0 → |1 transition

[200]:
```
## Drive pulse parameters

gap = 20 * MHz
frequency_min = rough_qubit_frequency - gap / 2
frequency_max = rough_qubit_frequency + gap / 2
frequency_step = 1 * MHz
# Construct an np array of the frequencies for our experiment
frequencies = np.arange(frequency_min,
                        frequency_max,
                        frequency_step)
```

[201]:
```
## create a pulse schedule

# Create the base schedule, start with drive pulse acting on the drive channel
freq = Parameter('freq')
```

```python
with pulse.build(backend=backend, default_alignment='sequential',␣
 ↪name='Frequency sweep') as sweep_sched:
    drive_duration = get_closest_multiple_of_16(pulse.
 ↪seconds_to_samples(drive_duration_sec))
    drive_sigma = pulse.seconds_to_samples(drive_sigma_sec)
    drive_chan = pulse.drive_channel(qubit)
    pulse.set_frequency(freq, drive_chan)
    # Drive pulse samples
    pulse.play(pulse.Gaussian(duration=drive_duration,
                              sigma=drive_sigma,
                              amp=pi_amp,
                              name='freq_sweep_excitation_pulse'), drive_chan)
```

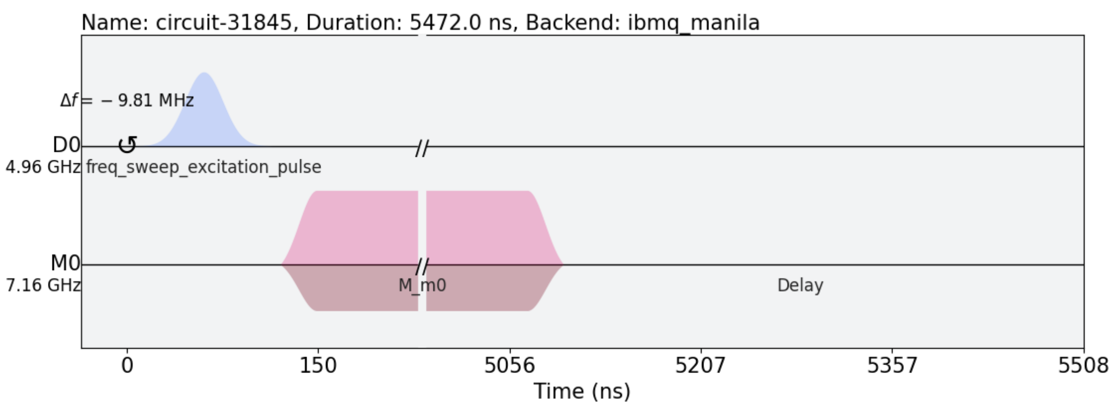
[202]:

```python
## create sweep

sweep_gate = Gate("sweep", 1, [freq])

qc_sweep = QuantumCircuit(1, 1)

qc_sweep.append(sweep_gate, [0])
qc_sweep.measure(0, 0)
qc_sweep.add_calibration(sweep_gate, (0,), sweep_sched, [freq])

exp_sweep_circs = [qc_sweep.assign_parameters({freq: f}, inplace=False) for f␣
 ↪in frequencies]
```

[203]:

```python
## draw sweep schedule

sweep_schedule = schedule(exp_sweep_circs[0], backend)
sweep_schedule.draw(backend=backend)
```

[203]:



[204]:

```
## run assembled program on backend

num_shots_per_frequency = 1024

job = backend.run(exp_sweep_circs,
                  meas_level=1,
                  meas_return='avg',
                  shots=num_shots_per_frequency)
```

[205]:

```
## monitor job status

job_monitor(job)
```

```
Job Status: job has successfully run
```

[210]:

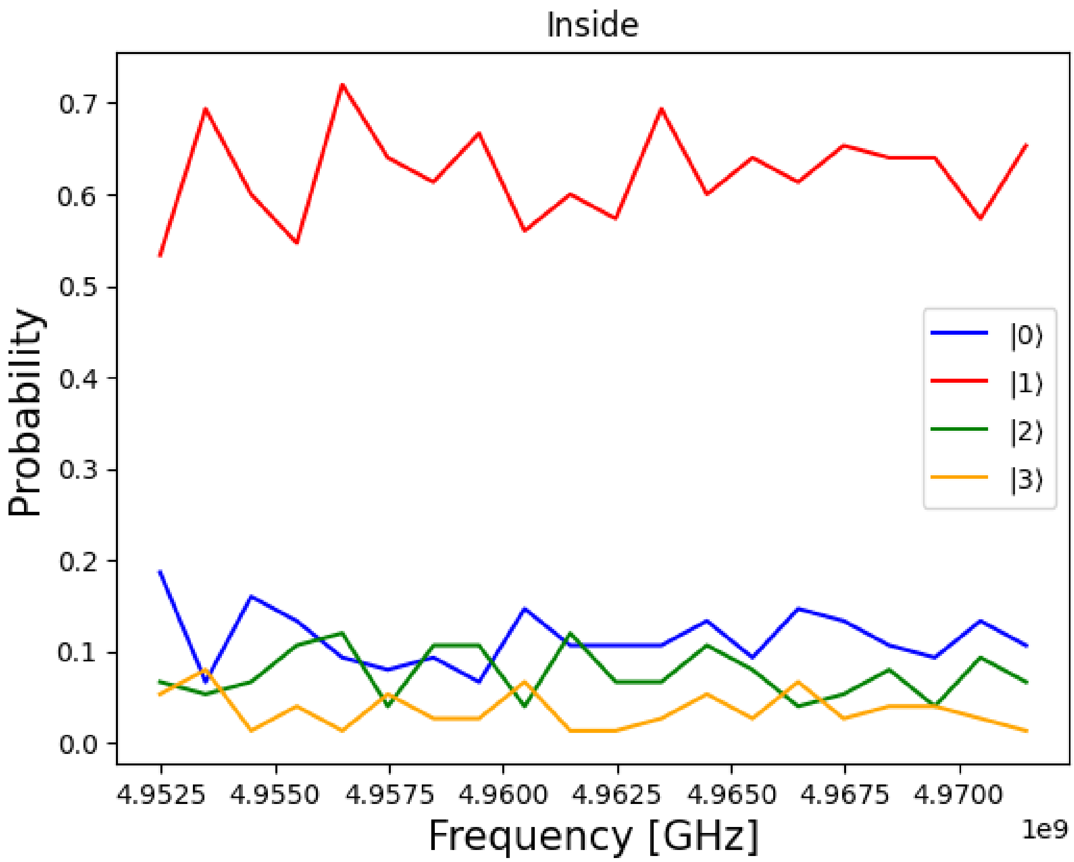
```
## retrieve job results

frequency_sweep_results = job.result(timeout=120)
```

[211]:

```
## classify results

ellipses = [(q_center_zero, i_center_zero, q_axis_zero, i_axis_zero),
            (q_center_one, i_center_one, q_axis_one, i_axis_one),
            (q_center_two, i_center_two, q_axis_two, i_axis_two),
            (q_center_three, i_center_three, q_axis_three, i_axis_three)]

freq_states = []
freq_classifications_inside = []
freq_classifications_closest = []

for j in range(len(frequencies)):
    states = []
    classifications_inside = []
    classifications_closest = []
    for i in range(num_rabi_points):
        state = rabi_results.get_memory(i)[j]
        states.append(state)
        classifications_inside.append(classify_inside(state * scale_factor,␣
 ↪ellipses))
        classifications_closest.append(classify_closest(state * scale_factor,␣
 ↪ellipses))

    freq_states.append(np.array(states))
    freq_classifications_inside.append(np.array(classifications_inside))
    freq_classifications_closest.append(np.array(classifications_closest))
```

[212]:
```
max_probability = 0
probabilities_inside = []
probabilities_closest = []
for j in range(len(freq_classifications_inside)):
    num_escaped_inside = np.count_nonzero(freq_classifications_inside[j] == -1)␣
 ↪+ np.count_nonzero(freq_classifications_inside[j] == 2)
    probability_inside = num_escaped_inside / num_rabi_points
    probabilities_inside.append(probability_inside)
for j in range(len(freq_classifications_closest)):
    num_escaped_closest = np.count_nonzero(freq_classifications_closest[j] ==␣
 ↪-1) + np.count_nonzero(freq_classifications_closest[j] == 2)
    probability_closest = num_escaped_closest / num_rabi_points
    probabilities_closest.append(probability_closest)
```

[213]:
```
## plot probabilities

zero_probabilities_inside = []
one_probabilities_inside = []
two_probabilities_inside = []
three_probabilities_inside = []
for j in range(len(freq_classifications_inside)):
    num_zero = np.count_nonzero(freq_classifications_inside[j] == 0)
    zero_probabilities_inside.append(num_zero / num_rabi_points)
    num_one = np.count_nonzero(freq_classifications_inside[j] == 1)
    one_probabilities_inside.append(num_one / num_rabi_points)
    num_two = np.count_nonzero(freq_classifications_inside[j] == 2)
    two_probabilities_inside.append(num_two / num_rabi_points)
    num_three = np.count_nonzero(freq_classifications_inside[j] == 3)
    three_probabilities_inside.append(num_three / num_rabi_points)

plt.plot(frequencies, zero_probabilities_inside, color='blue',␣
 ↪label=r'$|0\rangle$')
plt.plot(frequencies, one_probabilities_inside, color='red',␣
 ↪label=r'$|1\rangle$')
plt.plot(frequencies, two_probabilities_inside, color='green',␣
 ↪label=r'$|2\rangle$')
plt.plot(frequencies, three_probabilities_inside, color='orange',␣
 ↪label=r'$|3\rangle$')

plt.title('Inside')
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Probability", fontsize=15)
plt.legend()
plt.show()
```



[214]:

```
## plot probabilities

zero_probabilities_closest = []
one_probabilities_closest = []
two_probabilities_closest = []
three_probabilities_closest = []
for j in range(len(freq_classifications_closest)):
    num_zero = np.count_nonzero(freq_classifications_closest[j] == 0)
    zero_probabilities_closest.append(num_zero / num_rabi_points)
    num_one = np.count_nonzero(freq_classifications_closest[j] == 1)
    one_probabilities_closest.append(num_one / num_rabi_points)
    num_two = np.count_nonzero(freq_classifications_closest[j] == 2)
    two_probabilities_closest.append(num_two / num_rabi_points)
    num_three = np.count_nonzero(freq_classifications_closest[j] == 3)
    three_probabilities_closest.append(num_three / num_rabi_points)

plt.plot(frequencies, zero_probabilities_closest, color='blue',␣
 ↪label=r'$|0\rangle$')
```

```
plt.plot(frequencies, one_probabilities_closest, color='red',␣
 ↪label=r'$|1\rangle$')
plt.plot(frequencies, two_probabilities_closest, color='green',␣
 ↪label=r'$|2\rangle$')
plt.plot(frequencies, three_probabilities_closest, color='orange',␣
 ↪label=r'$|3\rangle$')

plt.title('Closest')
plt.xlabel("Frequency [GHz]", fontsize=15)
plt.ylabel("Probability", fontsize=15)
plt.legend()
plt.show()
```