Homework 6

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1.

(a)

$p(\sigma_1^2|Y, \sigma_2^2, ...\sigma_{10}^2, b) \sim InvGamma(a + \frac{1}{2}, \frac{1}{2}(Y_1^2 + 2b))$

$p(b|*) \sim Gamma(1, 1)$

(b)

Step 1: Select initial values for σ_1^2, σ_2^2, σ_3^2, σ_4^2, σ_5^2, σ_6^2, σ_7^2, σ_8^2, σ_9^2, σ_{10}^2

Step 2: For s = 1 ... S, iterate through the following:

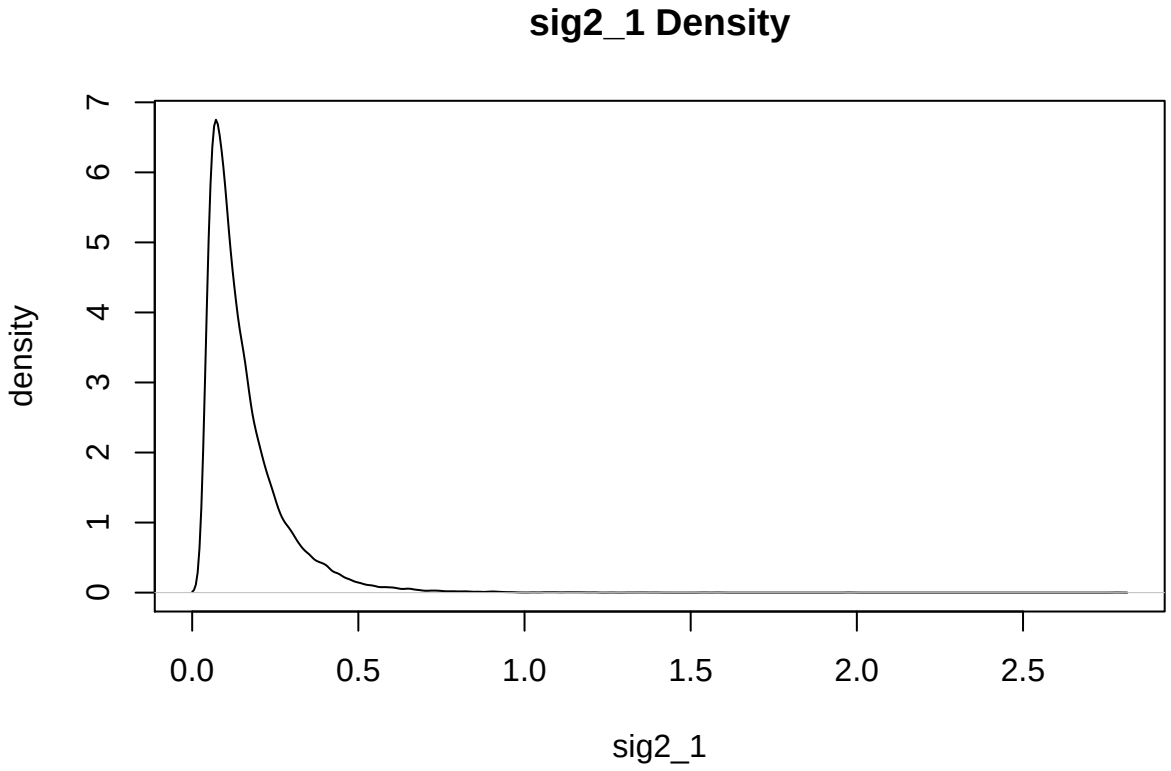
Step 2a: $p(b|*) \sim Gamma(1, 1)$

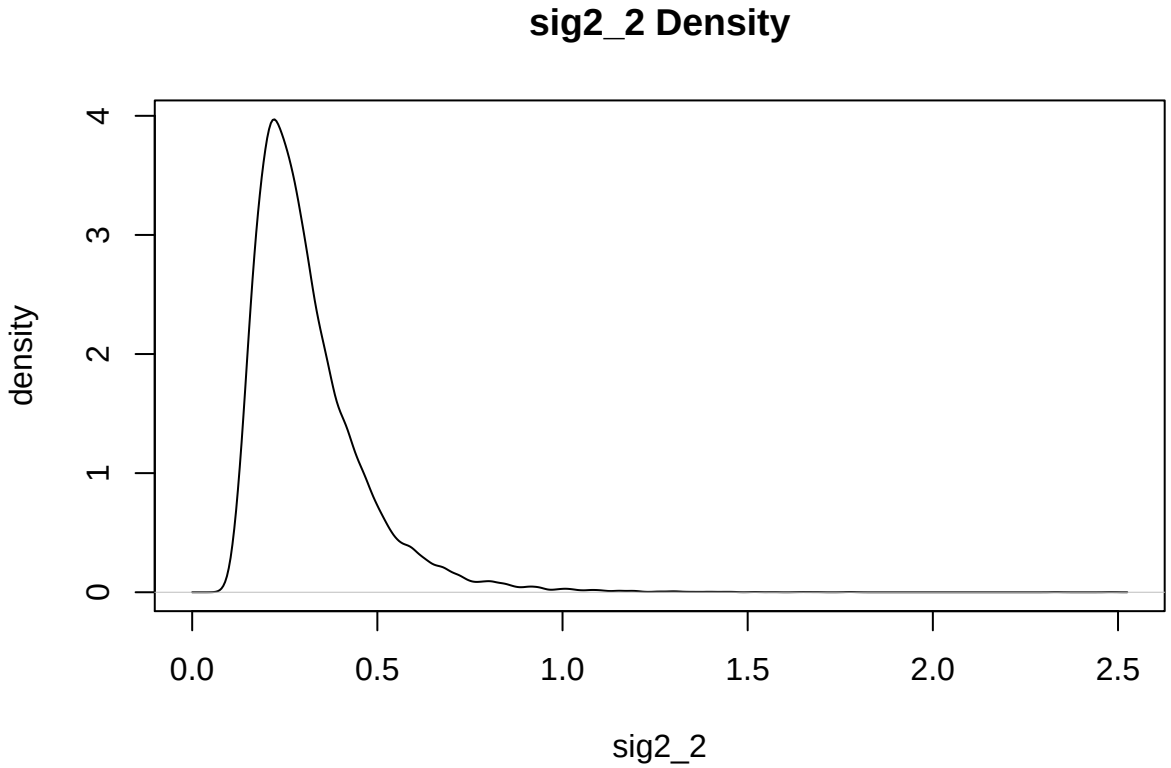
Step 2b: $p(\sigma_1^2|Y, \sigma_2^2, ..., \sigma_{10}^2, b) \sim InvGamma(a + \frac{1}{2}, \frac{1}{2}(Y_1^2 + 2b))$

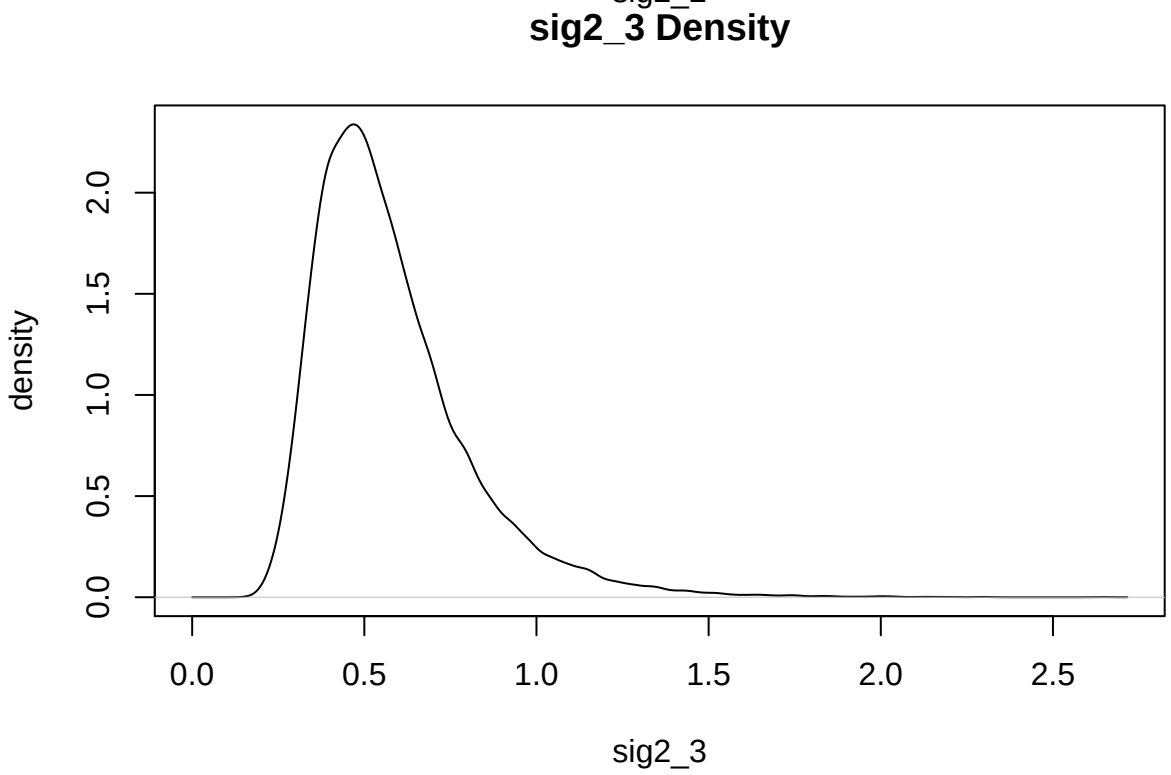
Step 2c: $p(\sigma_2^2|Y, \sigma_1^2, \sigma_3^2, ..., \sigma_{10}^2, b) \sim InvGamma(a + \frac{1}{2}, \frac{1}{2}(Y_2^2 + 2b))$

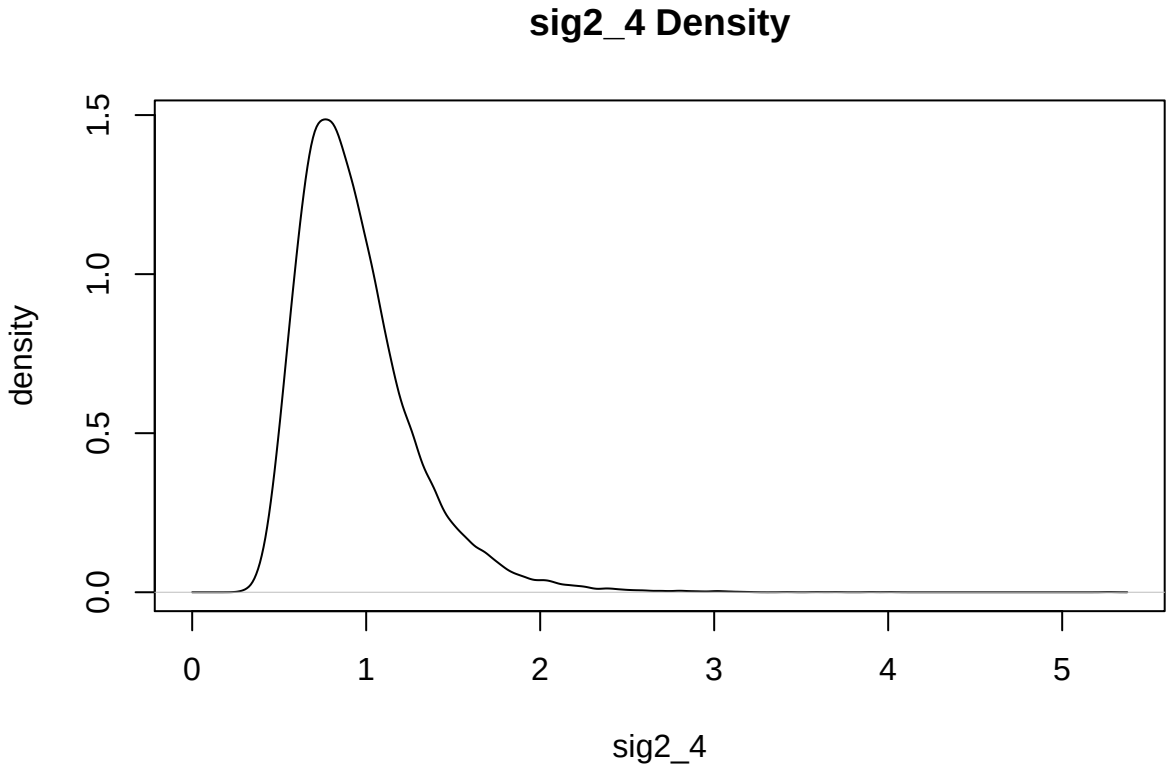
Step 2d-k: Similar to above for σ_3^2 through σ_{10}^2.

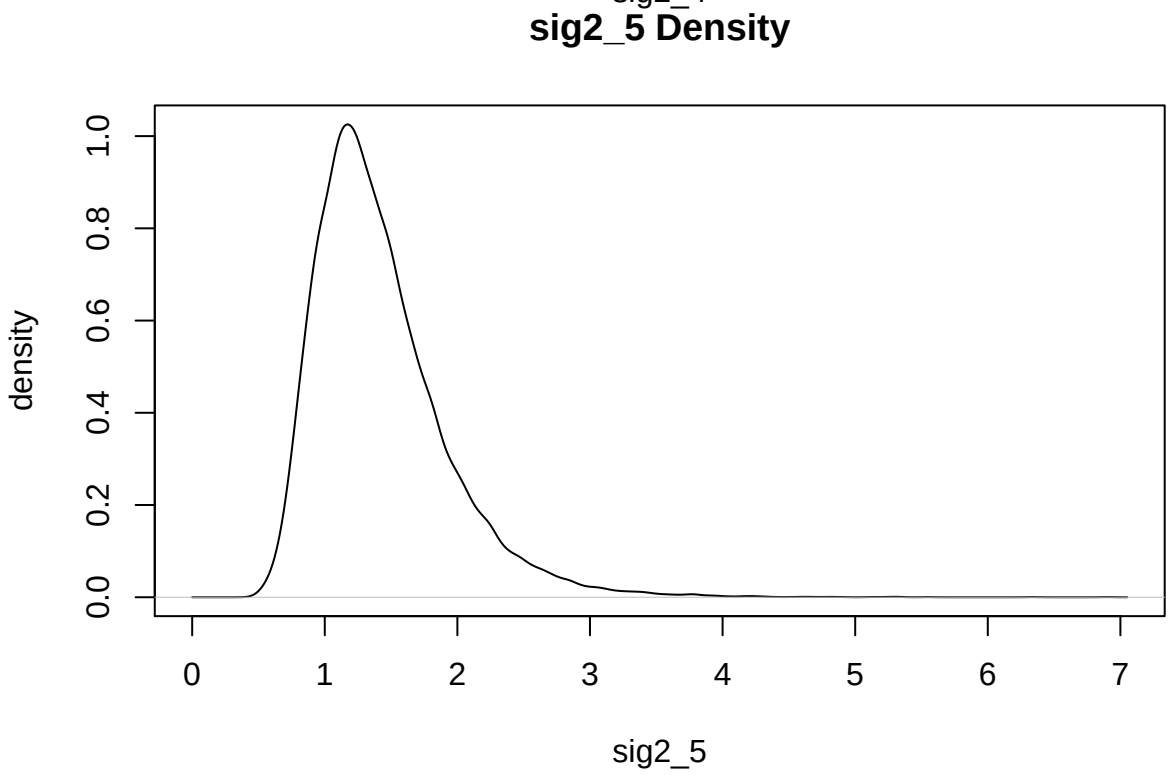
(c)

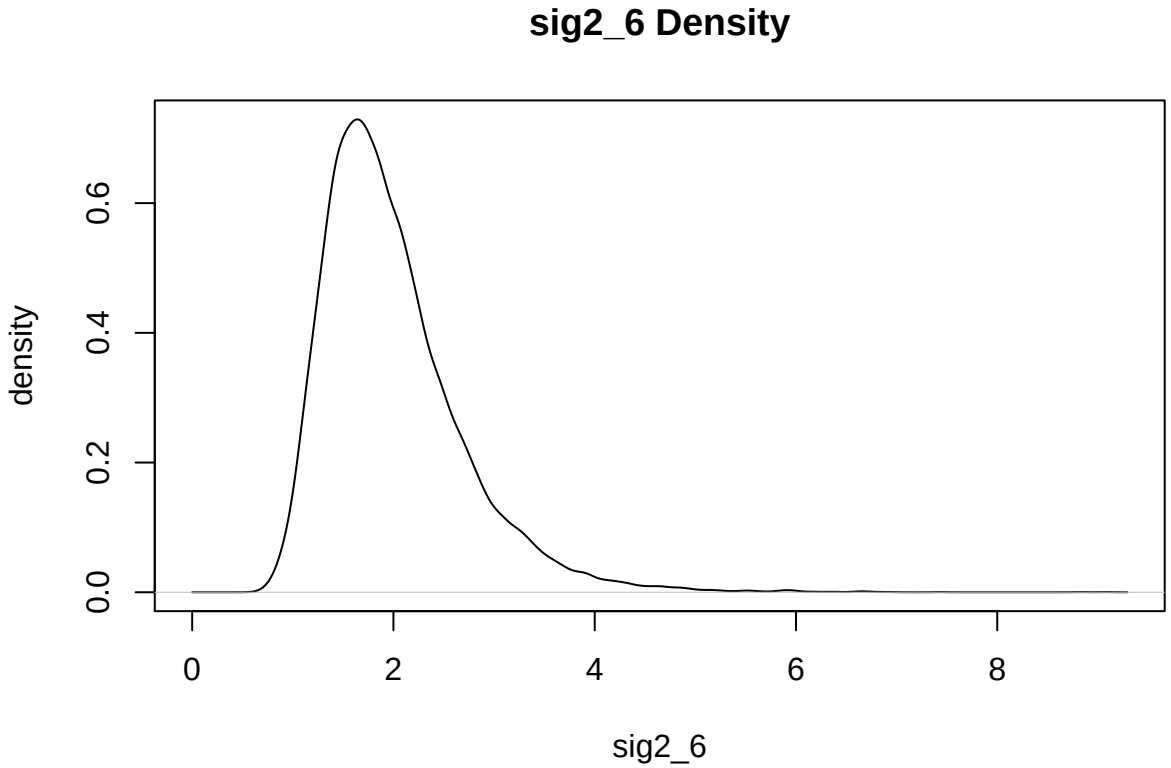


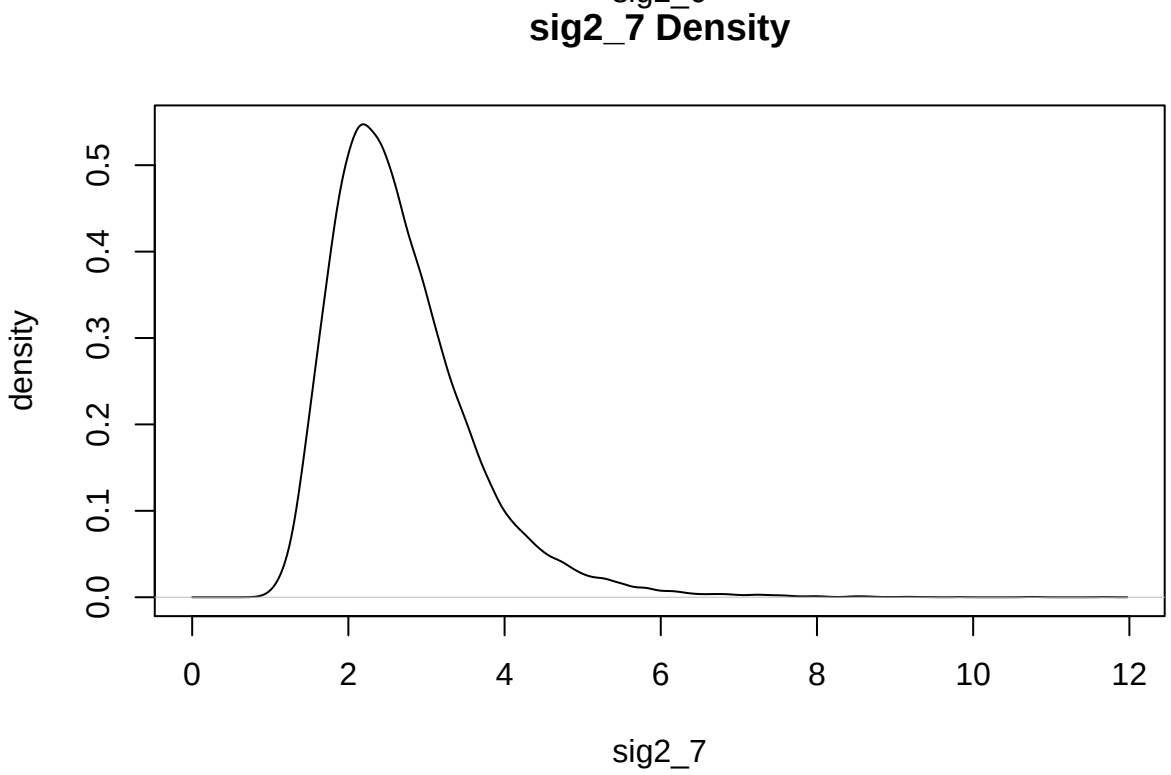

sig2_2 Density
density
0
1
2
3
4
0.0
0.5
1.0
1.5
2.0
2.5
sig2_2

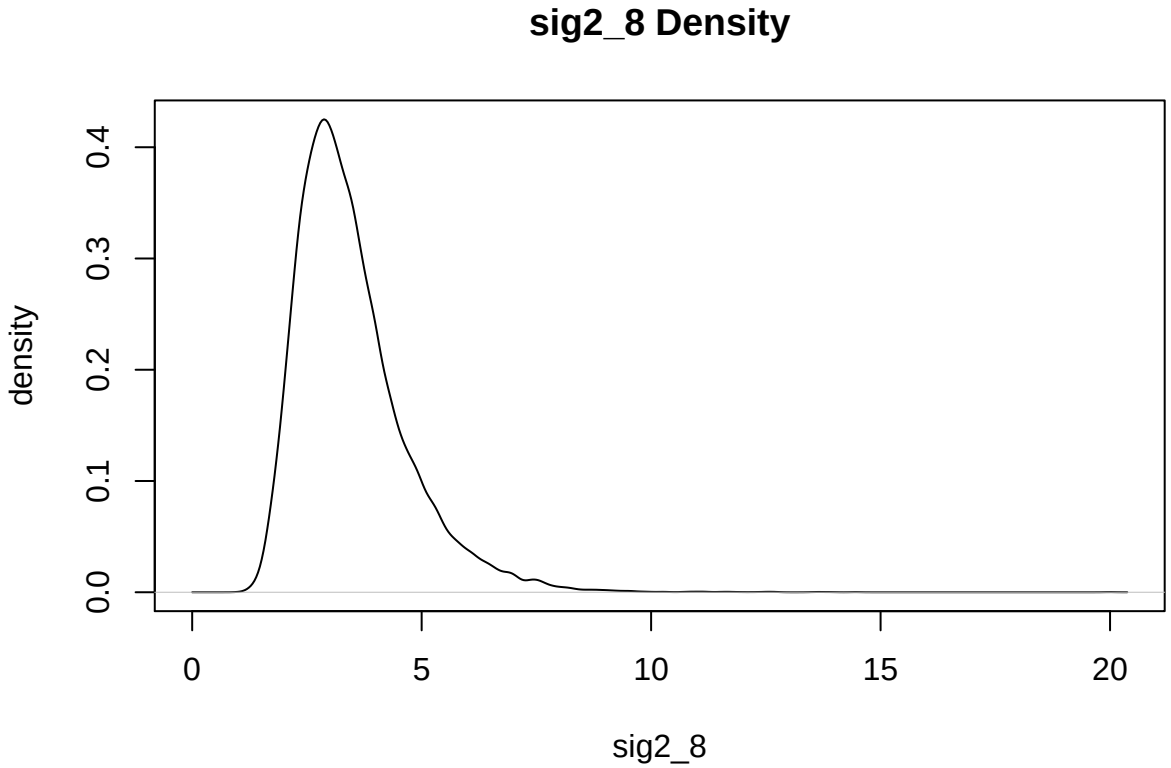

sig2_3 Density
density
0.0
0.5
1.0
1.5
2.0
0.0
0.5
1.0
1.5
2.0
2.5
sig2_3

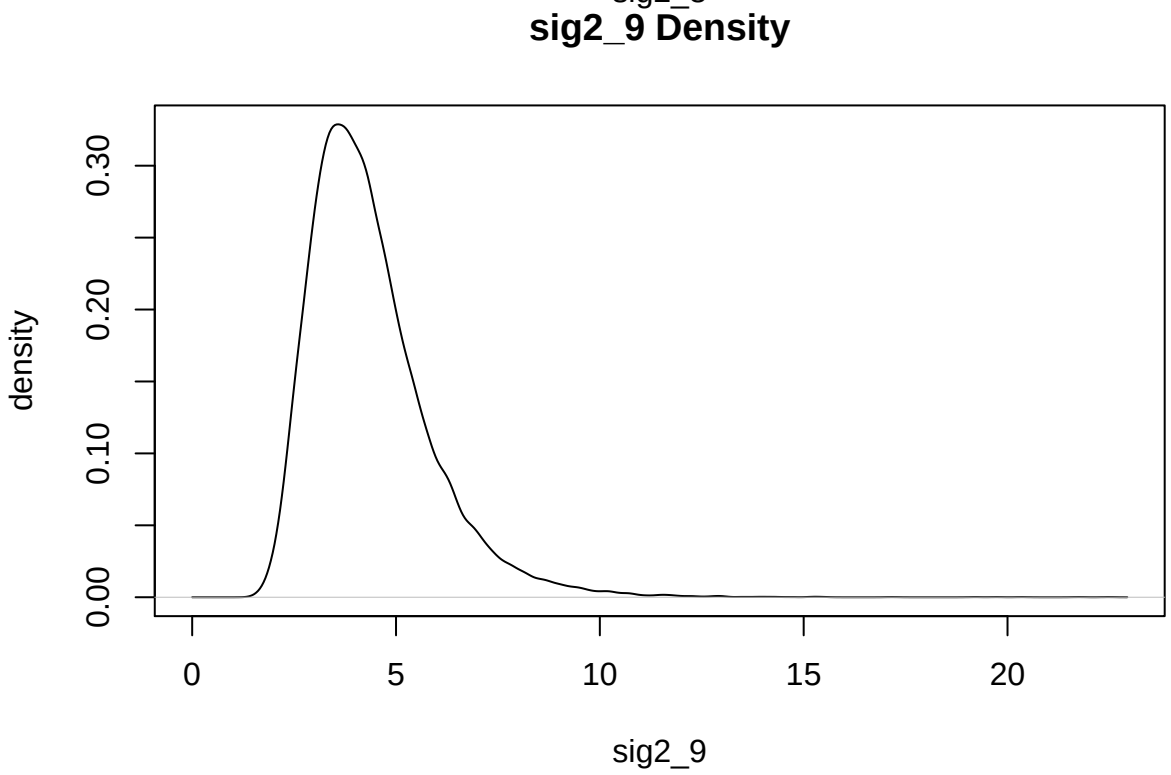

sig2_4 Density
density
0.0
0.5
1.0
1.5
0
1
2
3
4
5
sig2_4

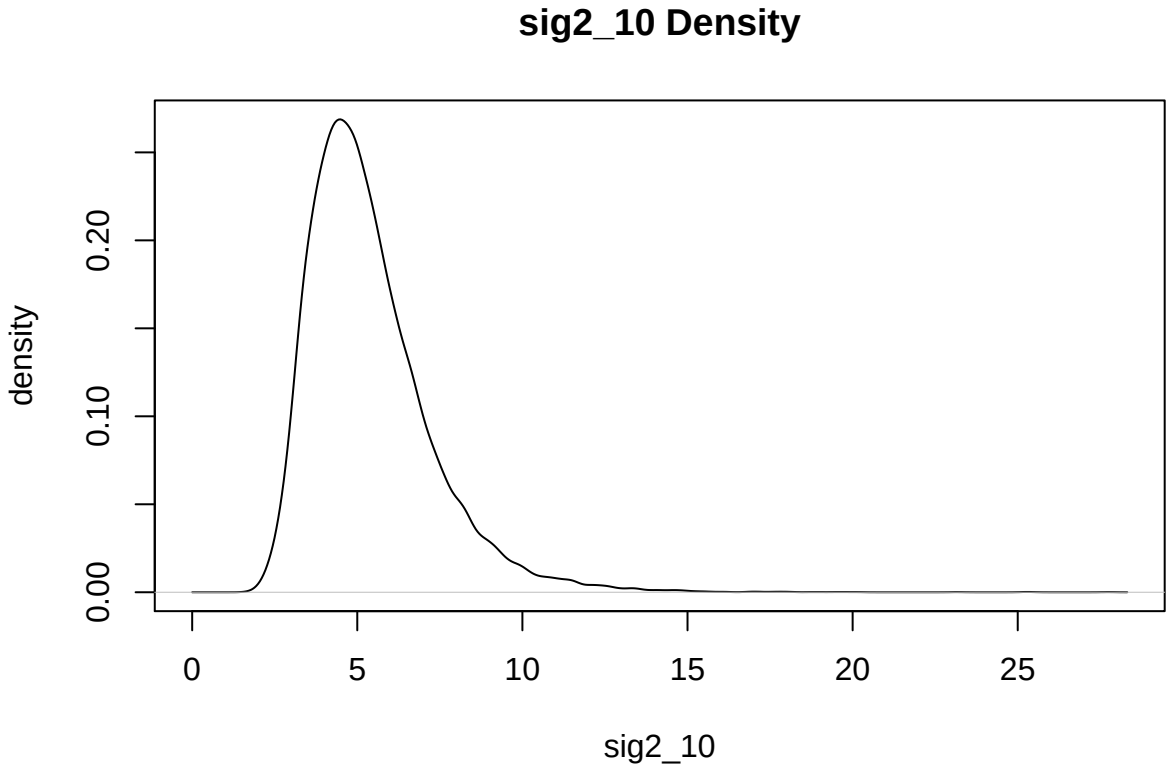

sig2_5 Density
density
0.0
0.2
0.4
0.6
0.8
1.0
0
1
2
3
4
5
6
7
sig2_5


sig2_6 Density
density
0.0
0.2
0.4
0.6
0
2
4
6
8
sig2_6


sig2_7 Density
density
0.0
0.1
0.2
0.3
0.4
0.5
0
2
4
6
8
10
12
sig2_7

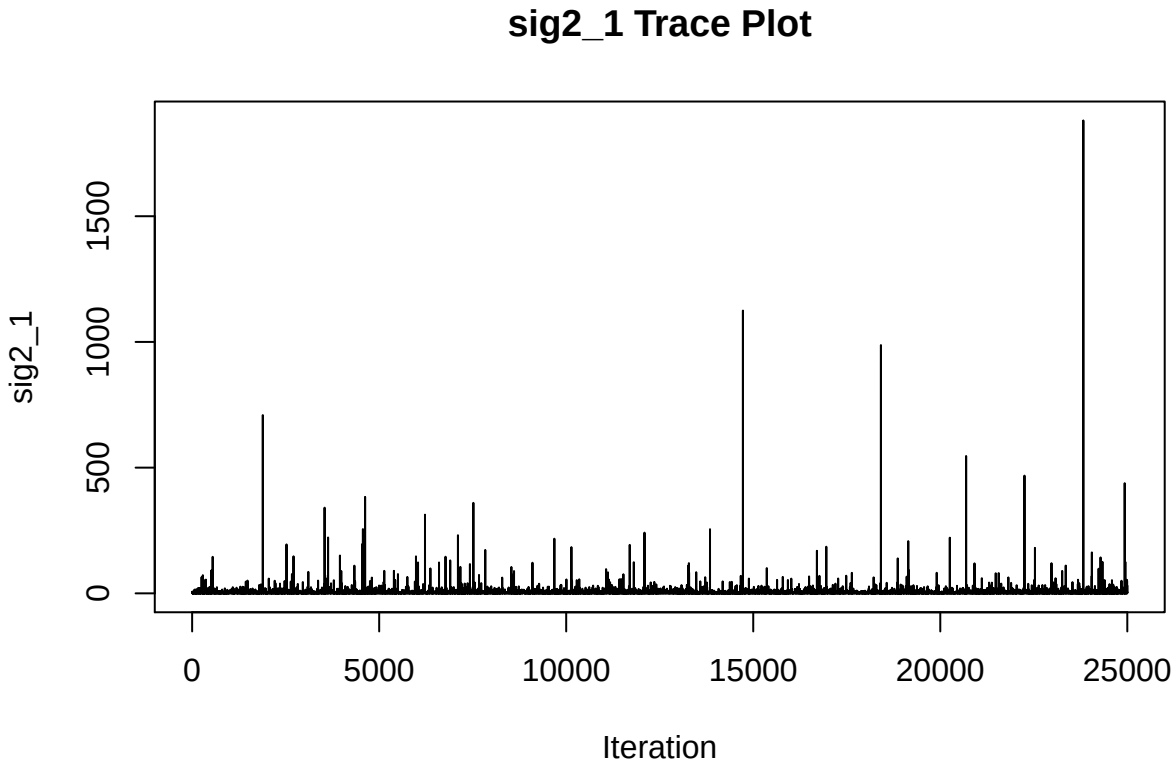

sig2_8 Density
density
0.0
0.1
0.2
0.3
0.4
0
5
10
15
20
sig2_8

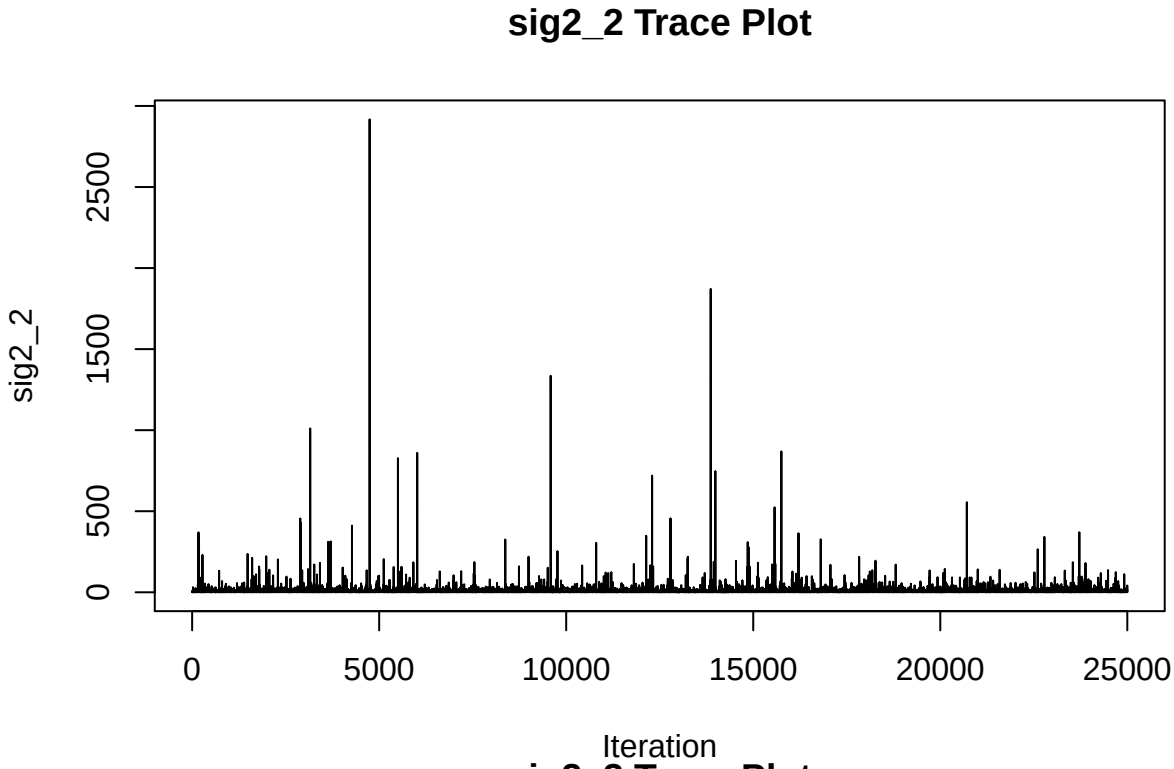

sig2_9 Density
density
0.00
0.10
0.20
0.30
0
5
10
15
20
sig2_9

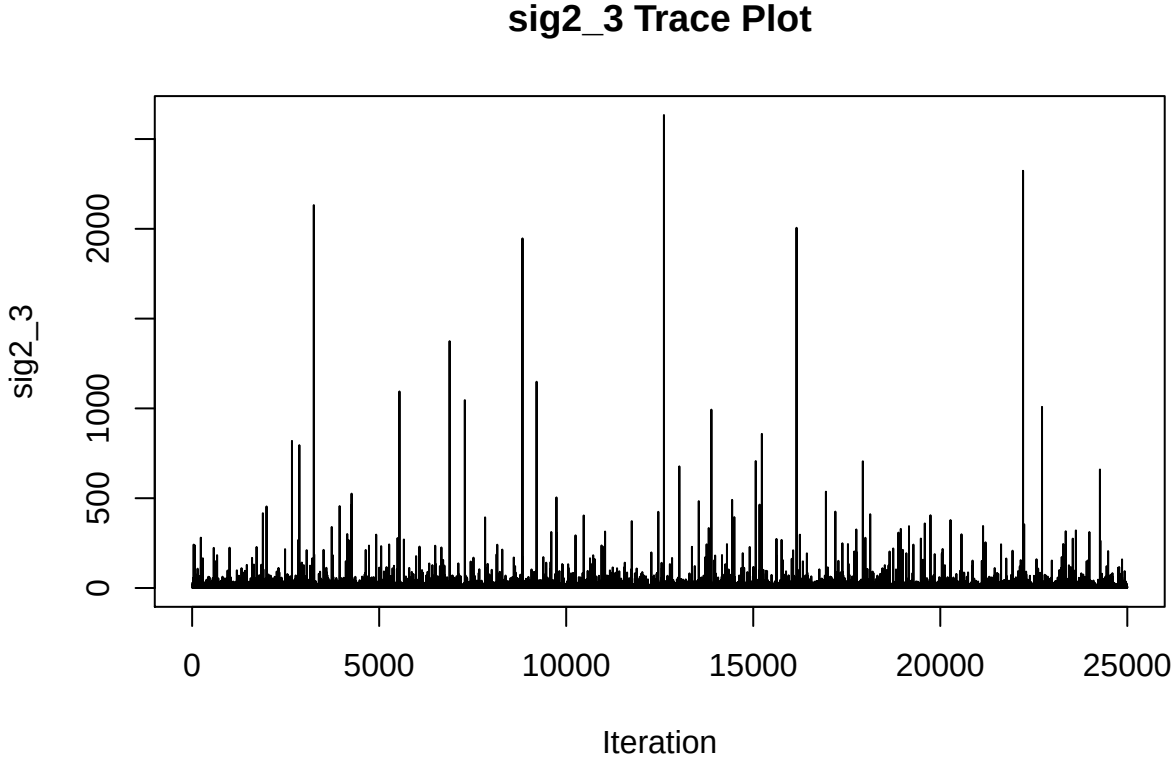


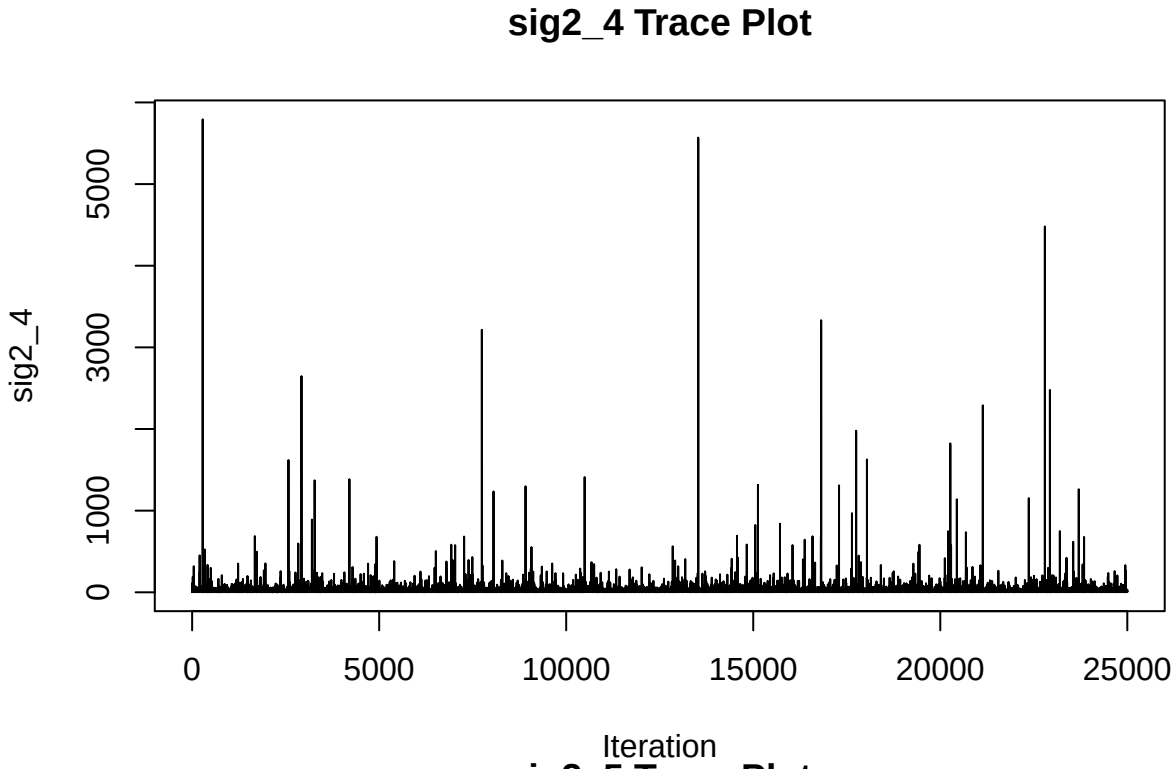
(d)

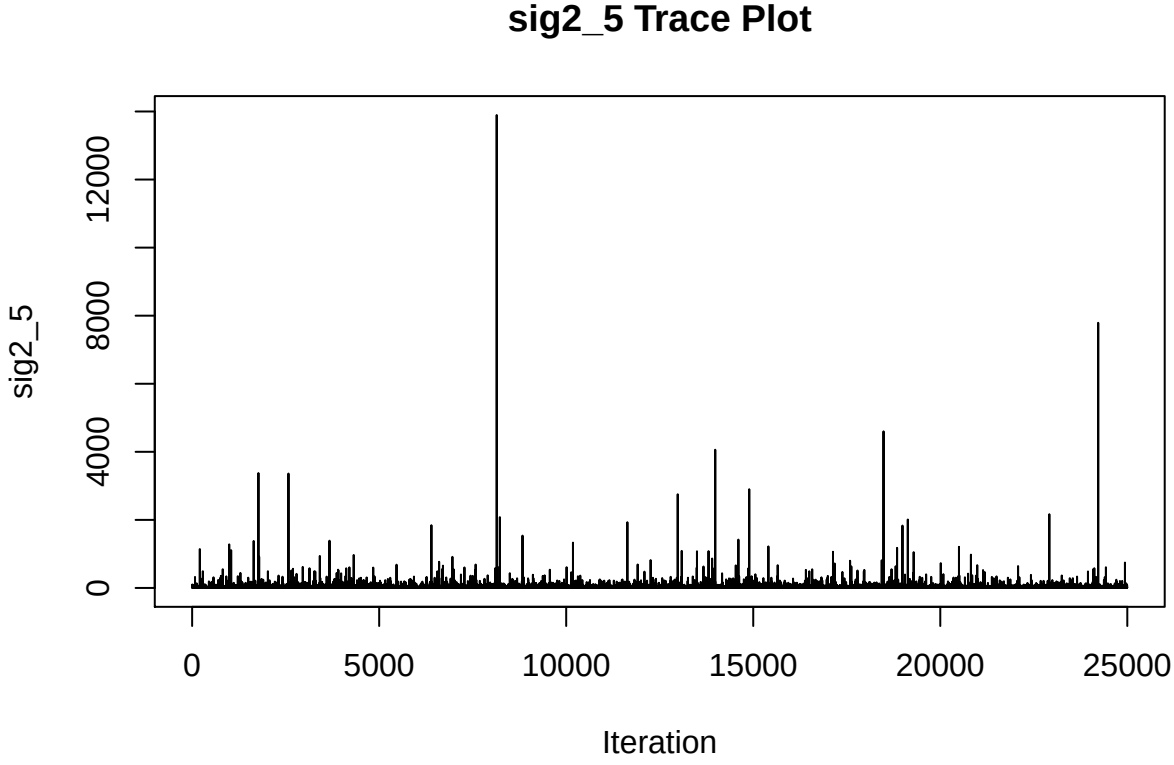
The trace plots raise questions about the convergence of the chains. They could maybe pass as a caterpillar that just got a fresh shave (i.e., no fuzz).

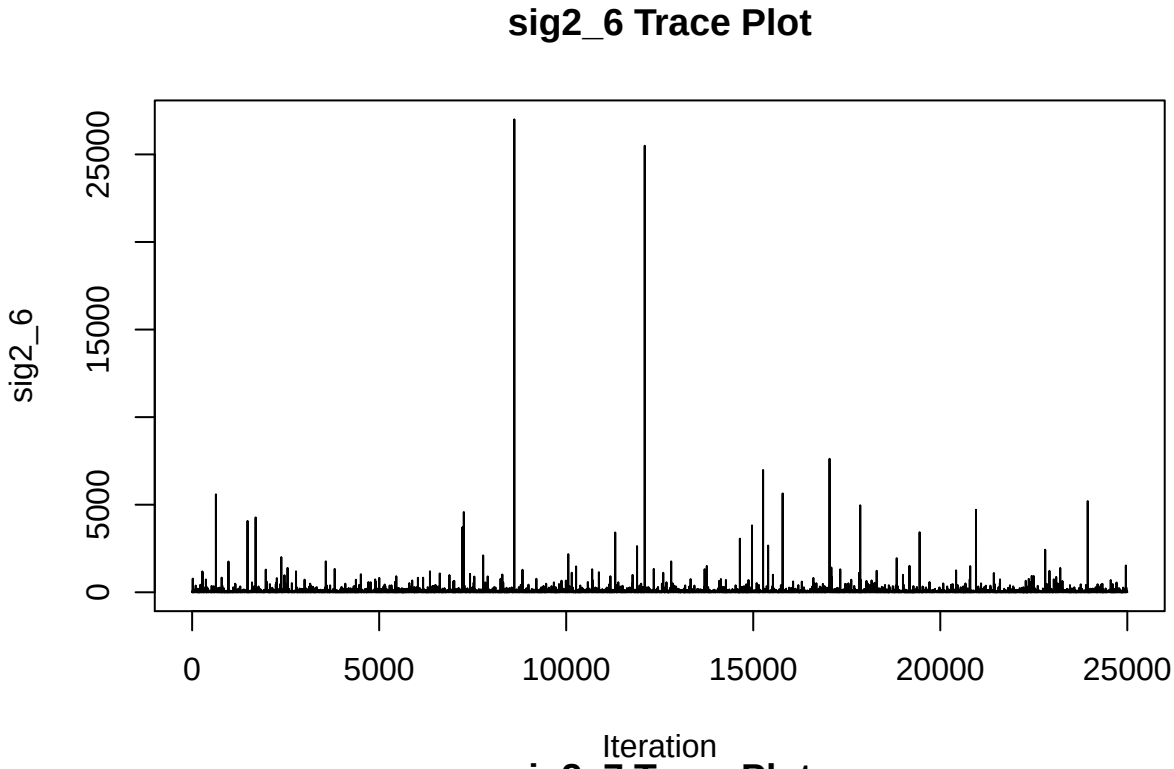


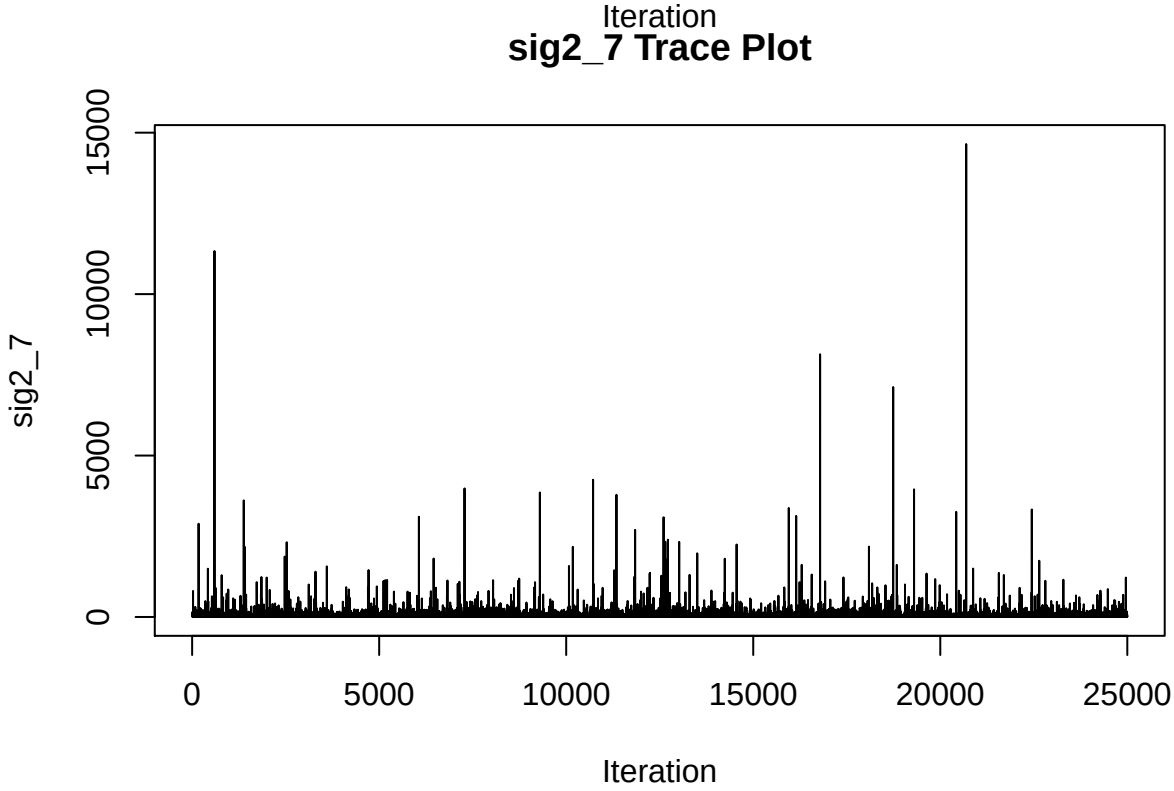

sig2_2 Trace Plot
sig2_2
0
500
1500
2500
0
5000
10000
15000
20000
25000
Iteration

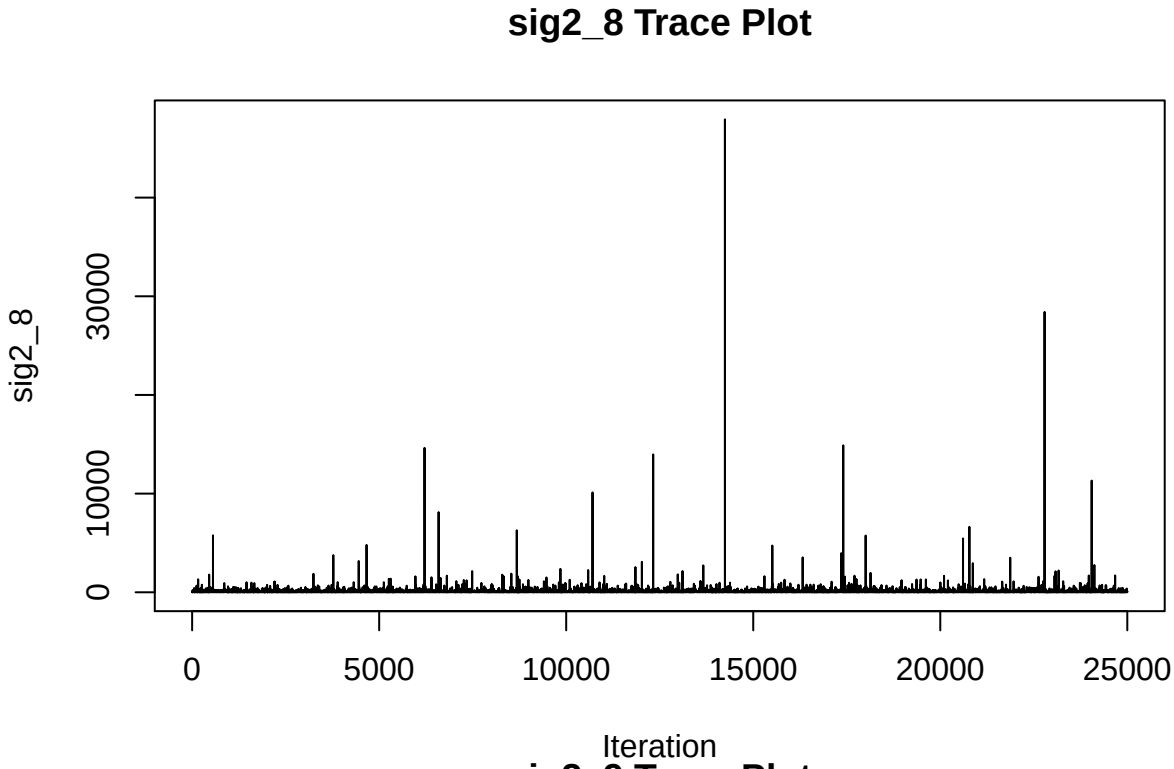

sig2_3 Trace Plot
sig2_3
0
500
1000
2000
0
5000
10000
15000
20000
25000
Iteration

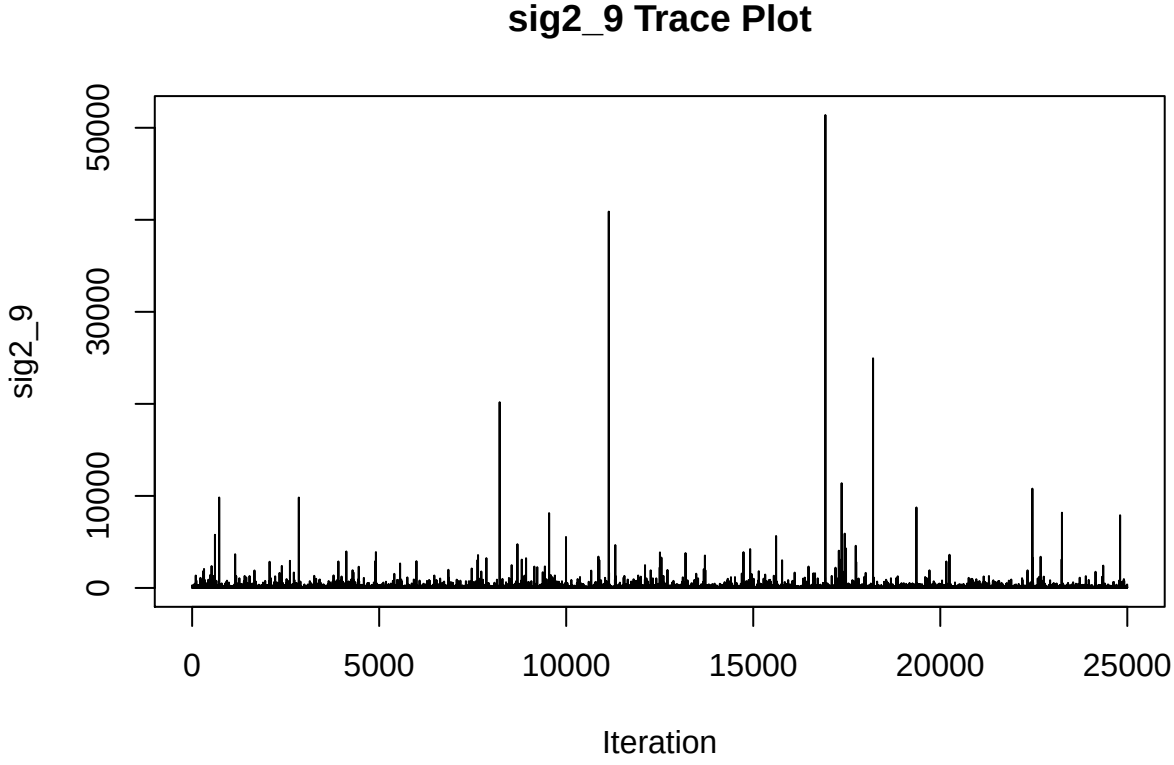

sig2_4 Trace Plot
sig2_4
0
1000
3000
5000
0
5000
10000
15000
20000
25000
Iteration

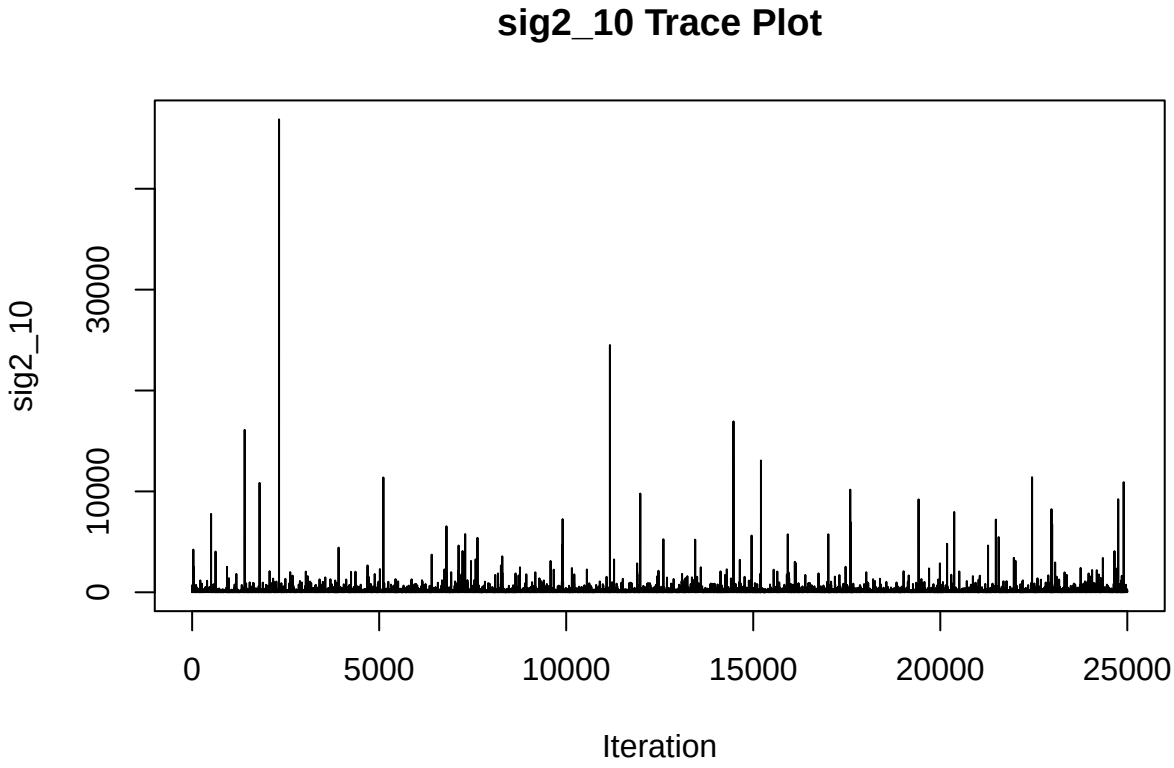

sig2_5 Trace Plot
sig2_5
0
4000
8000
12000
0
5000
10000
15000
20000
25000
Iteration


sig2_6 Trace Plot
sig2_6
0
5000
15000
25000
0
5000
10000
15000
20000
25000
Iteration


sig2_7 Trace Plot
sig2_7
0
5000
10000
15000
0
5000
10000
15000
20000
25000
Iteration


sig2_8 Trace Plot
sig2_8
0
10000
30000
0
5000
10000
15000
20000
25000
Iteration


sig2_9 Trace Plot
sig2_9
0
10000
30000
50000
0
5000
10000
15000
20000
25000
Iteration



(e)

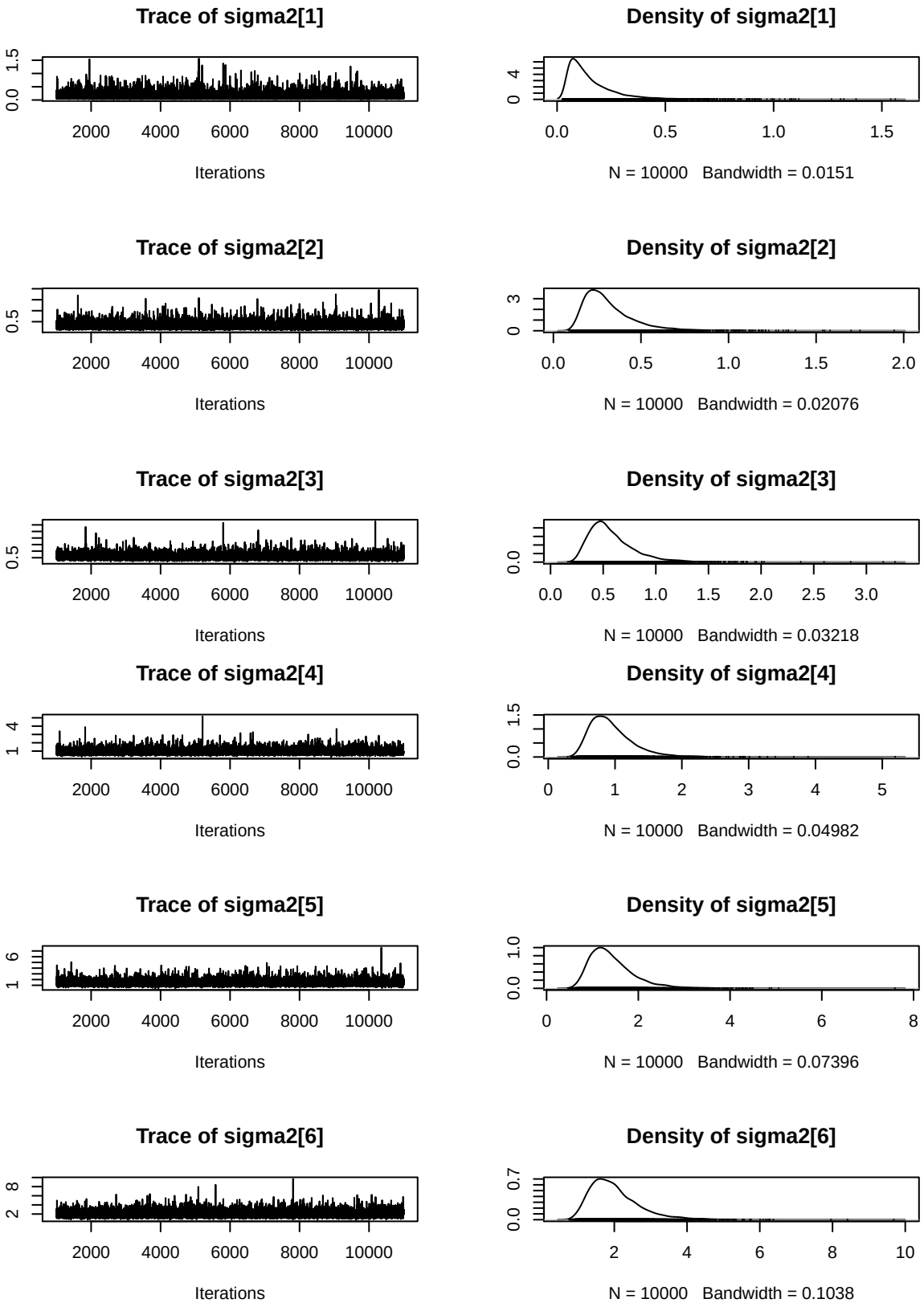
The results obtained via JAGS (posterior density & trace plots) are identical to those obtained in (c).

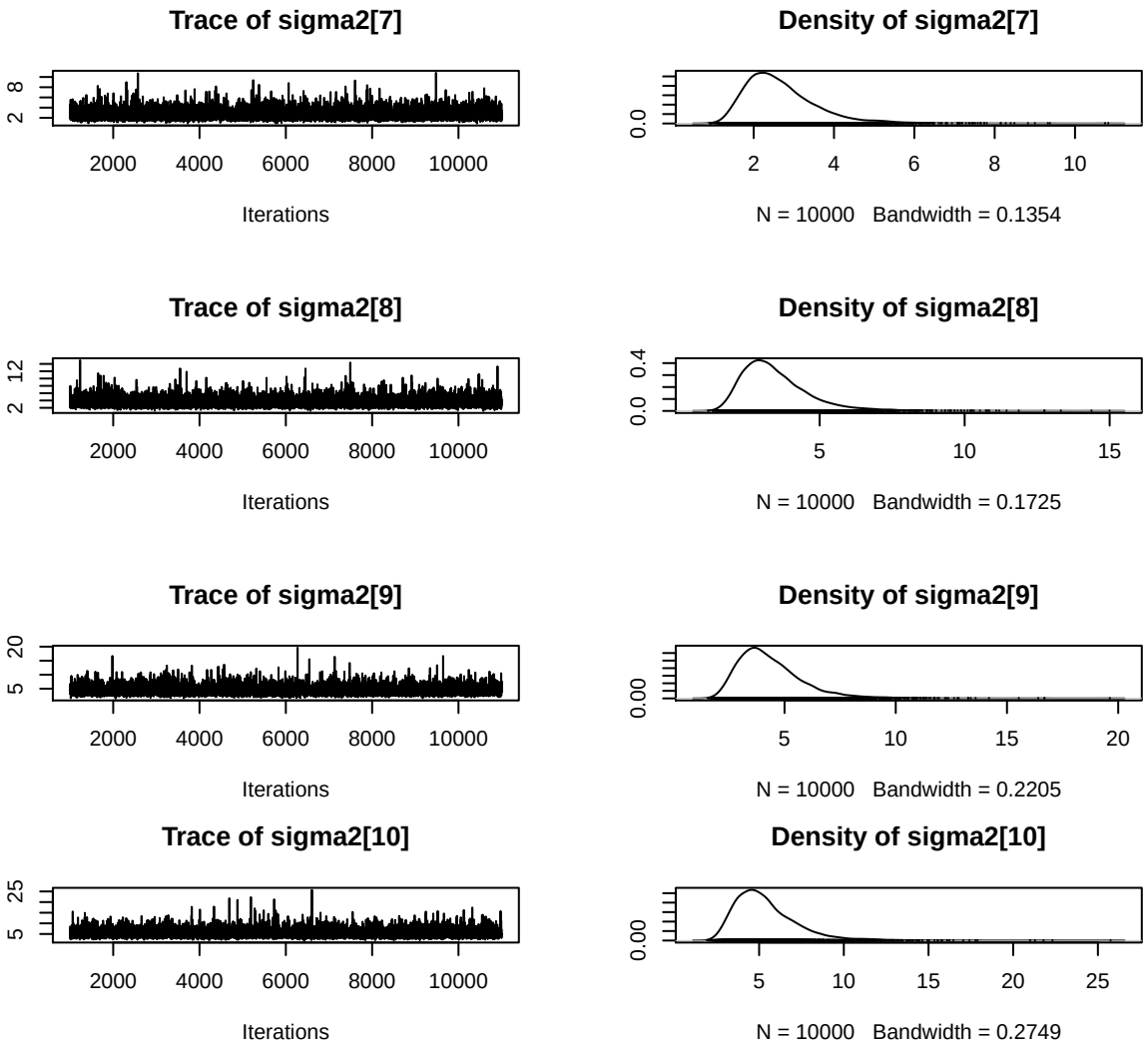
```
## Loading required package: coda
```

```
## Linked to JAGS 4.3.0
```

```
## Loaded modules: basemod,bugs
```

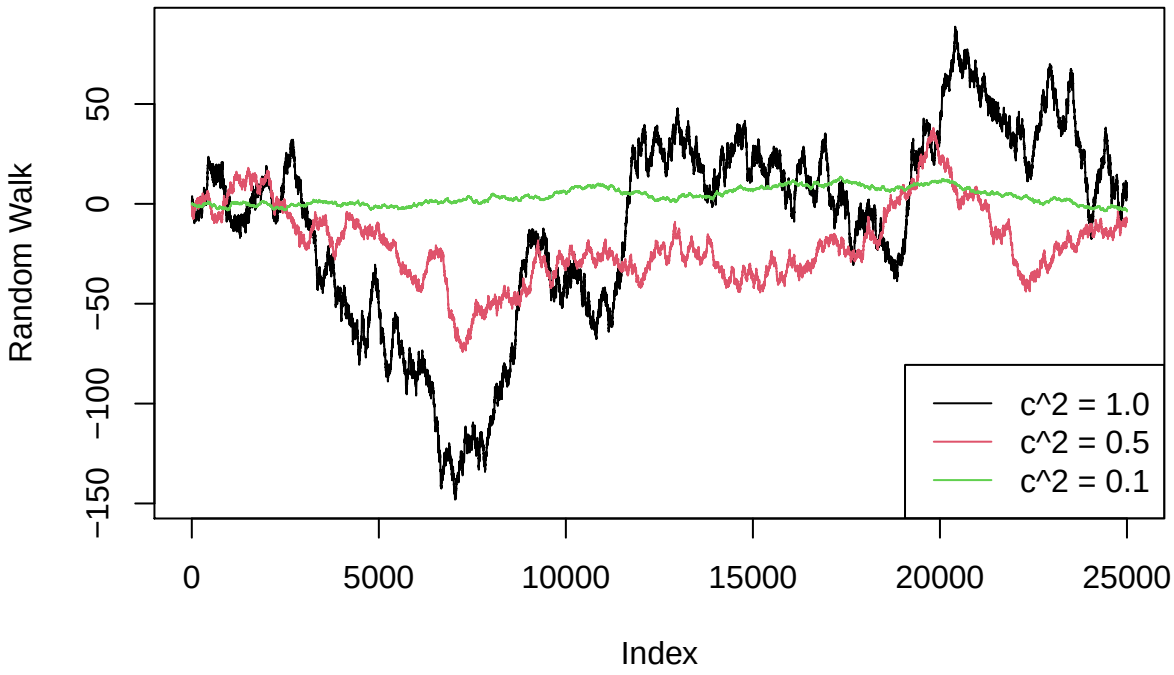
```
## Compiling model graph
##    Resolving undeclared variables
##    Allocating nodes
## Graph information:
##    Observed stochastic nodes: 0
##    Unobserved stochastic nodes: 11
##    Total graph size: 60
##
## Initializing model
```


Trace of sigma2[1]
Density of sigma2[1]
Iterations
N = 10000 Bandwidth = 0.0151
Trace of sigma2[2]
Density of sigma2[2]
Iterations
N = 10000 Bandwidth = 0.02076
Trace of sigma2[3]
Density of sigma2[3]
Iterations
N = 10000 Bandwidth = 0.03218
Trace of sigma2[4]
Density of sigma2[4]
Iterations
N = 10000 Bandwidth = 0.04982
Trace of sigma2[5]
Density of sigma2[5]
Iterations
N = 10000 Bandwidth = 0.07396
Trace of sigma2[6]
Density of sigma2[6]
Iterations
N = 10000 Bandwidth = 0.1038



(2)

The figure is about as expected. The random walk with the greatest c^2 value jumps around the chart more, whereas the one with the lowest c^2 value hardly moves at all in comparison.

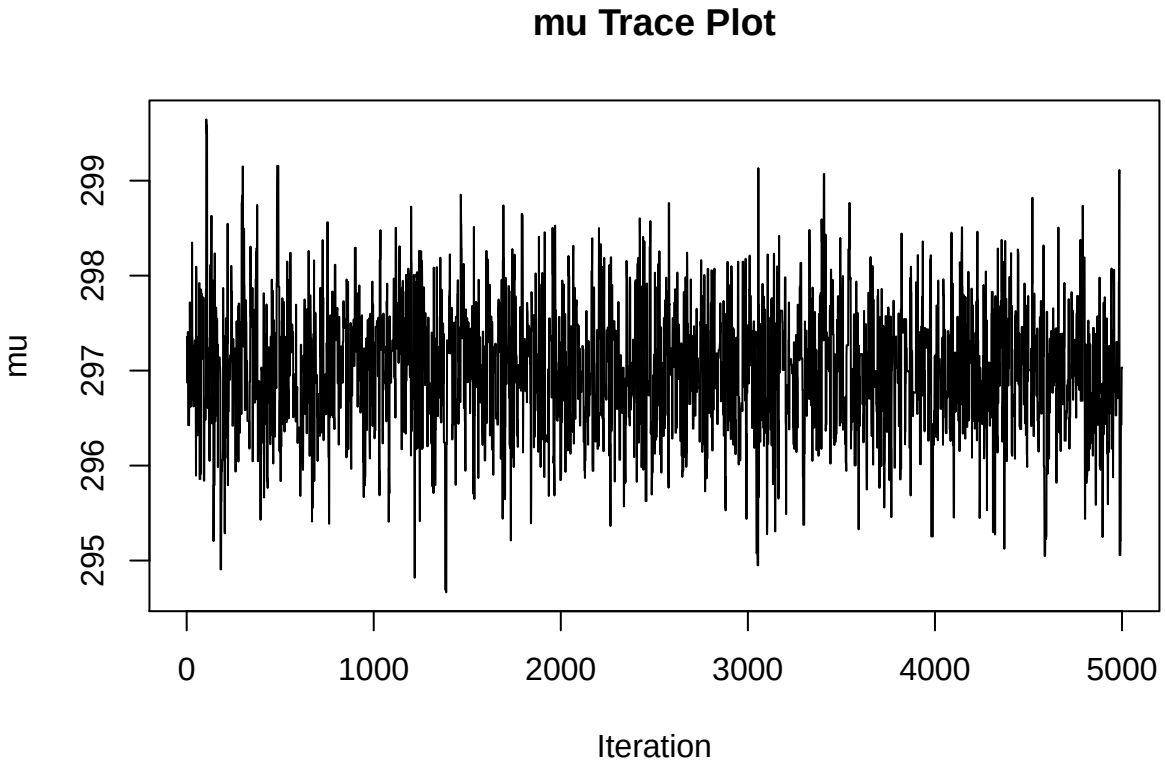


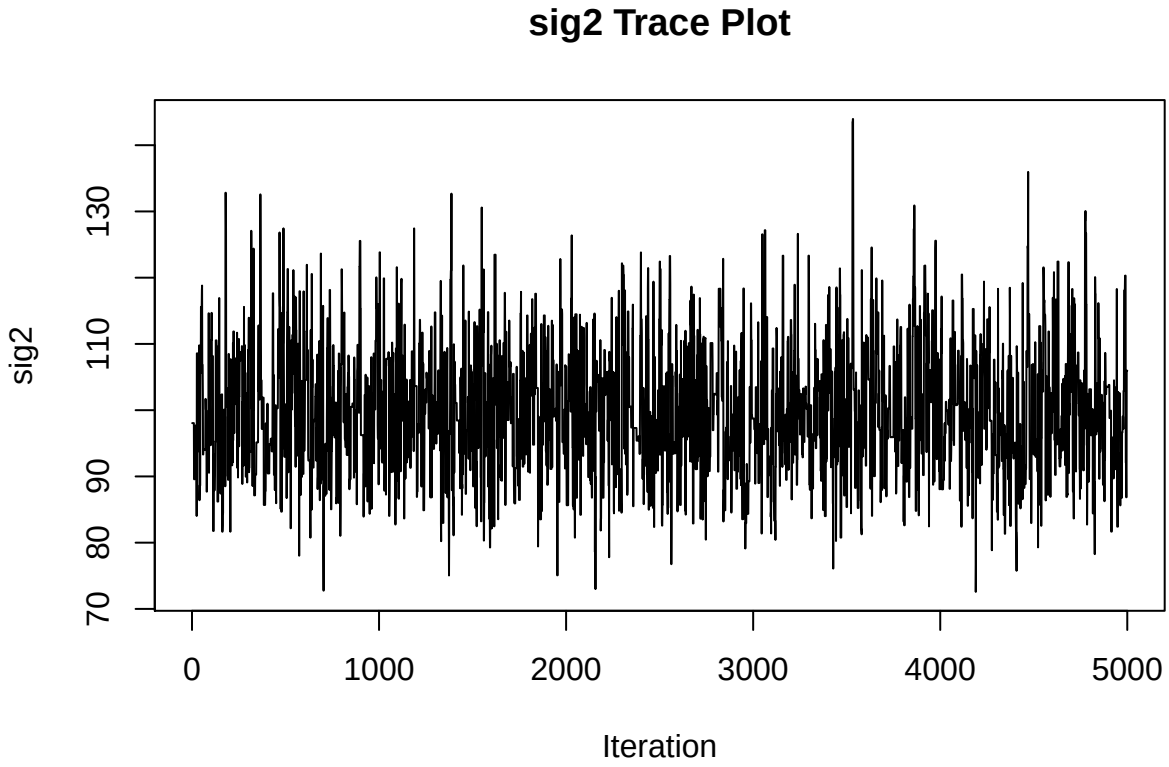
(3)

```
## Loading required package: MASS
## ##
## ## Markov Chain Monte Carlo Package (MCMCpack)
## ## Copyright (C) 2003-2022 Andrew D. Martin, Kevin M. Quinn, and Jong Hee Park
## ##
## ## Support provided by the U.S. National Science Foundation
## ## (Grants SES-0350646 and SES-0350613)
## ##
##
## Attaching package: 'MCMCpack'
## The following objects are masked from 'package:invgamma':
##
##     dinvgamma, rinvgamma
```

(a)

c^2 was set to 3, and d^2 was set to 600. The acceptance rates are 0.4182 for μ and 0.4084 for σ^2.





(b)

95% credible interval for μ = (295.6453372, 298.2930477).

95% credible interval for σ^2 = (83.3314509, 121.0733721)