Introduction

The notion of how to model equality in terms of net worth is a difficult concept to quantify. In recent years, economic equality has become a very prominent issue in policy debate around the world (Bourguignon, 2018). The most common form of policy that attempts to work towards lowering economic inequality is taxation, however, while common, taxation is not exceptionally beneficial when compared to GDP (Bourguignon, 2018). And as such, other options need to be taken to reduce the existing inequality. One idea that has been introduced recently has been income redistribution. If done properly, it allows for the reduction of social tensions that inequality causes and gives poor individuals the ability to devote more of their resources to capital accumulation (Bourguignon, 2018). In this paper we discuss and calculate the effects of two different policies on the net worth of wealthy and poor individuals.

Methods

In this paper we will consider two scenarios when it comes to inequality. The first is where wealth transfer between the wealthy and poor does occur, and the second where wealth transfer between the wealthy and poor does not occur. The dynamics of these are calculated slightly differently and we detail the differences below.

In both cases, we are given a difference equation and a recurrence equation will come from that. From these two, we can calculate the dynamics of our system and determine how the conditions given effect the overall outcome of the system. In most cases, we will look at the fixed points, which are a current state which the system cannot change from. To calculate fixed points, we set our difference equation equal to 0 and solve for the associated value. From there, we look at the stability of a fixed point. A fixed point can be either stable or unstable. A stable fixed point is a point that the system will converge to as time increases while an unstable fixed point is a point that the system will diverge from as time increases.

One dimensional system (Wealth transfer does not occur)

We first consider the scenario in which wealth transfer does not occur. In general, we are given a difference equation of the form:

$$\Delta a_n = f(a_n)$$

We can then use this to find a recurrence equation, which will be of the form:

$$a_{n+1} = g(a_n)$$

To find stability of fixed point find the derivative of the recurrence equation.

$$a_{n+1} = g(a_n)$$

A fixed point is labeled as a_n^* and is unstable if:

$$|g'(a_n^*)| > 1$$

And stable if:

$$|g'(a_n^*)| < 1$$

In our model, we are given the difference equations:

1. $\Delta w_n = kw_n + I - E$

Where w_n is the net worth of wealthy individuals at time n in thousands of dollars. k is the interest rate of wealthy individuals, I is the income of wealthy individuals, and E is the expenses of wealthy individuals.

2. $\Delta p_n = k' p_n + I' - E'$

Where p_n is the net worth of poor individuals at time n in thousands of dollars. k' is the interest rate of poor individuals, I' is the income of poor individuals, and E' is the expenses of poor individuals.

Two dimensional Systems (Wealth transfer does occur)

In the case of the scenario where wealth transfer does occur, we are given two difference equations of the form:

$$\Delta w_n = h(w_n, p_n)$$

$$\Delta p_n = k(w_n, p_n)$$

Using these difference equations, we set them equal to 0 and solve for the fixed points (labeled as (w_n^*, p_n^*).

$$0 = h(w_n, p_n)$$

$$0 = k(w_n, p_n)$$

To determine the stability of the fixed points requires the Jacobian matrix, which is a matrix of the form:

$J\begin{pmatrix} \frac{\partial h}{\partial w} & \frac{\partial h}{\partial p} \\ \frac{\partial k}{\partial w} & \frac{\partial k}{\partial p} \end{pmatrix}$ which is evaluated at the fixed points (w^*, p^*).

From the given matrix, we then solve for the eigenvalues using the formula:

$$\begin{pmatrix} \lambda - a & b \\ c & \lambda - d \end{pmatrix}$$

And solve for the eigenvalues using the formula:

$$(\lambda - a)(\lambda - d) - (b)(c) = 0$$

We are given two eigenvalues from this, and the stability of the fixed point is dependent on the value of the dominant eigenvalue. If the dominant eigenvalue is greater than 1, the fixed point is unstable, and if the dominant eigenvalue is less than 1, the fixed point is unstable.

In our model, we are given the difference equations:

1. $\Delta w_n = kw_n + I - K + 0.01p_n$

Where w_n is the net worth of wealthy individuals at time n in thousands of dollars. k is the interest rate of wealthy individuals, I is the income of wealthy individuals, E is the expenses of wealthy individuals, and p_n is the net worth of poor individuals at time n in thousands of dollars.

2. $\Delta p_n = k'p_n + I' - K' + 0.01w_n$

Where p_n is the net worth of poor individuals at time n in thousands of dollars. k' is the interest rate of poor individuals, I' is the income of poor individuals, E' is the expenses of poor individuals, and w_n is the net worth of poor individuals at time n in thousands of dollars.

In addition, we can use eigenvectors to determine the long run dynamics of this system. To find the eigenvectors of a system we can use the formula:

$$\vec{A}\vec{v} = \lambda\vec{v}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} w \\ p \end{pmatrix} = \lambda\begin{pmatrix} w \\ p \end{pmatrix}$$

Solve the resulting system of equations in terms of w or p.

The resulting vector is called the associated eigenvector of the system. Therefore if $\vec{v}$ is an eigenvector of $\vec{A}$ with an associated eigenvalue of λ then we have that:

$$\vec{A}^n\vec{v} = \lambda^n\vec{v} \text{ for all } n \geq 0$$

Therefore if $\vec{v}$ is an eigenvector of $\vec{A}$ with an associated eigenvalue of λ then we have that $\vec{A}^n\vec{v} = \lambda^n\vec{v}$ for all $n \geq 0$. And as $n \to \infty$, $\vec{A}^n\vec{v}$ will asymptotically approach $\lambda^n\vec{v}$.

Results

No Wealth Transfer

Utilizing the described above methods, we can now implement the original values given. Using the formula for wealthy net worth we have:

$$\Delta w_n = kw_n + I - E$$

We have values of $k = .1$, $I = 500, E = 400$, which translates to:

$$\Delta w_n = .1w_n + 500 - 400$$

Which simplifies to:

$$\varDelta w_n = .1w_n + 100$$

And the associated recurrence equation is:

$$w_{n+1} = 1.1w_n + 100, w_0 = 200$$

To find the fixed point of this system, we will set the difference equation to 0 and solve for w_n. This gives us:

$$0 = 0.1w_n + 100$$

$$w_n^* = -1000$$

To determine the stability of this 1D system we will look at the derivative of the recurrence equation evaluated at our fixed point. The derivative of our recurrence equation is:

$$w_{n+1}' = 1.1$$

The derivative is a constant, so we have that the fixed point will be unstable, since it is greater than 1. Below is a graph of the values of our recurrence equation, and as can be seen, this system will grow without bound.

We will now look at the system determining the net worth of the poor using the given formula:

$$\varDelta p_n = kp_n + I - E$$

We then apply the given values of $k = .1,\ I = 30, E = 29$, we have:

$$\varDelta p_n = 0.1p_n + 30 - 29$$

Which simplifies to:

$$\varDelta p_n = 0.1p_n + 1$$

The associated recurrence equation of this system, then, is:

$$p_{n+1} = 1.1p_n + 1 \text{ with } p_0 = -12$$

To find the fixed point of this system, we will set the difference equation to 0 and solve for w_n. This gives us:

$$0 = 0.1p_n + 1$$

$$p_n^* = -10$$

To determine the stability of this 1D system we will look at the absolute value of the derivative of the recurrence equation. The derivative of the recurrence equation is:

$$p'_{n+1} = 1.1$$

The derivative is a constant, so we have that the fixed point will be unstable since it is greater than 1. Below is a graph of the values of our recurrence equation, and as can be seen, this system will decay without bound.

As can be seen, if there is no transfer of wealth, there will be

Wealth Transfer

We will now look at the case of wealth transfer occurring. We are given the formulas:

$$\Delta w_n = 0.1w_n + 100 + 0.01p_n$$

$$\Delta p_n = 0.1p_n + 1 + 0.01w_n$$

And we will solve for the associated fixed points by setting these equations equal to 0 and solving.

$$0 = 0.1w_n + 100 + 0.01p_n$$

$$p_n = \frac{-100 - .1w_n}{0.01} = -10000 - 10w_n$$

$$0 = 0.1(10000 - 10w_n) + 1 + 0.01w_n$$

$$w_n = -1009.09$$

$$p_n = 90.9$$

The fixed points are $w_n = -1009.09$ and $p_n = 90.9$, so the fixed point will occur if wealthy people have a net worth of -$1,009.09 thousands of dollars while it will occur if poor individuals have a net worth of $90.9 thousands of dollars. We will now determine the stability of this fixed point.

We start by taking the recurrence equations of the difference equations listed above.

$$w_{n+1} = 1.1w_n + 100 + 0.01p_n$$

$$p_{n+1} = 1.1p_n + 1 + 0.01w_n$$

We then take the derivatives of these with respect to w and p, and place them into the Jacobian matrix.

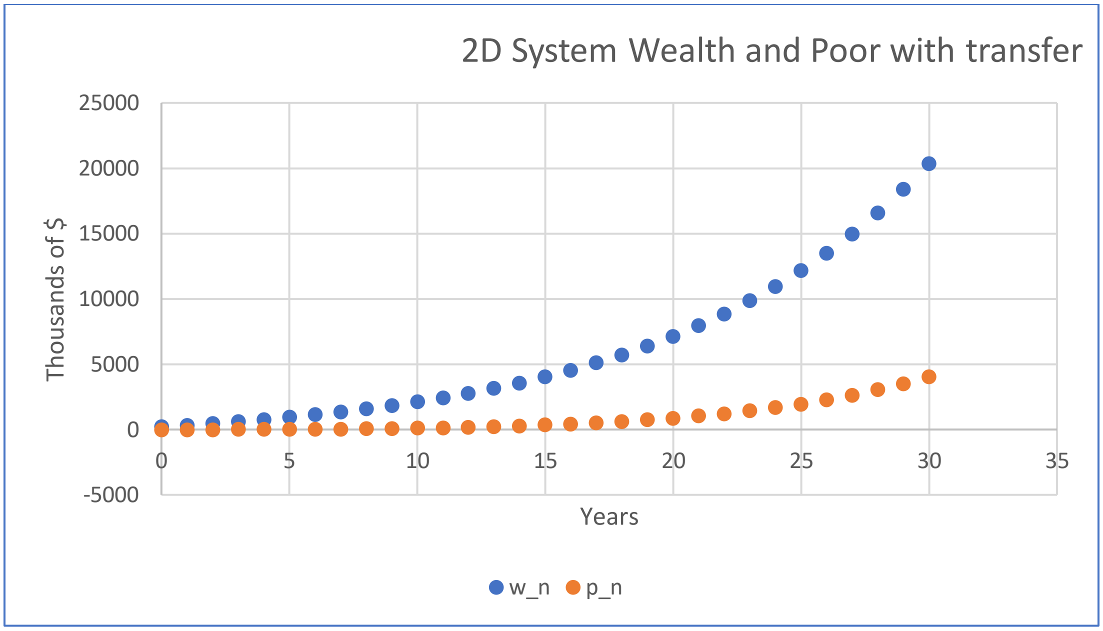
$$J = \begin{pmatrix} 1.1 & 0.01 \\ 0.01 & 1.1 \end{pmatrix}$$

We will now solve for the eigenvalues of the matrix:

$$\begin{pmatrix} 1.1 - \lambda & 0.01 \\ 0.01 & 1.1 - \lambda \end{pmatrix}$$

$$(\lambda - 1.1)(\lambda - 1.1) - (0.01)(0.01) = 0$$

The two eigenvalues are $\lambda = 1.11, \lambda = 1.09$. And the dominant eigenvalue is $\lambda = 1.1$ since it is greater than the other eigenvalue. Since the dominant eigenvalue is greater than 1, this is an unstable fixed point and there will be exponential growth in the system. Thus, indicating that if wealth transfer occurs, we will see the net worth of poor individuals and the net worth of wealthy individuals will grow or decay without bound. A graph of the system is below, and based on the initial conditions, we will see exponential growth for the net worth of all individuals if wealth transfer occurs.



We will also calculate the eigenvector of this system to determine the ratio of wealthy-to-poor net worth in the long run:

To find the Eigenvector of the fixed point, we must solve the matrix equation:

$$\begin{pmatrix} 1.1 & .01 \\ .01 & 1.1 \end{pmatrix} \begin{pmatrix} w \\ p \end{pmatrix} = \lambda \begin{pmatrix} w \\ p \end{pmatrix}$$

Where $\lambda = 1.11$

The above matrix system can be rewritten as the following system of equations:

$$1.1w + .01p = 1.1w$$

$$.01w + 1.1p = 1.1p$$

And the solution to this system of equations is:

$$w = 1$$

$$p = 1$$

Therefore, the eigenvector for this system as a whole is:

$$\vec{v}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Which shows that the ratio of wealthy-to-poor net worth in the long run is 1:1. Indicating that in the long run, the net worth of people will be equal across the entire society.

Discussion

In this paper, we viewed the effects of two policies on the net worth of wealthy and poor individuals. To understand how these policies came into effect, we modeled them using discrete dynamic models. What we found, was that in the case where wealth transfer does not occur, the net worth of poor individuals will negatively exponentially grow, indicating that they will go into debt and not be able to come out of it. In the same case, we found that the net worth of wealthy individuals will exponentially grow, indicating that their total assets will grow without bound. In the scenario where wealth transfer does occur, what we found was that the net worth of wealthy and poor individuals will positively exponentially grow, indicating that all people in the society are able to obtain assets because of the boost given to them from the wealth transfer. What was also found was that in the long run, the ratio of the net worth of wealthy individuals compared to the net worth of poor individuals was 1-to-1. This means that in the long-run, society will eventually arrive at total equality between all people.

Governments are given the choice of implementing certain public policies to allow for economic equality to occur. At any given time, if no policy is implemented, mathematical models show that economic inequality will grow until it reaches the upper limit that will occur in any society. But if some form of policy is implemented, it allows for the suffering in society to be reduced in the long run. This specific form of income redistribution indicates that total equality will be reached, however, it is more realistic to assume that the economic inequality will be reduced. Bourguignon mentions that it would be a serious mistake to not make use of public policy to reduce economic inequality, and our model agrees with this (2018). It is a mistake to allow economic inequality to continue to grow, a government should step in and implement policy to reduce that inequality, even if it is a small amount.

References

Bourguignon, F. (2018, March). Redistribution of income and reducing economic inequality - *IMF F&D Magazine*. IMF. Retrieved March 14, 2023, from https://www.imf.org/Publications/fandd/issues/2018/03/bourguignon