STAT 4640/7640 **Homework 1**

Due: January 27, 2022

- **Instructions**: Make sure your name is on your paper and your answers are clearly written.

1. Recall the stress fracture example from lecture. The prior probability of an athlete having a stress fracture for women was 9.7% and for men was 6.5%. The sensitity and specificity of the therapeautic ultrasound (TUS) is 64% and 63%, respectively. Suppose the athlete's TUS came back negative. For both men and women, fill in the entries to the following tables and compute the posterior probability that he/she has and doesn't have a stress fracture.

   For female athletes:

Model	Prior probability	Likelihood for $TUS-$	Prior × Likelihood	Posterior Probability
Stress fracture				
No stress fracture				

   For male athletes:

Model	Prior probability	Likelihood for $TUS-$	Prior × Likelihood	Posterior Probability
Stress fracture				
No stress fracture				

   Interpret the posterior probability results using complete sentences.

2. An experiment consists of flipping a fair coin three times independently. Let A be the event of "atleast one of the flips results in a head" and B be the event "all three flips have the same result."

   (a) List the sample space of the experiment.

   (b) List the outcomes in B and find $P(B)$.

   (c) List the outcomes in $A \cap B$ and find $P(A \cap B)$.

   (d) List the outcomes in $A \cup B$ and find $P(A \cup B)$.

   (e) Are A and B independent. Show why or why not.

3. I'm a golfer and playing a par 3 hole. Suppose the following statements are true.

   - The probability that my first shot lands on the green and I make par or better is 0.2.
   - The probability that my first shot lands on the green and I make worse than par is 0.1.
   - The probability that I make par or better but don't land my first shot on the green is 0.1.
   - The probability that I neither land my ball on the green nor make par or better is 0.6.

   Find the following probabilities.

   (a) My first shot lands on the green.

   (b) I land my first shot on the green or I make par or better or both.

   (c) I make par or better.

   (d) I make par or better given that I land my first shot on the green.

4. Let's assume the attempts of the Mizzou kickers for point after touchdown are independent and identically distributed such that each attempt is a Bernoulli trial with probability of success, θ. Assume a Beta(4,4) prior distribution on θ. In 2019, Mizzou kickers were successful on 16 out of 23 in point after attempts.

   Here is some example R code to help you in your homework.

```r
#create a set of values between 0 and 1 to compute the density at
theta = seq(0,1, length=100)
#plot the value vs the Beta(2,5) prior density
plot(theta, dbeta(theta, 2, 5), ylab="density", type ="l",
     col=2, ylim=c(0,3))
#add line for a second Beta(5,2) density
lines(theta, dbeta(theta, 5, 2), type ="l", col=4)
#add a legend at the top right
legend("topright", c("Beta(2,5)","Beta(5,2)"),lty=c(1,1),col=c(2,4))
```

   (a) What is the posterior distribution of θ, the probability of success? Give the posterior mean and posterior variance. Plot the prior and posterior densities of θ on the same graph. Comment on whether or not there is evidence of Bayesian learning and how you made your determination.
   Note: If you are new to R or need a refresher, check out the following link for some additional example code for plotting Beta densities: `https://stephens999.github.io/fiveMinuteStats/beta.html`.

   (b) Re-do part (a) with a Beta(1,1) prior distribution for θ.