A picture containing diagram

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A)

B) Verify the formula for for

. True mathematical Statement.

. True mathematical Statement.

. True mathematical Statement.

. True mathematical Statement.

. True mathematical Statement.

. True mathematical Statement.

. True mathematical Statement.

. True mathematical Statement.

. True mathematical Statement.

C) Prove:

Proof:

Let be the variable proposition “ ”

We will prove that by induction on .

Observe that holds:

Let be arbitrary and fixed. Suppose holds.

We want to show holds.

By the induction hypothesis = .

Thus,

*.*

Thus, by PMI it follows that .

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= =

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.

Thus, .

Prove that

Proof:

Let be the variable proposition “ . “,

where .

We will prove that by induction on .

Observe that holds: .

Let be arbitrary and fixed. Suppose holds.

We want to show holds .

Consider,

By the induction hypothesis,

Then,

Therefore, by PMI it follows that

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Proof:

Let be the variable proposition ).

We will prove that by induction on .

Observe that holds: ) =

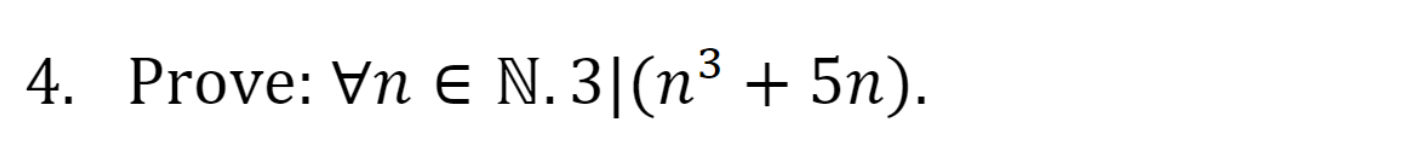
Let be arbitrary and fixed. Suppose holds.

We want to show holds.

By the induction hypothesis

Thus

Therefore, by PMI it follows that



Proof: let be the variable proposition “”

We will prove by induction on.

Observe holds:

3 divides 6 because there exists an integer such that by the definition of integer division..

Let be arbitrary and fixed. Suppose holds.

.

We want to show holds.

.

Distributing the cubic and the 5:

Rearranging terms:

By the induction hypothesis is divisible by 3.

We need to prove

Factoring:

Let for some integer

Thus

Which is true by the definition of integer division. Therefore is divisible by 3.

By PMI it follows that holds.

Text

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A)

B)

C)

Proof: Let be the variable proposition “”

We will prove by strong induction on.

Observe holds = 1.

Observe holds = 3.

Let be arbitrary and fixed. Suppose holds.

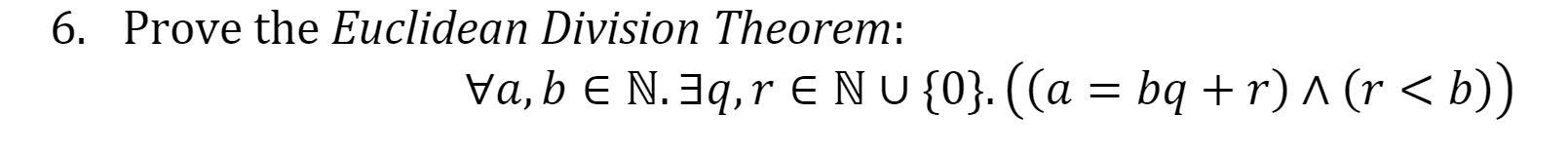
We want to show holds. That is

By the induction hypothesis

) )

) )

By PMI it follows that holds.



Proof:

Let 𝑏 ∈ℕ be arbitrary and fixed. Let 𝑃(𝑎) be the statement “.∃𝑞,𝑟 ∈ℕ∪  
{0}.((𝑎=𝑏𝑞+𝑟)∧(𝑟<𝑏))”, defined for 𝑎 ∈ℕ∪{0}. We will prove that 𝑃(𝑎) is true   
for all 𝑎 ∈ℕ∪{0} by strong induction on 𝑎.

Base Case: Suppose . Then and by worksheet 8. We know that so then

Induction hypothesis: Let 𝑘 ∈ℕ with 𝑘 ≥𝑏−1 be arbitrary and fixed and suppose the   
statement is true for all 𝑖 ∈[𝑘]∪{0}.

Induction step: We want to show that 𝑃(𝑎) is true for 𝑎 =𝑘+1. By our induction hypothesis, 𝑃(𝑎′) is true for 𝑎′=𝑘+1−𝑏.

Then

Then

Then

Therefore, by induction, 𝑃(𝑎) is true for all 𝑎 ∈ℕ∪{0}. Since 𝑏 ∈ℕ was arbitrary and fixed, the overall claim is true. ∎