hw8

2024-03-29

2a

```
library(dplyr)
##
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
##
       filter, lag
## The following objects are masked from 'package:base':
##
       intersect, setdiff, setequal, union
##
library(MASS)
##
## Attaching package: 'MASS'
## The following object is masked from 'package:dplyr':
##
##
       select
Y <- read.table("http://www2.stat.duke.edu/~pdh10/FCBS/Exercises/azdiabetes.dat", header=TRUE)
diabetics <- Y %>%
  filter(diabetes=="Yes")
Y_d <- diabetics[, 1:7]</pre>
nondiabetics <- Y %>%
  filter(diabetes=="No")
Y_n <- nondiabetics[, 1:7]</pre>
mu0_d <- colMeans(diabetics[, 1:7])</pre>
samp_cov_d <- cov(diabetics[, 1:7])</pre>
lambda0 d <- cov(diabetics[, 1:7])</pre>
S0_d <- samp_cov_d
nu0_d <- 9
mu0_n <- colMeans(nondiabetics[, 1:7])</pre>
samp_cov_n <- cov(nondiabetics[, 1:7])</pre>
lambda0_n <- cov(nondiabetics[, 1:7])</pre>
SO_n <- samp_cov_n
nu0_n <- 9
### Simulate multivariate normal vector
rmvnorm<-function(n,mu,Sigma)</pre>
p<-length(mu)
```

```
res<-matrix(0,nrow=n,ncol=p)</pre>
  if(n>0 & p>0)
    E<-matrix(rnorm(n*p),n,p)</pre>
    res<-t( t(E%*%chol(Sigma)) +c(mu))
  }
  res
}
### Simulate from the Wishart distribution
rwish<-function(n,nu0,S0)</pre>
{
  sS0 <- chol(S0)
  S<-array( dim=c( dim(S0),n ) )</pre>
  for(i in 1:n)
     Z <- matrix(rnorm(nu0 * dim(S0)[1]), nu0, dim(S0)[1]) %*% sS0
     S[,,i] \leftarrow t(Z)%*%Z
  }
  S[,,1:n]
### Gibbs sampler
Sigma_d <- samp_cov_d
n_d < -dim(Y_d)[1]
SO_d <- samp_cov_d
THETA_d <- NULL
SIGMA_d <- NULL
Sigma_n <- samp_cov_n
n n < -dim(Y n)[1]
SO_n <- samp_cov_n
THETA_n <- NULL
SIGMA_n <- NULL
S <- 500
for (s in 1:S){
    ###update theta_d
  Ln_d<-solve( solve(lambda0_d) + n_d*solve(Sigma_d) )</pre>
  mun_d<-Ln_d%*%( solve(lambda0_d)%*%mu0_d + n_d*solve(Sigma_d)%*%mu0_d )
  theta_d<-rmvnorm(1,mun_d,Ln_d)
  ###
    ###update Sigma_d
  Sn_d \leftarrow S0_d + (t(Y_d)-c(theta_d))%*%t(t(Y_d)-c(theta_d))
  Sigma_d<-solve( rwish(1, nu0_d+n_d, solve(Sn_d)) )
  ###
    ### save results
  THETA_d<-rbind(THETA_d,theta_d) ; SIGMA_d<-rbind(SIGMA_d,c(Sigma_d))</pre>
  ###
```

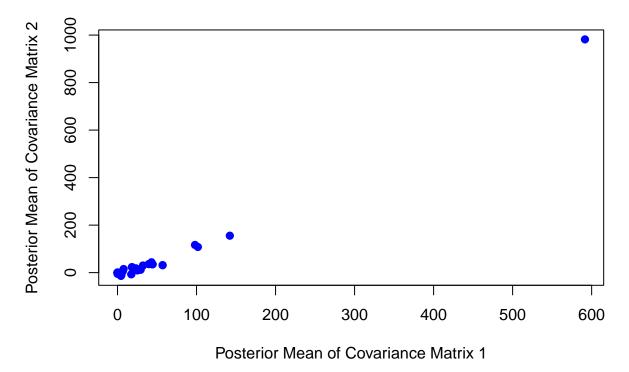
```
###update theta n
  Ln_n<-solve( solve(lambda0_n) + n_n*solve(Sigma_n) )</pre>
  mun_n < -Ln_n%%( solve(lambda0_n)%%mu0_n + n_n*solve(Sigma_n)%%mu0_n )
  theta_n<-rmvnorm(1,mun_n,Ln_n)</pre>
  ###
    ###update Sigma_n
  Sn_n < S0_n + (t(Y_n)-c(theta_n)) %*%t(t(Y_n)-c(theta_n))
  Sigma_n<-solve( rwish(1, nu0_d+n_n, solve(Sn_n)) )
  ###
    ### save results
  THETA_n<-rbind(THETA_n,theta_n); SIGMA_n<-rbind(SIGMA_n,c(Sigma_n))
  ###
  if (s \% 100 == 0){
    print(s)
  }
}
## [1] 100
## [1] 200
## [1] 300
## [1] 400
## [1] 500
colMeans(THETA_n) - colMeans(THETA_d)
##
                                      bp
                                                 skin
                                                               bmi
         npreg
                        glu
    -1.7816555 \ -32.9302286 \ -4.6869680 \ -5.7121708 \ -4.3873174 \ -0.1689197
##
##
           age
##
   -7.1778575
apply(THETA_n, 2, var) - apply(THETA_d, 2, var)
##
                                                         skin
                                                                          bmi
                             glu
           npreg
                                             bp
## -0.0625538205 -4.2937236289 -0.4560368370 -0.2792719934 -0.1101451668
             ped
## -0.0006127821 -0.3416572219
The glucose of diabetics seems to be, on average, much higher than that of non-diabetics. Additionally, the
variance of glucose is much greater for diabetics than it is for non-diabetics.
colMeans(THETA_d > THETA_n)
## npreg
           glu
                   bp
                      skin
                               bmi
                                     ped
             1
                    1
                          1
                                 1
                                       1
```

 $Pr(\theta_{d,j} > \theta_{n,j}|Y) = 1 \text{ for all } j \in \{1, 2, 3, 4, 5, 6, 7\}.$

```
colMeans(SIGMA_n) - colMeans(SIGMA_d)
##
    [1]
          -7.57570136
                         14.22913421
                                         0.68366192
                                                        7.78210451
                                                                       4.58786817
##
    [6]
           0.06127550
                         -4.86451002
                                        14.22913421 -390.28944104
                                                                      25.66920551
  [11]
##
           2.35791231
                         15.49342050
                                         0.44297895
                                                        7.71647816
                                                                       0.68366192
##
   [16]
          25.66920551
                        -13.27539500
                                        16.86205911
                                                        5.43727934
                                                                       0.07169206
##
   [21]
           3.63355464
                                                                      -6.26736091
                          7.78210451
                                         2.35791231
                                                       16.86205911
##
   [26]
           9.13864636
                         -0.47069163
                                        24.32173631
                                                        4.58786817
                                                                      15.49342050
   [31]
           5.43727934
                                        -0.64629268
##
                          9.13864636
                                                       -0.28028391
                                                                      17.89175550
##
   [36]
           0.06127550
                          0.44297895
                                         0.07169206
                                                       -0.47069163
                                                                      -0.28028391
                                                        7.71647816
   [41]
          -0.06819314
                          0.22429395
                                        -4.86451002
                                                                       3.63355464
##
## [46]
          24.32173631
                         17.89175550
                                         0.22429395
                                                      -18.49115330
```

The 2,2 entry corresponds to glucose, so this supports our observation that the variability in glucose is especially high in diabetics.

Posterior Means of Covariance Matrices



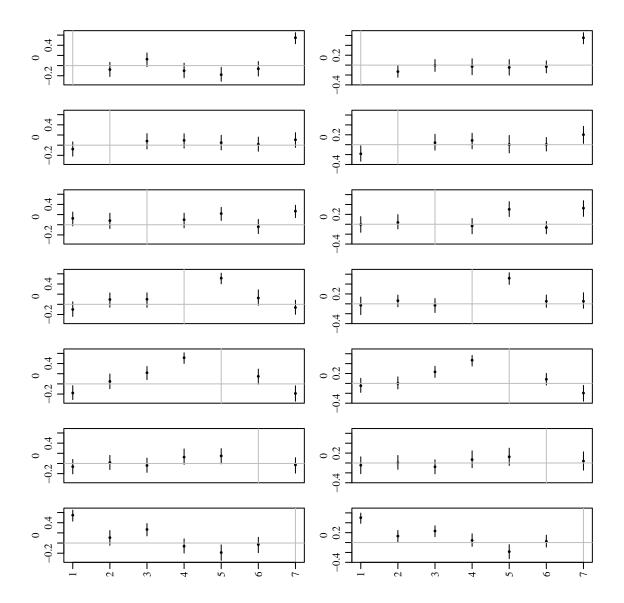
```
library(knitr)
#### Function for posterior quantile intervals for matrices
```

```
#### From the `sbgcop` package
plotci.sA<-function(sA, ylabs = colnames(sA[, , 1]), mgp = c(1.75, 0.75,
    0))
{
    qA \leftarrow qM.sM(sA)
    p \leftarrow dim(qA)[1]
    tmp \leftarrow c(qA)
    tmp <- tmp[tmp != 1]
    par(mgp = mgp)
    for (j in 1:p) {
         plot(0, 0, type = "n", ylim = range(c(tmp), na.rm = TRUE),
             xlim = c(1, p), ylab = ylabs[j], xaxt = "n", xlab = "")
         points((1:p)[-j], qA[j, -j, 2], pch = 16, cex = 0.6)
         segments(x0 = (1:p)[-j], y0 = qA[j, -j, 1], x1 = (1:p)[-j],
             y1 = qA[j, -j, 3])
         abline(h = 0, col = "gray")
         abline(v = j, col = "gray")
    axis(side = 1, at = 1:p, labels = colnames(qA[, , 1]), las = 2)
}
sR.sC<-function(sC)
{
    p <- dim(sC)[1]
    s \leftarrow dim(sC)[3]
    sR \leftarrow array(dim = c(p, p, s))
    dimnames(sR) <- dimnames(sC)</pre>
    for (1 in 1:s) {
         C \leftarrow sC[, , 1]
        R <- C * NA
         for (j in 1:p) {
             R[j, -j] \leftarrow C[j, -j] %*% solve(C[-j, -j])
         sR[, , 1] \leftarrow R
    }
    sR
}
qM.sM < -function (sM, quantiles = c(0.025, 0.5, 0.975))
    p1 <- dim(sM)[1]
    p2 \leftarrow dim(sM)[2]
    s \leftarrow dim(sM)[3]
    qM <- array(dim = c(p1, p2, length(quantiles)))
    dimnames(qM) <- list(dimnames(sM)[[1]], dimnames(sM)[[2]],</pre>
        paste(quantiles * 100, rep("% quantile", length(quantiles)),
             sep = ""))
    for (l in 1:length(quantiles)) {
         qM[, , 1] \leftarrow apply(sM, c(1, 2), quantile, prob = quantiles[1],
             na.rm = TRUE)
    }
    \mathtt{q}\mathtt{M}
}
```

```
######
p <- 7
COR <- array( dim=c(p,p,S) )</pre>
for(s in 1:S){
Sig <- matrix( SIGMA_d[s,] ,nrow=p,ncol=p)</pre>
COR[ , ,s] <- Sig/sqrt( outer( diag(Sig), diag(Sig) ) )</pre>
}
apply(COR, c(1,2), quantile, prob=c(.025,.975))
## , , 1
##
##
                  [,2]
                            [,3]
                                       [,4]
                                                 [,5]
       [,1]
        1 -0.21603624 -0.02333336 -0.23897292 -0.30874507 -0.20513279
## 2.5%
          1 0.06442201 0.24955069 0.04882643 -0.03512163 0.08022421
            [,7]
##
## 2.5% 0.4303916
## 97.5% 0.6476300
##
## , , 2
##
                          [,3]
                                                          [,6]
##
             [,1] [,2]
                                 [,4]
                                                [,5]
## 97.5% 0.06442201
                  1 0.2269402 0.22408940 0.19566058 0.1598039
             [,7]
## 2.5% -0.04348288
## 97.5% 0.24625907
##
## , , 3
##
                       [,2] [,3]
             [,1]
                                    [,4]
                                             [,5]
                                                        [,6]
## 2.5% -0.02333336 -0.0746227 1 -0.06026897 0.0831292 -0.1750032 0.1405919
## 97.5% 0.24955069 0.2269402 1 0.22708395 0.3422572 0.1055482 0.3832435
## , , 4
##
                        [,2]
                                  [,3] [,4]
                                               [,5]
                                                          [,6]
##
             [,1]
## 2.5% -0.23897292 -0.05773455 -0.06026897 1 0.4025493 -0.01769833
## 97.5% 0.04882643 0.22408940 0.22708395 1 0.6124939 0.28493195
##
             [,7]
## 2.5% -0.19629059
## 97.5% 0.08193227
##
## , , 5
##
                                       [,4] [,5]
                        [,2]
                                [,3]
                                                                   [,7]
##
             [,1]
                                                       [,6]
## 2.5% -0.30874507 -0.09412677 0.0831292 0.4025493 1 -0.007774265 -0.34022597
```

```
##
## , , 6
##
                                     [,3]
##
               [,1]
                          [,2]
                                                [,4]
                                                             [,5] [,6]
## 2.5% -0.20513279 -0.1197722 -0.1750032 -0.01769833 -0.007774265
## 97.5% 0.08022421 0.1598039 0.1055482 0.28493195 0.291489471
              Γ.71
## 2.5% -0.1900568
## 97.5% 0.1135090
##
## , , 7
##
                         [,2]
                                              [,4]
##
             [,1]
                                   [,3]
                                                          [,5]
                                                                     [,6] [,7]
## 2.5% 0.4303916 -0.04348288 0.1405919 -0.19629059 -0.34022597 -0.1900568
## 97.5% 0.6476300 0.24625907 0.3832435 0.08193227 -0.03690773 0.1135090
pdf("cor_graph.pdf",height=6,width=6,family="Times")
par(mfcol=c(7,2),mar=c(1,2.75,1,1),mgp=c(1.75,.75,0),oma=c(1.5,0,0,0))
plotci.sA(COR)
REG<-sR.sC(COR)
plotci.sA(REG)
dev.off()
## pdf
##
CQ<-apply(COR, c(1,2), quantile,prob=c(.025,.5,.975))
round(CQ[1,,],2)
##
        [,1] [,2] [,3] [,4] [,5] [,6]
## [1,] 1.00 -0.22 -0.02 -0.24 -0.31 -0.21 0.43
## [2,] -0.22 1.00 -0.07 -0.06 -0.09 -0.12 -0.04
## [3,] -0.02 -0.07 1.00 -0.06 0.08 -0.18 0.14
## [4,] -0.24 -0.06 -0.06 1.00 0.40 -0.02 -0.20
## [5,] -0.31 -0.09 0.08 0.40 1.00 -0.01 -0.34
## [6,] -0.21 -0.12 -0.18 -0.02 -0.01 1.00 -0.19
## [7,] 0.43 -0.04 0.14 -0.20 -0.34 -0.19 1.00
round(CQ[2,,],2)
                   [,3] [,4] [,5] [,6]
        [,1] [,2]
                                           [,7]
## [1,] 1.00 -0.08 0.13 -0.10 -0.18 -0.06 0.55
## [2,] -0.08 1.00 0.08 0.09 0.05 0.02 0.11
## [3,] 0.13 0.08 1.00 0.10 0.22 -0.04 0.27
## [4,] -0.10 0.09 0.10 1.00 0.51 0.12 -0.06
## [5,] -0.18 0.05 0.22 0.51 1.00 0.15 -0.19
## [6,] -0.06 0.02 -0.04 0.12 0.15 1.00 -0.03
## [7,] 0.55 0.11 0.27 -0.06 -0.19 -0.03 1.00
round(CQ[3,,],2)
        [,1] [,2] [,3] [,4] [,5] [,6] [,7]
##
```

```
## [1,] 1.00 0.06 0.25 0.05 -0.04 0.08 0.65
## [2,] 0.06 1.00 0.23 0.22 0.20 0.16 0.25
## [3,] 0.25 0.23 1.00 0.23 0.34 0.11 0.38
## [4,] 0.05 0.22 0.23 1.00 0.61 0.28 0.08
## [5,] -0.04 0.20 0.34 0.61 1.00 0.29 -0.04
## [6,] 0.08 0.16 0.11 0.28 0.29 1.00 0.11
## [7,] 0.65 0.25 0.38 0.08 -0.04 0.11 1.00
round(apply(COR,c(1,2),mean),2)
        [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## [1,] 1.00 -0.08 0.12 -0.10 -0.18 -0.06 0.55
## [2,] -0.08 1.00 0.08 0.09 0.05 0.02 0.11
## [3,] 0.12 0.08 1.00 0.10 0.22 -0.04 0.27
## [4,] -0.10 0.09 0.10 1.00 0.51 0.13 -0.06
## [5,] -0.18 0.05 0.22 0.51 1.00 0.14 -0.19
## [6,] -0.06 0.02 -0.04 0.13 0.14 1.00 -0.03
## [7,] 0.55 0.11 0.27 -0.06 -0.19 -0.03 1.00
knitr::include_graphics("cor_graph.pdf")
```



There do not seem to be many significant differences between the two groups. One difference we see is that

4

```
Y<-dget(url("http://www2.stat.duke.edu/~pdh10/FCBS/Inline/Y.pima.miss"))

Y=Y%>%
  filter(!is.na(glu))%>%
  filter(!is.na(bp))%>%
  filter(!is.na(skin))%>%
  filter(!is.na(bmi))

colMeans(Y)

## glu bp skin bmi
## 121.85039 70.61417 28.83465 31.67874
```

```
###
Y <- readRDS("hw8train.rds")

### prior parameters
n<-dim(Y)[1] ; p<-dim(Y)[2]
mu0<-c(rep(0,14))
sd0<-(mu0/2)
L0<-matrix(.1,p,p) ; diag(L0)<-1 ; L0<-L0*outer(sd0,sd0)
nu0<-p+2 ; S0<-L0
###

### starting values
Sigma<-S0
Y.full<-Y
0<-1*(!is.na(Y))</pre>
```