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Introduction

Housing instability is a topic that is widely discussed in attempts to understand and nullify the effects of it on those who experience it. It is often characterized as a number of challenges one might experience, but the aftereffects of it leave a person struggling to find a stable home for themselves. Most often, a few factors make it exceptionally difficult for someone when it comes to the transitions between being homeless and having a stable home. These factors can include but are not limited to the presence mental illness, knowing people in the local area, and race (HHS, 2021 & Eitzman et al., 2018). In this paper, we will focus on the first two factors mentioned, and their effects on someone's ability to go from being homeless to having a change in housing state. This is an especially important topic due to how prevalent homelessness can be and how negative the effects of it are (HHS, 2021).

Methods

For this paper we use a Markov chain model to depict transitions between a person experiencing homelessness, living in a shelter, or living in a home. A Markov Chain is defined as a finite set of states S, together with a collection of transition probabilities $p_{i,j}$ for each pair $i, j \in S$, where i, j are states in the chain. To represent the Markov chain mathematically, we use a transition matrix, where each entry depicts the probability of going from an ith state to a jth state. We will also use three separate models, a baseline model, a model that looks at the effect that a person has a serious mental illness, and a model that looks at the effect of someone being from the local area. The associated transition matrix of any two state Markov chain is:

$$\begin{bmatrix} X \\ Y \end{bmatrix} \begin{bmatrix} X & Y \\ P_{XX} & P_{XY} \\ P_{YX} & P_{YY} \end{bmatrix}$$

These matrices can then be extended for larger models. We can then use technology to simulate the long-term behavior of these matrices or calculate the stationary distribution. A stationary distribution for a Markov chain is a probability distribution over the states of the chain that is unchanged by the dynamics. It is a row vector that gives the probability of the various states of the Markov chain in the nth time step. It is found by solving the matrix equation:

$$\boldsymbol{\pi P} = \boldsymbol{\pi}$$

Where $\boldsymbol{P}$ is the transition matrix and $\boldsymbol{\pi}$ is a $1 \times n$ row vector $\boldsymbol{\pi} = (\pi_1, \dots, \pi_n)$ such that the entries of the vector are positive and sum to one. In addition, for each pair of states, $\lim_{n\to\infty} p_{ij}^n = \pi_j$, where π_j is the stationary distribution of state j. Lastly, for this paper, to simulate long-term dynamics, we will be using Microsoft Excel.

Our three models in this paper, as mentioned before, are a baseline model, a model that looks at the effect of mental illness on a person, and the effect of a person being from the local area. In general, the latter two models will cause changes to the transition matrix that will alter the simulated long-term dynamics of the model.

Baseline Model

Our baseline model begins with the Markov Chain described in Eitzman et al.'s paper. It is represented as such:

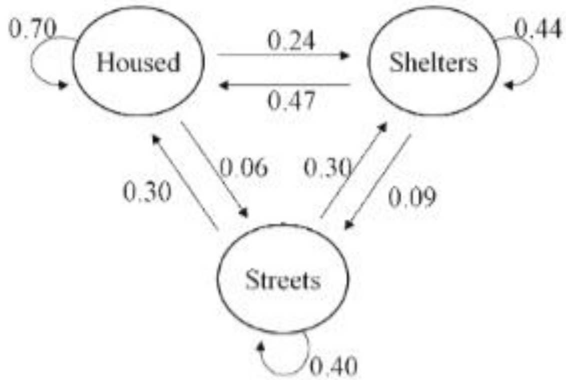


Fig 2: Base model of housing state transition probabilities from Eitzman, et al. (2018).

In this instance, the associated transition matrix would be:

$$\begin{array}{c c} & \begin{array}{ccc} Housed & Shelters & Streets \end{array} \\ \begin{bmatrix} Housed \\ Shelters \\ Streets \end{bmatrix} & \begin{bmatrix} 0.7 & 0.24 & 0.06 \\ 0.47 & 0.44 & 0.09 \\ 0.30 & 0.30 & 0.40 \end{bmatrix} \end{array}$$

This indicates that the probability of someone who is already on the streets to transition to being housed is 30%, based on the research that had been conducted. When simulating this, we will select a beginning state and simulate the effects of the model. However, there are other factors that can affect the housing status of someone, such as their mental health.

Effect of serious mental illness

This model is a modification of our baseline model. According to Etizman et al., significant mental illness (SMI) has a negative effect on the shelter-to-streets transition probability. Otherwise, there is no significant change in the other four transitions. We can use the formula $p' = \frac{1}{1+e^{-r}(p^{-1}-1)}$, where p' is the altered transition probability, p is the baseline transition probability ($p = 0.09$ here), and r is the non-exponentiated odds ratio ($r \approx -0.2$ here). We then calculate the new transition probability: $p' = \frac{1}{1+e^{0.2}(\frac{1}{0.09}-1)} \approx 0.075$. So the probability of someone with SMI going from being sheltered to being on the streets is reduced by approximately 2%. With the assumption that the Shelter-to-housed probability does not change, the remain-in-shelter transition probability would become 0.455. Making the new transition matrix:

$$\begin{bmatrix} Housed \\ Shelters \\ Streets \end{bmatrix} \begin{bmatrix} Housed & Shelters & Streets \\ 0.7 & 0.24 & 0.06 \\ 0.47 & 0.455 & 0.075 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

With the SMI transition matrix completed, we can now look at our last modeling framework.

Effect of being from the local area

The last thing mentioned in the paper is the effect of someone being from the local area. This tends to have a significant positive effect on remaining housed, a negative effect on housed-to-streets, and a negative effect on shelters-to-streets. We will complete a similar analysis as before, using the same formula. Giving us the following values:

Remaining Housed: $p' = \frac{1}{1+e^{-0.3}(0.7^{-1}-1)} = 0.76$

Housed-to-Streets: $p' = \frac{1}{1+e^{-(-0.5)}(0.06^{-1}-1)} = 0.04$

Shelters-to-Streets: $p' = \frac{1}{1+e^{-(-0.3)}(0.09^{-1}-1)} = 0.07$

This does result in the housed transition probabilities and the sheltered transition probabilities being changed. All rows must sum to one, so the new housed-to-sheltered probability is $1 - 0.76 - 0.04 = 0.2$. This, combined with the altered shelters-to-streets probability, makes the new transition matrix:

$$\begin{array}{c|c} & \begin{matrix} Housed & Shelters & Streets \end{matrix} \\ \begin{matrix} Housed \\ Shelters \\ Streets \end{matrix} & \begin{bmatrix} 0.76 & 0.2 & 0.04 \\ 0.48 & 0.45 & 0.07 \\ 0.30 & 0.30 & 0.40 \end{bmatrix} \end{array}$$

With our models complete, we can now calculate and use Microsoft Excel to simulate their long-term dynamics.

Results

Baseline:

Given the associated transition matrix described in *Methods: Baseline Model*

$$\begin{array}{c|c} & \begin{matrix} Housed & Shelters & Streets \end{matrix} \\ \begin{matrix} Housed \\ Shelters \\ Streets \end{matrix} & \begin{bmatrix} 0.70 & 0.24 & 0.06 \\ 0.47 & 0.44 & 0.09 \\ 0.30 & 0.30 & 0.40 \end{bmatrix} \end{array}$$

Using this associated transition matrix it is possible to calculate the stationary distribution of all states at any time. This is achieved by solving the matrix equation

$$\boldsymbol{\pi P} = \boldsymbol{\pi}$$

$$\begin{pmatrix} \pi_1 & \pi_2 & \pi_3 \end{pmatrix} \begin{pmatrix} 0.70 & 0.24 & 0.06 \\ 0.47 & 0.44 & 0.09 \\ 0.30 & 0.30 & 0.40 \end{pmatrix} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

Using matrix multiplication we arrive at the following:

$$\begin{pmatrix} 0.70\pi_1 + 0.47\pi_2 + 0.30\pi_3 & 0.24\pi_1 + 0.44\pi_2 + 0.30\pi_3 & 0.06\pi_1 + 0.09\pi_2 + 0.40\pi_3 \end{pmatrix} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

This can be re-written as a system of equations:

$$0.70\pi_1 + 0.47\pi_2 + 0.30\pi_3 = \pi_1$$

$$0.24\pi_1 + 0.44\pi_2 + 0.30\pi_3 = \pi_2$$

$$0.06\pi_1 + 0.09\pi_2 + 0.40\pi_3 = \pi_3$$

Moving all variables to LHS

$$-0.30\pi_1 + 0.47\pi_2 + 0.30\pi_3 = 0$$

$$0.24\pi_1 - 0.56\pi_2 + 0.30\pi_3 = 0$$

$$0.06\pi_1 + 0.09\pi_2 - 0.60\pi_3 = 0$$

Given the fact that transition probabilities sum to 1 we can deduce that

$$\pi_1 + \pi_2 + \pi_3 = 1$$

Thus we can find the vector form solution to the system of linear equations

$$A\boldsymbol{x} = \boldsymbol{b}$$

Where:

$$A = \begin{pmatrix} -0.30 & 0.47 & 0.30 \\ 0.24 & -0.56 & 0.30 \\ 0.06 & 0.09 & -0.60 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

$$\boldsymbol{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$A\boldsymbol{x} = \boldsymbol{b}$ yields

$$\begin{pmatrix} -0.30\pi_1 & 0.47\pi_2 & 0.30\pi_3 \\ 0.24\pi_1 & -0.56\pi_2 & 0.30\pi_3 \\ 0.06\pi_1 & 0.09\pi_2 & -0.60\pi_3 \\ \pi_1 & \pi_2 & \pi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Convert into Augmented Matrix

$$\begin{pmatrix} -0.30 & 0.47 & 0.30 & 0 \\ 0.24 & -0.56 & 0.30 & 0 \\ 0.06 & 0.09 & -0.60 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Calculate reduced row echelon form:

$$\begin{pmatrix} 1 & 0 & 0 & 0.58722 \\ 0 & 1 & 0 & 0.30787 \\ 0 & 0 & 1 & 0.10490 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore

$$\pi_1 = 0.58722$$

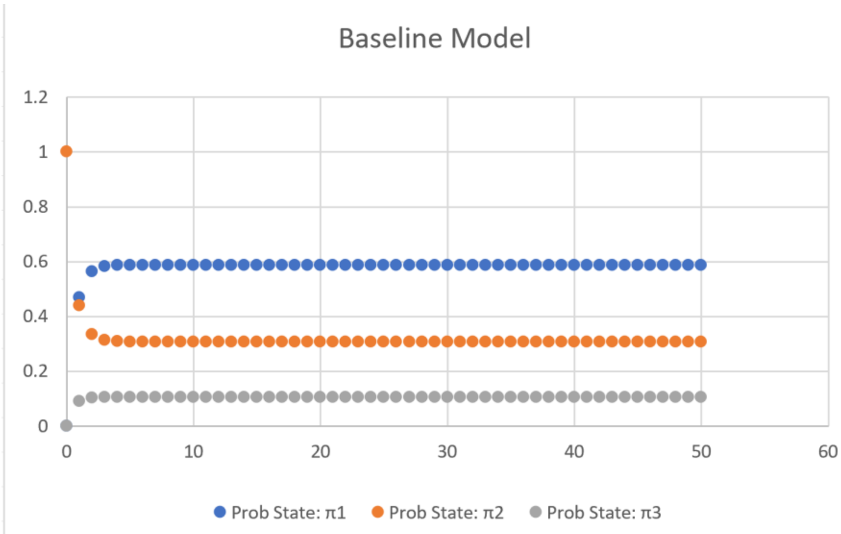
$$\pi_2 = 0.30787$$

$$\pi_3 = 0.10490$$

Thus

$$\boldsymbol{x} = \begin{pmatrix} 0.58722 \\ 0.30787 \\ 0.10490 \end{pmatrix}$$

Graphing the Baseline Model for 50 iterations yields:



Overtime it becomes clear that the stationary distributions approach their calculated values of

$$\pi_1 = 0\,.58722$$

$$\pi_2 = 0\,.30787$$

$$\pi_3 = 0.10490$$

The 100th iteration of the Baseline Model yields:

Time	Prob State: π1	Prob State: π2	Prob State: π3
100	0.58722919	0.30786773	0.10490308

Therefore $\pi_j = \begin{pmatrix} 0.58722 \\ 0.30787 \\ 0.10490 \end{pmatrix}$ is the stationary distribution of state j as $\lim_{n\to\infty} p_{ij}^n = \pi_j$ the baseline model.

Effect of serious mental illness:

Given the associated transition matrix described in *Methods: Effect of serious mental illness*

$$\begin{bmatrix} Housed \\ Shelters \\ Streets \end{bmatrix} \begin{bmatrix} Housed & Shelters & Streets \\ 0.7 & 0.24 & 0.06 \\ 0.47 & 0.455 & 0.075 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

Using this associated transition matrix it is possible to calculate the marginal distribution of all states at any time. This is achieved by solving the matrix equation

$$\boldsymbol{\pi P} = \boldsymbol{\pi}$$

$$(\pi_1 \quad \pi_2 \quad \pi_3) \begin{pmatrix} 0.70 & 0.24 & 0.06 \\ 0.47 & 0.44 & 0.075 \\ 0.30 & 0.30 & 0.40 \end{pmatrix} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

Using matrix multiplication

$$(0.70\pi_1 + 0.47\pi_2 + 0.30\pi_3 \quad 0.24\pi_1 + 0.455\pi_2 + 0.30\pi_3 \quad 0.06\pi_1 + 0.075\pi_2 + 0.40\pi_3) = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

This can be re-written as a system of equations:

$$0.70\pi_1 + 0.47\pi_2 + 0.30\pi_3 = \pi_1$$

$$0.24\pi_1 + 0.455\pi_2 + 0.30\pi_3 = \pi_2$$

$$0.06\pi_1 + 0.075\pi_2 + 0.40\pi_3 = \pi_3$$

Moving all variables to LHS

$$-0.30\pi_1 + 0.47\pi_2 + 0.30\pi_3 = 0$$

$$0.24\pi_1 - 0.545\pi_2 + 0.30\pi_3 = 0$$

$$0.06\pi_1 + 0.075\pi_2 - 0.60\pi_3 = 0$$

Given the fact that transition probabilities sum to 1 we can deduce that

$$\pi_1 + \pi_2 + \pi_3 = 1$$

Thus we can find the vector form solution to the system of linear equations

$$A\boldsymbol{x} = \boldsymbol{b}$$

Where:

$$A = \begin{pmatrix} -0.30 & 0.47 & 0.30 \\ 0.24 & -0.545 & 0.30 \\ 0.06 & 0.075 & -0.60 \end{pmatrix}$$
$$1 \quad 1 \quad 1$$

$$\boldsymbol{x} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

$$\boldsymbol{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$A\boldsymbol{x} = \boldsymbol{b}$ yields

$$\begin{pmatrix} -0.30\pi_1 & 0.47\pi_2 & 0.30\pi_3 \\ 0.24\pi_1 & -0.545\pi_2 & 0.30\pi_3 \\ 0.06\pi_1 & 0.075\pi_2 & -0.60\pi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$
$$\pi_1 \quad \pi_2 \quad \pi_3$$

Convert into Augmented Matrix

$$\begin{pmatrix} -0.30 & 0.47 & 0.30 & 0 \\ 0.24 & -0.545 & 0.30 & 0 \\ 0.06 & 0.075 & -0.60 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Calculate reduced row echelon form:

$$\begin{pmatrix} 1 & 0 & 0 & 0.58875 \\ 0 & 1 & 0 & 0.31323 \\ 0 & 0 & 1 & 0.09803 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore

$$\pi_1 = 0.58875$$

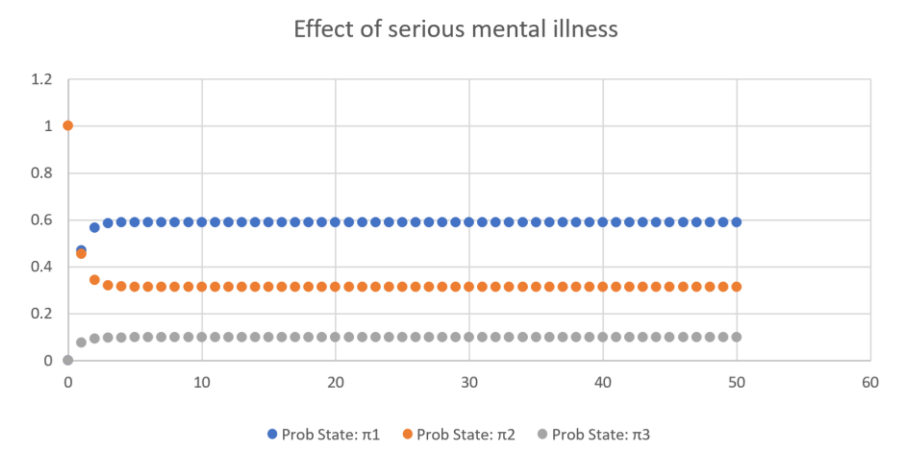
$$\pi_2 = 0.31323$$

$$\pi_3 = 0.09803$$

Thus

$$\boldsymbol{x} = \begin{pmatrix} 0.58876 \\ 0.31323 \\ 0.09803 \end{pmatrix}$$

Graphing the Effect of serious mental illness model for 50 iterations yields:



Overtime it becomes clear that the stationary distributions approach their calculated values of

$$\pi_1 = 0.58875$$

$$\pi_2 = 0.31323$$

$$\pi_3 = 0.09803$$

The 100th iteration of the effect of serious mental illness model yields:

Time	Prob State: π1	Prob State: π2	Prob State: π3
100	0.5887471	0.31322506	0.09802784

Therefore $\pi_j = \begin{pmatrix} 0.58876 \\ 0.31323 \\ 0.09803 \end{pmatrix}$ is the stationary distribution of state j as $\lim_{n\to\infty} p_{ij}^n = \pi_j$ for the model on the effect of serious mental illness.

Effect of being from the local area:

Given the associated transition matrix described in *Methods: Effect of being from the local area*

$$\begin{bmatrix} Housed \\ Shelters \\ Streets \end{bmatrix} \begin{bmatrix} Housed & Shelters & Streets \\ 0.76 & 0.20 & 0.04 \\ 0.48 & 0.45 & 0.07 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

Using this associated transition matrix it is possible to calculate the marginal distribution of all states at any time. This is achieved by solving the matrix equation

$$\boldsymbol{\pi P} = \boldsymbol{\pi}$$

$$(\pi_1 \quad \pi_2 \quad \pi_3)\begin{pmatrix} 0.76 & 0.20 & 0.04 \\ 0.48 & 0.45 & 0.07 \\ 0.30 & 0.30 & 0.40 \end{pmatrix} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

Using matrix multiplication

$$(0.76\pi_1 + 0.48\pi_2 + 0.30\pi_3 \quad 0.20\pi_1 + 0.45\pi_2 + 0.30\pi_3 \quad 0.04\pi_1 + 0.07\pi_2 + 0.40\pi_3) = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

This can be re-written as a system of equations:

$$0.76\pi_1 + 0.48\pi_2 + 0.30\pi_3 = \pi_1$$

$$0.20\pi_1 + 0.45\pi_2 + 0.30\pi_3 = \pi_2$$

$$0.04\pi_1 + 0.07\pi_2 + 0.40\pi_3 = \pi_3$$

Moving all variables to LHS

$$-0.24\pi_1 + 0.48\pi_2 + 0.30\pi_3 = 0$$

$$0.20\pi_1 - 0.55\pi_2 + 0.30\pi_3 = 0$$

$$0.04\pi_1 + 0.07\pi_2 - 0.60\pi_3 = 0$$

Given the fact that transition probabilities sum to 1 we can deduce that

$$\pi_1 + \pi_2 + \pi_3 = 1$$

Thus we can find the vector form solution to the system of linear equations

$$A\boldsymbol{x} = \boldsymbol{b}$$

Where:

$$A = \begin{pmatrix} -0.24 & 0.48 & 0.30 \\ 0.20 & -0.55 & 0.30 \\ 0.04 & 0.07 & -0.60 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\boldsymbol{x} = \begin{pmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \end{pmatrix}$$

$$\boldsymbol{b} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$A\boldsymbol{x} = \boldsymbol{b} \text{ yields}$$

$$\begin{pmatrix} -0.24\pi_1 & 0.48\pi_2 & 0.30\pi_3 \\ 0.20\pi_1 & -0.55\pi_2 & 0.30\pi_3 \\ 0.04\pi_1 & 0.07\pi_2 & -0.60\pi_3 \\ \pi_1 & \pi_2 & \pi_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

Convert into Augmented Matrix

$$\begin{pmatrix} -0.24 & 0.48 & 0.30 & 0 \\ 0.20 & -0.55 & 0.30 & 0 \\ 0.04 & 0.07 & -0.60 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Calculate reduced row echelon form:

$$\begin{pmatrix} 1 & 0 & 0 & 0.64780 \\ 0 & 1 & 0 & 0.27680 \\ 0 & 0 & 1 & 0.07547 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Therefore

$$\pi_1 = 0.64780$$

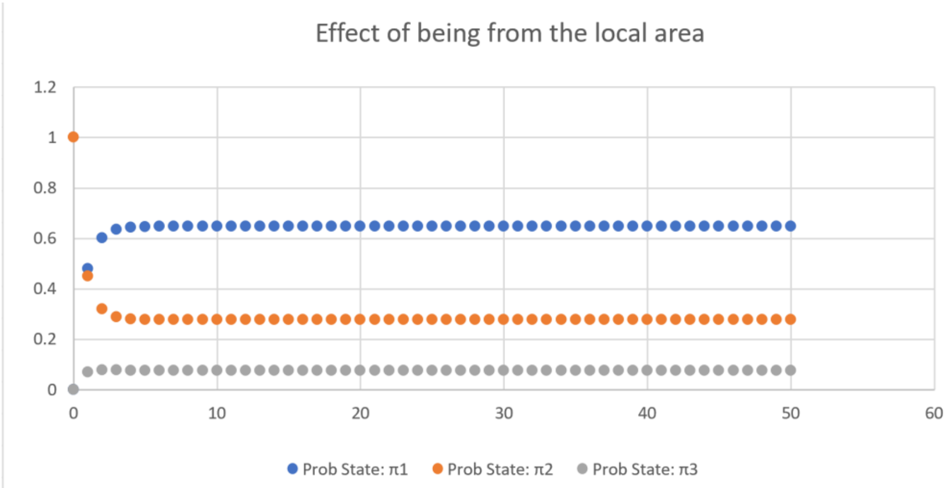
$$\pi_2 = 0.27680$$

$$\pi_3 = 0.07547$$

Thus

$$\boldsymbol{x} = \begin{pmatrix} 0.64780 \\ 0.27680 \\ 0.07547 \end{pmatrix}$$

Graphing the Effect of being from the local area model for 50 iterations yields:



Overtime it becomes clear that the stationary distributions approach their calculated values of

$$\pi_1 = 0.64780$$

$$\pi_2 = 0.27680$$

$$\pi_3 = 0.07547$$

The 100th iteration of the effect of being from local area model yields:

Time	Prob State: π1	Prob State: π2	Prob State: π3
100	0.64779874	0.27672956	0.0754717

Therefore $\pi_j = \begin{pmatrix} 0.64780 \\ 0.27680 \\ 0.07547 \end{pmatrix}$ is the stationary distribution of state j as $\lim_{n\to\infty} p_{ij}^n = \pi_j$ for the model on Effect of being from the local area.

Discussion

The stationary distribution tells us as we progress through time, the probability of being in certain states is more likely to occur than others. The stationary distribution will reach an equilibrium with an associated probability of being in each state. Thus if you apply the transition matrix to the stationary distribution, the resulting distribution is the same as the stationary distribution.

$$\boldsymbol{\pi P} = \boldsymbol{\pi}$$

Where $\boldsymbol{\pi}$ is the stationary distribution and $\boldsymbol{P}$ is the transition matrix.

Baseline Model:

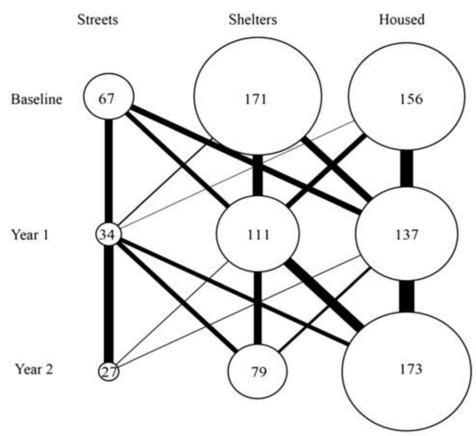
The baseline model has an associated transition matrix of

$$\begin{array}{c} \\ Housed \\ Shelters \\ Streets \end{array} \begin{array}{ccc} Housed & Shelters & Streets \\ \hline 0.70 & 0.24 & 0.06 \\ 0.47 & 0.44 & 0.09 \\ 0.30 & 0.30 & 0.40 \end{array}$$

With a stationary distribution of $\begin{pmatrix} 0.58722 \\ 0.30787 \\ 0.10490 \end{pmatrix}$

Therefore the associated probability of being housed over time is .58722, the associated probability of being sheltered over time is .30787 and the associated probability of being in the streets in .10490. Therefore given a population *n*, over time 58% of the population will be housed, 31% will be sheltered and 11% will be on the streets. We can compare the results of the stationary distribution of the associated transition matrix to the survey results in Figure 1 of Transitions between Housing States among Urban Homeless Adults: a Bayesian Markov Model.

Fig. 1 Number of participants that report their usual living situation as streets, shelters, or housed at each time point and transitions between housing states



By taking the weighted average of each category per year we can compare the results of Fig1 to our stationary distribution.

The weighted average of Fig 1

	Street	Shelters	Housed
Baseline	17%	43%	40%
y1	12%	39%	49%
y2	10%	28%	62%

At each time interval this is the portion of the population in each state. We can compare this to our results for stationary distribution.

	Street	Shelters	Housed
Baseline	17%	43%	40%
y1	12%	39%	49%
y2	10%	28%	62%
Stationary Distrobution	11%	31%	58%

As we can see from the baseline to y2 the probability of being in a certain state is approaching our stationary distribution model. If we take the baseline probabilities of Fig1 and use it as initial probabilities for our baseline model, we can compare survey results to our baseline model results for y1 and y2

Time	Prob. Housed	Prob. Sheltered	Prob. Street
0	0.17	0.43	0.4
1	0.4411	0.35	0.2089
2	0.53594	0.322534	0.141526

The model does not show a strong connection between the surveyed results. However, we are limited by the amount of data collected in the survey since we are only able to compute two iterations. Further survey results may conclude our baseline model is a good predictor of the weighted average per category. By year 11 our model predicts that the initial probabilities for weighted average will reach stationary distribution.

Effect of serious mental illness Model:

The baseline model has an associated transition matrix of

$$\begin{bmatrix} Housed \\ Shelters \\ Streets \end{bmatrix} \begin{bmatrix} Housed & Shelters & Streets \\ 0.7 & 0.24 & 0.06 \\ 0.47 & 0.455 & 0.075 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

With a stationary distribution of $\begin{pmatrix} 0.58876 \\ 0.31323 \\ 0.09803 \end{pmatrix}$

Therefore the associated probability of being housed over time is .58876, the associated probability of being sheltered over time is .31323 and the associated probability of being in the streets in .09803. Therefore given a population *n*, over time 59% of the population will be housed, 31% will be sheltered and 10% will be on the streets.

Compared to our baseline model of 58% of the population being housed, 31% being sheltered and 11% being on the streets. We can observe the given: that the effect of mental illness has a negative effect on the shelter-to-street transition. In our baseline model the stationary distribution probability of being on the streets is 11%, however accounting for serious mental illness will decrease the probability of being on the streets to 10%. This is a result of changing the transition matrix. In the baseline model the probability of going from shelter to street is 9%, but in the effect of serious mental illness model the probability of going from shelter to street is 7.5%. Furthermore in the effect of serious mental illness model the transition probability of staying in a shelter increases from 44% to 45.5%. Therefore introducing a negative effect on the shelter to street transition decreases the probability that an individual from a shelter will go to the streets. Which is the observed difference between our Baseline model and effect of serious mental illness model. This can be extended to the real world because a study of newly homeless people in NYC found that 35% had major depression (HHS, 2021). By identifying strategies to help reduce housing stability an emphasis should be put towards helping those with mental illness stay in shelters and off the streets. By reducing the negative effects of housing instability on health outcomes, those at risk may be able to achieve a healthier life.

Effect of being from the local area:

The baseline model has an associated transition matrix of

$$\begin{bmatrix} Housed \\ Shelters \\ Streets \end{bmatrix}\begin{bmatrix} Housed & Shelters & Streets \\ 0.76 & 0.2 & 0.04 \\ 0.48 & 0.45 & 0.07 \\ 0.30 & 0.30 & 0.40 \end{bmatrix}$$

With a stationary distribution of $\begin{pmatrix} 0.64780 \\ 0.27680 \\ 0.07547 \end{pmatrix}$

Therefore the associated probability of being housed over time is .64780, the associated probability of being sheltered over time is .27680 and the associated probability of being in the streets in .07547. Therefore given a population *n*, over time 65% of the population will be housed, 28% will be sheltered and 7% will be on the streets.

Compared to our baseline model of 58% of the population being housed, 31% being sheltered and 11% being on the streets. This is a result of changing our transition matrix. Being from the local area increases the transition probability of being housed from 70% to 76%. Being from the local area decreases the transition probability from housed to streets from 6% to 4%. Finally being from the local area decreases the transition probability from shelter to streets from 9% to 7%. Therefore the stationary distribution for being housed increases from 58% to 65%. The stationary distribution for being sheltered decreases from 31% to 28%. Finally the stationary distribution for being on the streets decreases from 11% to 7%. In *Housing instability* by the U.S Department of Health and Human Services found that neighborhood characteristics can strongly influence health. Which in turn can strongly influence homelessness. The Moving to Opportunity (MTO) is a program that provided low-income families with assistance to move from high-poverty to low-poverty neighborhoods (HHS, 2021). Research found that people that moved from high-poverty to low-poverty before the age of 13 we more likely to attend college, had higher income, less likely to be single parents, and lived in better neighborhoods as adults (HHS, 2021). Therefore providing families with the ability to move from high-poverty to low-poverty increases the ability of that family to stay away from shelters and streets. However instead of focusing on moving families from one area to another, work should be done to decrease the poverty in neighborhoods to create a better environment for children and adults to succeed.

References

Alexander-Eitzman, B., North, C. S., & Pollio, D. E. (2018). Transitions between housing states among urban homeless adults: A bayesian markov model. *Journal of Urban Health*, *95*(3), 423–430. https://doi.org/10.1007/s11524-018-0236-8

U.S. Department of Health and Human Services. (2021). *Housing instability*. Housing Instability - Healthy People 2030. Retrieved April 9, 2023, from https://health.gov/healthypeople/priority-areas/social-determinants-health/literature-summaries/housing-instability