

Homework 4

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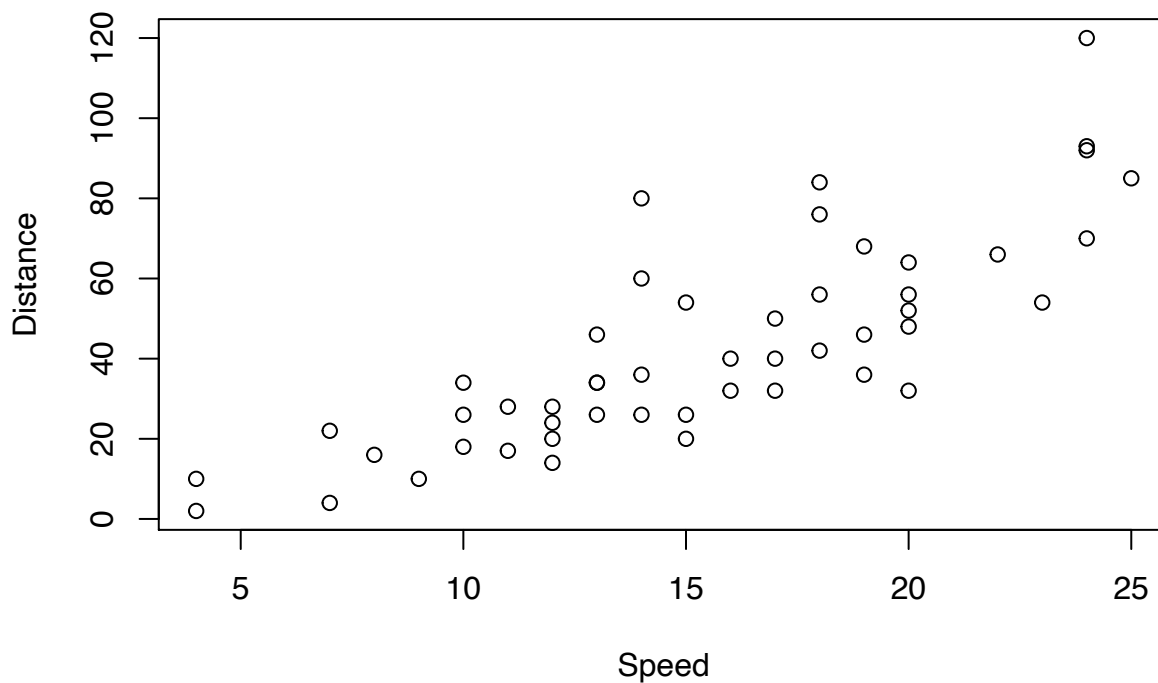
1. $\mathcal{P}(\mu|\psi) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\psi-\mu)^2} = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(\mu-\psi)^2} = \boxed{\mathcal{N}(\psi, 1)}$

2.

(a)

From the plot, there appears to be an approximate linear relationship between speed and distance.

Speed vs Distance



(b)

The fitted line is $\hat{y} = -17.57 + 3.93x_1$. We may interpret this to mean that there is a positive relationship between speed and distance.

```
##
## Call:
## lm(formula = dist ~ speed, data = cars)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -29.069  -9.525  -2.272   9.215  43.201
##
```

```
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -17.5791      6.7584  -2.601   0.0123 *
## speed        3.9324      0.4155   9.464 1.49e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.38 on 48 degrees of freedom
## Multiple R-squared:  0.6511, Adjusted R-squared:  0.6438
## F-statistic: 89.57 on 1 and 48 DF,  p-value: 1.49e-12
```

(c)

Posterior mean = (-17.5738074, 3.9321005), posterior variance = [2.9691391, -0.1728331, -0.1728331, 0.0112233]. The posterior mean estimates for $\hat{\beta}$ are within 2-3 decimal places of those in part (b), whereas the variance estimates are greatly reduced from part (b).

(d)

$P[\hat{\beta}_1 < 3] = 0$. This aligns with what would be expected from our regression fit since it has a coefficient est. of 3.93 with std. err. of 0.41. The interpretation for the problem is that the distance is roughly proportion to at least 3 times the speed.

(e)

Estimates for posterior mean = 218.4138664, posterior variance = 1908.1846812, 95% credible interval = (149.0643883, 319.1539497).

(f)

For many prior specifications, the resulting posterior distribution is very similar, having a mean around 200, about the same amount of variance, and with large overlap in credible intervals. I tried some priors (not shown) that are very informative (i.e. almost no variance) and those do shift the posterior considerably, as expected. However, most “sensible” priors I tried do not change the posterior very much, thus this model is decently robust to the choice of prior.

InvGamma Posteriors

