MATH-M415 CLASS NOTES

"COMPLEX VARIABLES AND HARMONIC/ANALYTIC FUNCTIONS"

"Elementary Complex Variables"

PLTS: Proof Left To Student.

- Semi-Qualitative Approach
- Specifics ignored/irrelevant
- Polar Coordinates 'used'; not explicitly in book
- Often 'Evaluating' Integrals
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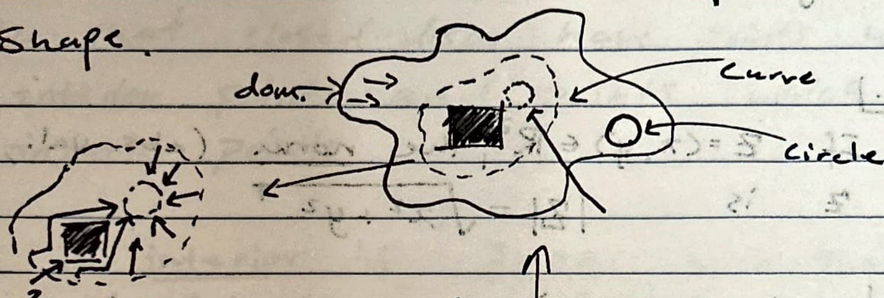
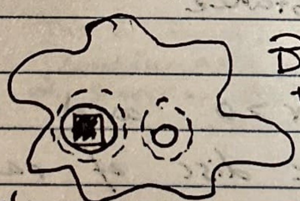
Ex.| If $f'(z)$ exists, significant restrictions are imposed on $f(Z)$.

Ex.| Circles are the most important shape.



Dashed circle and red shape may not be the same because of the square.

equiv.



Think of the red shapes as rubber bands. Can we have a red Rubber Band match the shape of the black circle without snagging?

Both of these though may be the same.

Algebra in the Plane

- real plane is denoted as $\mathbb{R}^2$
- The set of all ordered pairs $\{(x,y) : x,y \in \mathbb{R}\}$

Add/sub.

$$\begin{array}{r} (1,2) \\ + \ (-3,4) \\ \hline (-2,6) \end{array}$$

Sc. multiplication

$5 \cdot (1,2)$
$= (5\cdot 1, 5\cdot 2)$
$= (5,10)$

Greek letters:

ζ / ζ - zeta
η - eta
ξ / ξ - xi
Ω - omega
α - alpha
Γ - gamma

Def.) If $z=(x,y) \in \mathbb{R}^2$, the norm (abs. val., modulus) of z is $|z| = \sqrt{x^2+y^2}$

* Def.) Let $z=(x,y)$ and $z_0=(x_0,y_0)$. Then the distance between z and z_0 is
$|z-z_0| = \sqrt{(x-x_0)^2+(y-y_0)^2}$

distance $\Rightarrow$ size

size $\not\Rightarrow$ distance

Def.) Let $r>0$ and z_0 be a fixed point in $\mathbb{R}^2$. The disc of radius r with center z_0 is

$$D(z_0;r) = \{z \in \mathbb{R}^2 : |z-z_0| < r\}$$

Domains in the Plane II

Ex.

① $\Omega = \mathbb{R}^2$ — Normally use Ω to represent the domain.

② $\Omega = \{(x,y) : 0 \le x \le 1,\ 0 < y \le 1\}$

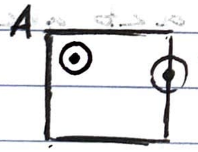
Def. Let $r > 0$ and z_0 be a fixed point in $\mathbb{R}^2$. The set

$$\bar{D}(z_0; r) = \{z \in \mathbb{R}^2 : |z - z_0| \le r\}$$

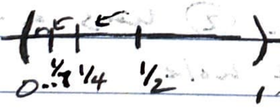
is the closed disc of radius r with center z_0.

- Note that closed discs have points which are interior points and points which are not interior points.

$x \in A$ is interior if $\exists r > 0$ such that $D(x; r) \subset A$.

Ex. 

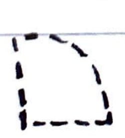
Ex. 

Ex. 

The 2 above, continuously getting closer is similar to this idea.

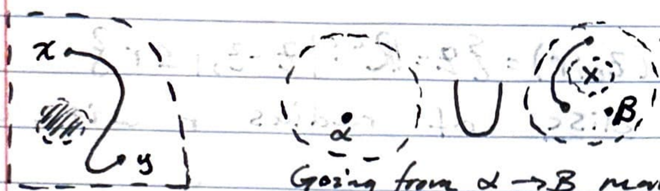
* - More so, a disc which is "partially closed" is not a

Def. A subset $\Omega \subset \mathbb{R}^2$ is open if and only if every point of Ω is an interior point.

 ✓  As long as the inside not included it is still open.

Def. A subset $S \subset \mathbb{R}^2$ is closed if and only if the complement set $\mathbb{R}^2 / S$ is open.

Def. An open subset $\Omega \subset \mathbb{R}^2$ is disconnected if and only if it can be written as $\Omega = \Omega_1 \cup \Omega_2$ where Ω_1, Ω_2 are non-empty and disjoint.



Going from $\alpha \to \beta$ makes that disconnected.

If a set is not disconnected it is connected.

Examples of Connected Sets

Ex. 1

① $D(z_0; r)$

* ② $D(z_0; r) \setminus \{z_0\}$ Disk where we ignore the center.

③ $A = \{z : a < |z| < b\}$ where $a < b$ are real numbers

A: 

Topologically ② and ③ are the same. ② has a tiny hole, ③ has a fat hole.

Def. A subset $\Omega \subset \mathbb{R}^2$ is a domain if and only if it is open and connected.

* If MS Paint could fill it in on one dump it is connected.