STAT 4640/7640 **Homework 1**

Due: January 27, 2022

- **Instructions**: Make sure your name is on your paper and your answers are clearly written.

1. Recall the stress fracture example from lecture. The prior probability of an athlete having a stress fracture for women was 9.7% and for men was 6.5%. The sensitity and specificity of the therapeautic ultrasound (TUS) is 64% and 63%, respectively. Suppose the athlete's TUS came back negative. For both men and women, fill in the entries to the following tables and compute the posterior probability that he/she has and doesn't have a stress fracture.

   For female athletes:

Model	Prior probability	Likelihood for $TUS-$	Prior × Likelihood	Posterior Probability
Stress fracture	.097	.36	.035	.058
No stress fracture	.903	.63	.569	.942

   For male athletes:

Model	Prior probability	Likelihood for $TUS-$	Prior × Likelihood	Posterior Probability
Stress fracture	.065	.36	.023	.038
No stress fracture	.935	.63	.589	.962

The results from the TUS further increased the probability of no stress fracture from .903 to .942 for women, and from .935 to .962 for men.

   Interpret the posterior probability results using complete sentences.

2. An experiment consists of flipping a fair coin three times independently. Let A be the event of "atleast one of the flips results in a head" and B be the event "all three flips have the same result."

   (a) List the sample space of the experiment.

   (b) List the outcomes in B and find $P(B)$.

   (c) List the outcomes in $A \cap B$ and find $P(A \cap B)$.

   (d) List the outcomes in $A \cup B$ and find $P(A \cup B)$.

   (e) Are A and B independent. Show why or why not.

2.

a) $S = \{(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T)\}$

b) $B = \{(H,H,H), (T,T,T)\}$

$P(B) = 2/8 = .25$

c) $A \cap B = \{(H,H,H)\}$

$P(A \cap B) = \frac{1}{8} = .125$

d) $A \cup B = S$

$P(A \cup B) = P(S) = 1$

e) $P(A \cap B) = .125 \neq .21875 = (.875)(.25) = P(A)P(B)$

Thus, A and B are <u>not independent</u>.

3.

a) $P(\text{Green}) = P(\text{Green} \cap \text{Par or better}) + P(\text{Green} \cap \text{Worse than par})$

$= 0.2 + 0.1 = \boxed{0.3}$

b) $P(\text{Green} \cup \text{Par}) = P(\overline{\text{Green}} \cap \overline{\text{Par}}) = 1 - 0.6 = \boxed{0.4}$

c) $P(\text{Par}) = P(\text{Green} \cap \text{Par}) + P(\text{No green} \cap \text{Par})$

$= 0.2 + 0.1 = \boxed{0.3}$

d) $P(\text{Par} \mid \text{Green}) = \dfrac{P(\text{Green} \mid \text{Par}) \cdot P(\text{Par})}{P(\text{Green})} = \dfrac{P(\text{Green} \cap \text{Par})}{P(\text{Green})} = \dfrac{0.2}{0.3} = \boxed{0.\overline{6}}$

3. I'm a golfer and playing a par 3 hole. Suppose the following statements are true.

   - The probability that my first shot lands on the green and I make par or better is 0.2.
   - The probability that my first shot lands on the green and I make worse than par is 0.1.
   - The probability that I make par or better but don't land my first shot on the green is 0.1.
   - The probability that I neither land my ball on the green nor make par or better is 0.6.

   Find the following probabilities.

   (a) My first shot lands on the green.
   (b) I land my first shot on the green or I make par or better or both.
   (c) I make par or better.
   (d) I make par or better given that I land my first shot on the green.

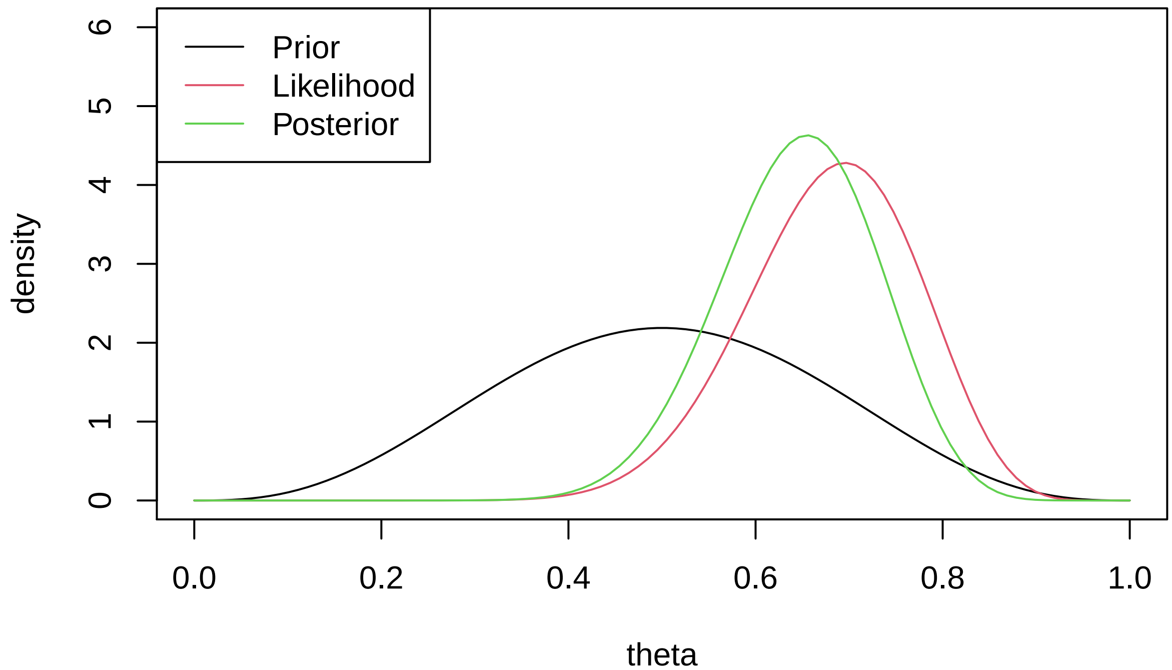
4. Let's assume the attempts of the Mizzou kickers for point after touchdown are independent and identically distributed such that each attempt is a Bernoulli trial with probability of success, θ. Assume a Beta(4,4) prior distribution on θ. In 2019, Mizzou kickers were successful on 16 out of 23 in point after attempts.

   Here is some example R code to help you in your homework.

```
#create a set of values between 0 and 1 to compute the density at
theta = seq(0,1, length=100)
#plot the value vs the Beta(2,5) prior density
plot(theta, dbeta(theta, 2, 5), ylab="density", type ="l",
     col=2, ylim=c(0,3))
#add line for a second Beta(5,2) density
lines(theta, dbeta(theta, 5, 2), type ="l", col=4)
#add a legend at the top right
legend("topright", c("Beta(2,5)","Beta(5,2)"),lty=c(1,1),col=c(2,4))
```

   (a) What is the posterior distribution of θ, the probability of success? Give the posterior mean and posterior variance. Plot the prior and posterior densities of θ on the same graph. Comment on whether or not there is evidence of Bayesian learning and how you made your determination.
   Note: If you are new to R or need a refresher, check out the following link for some additional example code for plotting Beta densities: `https://stephens999.github.io/fiveMinuteStats/beta.html`.
   (b) Re-do part (a) with a Beta(1,1) prior distribution for θ.

a)



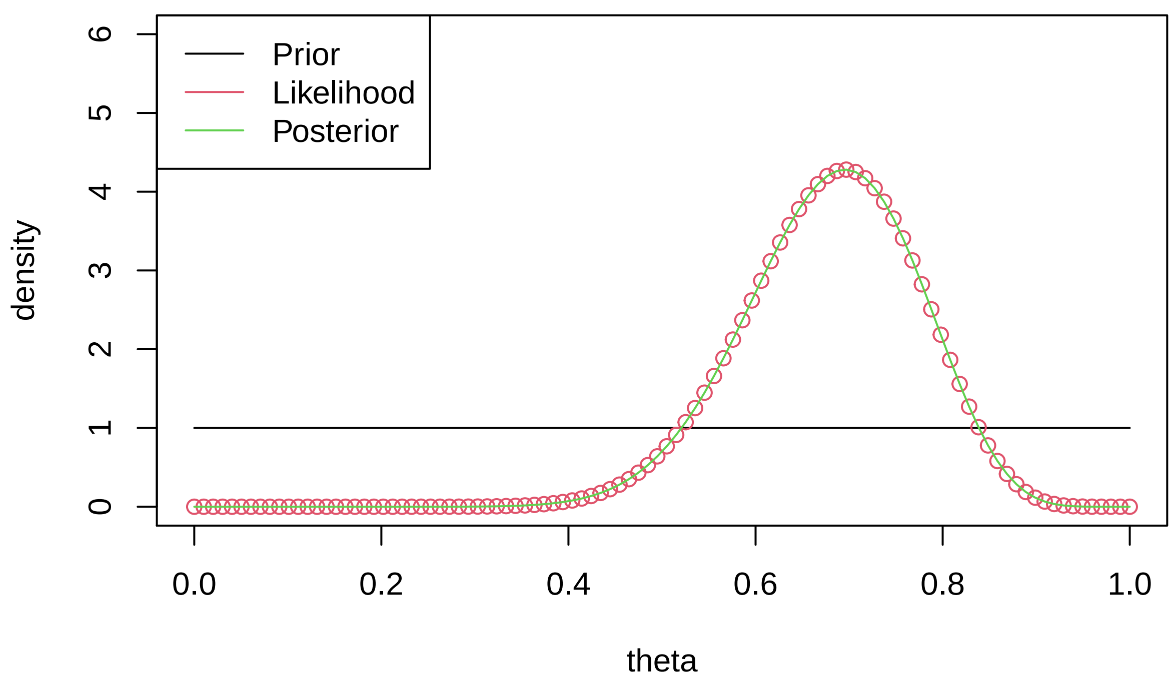
The posterior distribution of $\theta \sim \text{Beta}(20, 11)$, the probability of success is 0.6451613.

Posterior mean: 0.6451613

Posterior variance: 0.007154

Yes, there is evidence of Bayesian learning since our prior belief has been updated to reflect the new information we obtained from the collected data.

b)



The posterior distribution of $\theta \sim \text{Beta}(17, 8)$, the probability of success is 0.68.

Posterior mean: 0.68

Posterior variance: 0.0083692

No, there is no evidence of Bayesian learning since the posterior is exactly the same as the likelihood function before we applied our prior distribution.