

# STAT 4640/7640 Homework 3

**Due: February 17, 2022**

1. You are designing a very small experiment to determine the sensitivity of a lab test to be used to detect doping in Olympic athletes. You will inject 5 mice with banned substances and record the number of mice that test positive. Because the dataset will be small, you ask two experts for their opinions. Expert 1 expects the positive test probability to be 0.95 with a standard deviation of 0.05, Expert 2 expects 0.50 with standard deviation 1/12.
  - (a) Translate these two priors into Beta densities. Are either of these priors noninformative? Justify your answer.
  - (b) Now, suppose you conduct the experiment and all five mice test positive for banned substances. Plot the posterior of the test probability under each experts prior and comment on the results.
2. Problem 4 of Chapter 2 (modified): Assume the  $Y|\theta \sim \text{NegBinomial}(\theta, m)$  (see Appendix A.1) and  $\theta \sim \text{Beta}(a, b)$ .
  - (a) Describe the negative binomial distribution in words (e.g., what is it used for?).
  - (b) Derive the posterior of  $\theta$ .
  - (c) Plot the posterior of  $\theta$  and give its 95% credible interval assuming  $m = 5$ ,  $Y = 10$ , and  $a = b = 1$ .
3. Derive explicitly (like we did on the board) the posterior of  $\lambda$  under the Poisson-Gamma model where  $Y \sim \text{Poisson}(\lambda)$  and  $\lambda \sim \text{Gamma}(a, b)$ .
4. Problem 15 of Chapter 2: Say  $Y|\lambda \sim \text{Poisson}(\lambda)$ .
  - (a) Derive and plot the Jeffreys' prior for  $\lambda$ .
  - (b) Is this prior proper?
  - (c) Derive the posterior and give conditions on  $Y$  to ensure it is proper.