Production Peaks and Declines in NFL Players by Position: A Statistical Analysis

November 2024

1. Overview

1.1 Introduction

In the NFL, understanding and forecasting when a player's performance could start to decline is crucial for scouts and team executives to make informed decisions. There are outlier players like Tom Brady who have maintained peak performance into their 40s, while others at different positions experience a decline in their mid-20s. These production trends vary widely by position, with Running Backs (RBs) often experiencing performance declines in their mid-20s due to the physical demands of the role, while other positions such as Quarterbacks (QB) tend to maintain effectiveness into their 30s. This study explores productivity trends across age, years played, and cumulative snap counts for key offensive positions (Quarterback (QB), Running Back (RB), Wide Receiver (WR), and Tight End (TE)) with the aim of identifying common peak and decline points. By leveraging advanced statistical modeling, this study aims to reveal patterns and establish predictive models that can inform NFL management in contract negotiation, roster development, and player utilization.

1.2 Importance of the Study

Accurately predicting a player's peak and decline periods can give NFL team management a competitive edge in contract negotiations, draft strategies, and roster management. This prediction can also help player agents determine the best time to capitalize on their clients' productivity, securing favorable contracts while performance remains high. Understanding these patterns would help teams optimize player usage, minimize injury risks, and maximize player output during their prime years. Additionally, the insights derived from this study can benefit the rapidly growing fantasy football industry by providing a richer understanding of player dynamics to fans and analysts.

1.3 Research Objectives

The first aim of this analysis is to determine the age at which NFL players' production begins to decline, segmented by position (QB, RB, WR, TE). This will test the initial thought that there is an "age cliff" for NFL players, where athletes typically start to become less effective at a certain age based on their position. The second goal is to use players' years played to identify the point in their career where their production initially increases, reaches its peak, and eventually declines. This aim seeks to test whether the concept of an "age cliff" is more accurately described as a "years played cliff." Analyzing years played may offer a better framework for pinpointing the phases of peak performance and subsequent decline. The third goal is to investigate the cumulative snap counts at which production begins to peak and decline for each position. While cumulative snap counts may correlate with years played, this measure provides an alternative perspective, especially for players who miss games due to injury. It is expected that age, years played, and cumulative snaps will have some effect on production, so this study will analyze all three to gain insights into whether biological aging or career-long physical demands are more predictive of performance decline.

2. Approach and Methods

2.1 Data

Data were obtained from nflverse [5], using the 'nflreadr' package, which provides comprehensive datasets on NFL player statistics, rosters, Next Gen Stats, and snap counts from 2000 to 2023. The

final dataset is a longitudinal dataset that contains detailed player statistics for 3099 unique NFL players at QB, TE, WR, and RB positions. The player statistics included are variables such as completions, passing yards, rushing yards, receptions, targets, PPR fantasy points, and more, along with player demographics like age, height, weight, and years in the league. The data also contain snap counts for both offensive and defensive plays to evaluate cumulative workload. Using these base statistics, additional variables were created such as rush yards per carry, yards per target, touchdown-to-interception ratios, PPR fantasy points per game, target share per game, yards after catch per game, touchdowns per game, and many more. These metrics were evaluated for their reliability as response variables in assessing productivity, with particular focus on PPR fantasy points per game due to its comprehensive reflection of player performance across various game situations.

2.2 Data Preparation

Initial data cleaning involved addressing missing or anomalous values. Of the original 12,686 entries, 2,436 were removed due to missing birth dates, which are crucial for age calculations. Entries with fewer than two games or zero fantasy points per game were excluded to ensure the analysis focused on players contributing meaningfully to productivity trends. Observations lacking data on years played or cumulative snap counts were excluded from the models where these variables were used as predictors. The remaining dataset included 2515 unique players with 9156 total rows of player data where each row is a player's data for one season.

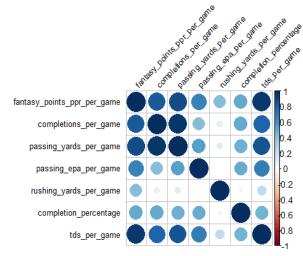
2.3 Statistical Methods

The methods of analysis on the dataset will begin with exploratory data analysis (EDA), utilizing summary statistics and visualizations such as box plots, histograms, and scatter plots to examine the distributions of age, years played, and snap counts by position. This step aims to identify data spread, outliers, and correlations among these variables. Correlation matrices and Principal Component Analysis (PCA) will be employed to determine the most representative productivity variable, focusing on metrics that effectively encapsulate player performance. The analysis proceeds by applying simple linear and quadratic models, alongside mixed-effects models, to examine the impact of age, years played, and cumulative snap counts on player productivity across the four key NFL offensive positions: Quarterback (QB), Running Back (RB), Wide Receiver (WR), and Tight End (TE). Mixed-effects models are explored to incorporate both fixed effects, such as age and position, and random effects to account for variability among individual players. This approach provides flexibility to capture player-specific performance baselines while effectively modeling non-linear productivity trajectories through quadratic terms.

3. Exploratory Data Analysis

3.1 Determining Response Variable

To identify which production statistic would be the best response variable to represent overall player performance, Principal Component Analysis (PCA) and Pearson correlation coefficient matrices were applied. The correlation matrices as seen in Figure 1, indicated that 'fantasy points ppr per game' was the best production encapsulating variable, as it had the strongest associations with all other production metrics for every position group. In the PCA, 'fantasy points ppr per game' was also the most significant contributor to the first principal component, which accounted for over 80% of the variance across all production variables (see Figure 2) which helped confirm that PPR fantasy points should be selected as the response variable for the subsequent analysis.



	Dim.1	ctr	cos2
yards_per_game	0.979	15.440	0.958
carries_per_game	0.054	0.047	0.003
rushing_epa_per_game	0.074	0.088	0.005
receptions_per_game	0.960	14.836	0.921
targets_per_game	0.873	12.275	0.762
receiving_yards_after_catch_per_game	0.686	7.578	0.470
receiving_epa_per_game	0.705	8.016	0.498
target_share_per_game	0.917	13.560	0.842
fantasy_points_ppr_per_game	0.990	15.781	0.980
yards_per_target	0.381	2.333	0.145

Figure 1: Example Pearson Correlation Matrix for QB data

Figure 2: PCA results for WR, RB, and TE Production Statistics

3.2 Distribution of the Response Variable

A key step in the EDA involved analyzing the distribution of the response variable, PPR fantasy points per game. As shown in Figure 3, the response exhibited a right-skewed distribution, with most players having relatively low scores and a few outliers achieving significantly higher values (which were significant and needed to be included). The skewness suggested the potential need for a transformation to improve model fit, and an initial linear model with age as the predictor revealed non-constant variance in the residuals, further supporting the need for transformation. A log transformation was considered for the models that were developed in the subsequent sections due to the ease of interpretation from back transforming the log.

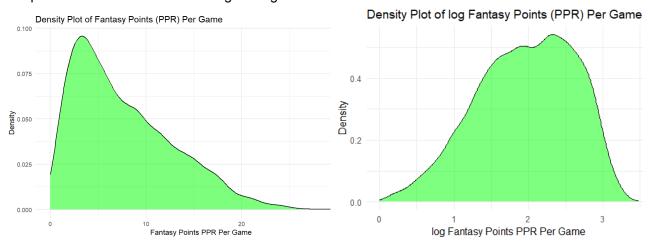


Figure 3: Example Density Plots of PPR Fantasy Points Per Game with and without a Log Transformation

3.3 Interactions Between Predictors

During this exploratory phase, strong Pearson correlation coefficients between age, years played, and cumulative snap counts were identified. These associations indicated the potential for multicollinearity if these variables were used together in a single model. This finding highlighted the importance of using either interaction terms or developing separate models for each predictor to accurately capture their influence on player performance. The Pearson correlation matrix in Figure 4 shows these relationships.

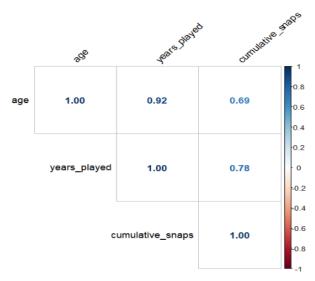


Figure 4: Predictor Variable Pearson Correlation Matrix

3.4 Discovering Trends

Exploratory Data Analysis revealed notable differences in PPR fantasy points trajectories and distributions across positions. As seen in Figure 5, the distribution of fantasy points varies significantly among the four position groups showing distinct differences in means, medians, and ranges. It was also found that the relationships between age, cumulative snaps, and years played follow different trajectories for each position group, as demonstrated in Figure 6 (age vs. fantasy points). The variation in fantasy points per game by age for individual players highlights the importance of accounting for positional differences. This suggested that separate models for identifying production peaks and declines are likely necessary for each position group. This ensures that the unique underlying characteristics of each position are captured in the analysis.

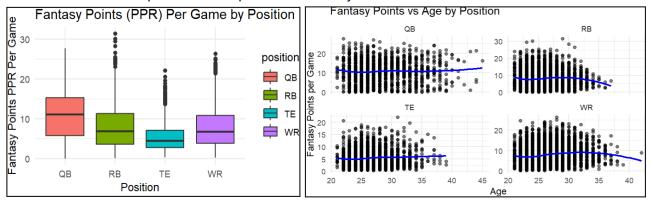


Figure 5: Box Plots of Fantasy Points Per Game By Position

Figure 6: Fantasy Points vs Age Relationship by Position

4. Modeling Preparation

4.1 Separate Models For Each Position

The decision was made to build separate models for each position rather than including position as a fixed effect. Each NFL position has distinct performance patterns due to different physical and skill demands. For instance, running backs often experience shorter careers due to the physical toll of their position, while quarterbacks may have longer careers and later peaks. Modeling each position separately captures these unique performance trajectories and avoids the complexity that would arise

from including interaction terms between position and predictors, which would have been necessary if position were treated as a fixed effect.

4.2 Handling Predictors and Multicollinearity

The primary predictors (age, years played, and cumulative snap counts) were found to be highly correlated, with the Variance Inflation Factors (VIFs) for 'age' and 'years played' exceeding 5, indicating problematic multicollinearity when included together. Data availability is also limited for 'years_played' and 'cumulative_snaps', as the data is only available from 2012 onwards, while age data spans back to 2000. To mitigate multicollinearity and ensure practical application, separate models were developed for each predictor. This led to a total of 12 models, three for each position, with one model having 'age' as a fixed effect, another with 'years played', and the third with 'cumulative snaps'. This approach allows for a clearer, independent evaluation of each predictor's effect on player performance, providing more precise insights into productivity peaks and declines that NFL management and agents can realistically understand and apply.

4.3 Response Variable Transformation

The response variable, PPR fantasy points per game, exhibited a right-skewed distribution for the RB, WR, and TE positions, with most players having lower points and a few outliers achieving significantly higher values. Simple linear and quadratic models, as well as linear and quadratic mixed-effects models showed non-constant variance in the residuals for all nine models involving these positions, consistently displaying a cone-shaped pattern in the residuals versus fitted plots that indicated heteroscedasticity. Figure 7 shows an example of this consistent result for the linear mixed-effect model. To address this issue, a logarithmic transformation of 'fantasy points ppr per game' was applied. The log transformation stabilized the variance (see Figure 7), leading to better model fit for all 9 models and adherence to model assumptions.

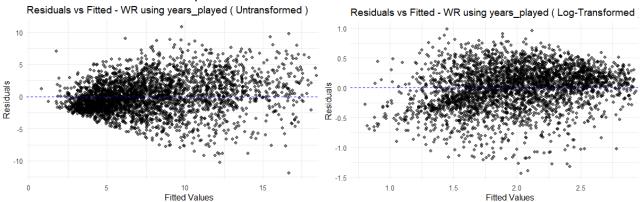


Figure 7: Residuals vs Fitted Plot before and after log transformation for the Linear Mixed-Effects years played WR Model For the QB position, performance metrics are more consistent and less skewed due to the nature of their role. Diagnostic checks showed that applying a log transformation did not improve the variance or normality of residuals for QBs and Q-Q plots showed that it caused the distribution to deviate further from normality. Also, the residuals versus fitted plots for QB did not exhibit as much of a cone-shaped pattern that was observed in other positions. Based on these results, it was decided not to log-transform the response for the QB models. By applying back-transformations on the log-transformed responses for other positions, interpretability between models will be maintained despite using different transformations across models.

5. Quadratic Mixed-Effects Model Approach

5.1 Model Selection

To determine the best model for the relationships between the predictors and player performance, multiple linear and quadratic models were fitted for each position and predictor. Likelihood ratio tests (LRT), along with comparisons of Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC), consistently indicated that quadratic models provided a better fit across all positions and predictors, evidenced by the lower AIC and BIC values and significant p-values for the likelihood ratio tests that occurred for each model. Figure 8 presents a representative example of the test results that were consistently outputted.

Figure 8: AIC, BIC, and Likelihood ratio results for the model with 'wr_data" and 'age' as the fixed predictor

Through exploring different models, it was also determined that mixed-effects models should be used to account for the hierarchical structure of the data and potential variability between players and seasons. Random intercepts for 'player id' accounted for individual differences between players, capturing player-specific baseline performance levels. Random intercepts for 'season' accounted for differences between seasons, such as rule changes, overall league trends, or environmental factors impacting all players similarly. Including both of these random effects also improved the model fit, as seen in Figure 9 with the lower AIC and BIC values and significant likelihood ratio tests when compared to models without these random effects. However, Figure 9 also shows that the improvement made by adding the 'season' random effect is very minimal and model results showed that the season-level variance component which quantifies the degree to which the baseline productivity (intercept) varies across different seasons was less than 0.01 for each model (e.g., 0.0032 for the WR Age Model). Consequently, the decision was made to remove the 'season' random effect due to its low contribution.

```
ATC
                                                          BIC LRT_pvalue
Linear with Season & Player Random Effect
                                              5304.87
                                                      5335.77
                                                                      NA
Quadratic with Season & Player Random Effect 5221.21
                                                      5258.29
                                                                   0.0000
Linear Only Player Random Effect
                                              5315.69 5340.41
                                                                       NA
Quadratic Only Player Random Effect
                                              5239.35 5270.25
                                                                   0.0000
Linear no Random Effects
                                              6522.40 6540.94
                                                                       NA
Quadratic no Random Effects
                                              6522.31 6547.03
                                                                   0.1483
```

Figure 9: Table showing AIC, BIC, and LRT comparison for models with & without random effects for 'season' and 'player id'

5.2 Model Specification

To capture the unique performance trajectories of each position, twelve quadratic mixed-effects models were developed with three predictors for each of the four positions: age, years played, and cumulative snaps (scaled by dividing by 1,000 for interpretability). Separate models were built for each position to avoid the complexity of interaction terms and to allow for a more precise analysis of position-specific performance patterns. For RBs, WRs, and TEs, the response variable (PPR fantasy points per game) was log-transformed due to its right-skewed distribution and heteroscedasticity in residuals. For QBs, the response variable remained untransformed as the log transformation did not improve model diagnostics. The final quadratic mixed-effects models were:

For RBs, WRs, and TEs (log-transformed response):

$$log(PPR Fantasy Points_{ij}) = \beta_0 + \beta_1 X_{ij} + \beta_2 X_{ij}^2 + u_{0i} + \epsilon_{ij}$$

For QBs (untransformed response):

 $PPR \ Fantasy \ Points_{ij} = \beta_0 + \beta_1 X_{ij} + \beta_2 X_{ij}^2 + u_{0i} + \epsilon_{ij}$ Where β_0 , β_1 , β_2 are fixed effect coefficients, X_{ij} is the predictor variable (age, years played, or cumulative snaps) for player i in season j, $\mu_{0i} \sim N(0, \sigma_u^2)$ is the random intercept for player i, and $\epsilon_{ij} \sim N(0, \sigma_e^2)$ is the residual error term for player i in season j.

5.3 Analysis and Interpretation

Analysis was performed on each model to check significance and interpret results. To give an illustrative example, consider the R model output in Figure 10 for Wide Receivers with age as the predictor. The random-effects variances are , σ_u^2 =0.2236 for player-level variability and a residual variance of σ_e^2 =0.1620. The Intraclass Correlation Coefficient (ICC), calculated as $\frac{.2236}{.2236+.1620} \approx .579$, suggests that 57.9% of the total variability in the response variable is attributable to differences between players, which justifies the inclusion of random intercepts at the player level. Figure 11 below displays the intercepts and standard errors for the fixed effects in each model. Focusing again on the WR Age model, the fixed-effects estimates are as follows: Intercept (β_n) is -1.7207 with a standard error of 0.4647; Age (β_1) is 0.2731 with a standard error of 0.0342; and Age Squared (β_2) is -0.0052 with a standard error of 0.0006. The intercept serves as a baseline in the mode for age = 0, however age, years played, or cumulative snaps value of zero is not meaningful in this context, so the intercept is mainly an artifact of the model and does not have a practical interpretation on its own. The positive coefficient for Age indicates that performance increases with age, while the negative coefficient for Age Squared signifies that this improvement slows and eventually reverses, creating an inverted U-shaped relationship. This means WR performance increases up to a certain age before declining. The t-values (not shown) for all coefficients are significant, and the corresponding p-values are less than 0.001 (p<0.001), confirming that Age and Age Squared are significant predictors.

```
Random effects:
Groups Name Variance Std.Dev.
player_id (Intercept) 0.2236 0.4729
Residual 0.1620 0.4025
Number of obs: 3568, groups: player_id, 1001
```

Figure 10: R Output showing the Restricted Maximum Likelihood (REML) Summary for the Age WR Mixed-Effects Model

Position	Predictor	Term	Estimate	Std. Error	t value
Wide Receivers	Age	(Intercept)	-1.7207	0.4647	-3.703
Wide Receivers	Age	age	0.2731	0.0342	7.996
Wide Receivers	Age	I(age^2)	-0.0052	0.0006	-8.369
Running Backs	Age	(Intercept)	-1.9590	0.7968	-2.459
Running Backs	Age	age	0.3080	0.0599	5.140
Running Backs	Age	I(age^2)	-0.0062	0.0011	-5.556
Quarterbacks	Age	(Intercept)	-1.6297	5.0667	-0.322
Quarterbacks	Age	age	0.7813	0.3398	2.299
Quarterbacks	Age	I(age^2)	-0.0142	0.0056	-2.547
Tight Ends	Age	(Intercept)	-1.9024	0.5955	-3.195
Tight Ends	Age	age	0.2577	0.0432	5.965
Tight Ends		I(age^2)	-0.0047	0.0008	-6.077
Wide Receivers		(Intercept)	1.7241	0.0265	65.180
Wide Receivers	Years Played	years_played	0.0695	0.0088	7.908
Wide Receivers	Years Played	I(years_played^2)	-0.0069	0.0007	-9.932
	Cumulative Snaps	(Intercept)	1.6627	0.0312	53.249
	Cumulative Snaps	scaled_predictor	0.2711	0.0232	11.688
		I(scaled_predictor^2)	-0.0319	0.0033	-9.757
Running Backs		(Intercept)	1.7316	0.0370	46.848
Running Backs		years_played	0.0781	0.0140	5.589
Running Backs		I(years_played^2)	-0.0092	0.0012	-7.504
	Cumulative Snaps	(Intercept)	1.6573	0.0392	42.252
	Cumulative Snaps	scaled_predictor	0.5987	0.0542	11.048
		I(scaled_predictor^2)	-0.1151	0.0136	-8.439
Tight Ends		(Intercept)	1.4728	0.0323	45.591
Tight Ends		years_played	0.0551	0.0104	5.321
Tight Ends	,	I(years_played^2)	-0.0048	0.0008	-6.071
	Cumulative Snaps	(Intercept)	1.4712	0.0406	36.263
	Cumulative Snaps	scaled_predictor	0.1653	0.0302	5.470
		I(scaled_predictor^2)	-0.0149	0.0048	-3.085
Quarterbacks		(Intercept)	8.5285	0.4590	18.581
Quarterbacks		years_played	0.2380	0.1076	2.211
Quarterbacks	Years Played	I(years_played^2)	-0.0168	0.0061	-2.777
•	Cumulative Snaps	(Intercept)	8.8053	0.5859	15.030
	Cumulative Snaps	scaled_predictor	2.2618	0.3353	6.746
Quarterbacks	Cumulative Snaps	I(scaled_predictor^2)	-0.1763	0.0352	-5.003

Figure 11: Table displaying the Intercept and Standard Errors for the Predictors in Each Model

6. Results and Discussion

6.1 Age

To identify the peak performance point, we calculate the vertex of the quadratic equation. For example, using the coefficients shown in Figure 11: Peak Age = $-\frac{\beta_1}{2\beta_2} = \frac{-.2731}{2(-.0052008)} \approx 26.26$. Thus, Wide

Receivers reach their peak performance at approximately 26.26 years old, as confirmed by the bootstrap analysis in Figure 12, which provides a standard error of 0.34 for this peak estimate. To determine the drop-off points, we analyze the absolute differences in predicted performance between consecutive ages, years played, and snaps. The drop-off point is the first age after the peak where the absolute change exceeds the overall average absolute change. For WRs, this drop-off occurs at about 30.61 years old, indicating the start of productivity decline as shown in Figure 12. Running Backs (RBs), who face a high physical workload, display an earlier peak age of 24.8 years old, with a drop-off observed around 28.61 years old. This trend suggests that the physical demands of the RB position accelerate performance declines. In contrast, Quarterbacks (QBs) tend to peak later, around an age of 27.4, with significant declines starting only after 34.74 years old, highlighting their extended career longevity. Tight Ends (TEs) exhibit peak performance at an age of approximately 27.3, with declines starting around 31.45 years old. Figure 13 visualizes these age-based production curves, showcasing specific peak and decline trajectories. These age-related trends underscore the importance of understanding position-specific demands, as different positions have distinct windows of peak productivity.

Position	Predictor	Peak_Point	Standard_Error	Acceleration_Point	Drop_Off_Point
Wide Receivers	Age	26.26	0.35	21.02	30.61
Wide Receivers	Years Played	5.01	0.27	1.02	8.78
Wide Receivers	Cumulative Snaps	4253.31	237.92	26.34	6144.26
Running Backs	Age	24.80	0.51	21.02	28.61
Running Backs	Years Played	4.27	0.32	1.02	7.53
Running Backs	Cumulative Snaps	2599.80	195.33	7.13	3650.21
Tight Ends	Age	27.30	0.44	21.02	31.45
Tight Ends	Years Played	5.75	0.46	1.02	9.87
Tight Ends	Cumulative Snaps	5544.58	1556.11	16.13	7825.69
Quarterbacks	Age	27.42	16.66	21.02	34.74
Quarterbacks	Years Played	7.08	2.78	1.02	13.71
Quarterbacks	Cumulative Snaps	6413.59	703.30	29.02	9673.88

Figure 12: Peak, Acceleration, and Drop-off Points by Position and Predictor with Standard Errors based on Bootstrapped Peak Point values

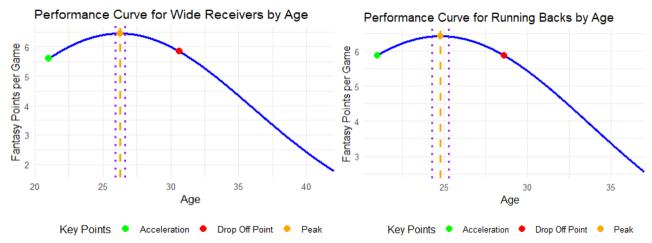


Figure 13: Plot of Age vs Expected Fantasy Points for WRs and RBs as predicted by the WR and RB Age Models, showcasing Production Peaks and Drop-off Points

6.2 Years Played

Analyzing years played as a predictor reveals position-specific career arcs. For RBs, productivity typically peaks within the first 4 to 5 years, with a sharp decline afterward, as indicated in Figure 12. WRs and TEs reach peak productivity slightly later, around 5 to 6 years into their careers. Bootstrap analysis supports these values, with WRs peaking at approximately 5.01 years (standard error: 0.27 years) and TEs at 5.75 years (standard error: 0.46 years). QBs generally reach their productivity peak around 7 to 8 years into their careers, with a more gradual decline emerging after approximately 13 years. This extended peak duration for QBs emphasizes the longevity typical for positions with less physical wear and suggests that QBs can sustain productive careers longer than other positions. These trends inform teams' long-term planning decisions, enabling strategic alignment of player contracts and development plans with expected career trajectories based on years played.

6.3 Cumulative Snaps

Cumulative snaps offer an alternative view of productivity by reflecting accumulated physical workload. For RBs, productivity decline correlates with a cumulative snap count of around 3,650 snaps, reflecting the toll of repeated contact over time. For WRs and TEs, declines begin around 6,100 and 7,800 snaps, respectively. QBs show the most resilience to workload, with declines observed only after accumulating approximately 9,600 snaps, suggesting that lower physical impact allows for sustained performance across a greater workload. Figure 12 details these cumulative snap thresholds and associated drop-off

points, providing NFL teams with insights into managing player workloads. For high-contact positions like RB, teams could use these snap count thresholds to monitor player usage closely, thereby prolonging player performance longevity.

6.4 Applications

Based on these findings, NFL teams can develop position-specific strategies to optimize talent development and resource allocation. For RBs, early investment is critical due to their earlier peak and shorter career span, while QBs, with longer peak productivity, may benefit from extended contract terms and development plans. By understanding these position-specific trajectories, teams can structure contracts to align with expected performance and avoid long-term commitments to players likely to experience performance declines. Players and agents can leverage these insights to time contract renewals effectively, ensuring financial stability prior to significant performance drop-offs.

7. Conclusion and Future Work

This study identified key productivity patterns across NFL offensive positions, demonstrating that RBs tend to reach peak performance earlier than other positions and experience declines sooner, while QBs enjoy longer peak periods. These findings highlight the value of tailored investment strategies for each position, with important implications for contract negotiations, roster planning, and player health management. By understanding these typical productivity trajectories, NFL teams can make informed decisions to maximize player value and career longevity. Future research could build upon these findings by integrating injury data, examining how cumulative injuries or specific types of injuries, such as concussions, ligament tears, and other common ailments, impact productivity decline. This would offer insights into the role of physical wear beyond general playtime. Expanding the analysis to include defensive positions could also provide a more comprehensive view of productivity patterns across the entire NFL roster, further helping teams optimize player management strategies for both offensive and defensive roles.

8. References

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