Homework 2

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2/3/2022

1.

(a) $\theta = 16/23 = 0.6956522$. The MLE of θ is slightly greater than the posterior mean estimate under a Beta(1,1) prior which was 0.68.

(b) w = 2/25 = 0.08. This suggests more weight from the data.

(c) 95% CI = (0.5076022, 0.8837022)

(d) Using a Beta(1,1) prior, the 95% credible interval = (0.4890522, 0.8436977).

(e) The Monte Carlo sampled estimates are often very near the analytic results we would expect. Both sample means are often correct to two decimal places, however when S=100 the sample variance and 95% credible interval tend to vary quite a lot which doesn't happen as often when S=10000.

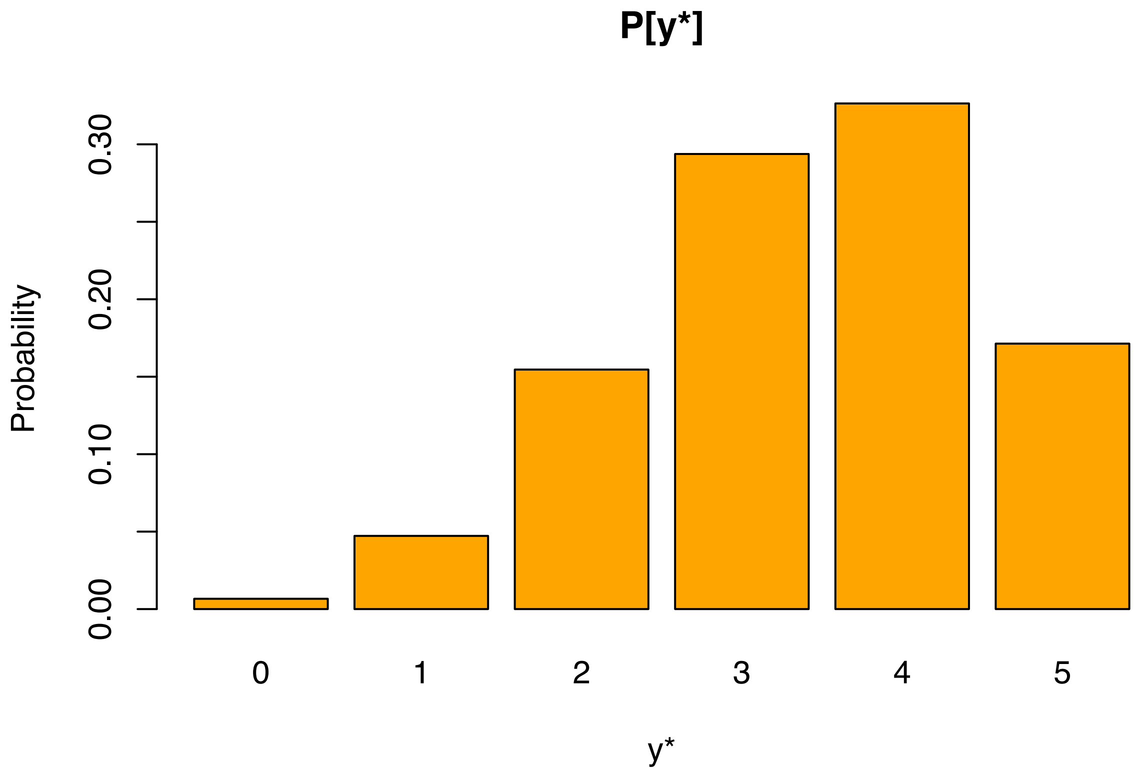
	mean	variance	95% credible interval
S = 100	0.6430163	0.0069842	(0.4939099, 0.81603)
S = 10000	0.6456745	0.0070643	(0.4718217, 0.7996407)
Analytic	0.6451613	0.007154	(0.47188, 0.8007014)

(f) The 95% HPD interval is very slightly shifted to the right on both sides when compared to the 95% central interval.

```
##     lower     upper
## 0.4866890 0.8114397
## attr(,"credMass")
## [1] 0.95
```

(g) $y^* = 0, 1, 2, 3, 4, 5.$

(h)



(i)

```
## Y.star
##      0      1      2      3      4      5
## 0.0067 0.0500 0.1495 0.2885 0.3342 0.1711
```

2.

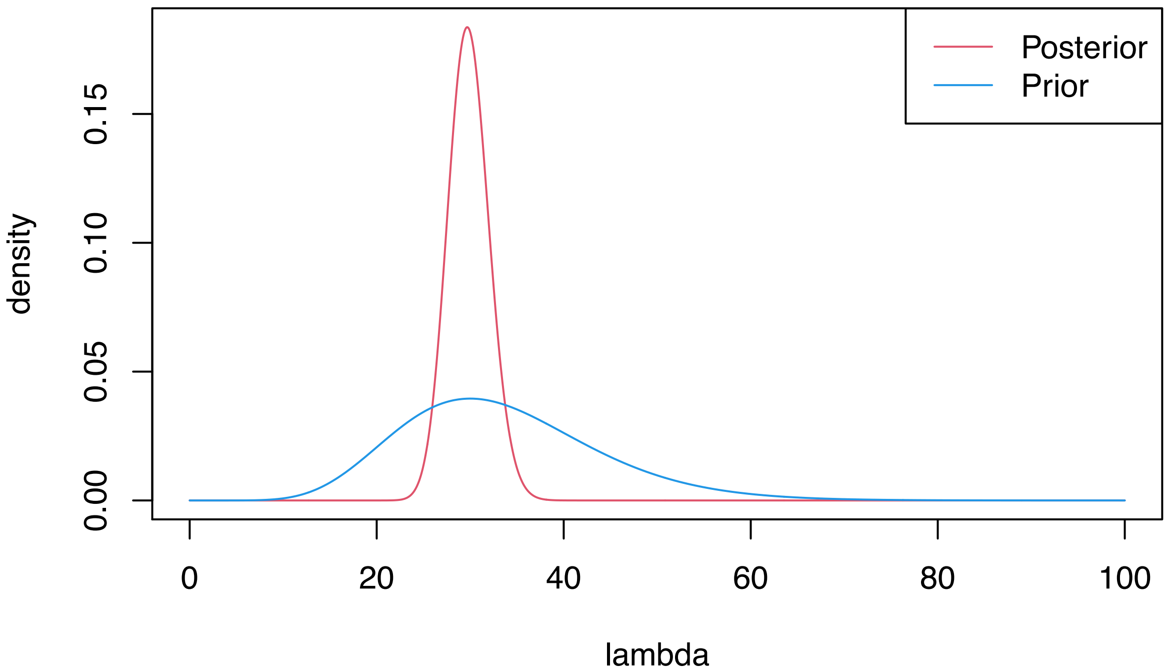
(a) Gamma(188, 6.3)

$$= \frac{\lambda^{y_0} e^{-\lambda}}{\cancel{y_0!}} \cdots \frac{\lambda^{y_5} e^{-\lambda}}{\cancel{y_5!}} \cdot \cancel{\frac{b^{a}}{\Gamma(a)}} \lambda^{a-1} e^{-b\lambda}$$

$$\propto \lambda^{a-1+\sum y_i} e^{-n\lambda - b\lambda}$$

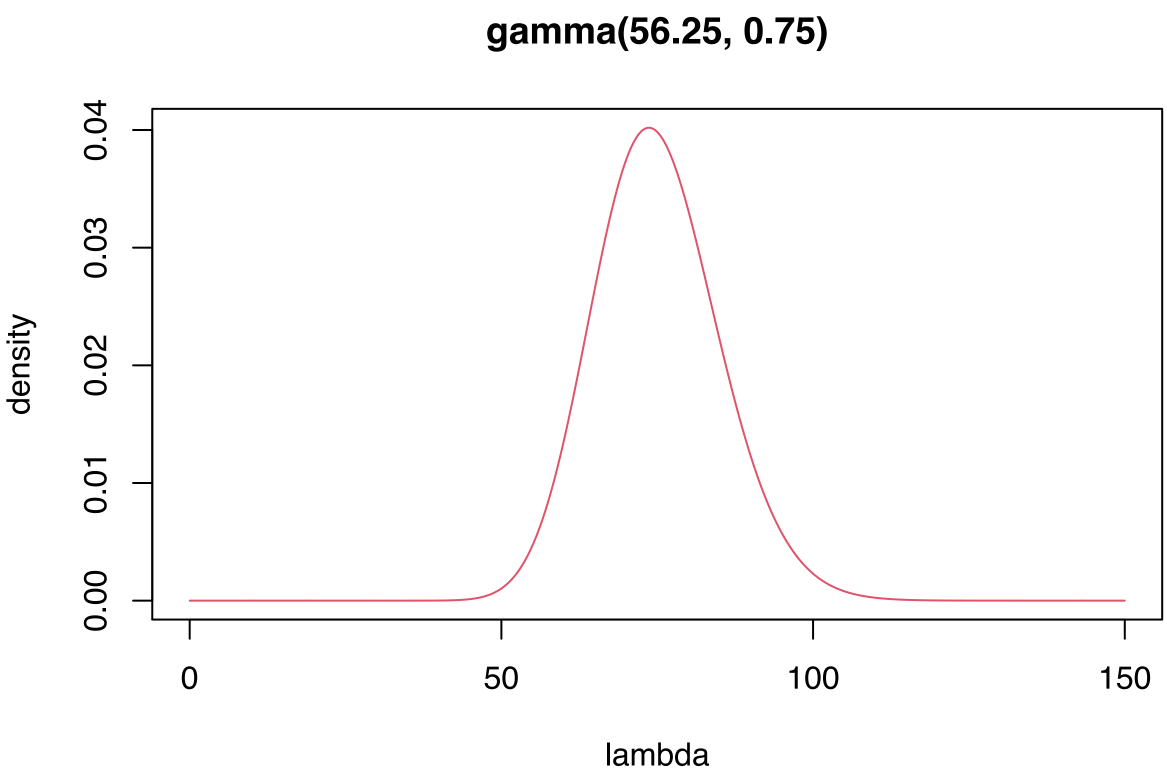
$$= \lambda^{(a+\sum y_i)-1} e^{-(n+b)\lambda} = \text{Gamma}(a+\sum y_i, n+b)$$

(b) Yes, there is evidence of Bayesian learning. We have become much more certain about what the true value of λ is.



(c) Posterior mean = 29.8412698, variance = 4.7367095, 90% central credible interval = (0.9443522, 0.9853581), 90% HPD interval = (0.9476744, 0.9873332).

3.



4.

(a) c = 1 and a = b = 0.2

$P[\sigma^2 > 1] = 0.2263547$

(b) c = 1 and a = b = 2.0

$P[\sigma^2 > 1] = 0.2480104$

(c) c = 2 and a = b = 0.2

$P[\sigma^2 > 2] = 0.0051231$

(d) c = 2 and a = b = 2.0

$P[\sigma^2 > 2] = 0.0036806$

c = 1, a = b = 0.2 / a = b = 2.0 : 0.9126822

c = 2, a = b = 0.2 / a = b = 2.0 : 1.3919144

The results for c = 2 are sensitive to the choice of prior since the ratio of probabilities between the two choices of prior is approx. 1.4. Whereas the ratio of probabilities for c = 1 is pretty close to 1.

const.

$$= \overbrace{\frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{y_1-\mu}{\sigma}\right)^2} \cdots \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{1}{2}\left(\frac{y_n-\mu}{\sigma}\right)^2}}^{\times n} \cdot \frac{\beta^\alpha}{\Gamma(\alpha)} (\sigma^2)^{-\alpha-1} \cdot e^{-\frac{\beta}{\sigma^2}}$$

$$\propto \left(\frac{1}{\sigma\sqrt{2\pi}}\right)^n (\sigma^2)^{-\alpha-1} \exp\left(\frac{-\frac{1}{2}\sum(y_i-\mu)^2 - \beta}{\sigma^2}\right)$$

const.

$$= (2\pi)^{-\frac{n}{2}} \sigma^{-n} \sigma^{-2\alpha-2} \exp\left(\frac{-\left(\beta + \frac{1}{2}\sum(y_i-\mu)^2\right)}{\sigma^2}\right)$$

$$= \sigma^{-n-2\alpha-2} \exp\left(\frac{-\left(\beta + \frac{1}{2}\sum(y_i-\mu)^2\right)}{\sigma^2}\right)$$

$$= (\sigma^2)^{-\left(\alpha+\frac{n}{2}\right)-1} \exp\left(\frac{-\left(\beta + \frac{1}{2}\sum(y_i-\mu)^2\right)}{\sigma^2}\right)$$

$$= \boxed{\text{InvGam}\left(\alpha + \frac{n}{2}, \beta + \frac{1}{2}\sum_{i=1}^{n}(y_i-\mu)^2\right)}$$