Introduction:

From 2008-2009, the world suffered the worst financial and economic crisis since the Great Depression. Hundreds of banks failed, millions of homes went into foreclosure, and Americans lost over $14 trillion. The social impact of the 2008 financial crisis resulted in increased unemployment, loss of income, and increased vulnerability(United Nations). Understanding the three main causes of the housing crisis can help us predict the possible volatility of the current U.S housing market and economy. The 2008 bubble and subsequent financial meltdown can be attributed to having three main causes: relaxed mortgage lending standards, low interest rate policies, and increased leveraging by investment banks.

A housing bubble is a rapid increase in the valuation of property such as homes and land. This valuation increases to levels that are incapable of being maintained over a long period of time relative to economic conditions (Aziz, 2012). Normally housing increased slightly faster than inflation which is what happened from 1975 to 1995 (Aziz, 2012). In these 20 years the housing market went through two cyclical waves. The trough of the last wave was in 1990 and from 1990 to 1997 housing increased 8.3% (Median sales price of houses sold for the United States, 2021). However, from this point on housing prices began to surge. In 2006 home prices were 132% higher than in 1997 (Holt, 2009). From 2000-2005, home prices rose 17% which was more than double any 5-year rate in the past 30 years (Aziz, 2012). The price to rent ratio in 2006 was 14.5% (Aziz, 2012). Usually, house prices are 9-11% the annual level of rent paid. All these factors pointed towards the existence of a housing bubble, however few at this time saw it.

At the same time the U.S government was lowering the short-term interest rate. To recover from the 2001 recession, the federal funds rate was lowered 11 times from 6.50% to 1.75% to create expansionary monetary policy (Holt, 2009). As no significant inflation appeared, the federal funds rate continued to be lowered to around 1% in 2003 (Holt, 2009). A lower federal fund rates heavily influences the prime rate which is the interest rate banks charge to customers.

As short-term interest rates were lowered this encouraged home buyers to use adjustable-rate mortgages (ARMS). An ARMS is a loan based on the federal funds rate that will use an advertised rate for a certain period and then change that rate at certain intervals. This means that the interest rate on your mortgage can rise at each interval depending on if the interest rate rises. In the 2000's as home prices began to rise faster than income, many buyers opted to use ARMS because they could not afford the fixed rate. As ARMS made monthly mortgage payments more affordable to more buyers it contributed to the rising prices of homes. Then as the interest rates rose, the monthly mortgages became more expensive, which homebuyers could no longer afford. Most of the ARMS were subprime, meaning that the loans were given to people with poor credit risk. The subprime loan market grew from 190 billion in 2001 to 625 billion in 2005 (Aziz, 2012). Thus, contributing to the number of foreclosures when interest rates rose.

Methods:

To model the housing bubble, this paper aims to use difference equations, decreasing interest rates and increase in the expected value of the housing market. I will define the variables in this section:

E_t: Expected value

$\Delta E_t = xE_t$ where x is a constant pertaining to expected value growth or decay.

I_t: Short term interest Rate

$I_t = yI_t$ where y is a constant pertaining to interest rate growth or decay.

V_t: Real valuation of housing market

$\Delta V_t = V_t E_t I_t$

ΔM_t: Market value of housing market.

$\Delta M_t = V_t E_t$

A difference equation is a mathematical equality involving the differences between successive values of a function of a discrete variable. Difference equations have the form

$$\Delta a_n = f(a_n)$$

Where,

$$\Delta a_n = a_{n+1} - a_n$$

We can then use this to find a recurrence equation, which will be of the form:

$$a_{n+1} = g(a_n)$$

To find stability of fixed point find the derivative of the recurrence equation.

$$a_{n+1} = g(a_n)$$

A fixed point is labeled as a_n^* and is unstable if:

$$|g'(a_n^*)| > 1$$

And stable if:

$$|g'(a_n^*)| < 1$$

Now to quantify the value of the housing bubble, we will take the integral of V_t and subtract it from the integral of M_t. To take the integral of V_t and M_t we will construct a polynomial of best fit curve using MATLAB. We will end up with a quantifiable unit of the Bubble, because we are integrating a rate of change. The units of the independent variable will cancel out and we will end up with units of the original function.

In this section I will define initial conditions:

I_t: Interest Rate

We know that the short-term interest rate was lowered to recover from the 2001 recession. We will choose $t = 0$ to be July 3rd, 2000, due to the constraint of the data set. The short-term interest rate at this time was 7.03%. We will calculate the average change in short-term interest rates from January 1st, 2000, to June 21st, 2004. The interest rate June 21st, 2004 was 1%. This average change will give y where y is a constant pertaining to interest rate growth or decay.

To calculate the average rate of change of short-term interest rates I first began by downloading the short-term interest rate from FRED for the specified dates. I then took the difference between each change in interest rate, summed them together, and divided them by the number of changes. Sample Code:

```
% Read the data
%%%%%%%%%%%%%%%
data = csvread('short.term.csv',2);
%%%%%%%%%%%%%%%
 %Formatting%
t = data(:,1) ;
x = data(:,2) ;
t = transpose(t);
x = transpose(x);
x = flip(x,2);
plot(t,x);
y = [];
%%%%%%%%%%%%%%%%%%%%%%%
%Calcuting difference%
for i = 2:length(x)
y(i) = ((x(i)-x(i-1)));
end

%%%%%%%%%%%%%%%%%%%%%%%
%Calculating Average%
b= length(y);
avg = sum(y)/b;
```

$I_0 = .07$

$y = -.0060$

E_t: Expected value

We will choose the expected value time frame t to be from July 1st, 2000, to June 31st, 2004. To calculate the expected value we will use the Consumer Price Index (CPI) to measure the expected value. The CPI is the average change over time in the prices paid by urban consumers for market basket of consumer goods and services (U.S. Bureau of Labor Statistics). To calculate the initial value and the average change of the expected value we will download a data set from the U.S Bureau of Labor Statistics.

We will calculate the average change in CPI from July 3rd, 2000, to June 21st, 2004. This will the initial condition for E_t, and give x where x is a constant pertaining to expected growth or decay.

To do this I used the same code posted in the calculation of the Interest Rate.

$E_0 = .10$

$x = .3292$

V_t: Valuation of housing market

Our initial condition for $t = 0$ for the honest housing market valuation will be the median price of a home July 3rd, 2000.

Now we can lay out the initial conditions

$V_0 = 172{,}900$ USD

$I_0 = .07\%$

$E_0 = .10$

$x = .3292$

$y = .0060$

Here is provided sample code to calculate all values:

```
function [V, M, t, I, E] = Bubble( iI, iE, iV, ix, iy)
t= [0];
% x = [iV];
E = [iE];  % Expected Value
I = [iI];  % Interest Rate
V = [iV];  % Valuation at t =  0
M = [iV];    % Market Value
x = (-ix); %how mcuh interest rate decreases
y = (iy);  %how much expected value increases
i = 1; |

      for i = 2:5
% %                           i = i + 1;
t(i) = t(i-1) + 1;
E(i) = (E(i-1)*y) + E(i-1);  %Choose the expected value to incresae by x%
I(i) =  (I(i-1) * x) + I(i-1) ;   %Choose Interest rate to decrease by y%
M(i) = (V(i-1)*E(i-1))+M(i-1);
V(i) = V(i-1)+  ( ((V(i-1))) * (E(i-1) * I(i-1)));


      end


end
```

Results:

Utilizing the described above methods, we can now implement the original values given. Using the formula for Expected value we have:

$\Delta E_t = xE_t$

We have values $x = .3292$ and $E_0 = .10$

$\Delta E_t = (.3292\ )E_t$

Which simplifies to

$E_{t+1} = ((.3292)E_t) + E_t$

And the associated recurrence equation is
$E_{t+1} = 1.3292E_t, \quad E_0 = .10$

To find the fixed point of this system, we will set the difference equation to 0 and solve for E_t. This gives us:

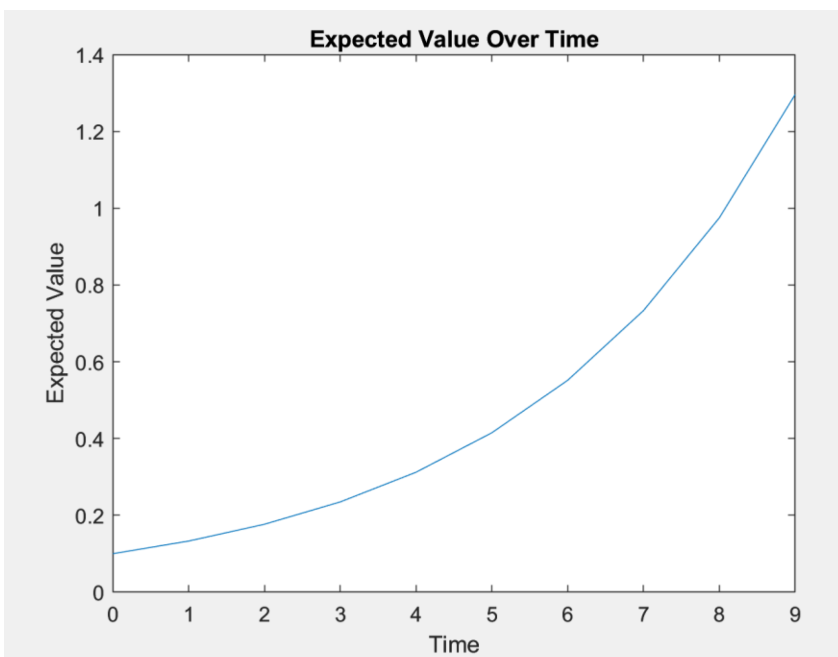
$0 = (.3291)E_t$

$E_n^* = 0$

To determine the stability of this 1D system we will look at the derivative of the recurrence equation evaluated at our fixed point. The derivative of our recurrence equation is:

$$E'_{n+1} = 1.3292$$

The derivative is a constant, so we have that the fixed point will be unstable, since it is greater than 1. Below is a graph of the values of our recurrence equation, and as can be seen, this system will grow without bound.



Utilizing the described above methods, we can now implement the original values given. Using the formula for short term Interest rate:

$I_t = yI_t$

We have values $y = -.0060$ and $I_0 = .07$

$\varDelta I_t = (-.0060)I_t$

Which simplifies to

$I_{t+1} = ((-.0060)I_t) + I_t$

And the associated recurrence equation is
$I_{t+1} = .994I_t, \quad I_0 =..07$

To find the fixed point of this system, we will set the difference equation to 0 and solve for I_t. This gives us:

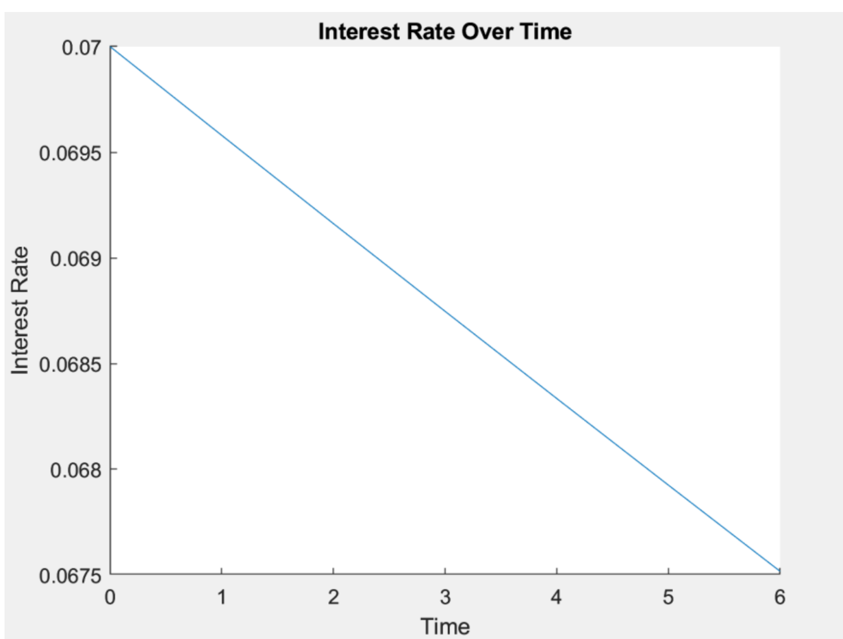
$0 = (-.006)I_t$

$I_n^* = \ 0$

To determine the stability of this 1D system we will look at the derivative of the recurrence equation evaluated at our fixed point. The derivative of our recurrence equation is:

$$I'_{n+1} = .994$$

The derivative is a constant, so we have that the fixed point will be stable, since it is less than 1. Below is a graph of the values of our recurrence equation, and as can be seen, this system will grow with a bound.



Utilizing the described above methods, we can now implement the original values given. Using the formula for real valuation of the market:

$\Delta V_t = V_t E_t I_t$

We have values $V_0 = 172{,}900, I_0 = .007, E_0 = .10$

$\Delta V_t = V_t(.0007)$

Which simplifies to

$V_{t+1} = 1.0007V_t$

And the associated recurrence equation is
$V_{t+1} = 1.0007V_t, \quad V_0 = 172{,}900$

To find the fixed point of this system, we will set the difference equation to 0 and solve for V_t. This gives us:

$0 = V_t E_t I_t$

$V_n^* = 0$

To determine the stability of this 1D system we will look at the derivative of the recurrence equation evaluated at our fixed point. The derivative of our recurrence equation is:

$$V'_{n+1} = 1.0007$$

The derivative is a constant, so we have that the fixed point will be unstable, since it is greater than 1. Below is a graph of the values of our recurrence equation, and as can be seen, this system will grow with a bound.

Utilizing the described above methods, we can now implement the original values given. Using the formula for the market valuation:

$\Delta M_t = V_t E_t$

We have values $M_0 = 172{,}900, V_0 = 172{,}900, E_0 = .10$

$\Delta M_t = M_t(17{,}290)$

Which simplifies to

$M_{t+1} = (17{,}290) + M_t$

And the associated recurrence equation is
$M_{t+1} = (17{,}291)M_t, \quad M_0 = 172{,}900$

To find the fixed point of this system, we will set the difference equation to 0 and solve for V_t. This gives us:

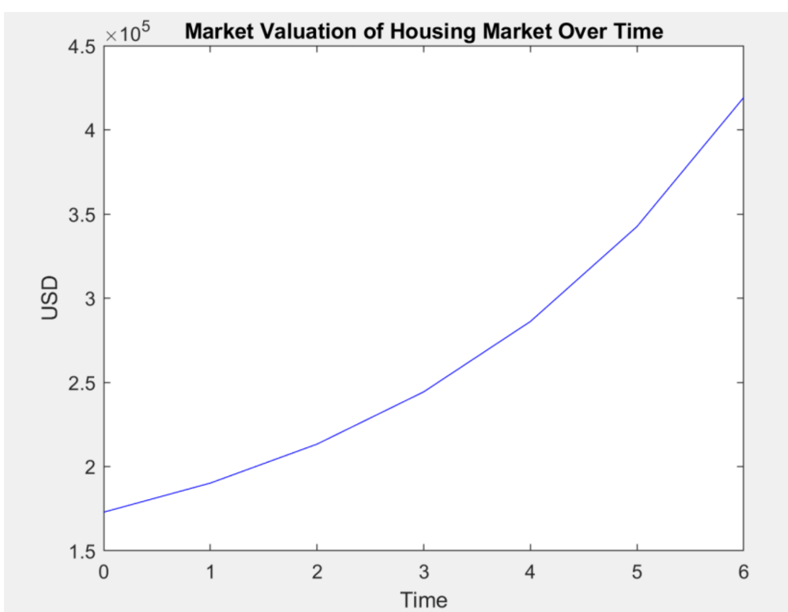
$0 = M_t V_t E_t$

$M_t^* = 0$

To determine the stability of this 1D system we will look at the derivative of the recurrence equation evaluated at our fixed point. The derivative of our recurrence equation is:

$$M'_{n+1} = 17{,}291$$

The derivative is a constant, so we have that the fixed point will be unstable, since it is greater than 1. Below is a graph of the values of our recurrence equation, and as can be seen, this system will grow with a bound.



Now we can plot M_t and V_t on the same graph to visualize the bubble of the housing market. Since the data set is limited, we will be looking at the bubble from 2000-2004.

Now we can construct a polynomial of best fit curve for Market Valuation and Actual Valuation using MATLAB.

Using a cubic approximation for Market Value:

$$M_t = 406.2(t^3) + 1626(t^2) + (1.531 * 10^4) + (1.729 * 10^5)$$

And for Actual Valuation we will use a linear approximation.

$$V_t = 1944t + (1.724 * 10^5)$$

We can approximate the value of the Housing bubble by subtracting the integral of V_t from the integral of M_t

$$\int_0^4 M_t - \int_0^4 V_t$$

$$\int_0^4 406.2(t^3) + 1626(t^2) + (1.531 * 10^4) + (1.729 * 10^5) - \int_0^4 1944t + (1.724 * 10^5)$$

$$874{,}765 - 705{,}152 = 169{,}613$$

Thus the Value of the Housing Bubble from 2000-2004 was 169,613 USD.

Analysis:

The Values my model calculated for the Market Valuation were

2000	2001	2002	2003	2004
172900	190190	213000	244000	286000

The Median sale price of a home from 2000-2004 was

2000	2001	2002	2003	2004
205375	211050	226700	244550	272125

Therefore we can calculate the relative error of my model:

2000	2001	2002	2003	2004
-15.8125	-9.88391	-6.04323	-0.2249	5.09876

For a good measurement system accuracy should be 5% so my model is not perfect.

Discussion:

From 2000-2005, home prices rose 17% which was more than double any 5-year rate in the past 30 years. The 2008 bubble and subsequent financial meltdown can be attributed to having three main causes: relaxed mortgage lending standards, low interest rate policies, and increased leveraging by investment banks. The aim of this paper was to model the growth of the housing market bubble from 2000-2004, due to a limited data set. Specifically, the aim of the paper was to understand how lower interest rates and an increase in the expected value of the market measured by the CPI may have led to the rise of this housing bubble. In the model, the market valuation of the housing market grew by an average of 13%, which is around 4% lower than the true percent increase. My model is also only used to fit 2000-2004 specifically due to the limit of the data set. If we extend the model to encompass 2000-20009, the results are insignificant. Model the Housing market bubble is important because it will allow the general population to understand how and why these housing bubbles occur. After all, the social impact of the 2008 financial crisis resulted in increased unemployment, loss of income, and increased vulnerability (United Nations). All of which may disproportionately affect subgroups of the population.

References

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