STAT 4640/7640 **Homework 6**

Due: March 24, 2022

- **Instructions**: Make sure your name is on your paper and your answers are clearly written.

1. Problem 4 of Chapter 3. Consider the model $Y_i|\sigma_i^2 \stackrel{indep}{\sim} Normal(0, \sigma_i^2)$ for $i = 1, \ldots, n$ where $\sigma_i^2 \sim InvGamma(a, b)$ and $b \sim Gamma(1, 1)$.
   (a) Derive the full conditional posterior distributions for σ_1^2 and b
   (b) Write pseudocode for Gibbs sampling, i.e., describe in detail each step of the Gibbs sampling algorithm.
   (c) Write your own Gibbs sampling code (not in JAGS) and plot the marginal posterior density for each parameter. Assume $n = 10$, $a = 10$, and $Y_i = i$ for $i = 1, \ldots, 10$.
   (d) Repeat the analysis of part (c) with $a = 1$ and comment on the convergence of the MCMC chain.
   (e) Implement the model in part (c) using JAGS and compare the results to those in (c).

2. Generate a time series from a random walk distribution with

$$\phi^{(s)} \sim N(\phi^{(s-1)}, c^2)$$

   for three different values for c^2. Plot the time series on the same figure and compare the chains.

3. Load the dataset "drive.Rdata" as well as the following libraries.

```r
library(truncnorm)
library(MCMCpack)
load("drive.Rdata")
```

   The data conists of the average driving distance of 228 players on the PGA Tour from tournaments held between January 1, 2020 and March 1, 2020. We assume that average driving distance follows a normal distribution such that $Y_i \stackrel{iid}{\sim} Normal(\mu, \sigma^2)$ for $i = 1, \ldots, 228$. The goal is to obtain parameter inference on μ and σ^2. For prior distributions, let's assign $\mu \sim N(300, 100^2)$ and $\sigma^2 \sim Inv.Gamma(2, 20)$. Now, we know that the full conditional distributions are known in closed form. The goal of this exercise, however, is to implement a Metropolis-Hastings algorithm for both μ and σ^2 (knowing that we can check our work). Write your own Metropolis-Hastings sampling code (not in JAGS) using Listings 3.5 and 3.6 as well as the example from class as your guide. Let's assume the following proposal distribtuions

$$q(\mu^*|\mu^{(s-1)}) \sim N(\mu^{(s-1)}, c^2)$$
$$q(\sigma^{2*}|\sigma^{2(s-1)}) \sim TN(\sigma^{2(s-1)}, d^2, 0, \infty)$$

where TN denotes a truncated normal distribution. The truncated normal distribution with lower bound 0 and upper bound ∞ will ensure that $\sigma^2 \in \mathbb{R}^+$. Note that you will have to tune your proposal distributions by specifying c^2 and d^2. Once you are confident with your sampling algorithm, generate 5,000 samples from the posterior and report the following:

(a) Produce trace plots for both μ and σ^2, report the values c^2 and d^2 as well as your acceptance rate.

(b) Compute 95% credible intervals for μ and σ^2.