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Course: EC 484/584
Vertical / Horizontal Foreign Direct Investment Problem Set

All calculations presented here are implemented in analysis...ipynb using model.py. The notebook contains the same scenarios with visualizations of profits, quantities, and price outcomes.
https://github.com/drewlesh/FDI-Strategy

Vertical FDI

Two Firms:

Slovenia (upstream firm):
- Ball bearings cost of production: 6 per unit

Greece (downstream firm):
- One machine, Greek must by 2 ball bearings
- Production cost of 4 per machine

Machines sold according to demand curve:

$$P = 240 - 2Q$$

Scenario: Separated (Independent Firms)

1.) Profit Function - Greek Firm (Downstream)

$$\pi^G = TR - TC$$
$$TR = P * Q = (240 - 2Q)Q$$
$$TC = 4Q + P_b * 2Q$$

The Profit Function for Greece is:

$$\pi^G = (240 - 2Q)Q - 4Q - P_b * 2Q$$

2.) Optimal Quantity of Machines produced/sold by the Downstream Firm:

Choose Q to maximize profit function:

$$\frac{\partial \pi^G}{\partial Q} = 0 \Rightarrow 240 - 4Q - 4 - 2P_b = 0 \Rightarrow \quad Q = \frac{236 - 2P_b}{4}$$

The optimal quantity of machines produced and sold by the Greek firm, taking price of ball bearings as given is $\frac{236-2P_b}{4}$.

3.) Profit Function - Slovenian Firm (Upstream)

$$\pi^S = TR - TC$$
$$TR = P_b * 2q$$
$$TC = 6Q$$

Profit Function for Slovenia:

$$\pi^S = P_b * 2q - 6q \Rightarrow (2P_b - 6)q$$

Substituting quantity demanded from Greece:

$$(2P_b - 6) * \frac{236-2P_b}{4} \Rightarrow \frac{1}{4}(472P_b - 4P_b^2 - 1416 + 12P_b) \Rightarrow$$
$$\pi^S = 121P_b - P_b^2 - 354$$

4.) Independent Firm Equilibriums:

a) **Equilibrium Price of ball bearings: 60.5**

$$\frac{\partial \pi^S}{\partial P_b} = 0 \Rightarrow 121 - 2P_b = 0$$
$$P_b = 60.5$$

b) **Equilibrium Price of a machine: 182.5**

$$P = 240 - 2Q$$
$$P = 240 - 2(28.75)$$
$$P = 182.5$$

c) **Quantity of machines produced/sold: 28.75**

$$Q = \frac{236-2(60.5)}{4} \Rightarrow 28.75$$

d) **Quantity of ball bearings produced/sold: 57.5**

$$q = 2Q$$
$$q = 57.5$$

e) **Profit - Slovenian firm: 3306.25**

$$\pi^S = 121(60.5) - 60.5^2 - 354 \Rightarrow 7320.5 - 3660.25 - 354$$
$$\pi^S = 3306.25$$

f) **Profits - Greek firm: 1653.13**

$$\pi^G = (240 - 2Q)Q - 4Q - P_b 2Q \Rightarrow$$
$$(240 - 2(28.75))28.7 - 4(28.75) - 60.5 * 2 * 28.75$$
$$\pi^G = 1653.13$$

Scenario: Integrated MNC (Full Integration)

5. Profit Function - Greek Multinational Firm(MNC):

$$\pi^{MNC} = \pi^{G} + \pi^{S} - FC$$

$$\pi^{MNC} = (240 - 2Q)Q - 16Q + P_b * q - 6q - 1000$$

Slovenia:

$$\pi^{S} = P_b * q - 6q - 1000$$

$$P_b = 6$$

Greece:

$$\pi^{G} = (240 - 2Q)Q - 4Q - 6 * 2Q - 1000$$

$\Rightarrow$ (240 - 2Q)Q - 16 Q - 1000

6. Optimal Quantity of machines produced(MNC):

240 - 4Q - 16 = 0

4Q = 224

Q = 56 *Optimal Quantity Machines*

P = 240 - 2(56) = 128 *Equilibrium price of Machines*

Total Profit:

$$\pi^{MNC} = (128)56 - 16(56) + P_b * 0 - 6(0) - 1000$$

$\pi^{MNC} = 5272$ *Total Profit(Multinational Corporation)*

Compared to the two firms acting as independent entities–the quantity of machines produced has risen, the price of machines has declined, and profits are greater.

Scenario: Transfer Pricing(Internal Pricing MNC)

7.) **Price of ball bearings to ensure that Slovenian profits are the equal to Greek monopolist profits(Independent Firm):**

$\pi^{S} = P_b * q - 6q = 3978.25$ *Slovenian Profit Function*

$$P_b * 112 - 6 * 112 = 3978.25$$

$P_b = 35.52$ *Price Ball Bearings*

Pricing ball bearings at \$35.52, ensures the Slovenian Subsidiary earns the same profit the Greek Firm earned(when acting as an independent firm).

8.) Greek Profit when ball bearings are priced at profit level of downstream firm when it was independent:

$$\pi^G = (P - cd - kPb)Q - FC$$
$$\pi^G = 1965.76$$

Downstream Firm earns 1,965.76 in profit when ball bearings are priced at 35.52

Scenario: Buy-at-Cost (MNC)

9.)

Assume the Greek machine producer buys the ball bearings from the Slovenian subsidiary at cost.

So, $P_b = c_u = 6$ *Price ball bearings*

a.) Quantity of Machines produced:

$$\pi^G = (240 - 2Q)Q - 4Q - 6 * 2Q$$ *Greek Profit Function*

MC = 4 + 12 = 16 *Marginal Cost*

240 - 4Q - 16 = 0 *First Order Condition*

4Q = 224 ⇒ Q = 56 *Same Quantity as Q6*

b.) Equilibrium Price of Machines:

P = 240 - 2Q = 240 - 2(56) = 128

c.) Greek Producer Profit:

TR = 128 * 56 = 7168

TC = 16*56 = 896

π^G = 7168 - 896 = 6272 *Profit*

π^G = 6272

π^G = 6272

d.) Slovenian Subsidiary Profit:

$$\pi^S = (6 - 6) * 2Q = 0$$

e.) Total Profit Earned by the MNC:

$$\pi^{MNC} = \pi^G + \pi^S - FC$$ *Fixed Costs*

$$\pi^{MNC} = 6272 + 0 - 1000$$

π^{MNC} = 5272

The buy-at-cost scenario and the integrated MNC scenario yield similar total profits because both remove the double marginalization problem. The difference lies in the allocation of profit: buy-at-cost shifts all profit to the downstream firm, while the integrated MNC may distribute some profit internally to the upstream plant.

Horizontal FDI

Two Firms:

Ireland Firm - Produces / Sells bicycles in Ireland and in England.

Demand in Ireland:

$P = 280 - Q$

P = Irish price

Q = Quantity sold in Ireland

Demand in England:

$P^* = 400 - Q^*$

P = English price*

P = Quantity sold in England*

Cost of production in Ireland = Y^2*, where Y is quantity produced*

Cost of production in England = Z^2*, where Z is quantity produced*

Cost of transportation = 20 per unit

Fixed cost of 8,000 incurred by all the active production sites

Scenario: Export Only

1.) **Total Revenue from selling bicycles to Irish and British markets:**

$$TR = P\times Q+P^*\times Q^*$$
$$TR = (280 - Q)Q + (400 - Q^*)Q^*$$

2.) **Total cost of production and trade for the Irish firm**

$$TC = Y^2 + 20\times Q^* + F$$
$$TC = (Q + Q^*)Q^* + 20Q^* + 8000$$

3.) **Profit Function + Equilibrium Quantities sold in Irish/British markets:**

$$\pi^{exp} = TR - TC$$
$$\pi^{exp} = (280 - Q)Q + (400 - Q^*)Q^* - (Q + Q^*)Q^* - 20Q^* - 8000$$

Equilibrium quantity of bikes produced:

$$\frac{\partial \pi^{exp}}{\partial Q} = 280 - 2Q - 2(Q + Q^*) = 0$$
$$\frac{\partial \pi^{exp}}{\partial Q^*} = 400 - 2Q^* - 2(Q + Q^*) = 0$$
$$\Rightarrow 280 - 4Q = 2Q^*$$
$$\Rightarrow 400 - 4Q^* - 2Q = 20$$

Ireland: Q=30, England: $\Rightarrow$ Q*=80 *Equilibrium Quantities produced/sold*

4.) **Equilibrium Prices + Total Profit:**

Ireland:

$$P = 280 - Q$$
$$P = 250$$

England:

$$P^* = 400 - Q^*$$
$$P^* = 320$$
$$\pi^{exp} = 14{,}700$$

Scenario: Production in England

5.) **Total Revenue earned by the Irish multinational. Express as a function of quantities produced and exported by the two plants.**

$$TR = P \times Q + P^* \times Q^* = (280 - Q)Q + (400 - Q^*)Q^*$$
$$TR = [280 - (y - x)](y - x) + [400 - (z + x)](z + x)$$

6.) **Total cost of production and trade associated with operating the two plants:**

$$TC = Y^2 + Z^2 + t \times x + 2F$$
$$TC = Y^2 + Z^2 + 20x + 16000$$

7.) **Profit Function - Irish MNC + Equilibriums**

$$\pi^{exp} = [280 - (y - x)](y - x) + [400 - (z + x)](z + x) - Y2 - Z2 + 20x - 16000$$

Maximize π^{exp}:

(1) $\frac{\partial \pi^{exp}}{\partial Y} = 0 \Rightarrow 280 - 2(Y-X) = 2Y \rightarrow 140 + x = 2Y$

(2) $\frac{\partial \pi^{exp}}{\partial Z} = 0 \Rightarrow 400 - 2(Z+X) = 2Z \rightarrow 200 - x = 2Z$

(3) $\frac{\partial \pi^{exp}}{\partial X} = 0 \Rightarrow 2Y + 20 = 2Z$

Plug (1) and (2) into (3):

$$140 + x + 20 = 200 - x$$
$$x = 20$$
$$2Y = 140 \Rightarrow Y = 70$$
$$2Z = 200 \Rightarrow Z = 100$$

Equilibrium Production:

Ireland: 70

England: 100
Exports from the Irish plant to the British plant: 20

8.) Horizontal FDI Equilibriums:

a.) Equilibrium Price of bicycles:

Ireland: 230
$Q = 70 - 20 = 50$
$P = 280 - 50 = 230$
England: 280
$Q^* = 100 + 20 = 120$
$P^* = 400 - 120 = 280$

b.) Total Profit

$$\pi^{exp} = 13,800$$

9.) Reason for producing more in the home country than domestic demand:

It is optimal for the Irish firm to produce more bicycles in Ireland than what is demanded by Irish consumers because the marginal cost of production in Ireland is lower than in England, even after accounting for transportation costs. By producing a surplus at the lower-cost home plant and exporting it to England, the firm can take advantage of cost differentials between the two locations.

This strategy allows the firm to:

1. **Minimize total production costs** by producing as much as possible where production is cheapest.
2. **Serve higher-priced foreign markets** efficiently, increasing total revenue.
3. **Maximize profits** by balancing the cost of exporting with the higher prices obtainable abroad.

The firm uses arbitrage of production costs across borders: producing in the lower-cost location and selling in the higher-price market maximizes overall profit, which is the goal of horizontal FDI.

10.)
The English government is charging the Irish firm a flat fee β in order to produce in England. How much would β have to be for the firm to choose to only produce in Ireland and avoid the fee?

Integrated Production:

$Y = 70$
$Z = 100$
Exports = 20

Export-Only:

$Q = 30$
$Q^* = 80$

Total Profit = 11,400

Avoid production in England if Profit w/ fee ≤ Profit via export only.
Where

$$\pi_{fee} = \pi_{with\ England} - \beta$$

So,

$$\pi_{with\ England} - \beta = \pi_{export\ only}$$
$$\beta \geq \pi_{with\ England} - \pi_{export\ only}$$
$$\beta \geq 13{,}800 - 11{,}400$$
$$\beta \geq 2{,}400$$

If the English government charges any fee $\beta \geq 2400$, the Irish firm will find it more profitable to produce everything in Ireland and export rather than incur the fee.