

STARTING TREE For LL and LR

```
graph TD; B((B)) --> A((A)); B --> T3((T3)); A --> T1((T1)); A --> T2((T2));
```

Before an insertion, $height(C, left)$ and $height(C, right)$ differ by 1.

Case 1: Left-Left Imbalance (Single Rotation)

```
graph TD; B((B)) --> A((A)); B --> T3((T3)); A --> T1((T1)); A --> T2((T2)); T2 --> C((C)); C --> B;
```

Then, there is an insertion somewhere in T1 that causes A's height to increase by 1. Also, $height(C, left)$ and $height(C, right)$ differ by 2.

```
function rotateLL(Node C, parent):  
    A = C.left  
    C.left = A.right  
    A.right = C  
  
    if C == root of tree:  
        root of tree = A  
    else:  
        if parent.left == C:  
            parent.left = A  
        else:  
            parent.right = A  
  
    updateHeight(C)  
    updateHeight(A)
```

We can adjust for this imbalance with a **RIGHT-ROTATION** by rotating the node of imbalance with its left child.

After the re-balancing, A has the same height as C did in the original tree.

Case 2: Left-Right Imbalance (Double Rotation)

```
graph TD; A((A)) --> T1((T1)); A --> T2((T2)); A --> T3((T3)); T2 --> B((B)); B --> A;
```

As insertion in T2 causes C to become the node of imbalance.

```
graph TD; A((A)) --> T1((T1)); A --> T2((T2)); A --> T3((T3)); T2 --> B((B)); B --> A;
```

Step 1: Rotate A with Right Child B.

```
function doubleRotateLR(Node C, parent):  
    rotateRL(C.left, C, parent)  
    rotateLL(C, parent)
```

Step 2: Rotate C with Right Child B.

After this rotation, either T2L or T2R will be as deep as T1 and T3, but not both.

STARTING TREE For RR and RL

```
graph TD; A((A)) --> T1((T1)); A --> C((C)); C --> T2((T2)); C --> T3((T3));
```

Before an insertion, $height(A, left)$ and $height(A, right)$ differ by 1.

Case 4: Right-Right Imbalance (Single Rotation)

```
graph TD; B((B)) --> A((A)); B --> T3((T3)); A --> T1((T1)); A --> T2((T2)); T2 --> C((C)); C --> B;
```

Insertion into T3 causes it to be 2 levels deeper than T1, making A the node of imbalance.

Remember: All values in T2 fall between A and C, so T2 could be connected to the left of C or the right of A.

Case 3: Right-Left Imbalance (Double Rotation)

```
graph TD; B((B)) --> A((A)); B --> T3((T3)); A --> T1((T1)); A --> T2((T2)); T2 --> C((C)); C --> B;
```

As with Case 2, an insertion into T2 cannot be solved with a single rotation. So, we need to explicitly consider the root of T2, labeled as B here.